What lattice QCD spectral functions can tell us about heavy quarkonium in the QGP

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References:
With Y.Burnier  PRL 111 (2013) 182003,
PLB753 (2016) 232-236
With Y.Burnier and O.Kaczmarek PRL 114 (2015) 082001,
With S.Kim and P.Petreczky PRD91 (2015) 054511
Physics Motivation

- From run1 and ongoing run2 at LHC: unprecedented amount of precision data

Bound states of $c\bar{c}$ or $b\bar{b}$:  Heavy quarkonium  \( M_Q \gg T_{\text{med}} \)

- Theory goal: 1st principles insight into in-medium $Q\bar{Q}$ in heavy-ion collisions
A two-pronged approach to $c\bar{c}$

I. $Q\bar{Q}$ potential from Wilson loop lattice spec. func.
(static potential)

In-medium meson spectra

Assume full kinetic thermalization of $c\bar{c}$

&

Static medium from Lattice QCD

II. Direct reconstruction of lattice meson spectra
(limited resolution due to finite $N_t$)

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Reconstructing spectral functions

1. $N_\omega$ parameters $\rho_l \gg N_\tau$ datapoints
2. Simulation data $D_i$ has finite precision

Inversion of Laplace transform required to obtain spectra: Inherently Ill-defined

Give meaning to problem by incorporating prior knowledge: Bayesian approach

- Bayes theorem: Regularize the naïve $\chi^2$ functional $P[D|\rho]$ through a prior $P[\rho|I]$

$$P[\rho|D, I] \propto P[D|\rho] P[\rho|I]$$

Methods progress: Novel Bayesian reconstruction improves on Maximum Entropy Method

$$P[\rho|I] \propto e^S \quad S = \alpha \sum_{l=1}^{N_\omega} \Delta \omega_l \left( 1 - \frac{\rho_l}{m_l} + \log \left[ \frac{\rho_l}{m_l} \right] \right)$$

Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459
Y.Burnier, A.R. PRL 111 (2013) 18, 182003
Part I

Static $Q\bar{Q}$ potential from Wilson loop spectral functions
Defining the T>0 Q\bar{Q} potential

- Effective field theory
  \[ \frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{p}{m_Q} \ll 1 \]

- Relativistic thermal field theory

- Matching between QCD and pNRQCD in the static limit
  \[ V^{QCD}(R) = \lim_{t \to \infty} \frac{i\partial_t W_{\Box}(R, t)}{W_{\Box}(R, t)} \in \mathbb{C} \]

- No more need for model potentials, such as free/internal energies used previously models started with S. Nadkami, Phys. Rev. D 34, 3904 (1986)

- Beware: real-time wilson loop not directly accessible in lattice QCD simulations

Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017
Extracting the real-time $V_{QQ}$

- Spectral functions as bridge between the Euclidean and real-time Wilson loop

$$W_{\square}(R, t) = \int_{-\infty}^{\infty} d\omega \, e^{-i\omega t} \rho_{\square}(R, \omega) \quad \leftrightarrow \quad W_{\square}(R, \tau) = \int_{-\infty}^{\infty} d\omega \, e^{-\omega \tau} \rho_{\square}(R, \omega)$$

- With the novel Bayesian method: robust lattice determination of Re[V] & Im[V]

- Phenomenology: analytic parametrization available from generalized Gauss Law

Y. Burnier, A.R. PLB753 (2016) 232
Part II

Heavy quarkonium spectra from the complex lattice $T>0$ potential
Charmonium spectral function

Solve a Schrödinger equation for the meson spectral function:

\[ \left[ 2m_Q - \frac{\nabla^2}{2m_Q} + \text{Re}[V] \mp i|\text{Im}[V]| \right] D^>(t, r, r') = i\partial_t D^>(t, r, r'), \quad t \geq 0 \]

\[ \tilde{D}(\omega, r, r') \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} D^>(t, r, r') \quad \Rightarrow \quad \rho^V(\omega) = \lim_{r, r' \to 0} \frac{1}{2} \tilde{D}(\omega, r, r') \]

Melting temperatures from condition \( E_{\text{bind}} = \Gamma \)

\[
\begin{array}{c|cc}
\text{state} & J/\Psi(1S) & \Psi'(2S) \\
\hline
T_{\text{melt}} & 213^{+13}_{-11} & < 147 \\
\end{array}
\]

[MeV]
In-medium heavy quarkonium from lattice QCD spectral functions

Estimating the $\psi'$ to $J/\psi$ ratio

- In-medium dilepton emission from area under spectral resonance peaks
  $$R_{\ell\bar{\ell}} \propto \int dp_0 \int \frac{d^3p}{(2\pi)^3} \frac{\rho(P)}{p^2} n_B(p_0)$$
  (to leading order $\rho(P) = \rho(p_0^2 - p^2)$)

- Number density: divide by $T=0$ dimuon rate assuming all $cc$ in peak become real particles
  $$\frac{N_{\psi'}}{N_{J/\psi}} = \frac{R_{\ell\bar{\ell}}^{\psi'}}{R_{\ell\bar{\ell}}^{J/\psi}} \frac{M_{\psi'}^2 |\Phi_{J/\psi}(0)|^2}{M_{J/\psi}^2 |\Phi_{\psi'}(0)|^2} = 0.052 \pm 0.009$$

adapted from: Andronic, Beutler, Braun-Munzinger, Redlich, Stachel, PLB 678, 350 (2009)

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Part III

Direct Lattice QCD
heavy quarkonium spectral functions
Direct lattice Q̅Q spectra

Relativistic treatment of light and heavy d.o.f.

Kin. eq. non-relativistic Q̅Q in a background of light medium d.o.f.

- Full Lattice QCD simulation incl. QQ (still too costly)

- Lattice Non-Relativistic QCD (NRQCD) well established at T=0, applicable at T>0
  - no modeling, systematic expansion of QCD action in 1/m_Q a, includes v≠0 contributions

- State-of-the-art: realistic simulations of the QCD medium by the HotQCD collab.
  - 48^3x12 N_f=2+1 HISQ action  m_π=161MeV  T= [ 140 – 249 ] MeV  m_c a= [ 0.757 – 0.427 ]

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Correlation functions in NRQCD

Non-rel. propagator of a single heavy quark $G$


QQ propagator projected to a certain channel

„correlator of QQ wavefct. $D_{J/\psi}(\tau) \triangleq \langle \psi_{J/\psi}(\tau) \psi_{J/\psi}^\dagger(0) \rangle$“

Brambilla et al. Rev.Mod.Phys. 77 (2005) 1423

Ratio of $T>0$ and $T\approx0$ correlators: estimate of overall in-medium effects

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Due to limited number of data points (Nt=12) only ground state reliably reconstructed

GS strength decreases with temperature, quantitative features (m, Γ) are work in progress

Consistent with potential based spectra: survival of GS peak remnant up to T=1.6T_C

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Conclusions

- **Methods progress**: Novel Bayesian approach to spectral reconstruction

- Combining EFT methods (pNRQCD/NRQCD) and lattice QCD spectral functions
  - **Progress I**: First principles definition of static \( T > 0 \) potential from QCD spectral functions
  - **Progress II**: Predictions for \( \psi' / J/\psi \) from meson spectra based on the static potential
  - **Progress III**: Direct reconstruction of NRQCD meson spectra

- Physics results for in-medium quarkonium:
  - Confirm **sequential** in-medium **modification** of quarkonium states
  - Observation of **mass shifts** for quarkonia to lower values close to melting
  - \( \psi' / J/\psi \) ratio at freezeout **larger** than predicted from statistical model
  - **Melting T**: \( \Upsilon(1S): 412(40) \text{MeV} \), \( \chi_b(1P): 266(20) \text{MeV} \), \( J/\psi: 213(12) \text{MeV} \), \( \chi_c(1P): 182(12) \text{MeV} \)

Thank you for your attention