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What lattice QCD spectral functions can tell us about heavy quarkonium in the QGP

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References:

With Y.Burnier PRL 111 (2013) 182003,
PLB753 (2016) 232-236
With Y. Burnier and O.Kaczmarek PRL 114 (2015) 082001,
JHEP 1512 (2015) 101, arXiv:1606.06211
With S.Kim and P. Petreczky PRD91 (2015) 054511

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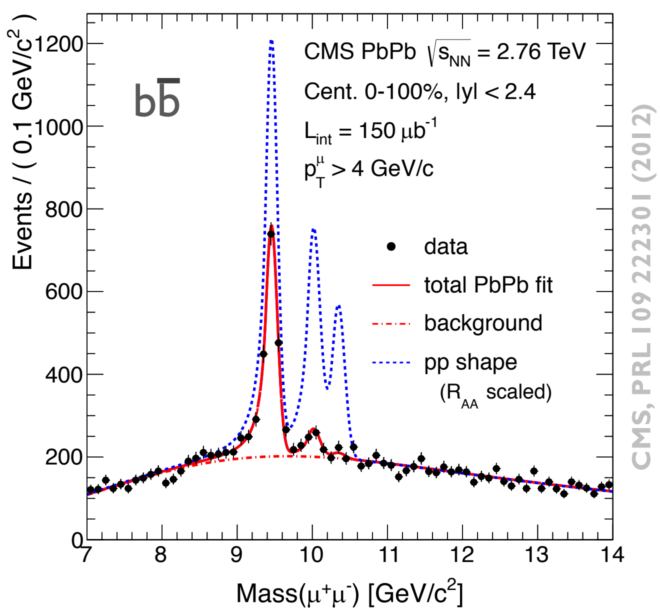
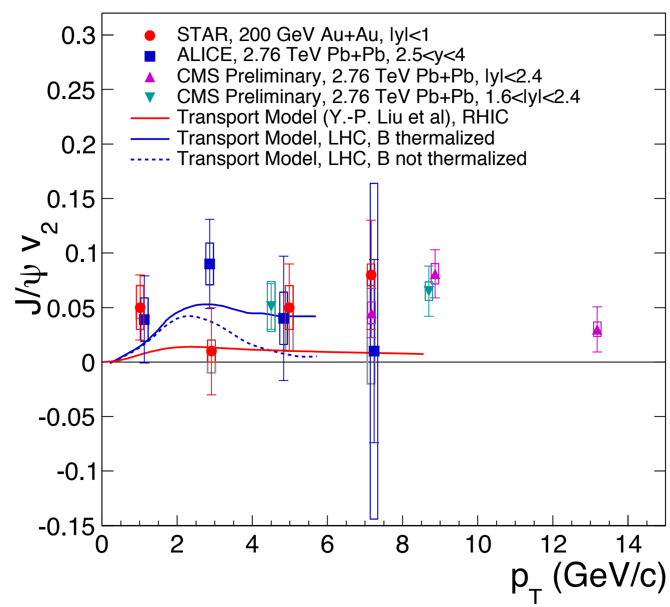
Physics Motivation

- From run1 and ongoing run2 at LHC: unprecedented amount of precision data

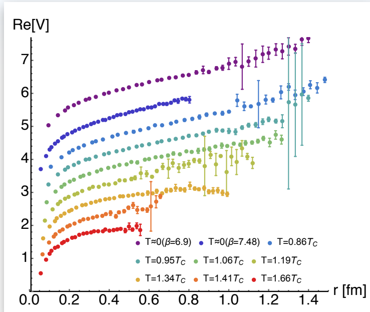
Bound states of $c\bar{c}$ or $b\bar{b}$: **Heavy quarkonium** $M_Q >> T_{med}$



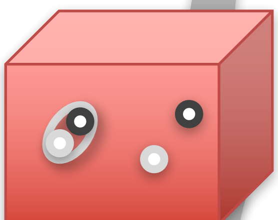


- Theory goal: 1st principles insight into in-medium $Q\bar{Q}$ in heavy-ion collisions

A two-pronged approach to $c\bar{c}$

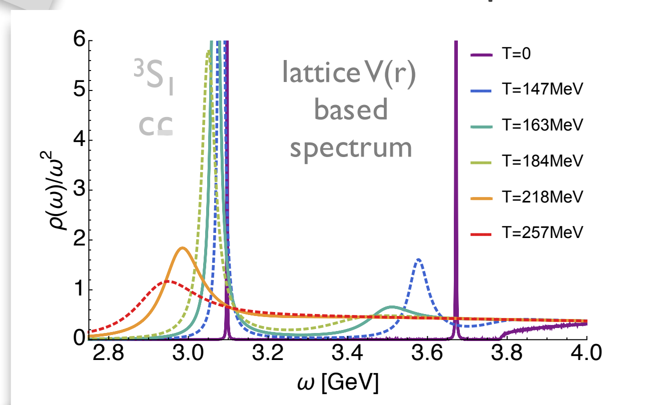


I. $Q\bar{Q}$ potential from Wilson loop lattice spec. func. (static potential)



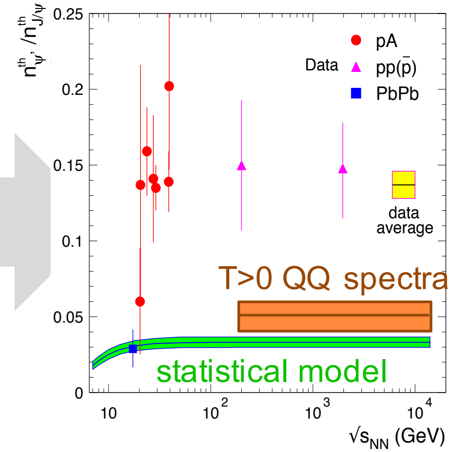
Assume full kinetic themalization of cc
&
Static medium from Lattice QCD

In-medium meson spectra

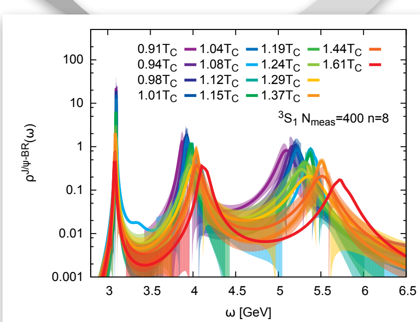


Y.Burnier, O. Kaczmarek, A.R. JHEP 1512 (2015)

ψ' / J/ψ ratio



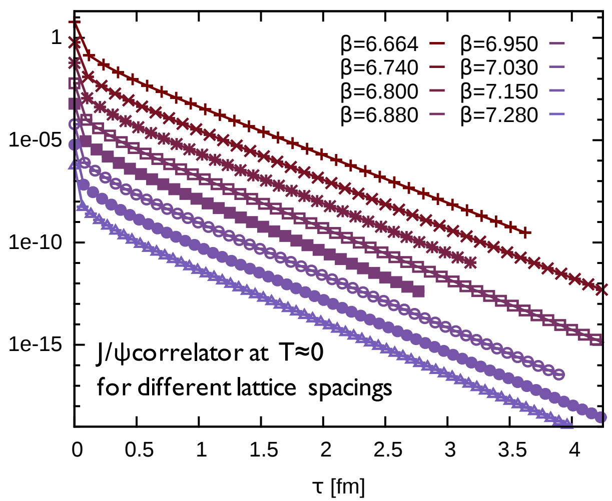
Y.Burnier, O. Kaczmarek, A.R. JHEP 1512 (2015) 101



S. Kim, P. Petreczky, A.R. in progress

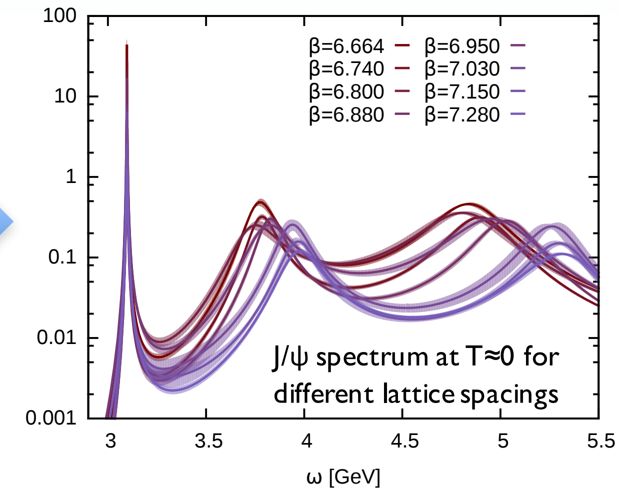
II. Direct reconstruction of lattice meson spectra (limited resolution due to finite Nt)

Reconstructing spectral functions



$$D_i = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\, \rho_l\, \Delta\omega_l$$

1. N_ω parameters ρ_l >> N_τ datapoints
2. Simulation data D_i has finite precision



- Inversion of Laplace transform required to obtain spectra: Inherently Ill-defined
- Give meaning to problem by incorporating prior knowledge: Bayesian approach
  - Bayes theorem: Regularize the naïve χ^2 functional $P[D|\rho]$ through a prior $P[\rho|I]$

$$P[\rho|D, I] \propto P[D|\rho]\; P[\rho|I]$$

Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459

- Methods **progress**: Novel Bayesian reconstruction improves on Maximum Entropy Method

$$P[\rho|I] \propto e^S \qquad S = \alpha \sum_{l=1}^{N_\omega} \Delta\omega_l \left(1 - \frac{\rho_l}{m_l} + \log\left[\frac{\rho_l}{m_l}\right]\right)$$

Y.Burnier, A.R. PRL 111 (2013) 18, 182003

Part I

Static $Q\bar{Q}$ potential from Wilson loop spectral functions

Defining the T>0 $Q\bar{Q}$ potential

- Effective field theory $\frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{p}{m_Q} \ll 1$

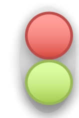
Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Relativistic thermal field theory

QCD	NRQCD	pNRQCD
Dirac fields	Pauli fields	Singlet/Octet
$\bar{Q}(x), Q(x)$	$\chi^\dagger(x), \chi(x)$	$\psi_S(R,t), \psi_O(R,t)$

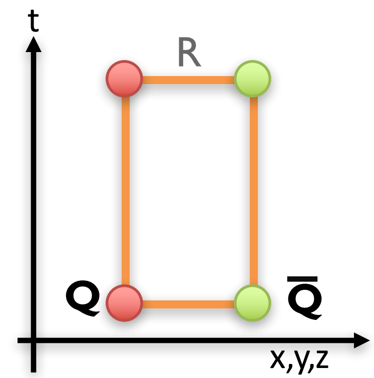
Quantum mechanics



$$i\partial_t \psi_S = \left(V^{QCD}(R) + \mathcal{O}(m_Q^{-1})\right)\psi_S$$

- Matching between QCD and pNRQCD in the static limit

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{i\partial_t W_\square(R,t)}{W_\square(R,t)} \in \mathbb{C}$$



- No more need for model potentials, such as free/internal energies used previously

models started with S. Nadkarni, Phys. Rev. D 34, 3904 (1986)

- Beware: real-time wilson loop not directly accessible in lattice QCD simulations

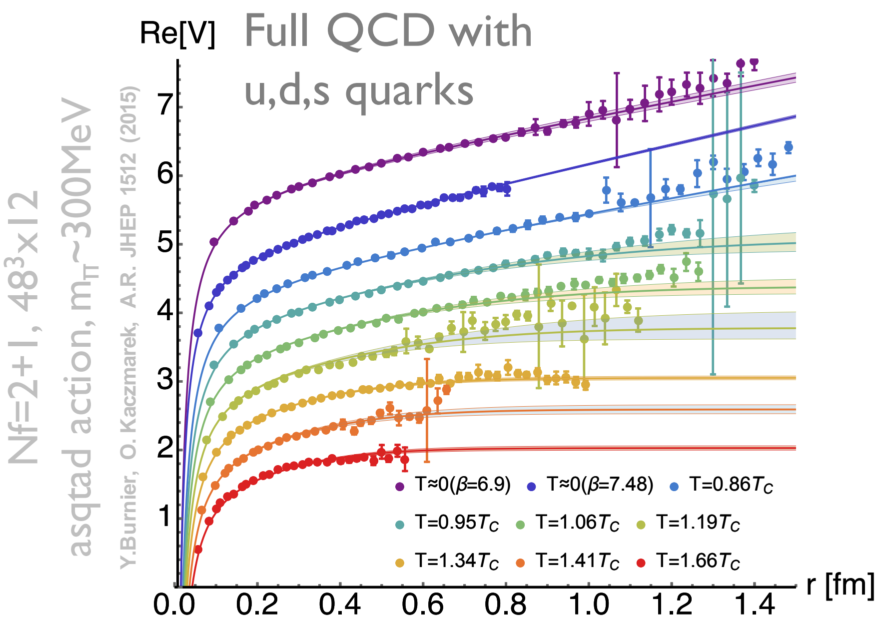
Extracting the real-time V_{QQ}

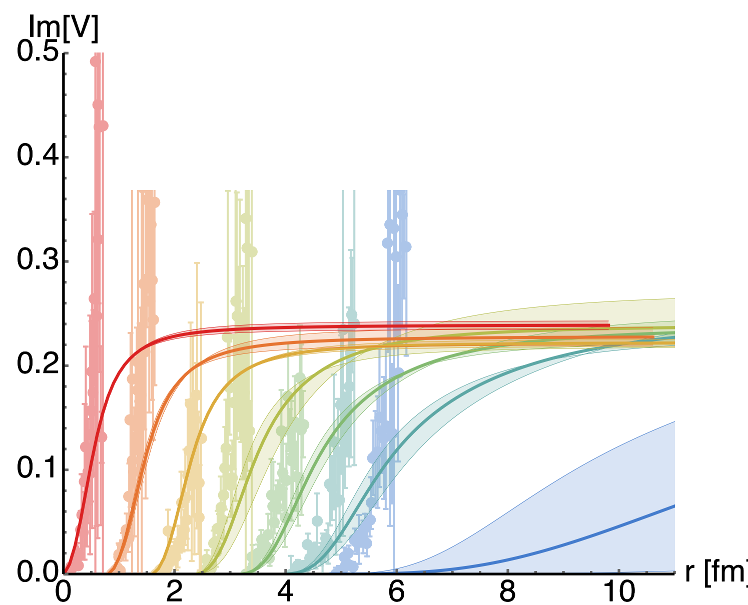
- Spectral functions as bridge between the Euclidean and real-time Wilson loop

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\rho_\square(R,\omega) \Longleftrightarrow W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau}\rho_\square(R,\omega)$$

A.R., T.Hatsuda & S.Sasaki , PRL 108 (2012) 162001

- With the novel Bayesian method: robust lattice determination of Re[V]&Im[V]





- Phenomenology: analytic parametrization available from generalized Gauss Law

Y.Burnier, A.R. PLB753 (2016) 232

Part II

Heavy quarkonium spectra from the complex lattice T>0 potential

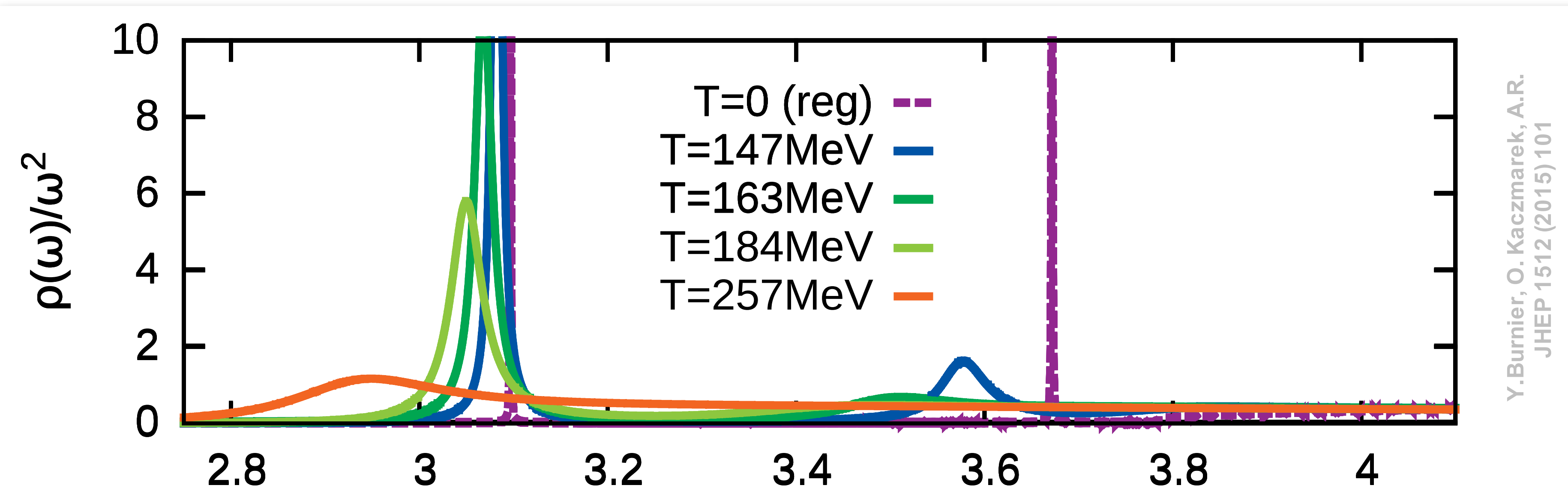
Charmonium spectral function

- Solve a Schrödinger equation for the meson spectral function:

Y. Burnier, M. Laine and M. Vepsalainen, JHEP 0801, 043 (2008)

$$\left[2m_Q - \frac{\nabla^2}{2m_Q} + \mathrm{Re}[V] \mp i|\mathrm{Im}[V]|\right] D^>(t,\mathbf{r},\mathbf{r}') = i\partial_t D^>(t,\mathbf{r},\mathbf{r}'), \quad t \gtrless 0$$

$$\tilde{D}(\omega,\mathbf{r},\mathbf{r}') \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} D^>(t,\mathbf{r},\mathbf{r}') \quad \Rightarrow \quad \rho^V(\omega) = \lim_{\mathbf{r},\mathbf{r}'\to 0} \frac{1}{2}\tilde{D}(\omega,\mathbf{r},\mathbf{r}')$$



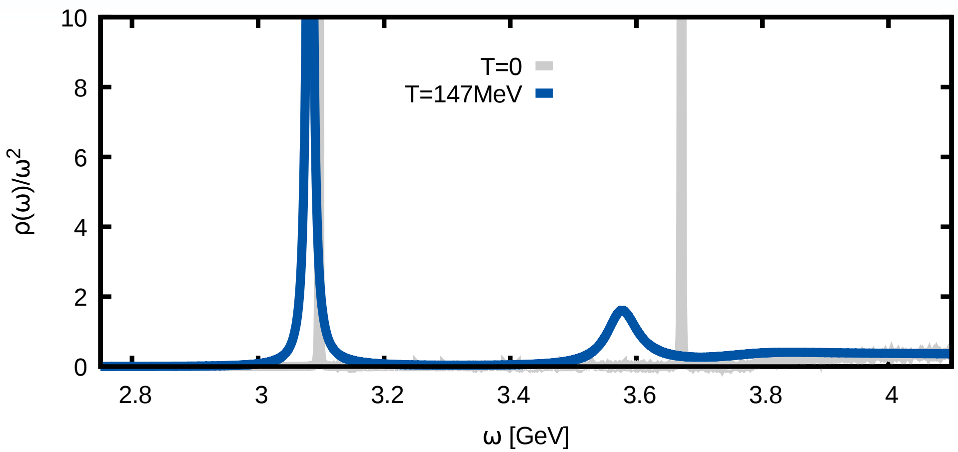
- Melting temperatures from condition $E_{bind}=\Gamma$

  ($E_{bind}(T) = E_{thresh}(T) - M_{Q\bar{Q}}(T)$)

state	$J/\Psi(1S)$	$\Psi'(2S)$
T_{melt}	213^{+13}_{-11}	< 147

[MeV]

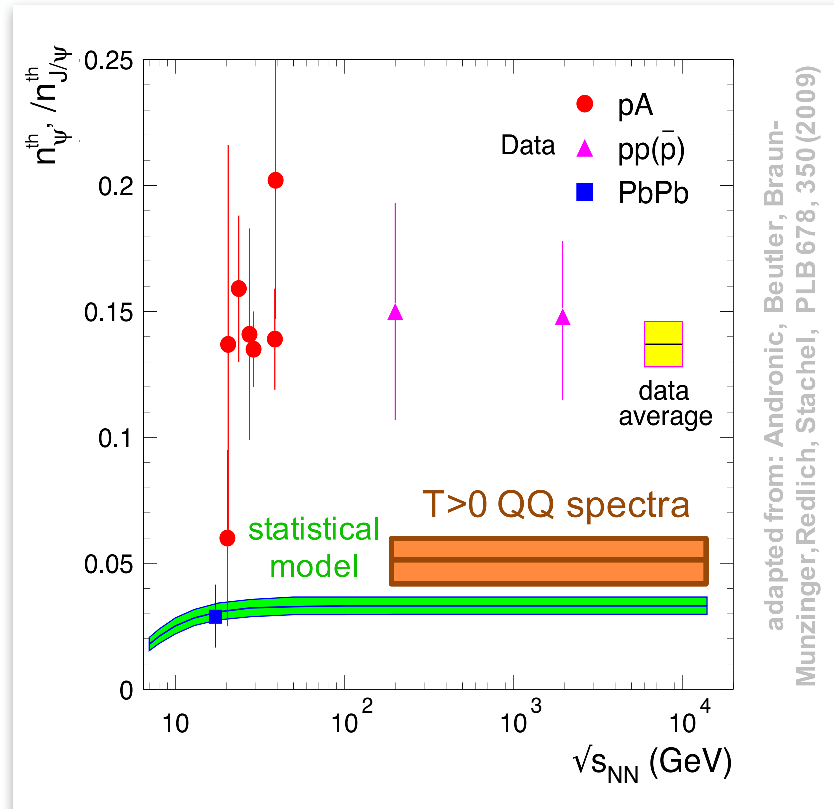
Estimating the ψ' to J/ψ ratio



- In-medium dilepton emission from area under spectral resonance peaks McLerran, Toimela PRD31 (1985) 545

$$R_{\ell\bar{\ell}} \propto \int dp_0 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\rho(P)}{P^2} n_B(p_0)$$

(to leading order $\rho(P) = \rho(p_0^2 - \mathbf{p}^2)$)



- Number density: divide by T=0 dimuon rate assuming all cc in peak become real particles

$$\frac{N_{\Psi'}}{N_{J/\Psi}} = \frac{R^{\Psi'}_{\ell\bar{\ell}}}{R^{J/\Psi}_{\ell\bar{\ell}}} \frac{M^2_{\Psi'}|\Phi_{J/\Psi}(0)|^2}{M^2_{J/\Psi}|\Phi_{\Psi'}(0)|^2} = 0.052 \pm 0.009$$

Y. Burnier, O. Kaczmarek, A.R. JHEP 1512 (2015) 101

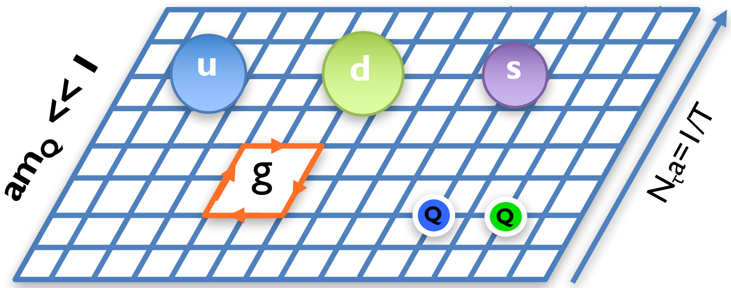
Part III

Direct Lattice QCD heavy quarkonium spectral functions

Direct lattice $Q\bar{Q}$ spectra

Relativistic treatment of light and heavy d.o.f.

Kin. eq. non-relativistic $Q\bar{Q}$ in a background of light medium d.o.f.

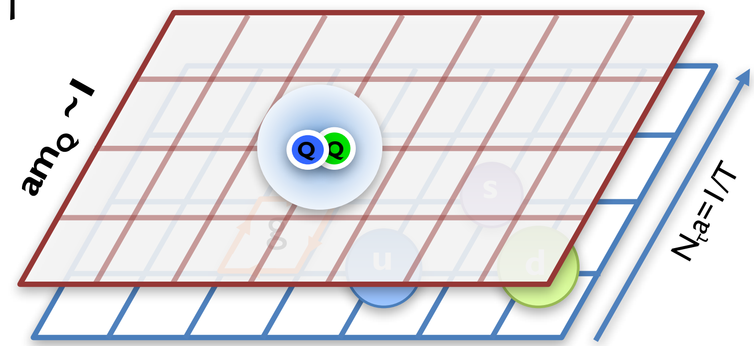


Full Lattice QCD simulation incl. QQ
(still too costly)

$$\frac{\Lambda_{QCD}}{m_Q} \ll 1$$

$$\frac{T}{m_Q} \ll 1$$

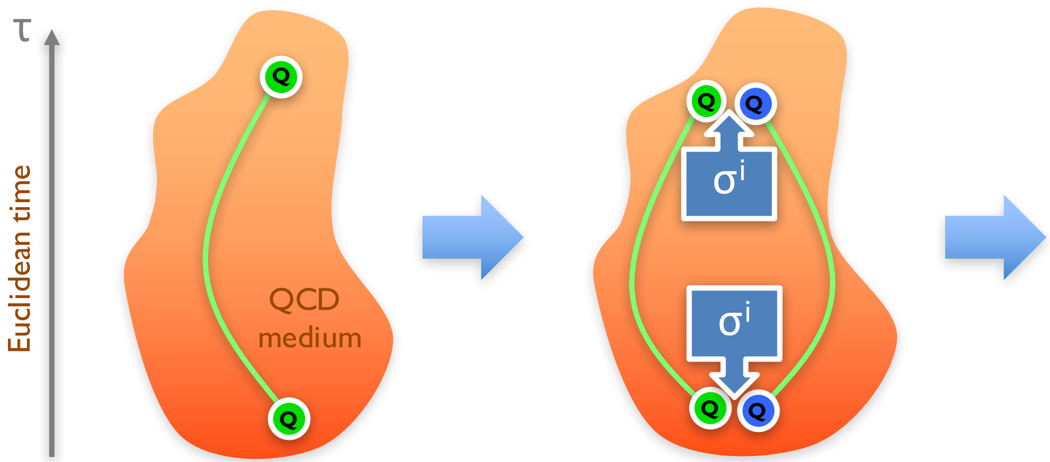
QQ in NRQCD effective theory

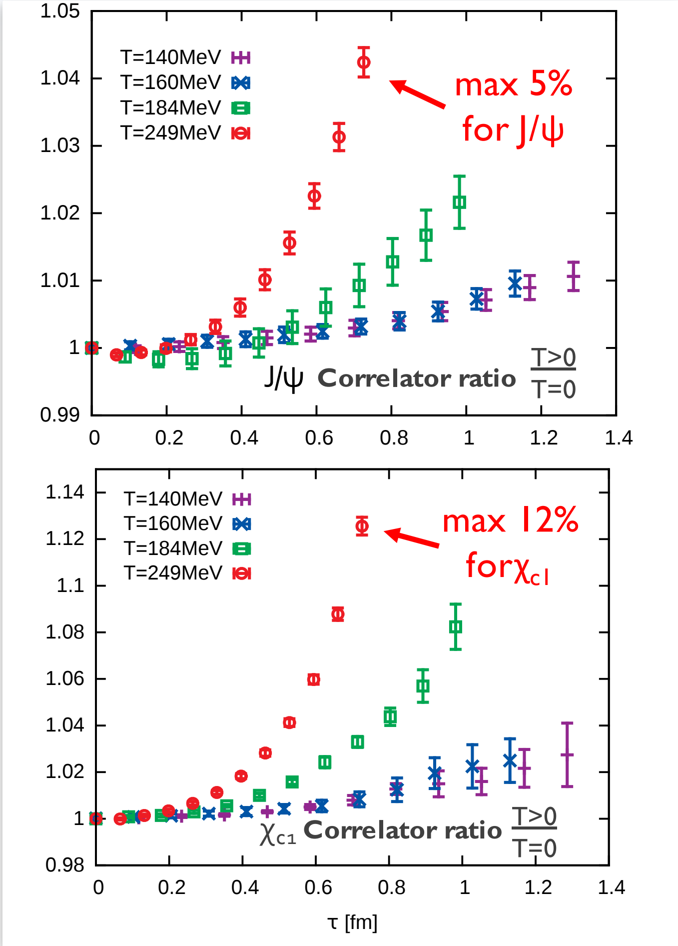


Lattice QCD simulation without QQ

- Lattice Non-Relativistic QCD (NRQCD) well **established** at T=0, applicable at T>0
  - no modeling, systematic expansion of QCD action in $1/m_Q a$, includes $v \neq 0$ contributions
    Thacker, Lepage Phys.Rev. D43 (1991) 196-208
- State-of-the-art: realistic simulations of the QCD medium by the HotQCD collab.
  HotQCD PRD85 (2012) 054503, PRD90 (2014) 094503
  - 48^3x12 N_f=2+1 HISQ action m_π=161MeV T= [140 – 249] MeV $m_c a$= [0.757 – 0.427]

Correlation functions in NRQCD





Non-rel. propagator of a single heavy quark G

Davies, Thacker Phys.Rev. D45 (1992)

QQ propagator projected to a certain channel

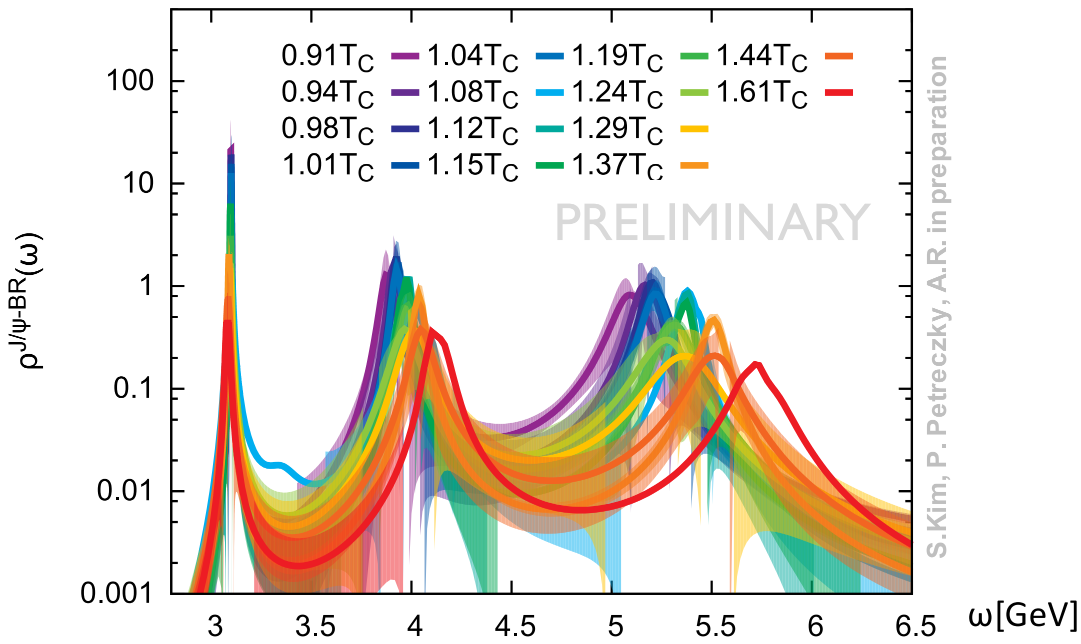
„correlator of QQ wavefct. $D_{J/\psi}(\tau) \triangleq <\psi_{J/\psi}(\tau)\psi^{\dagger}_{J/\psi}(0)>$ "

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

Ratio of T>0 and T≈0 correlators: estimate of overall in-medium effects

Direct Charmonium spectra

Lattice NRQCD Charmonium T>0 spectrum



- Due to limited number of data points (Nt=12) only ground state reliably reconstructed
- GS strength decreases with temperature, quantitative features (m , Γ) are work in progress
- Consistent with potential based spectra: survival of GS peak remnant up to $T=1.6T_C$

Conclusions

- **Methods progress**: Novel Bayesian approach to spectral reconstruction
- Combining EFT methods (pNRQCD/NRQCD) and lattice QCD spectral functions
  - **Progress I:** First principles definition of static T>0 potential from QCD spectral functions
  - **Progress II:** Predictions for ψ' / J/ψ from meson spectra based on the static potential
  - **Progress III:** Direct reconstruction of NRQCD meson spectra
- Physics results for in-medium quarkonium:
  - Confirm **sequential** in-medium **modification** of quarkonium states
  - Observation of **mass shifts** for quarkonia to lower values close to melting
  - **ψ' / J/ψ** ratio at freezeout **larger** than predicted from statistical model
  - **Melting T:** Υ(1S): 412(40)MeV, χ_b(1P): 266(20)MeV J/ψ: 213(12)MeV χ_c(1P):182(12) MeV

Thank you for your attention