Resonances in Coupled-Channel Scattering from Lattice QCD

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for the Hadron Spectrum Collaboration

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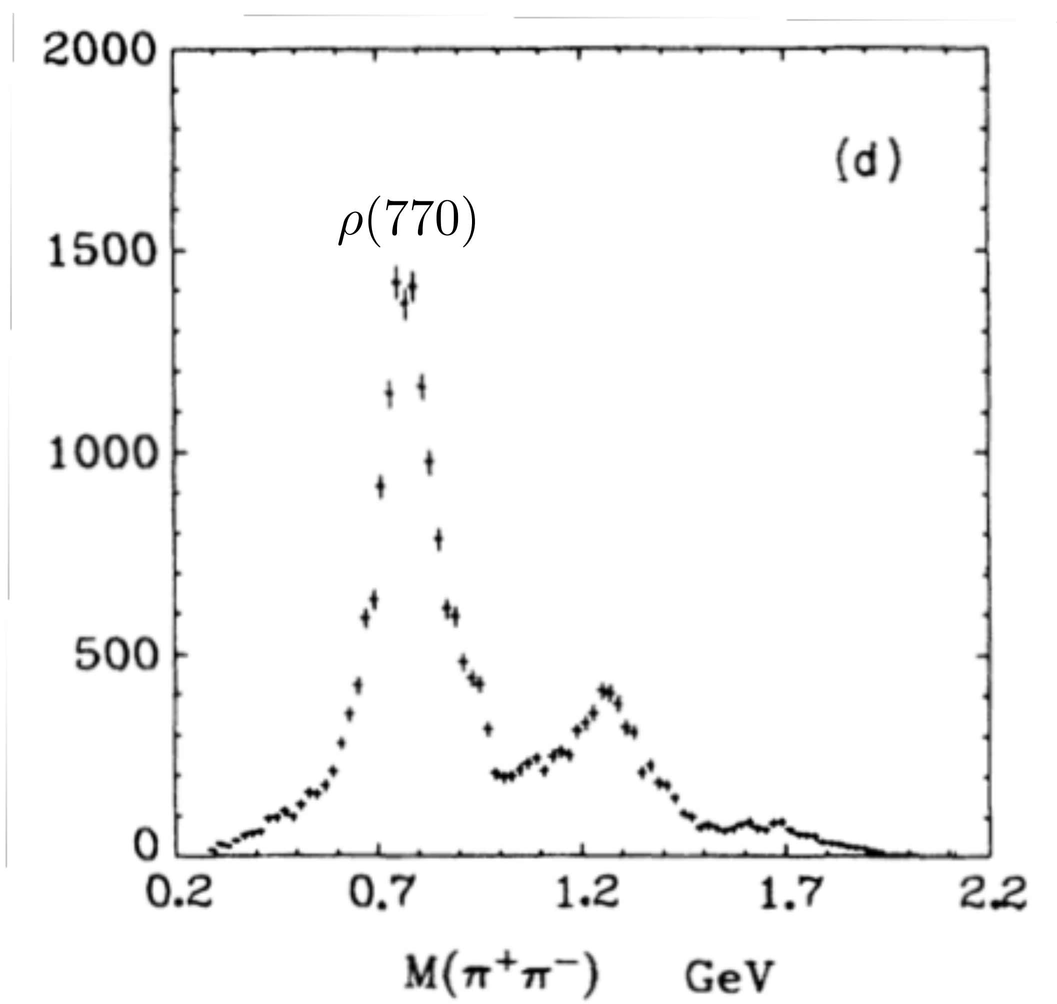
3rd-10th August

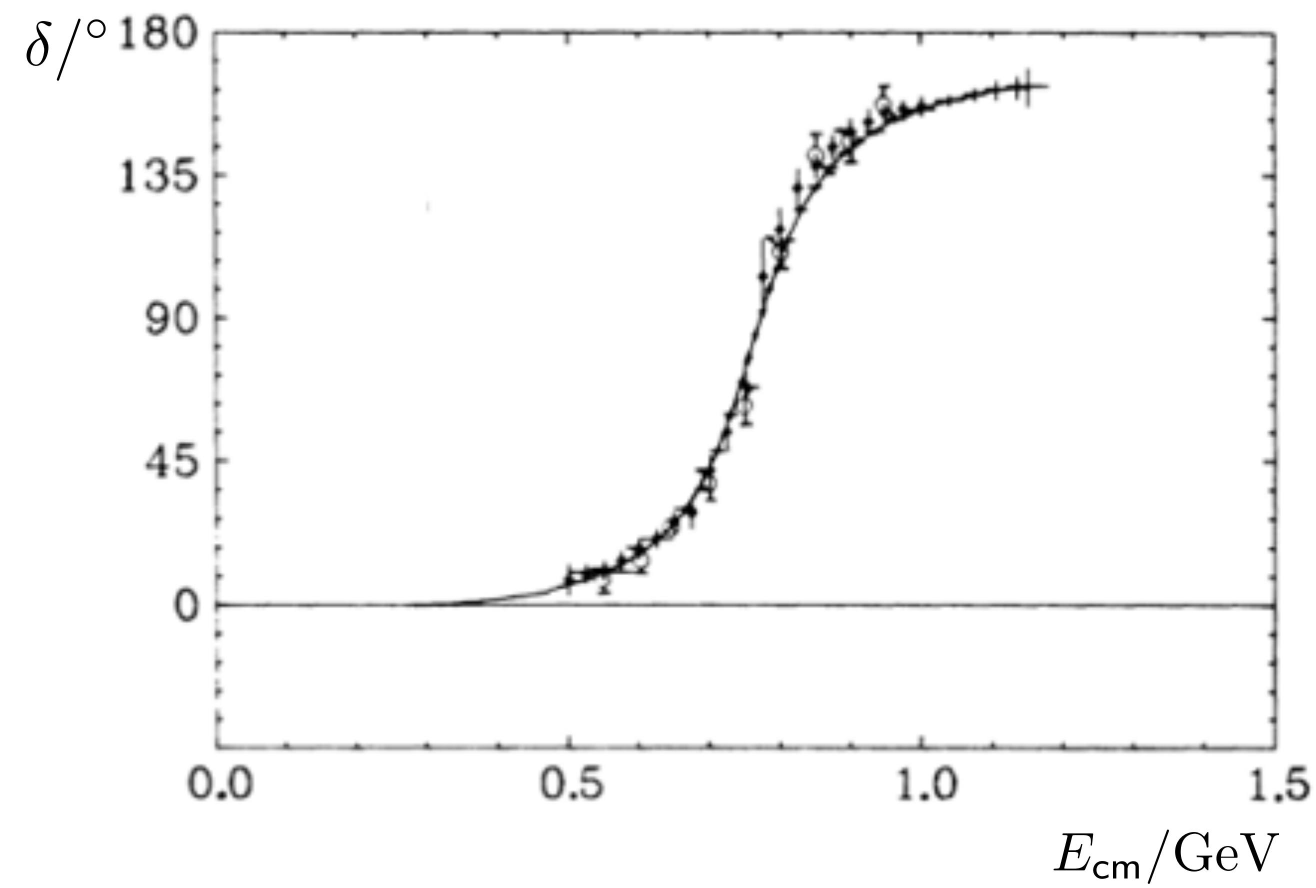
UNIVERSITY OF CAMBRIDGE

Extracting resonance properties

excited states seen as resonant enhancements
in the scattering of lighter stable particles

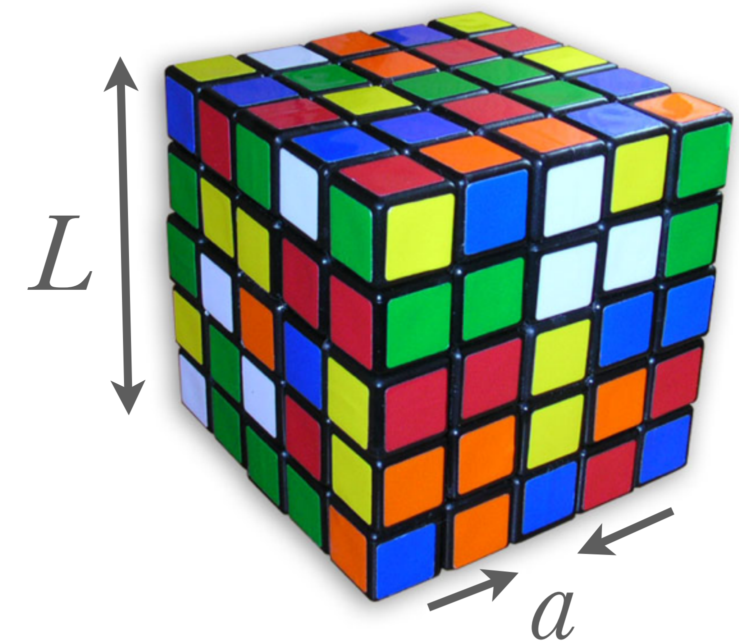


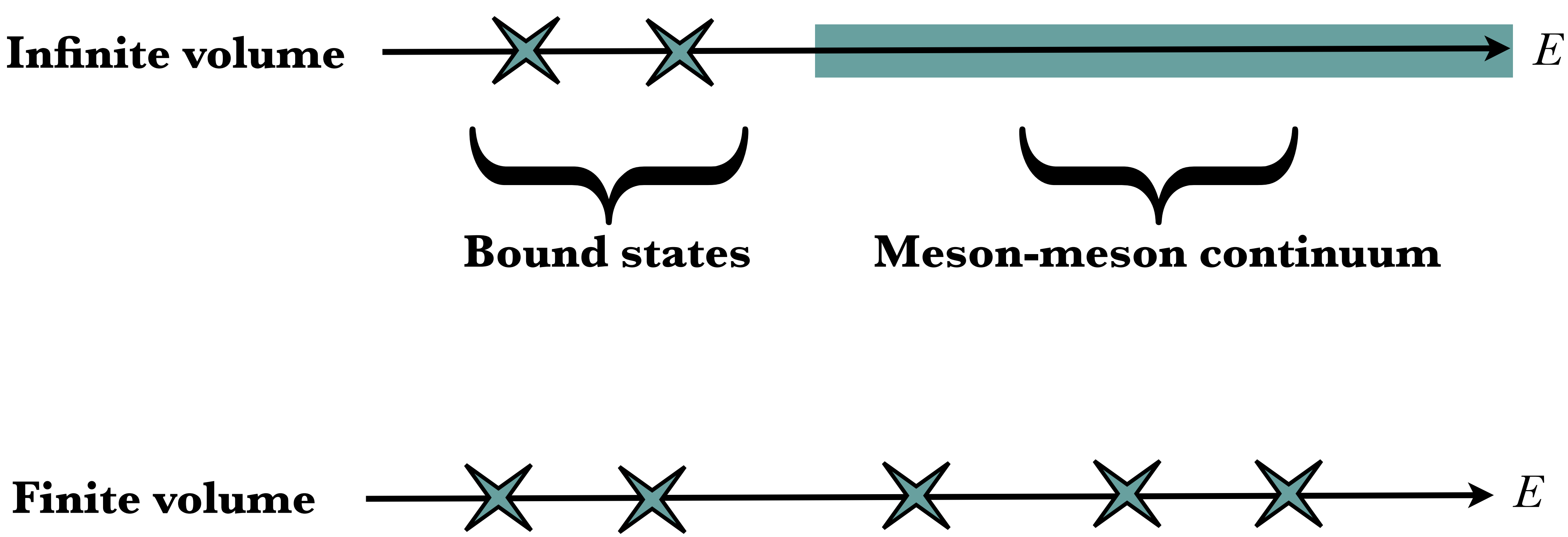
Protopopescu *et al*, PRD7, 5 (1973).



Extracting resonance properties

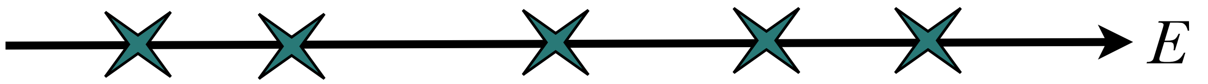
excited states seen as resonant enhancements
in the scattering of lighter stable particles

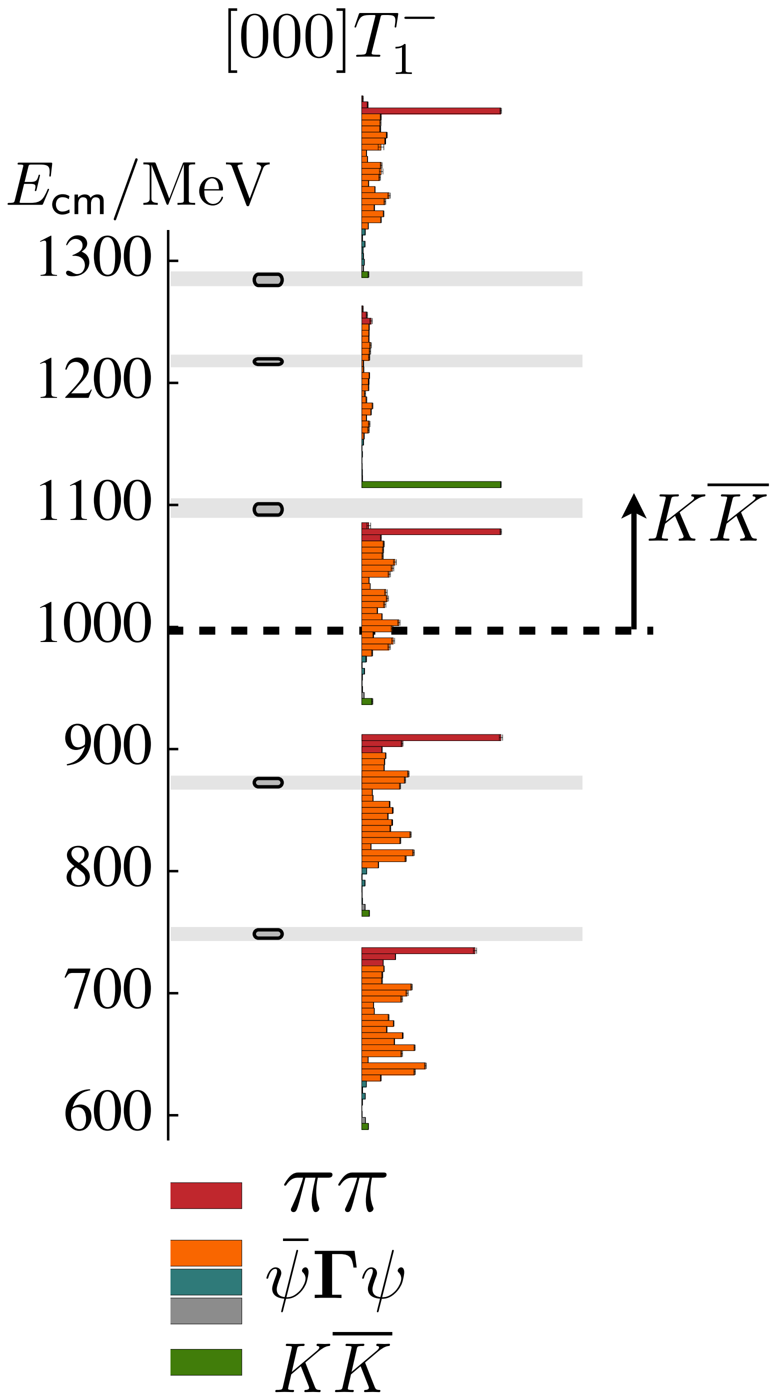


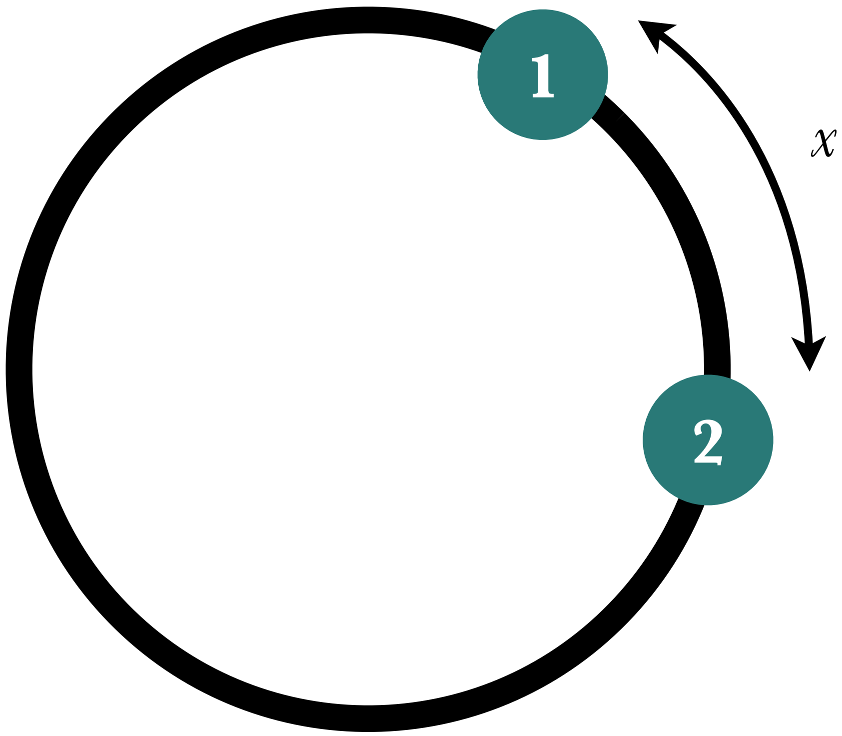


ρ resonance

Infinite volume phase shifts from a finite volume







$$\psi(0) = \psi(L), \quad \left.\frac{\partial\psi}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi}{\partial x}\right|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

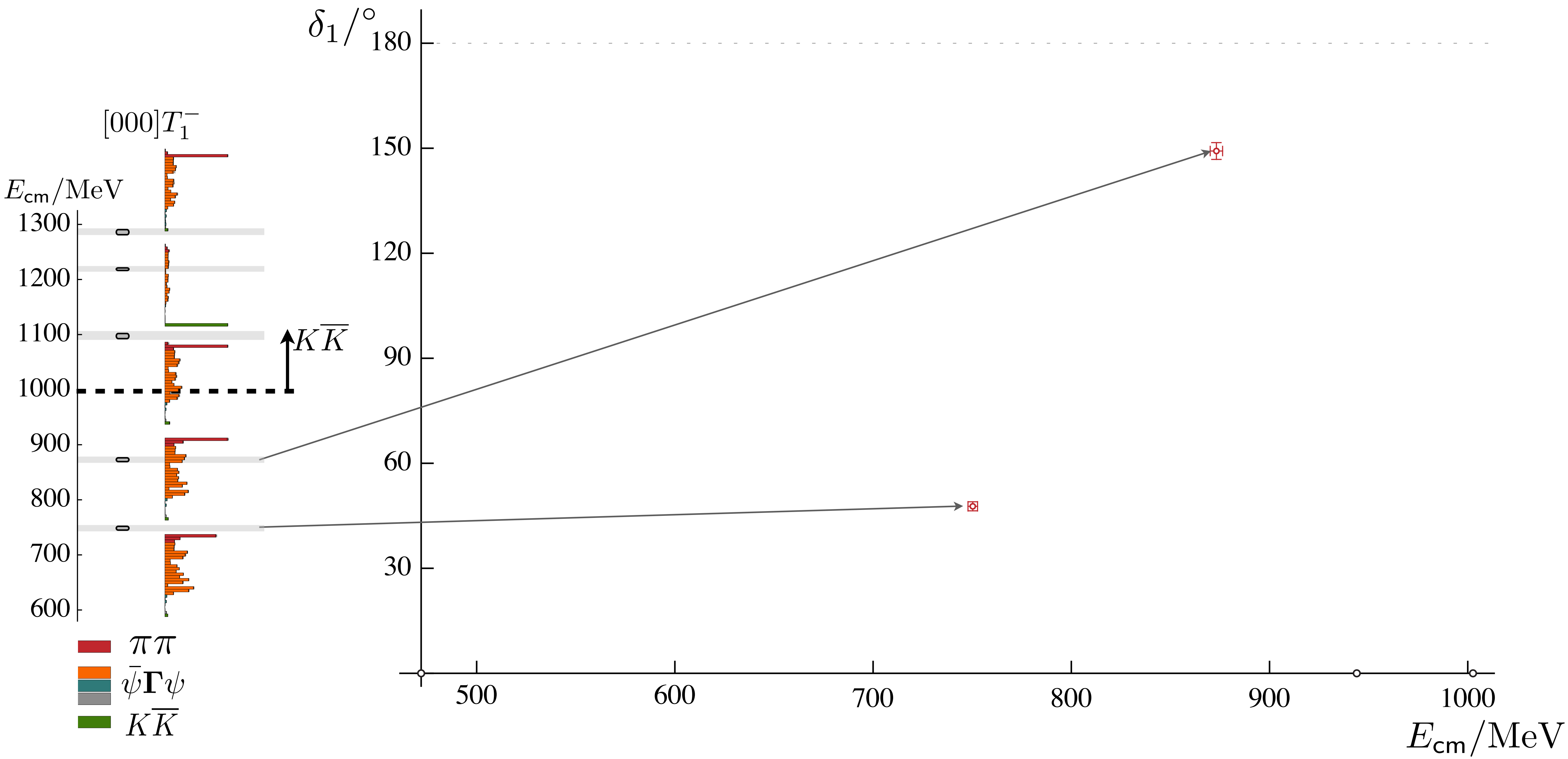
$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$

$m_\pi = 236$ MeV

ρ resonance

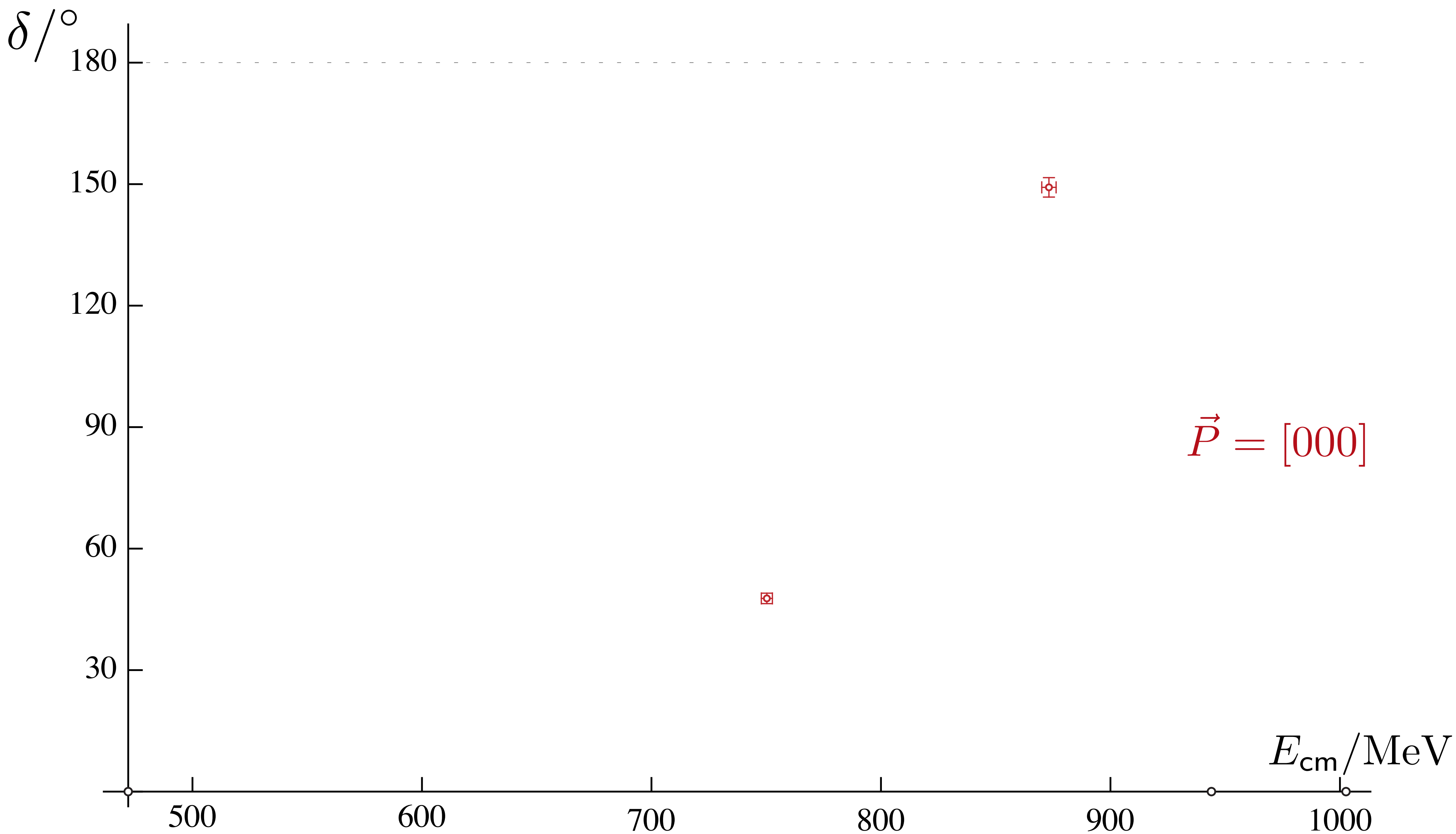
Phase shifts via the Lüscher method: $\tan\delta_1 = \dfrac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$



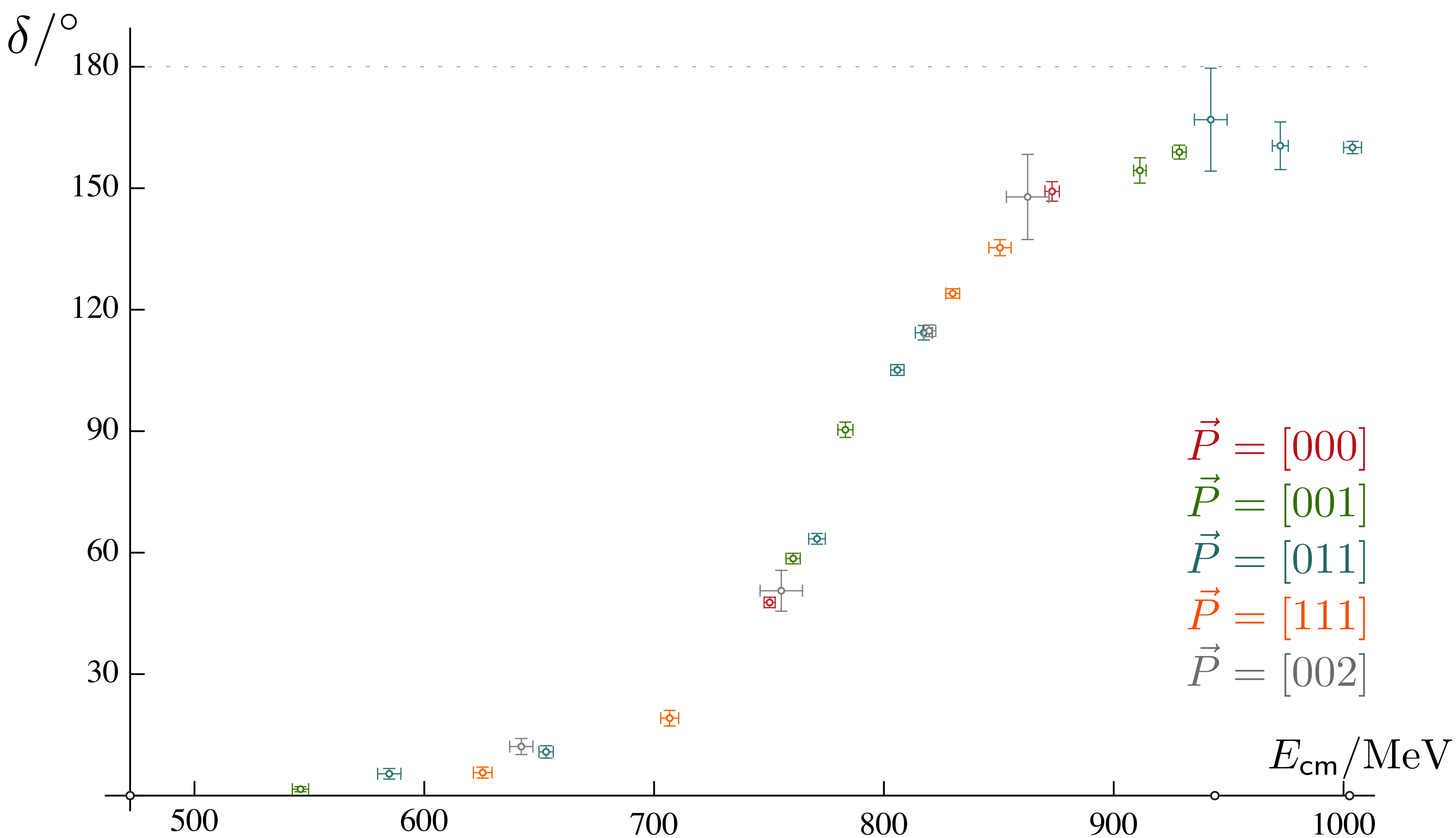
$m_\pi = 236$ MeV

ρ resonance with moving frames



$m_\pi = 236$ MeV

ρ resonance with moving frames



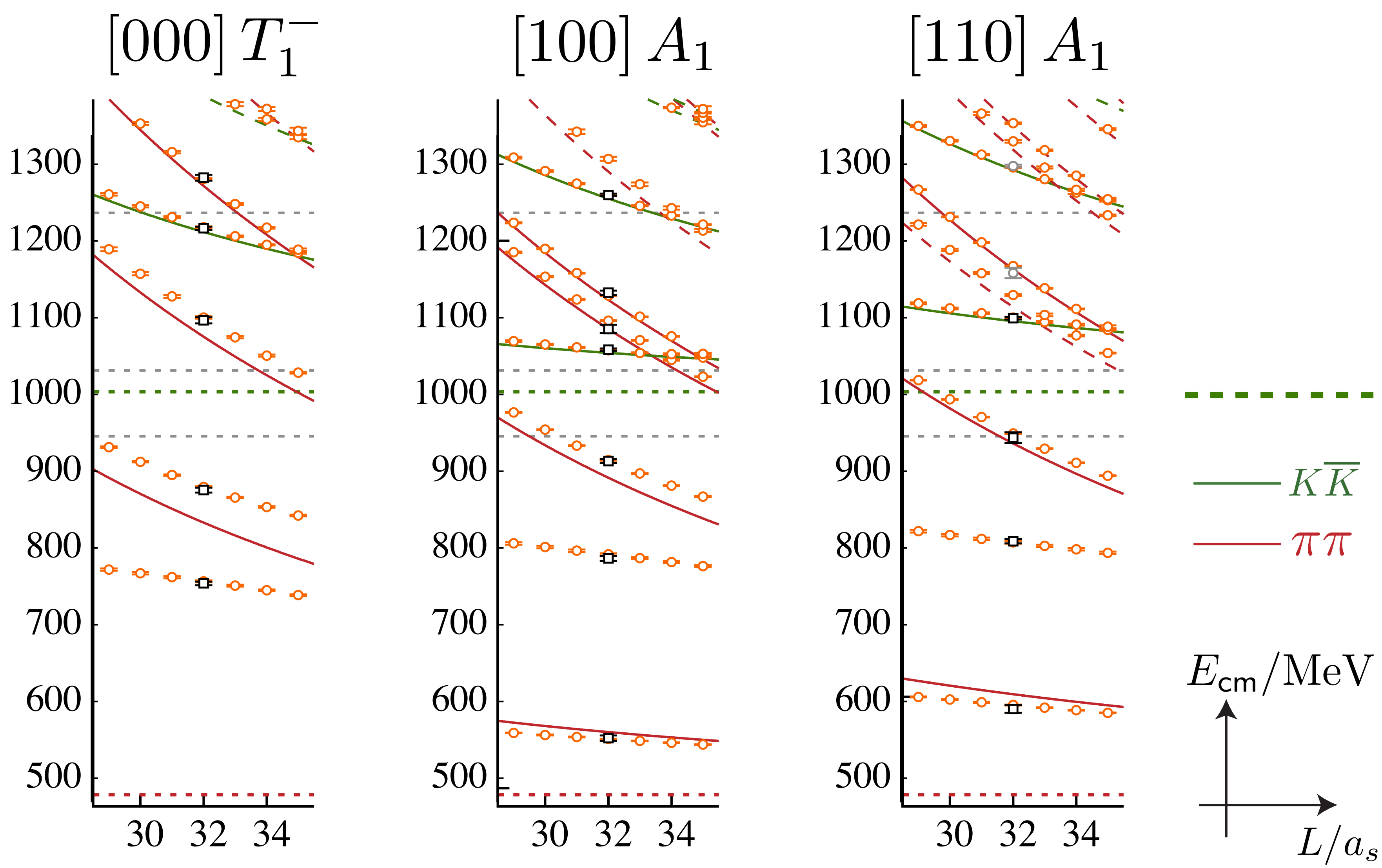
$m_\pi = 236\ \mathrm{MeV}$

ρ resonance into the coupled-channel region

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

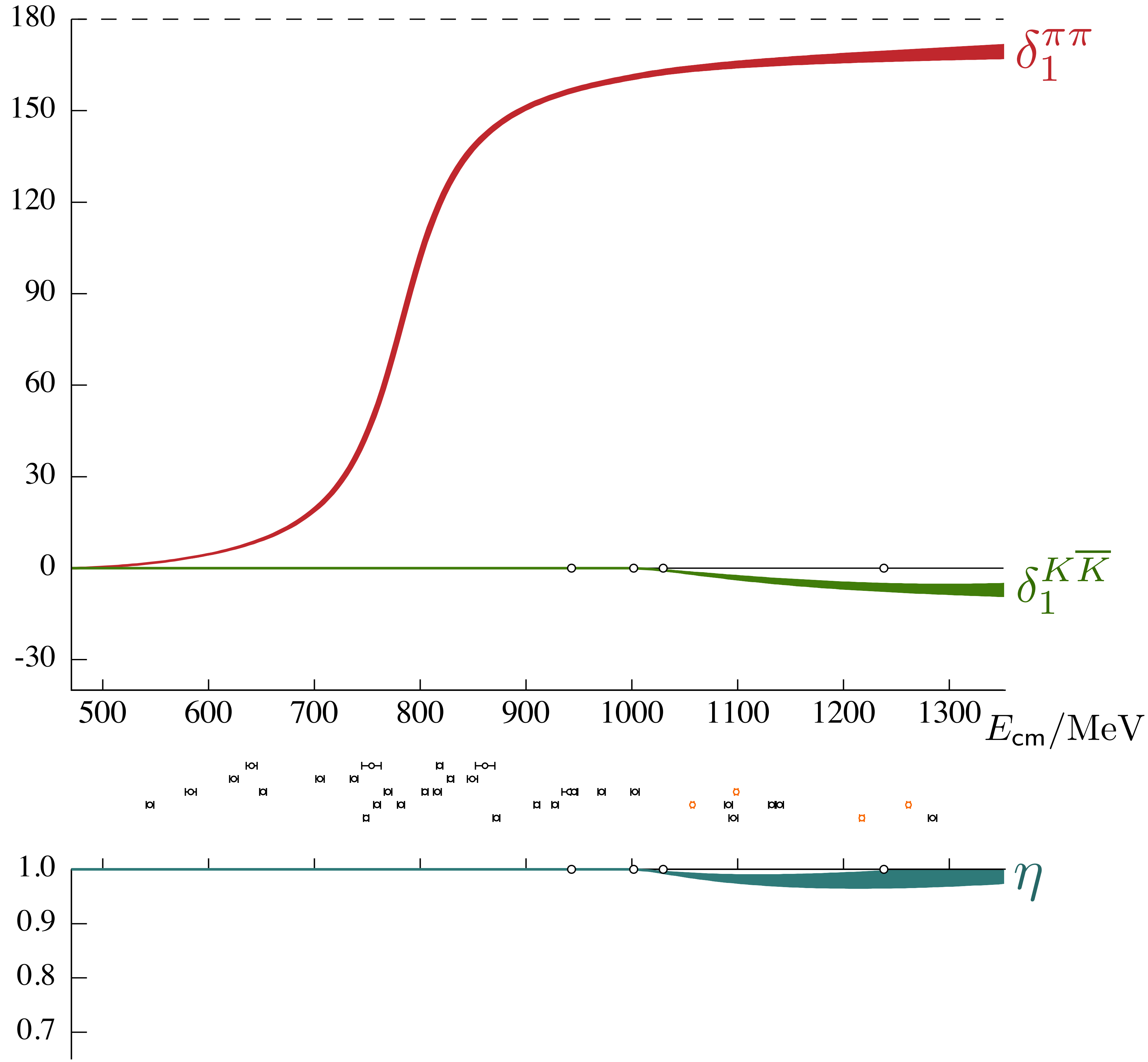
$$\text{e.g.: } K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$



$m_\pi = 236$ MeV

ρ resonance into the coupled-channel region

PRD 92 094502, arXiv:1507.02599



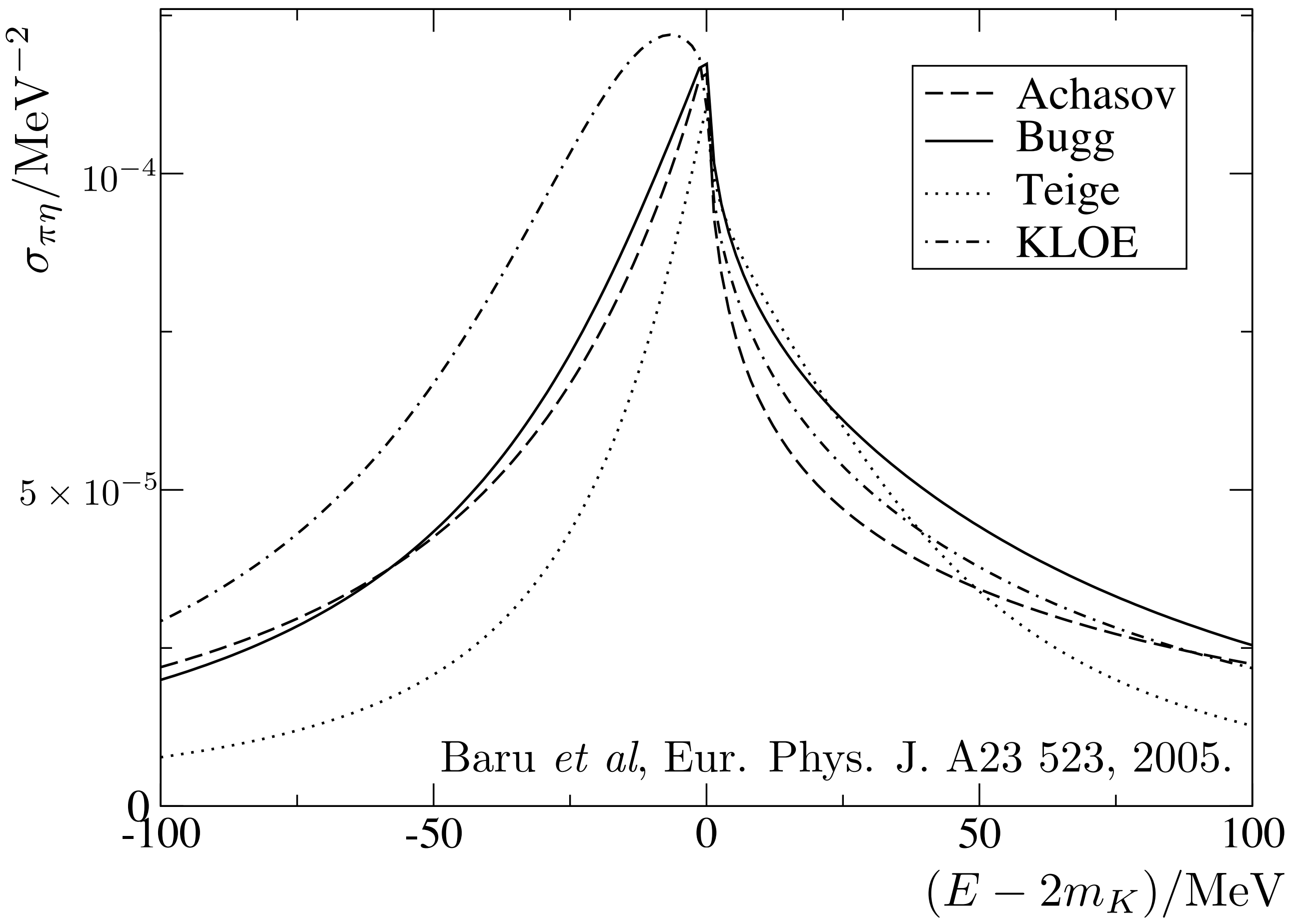
$m_\pi = 236$ MeV

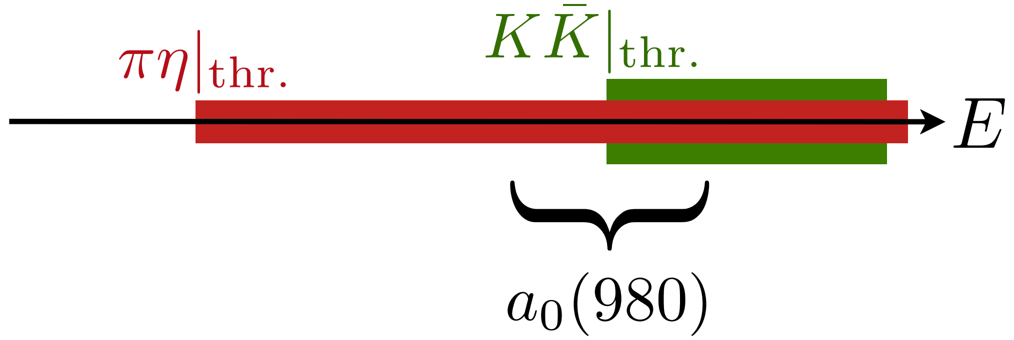
An a_0 resonance

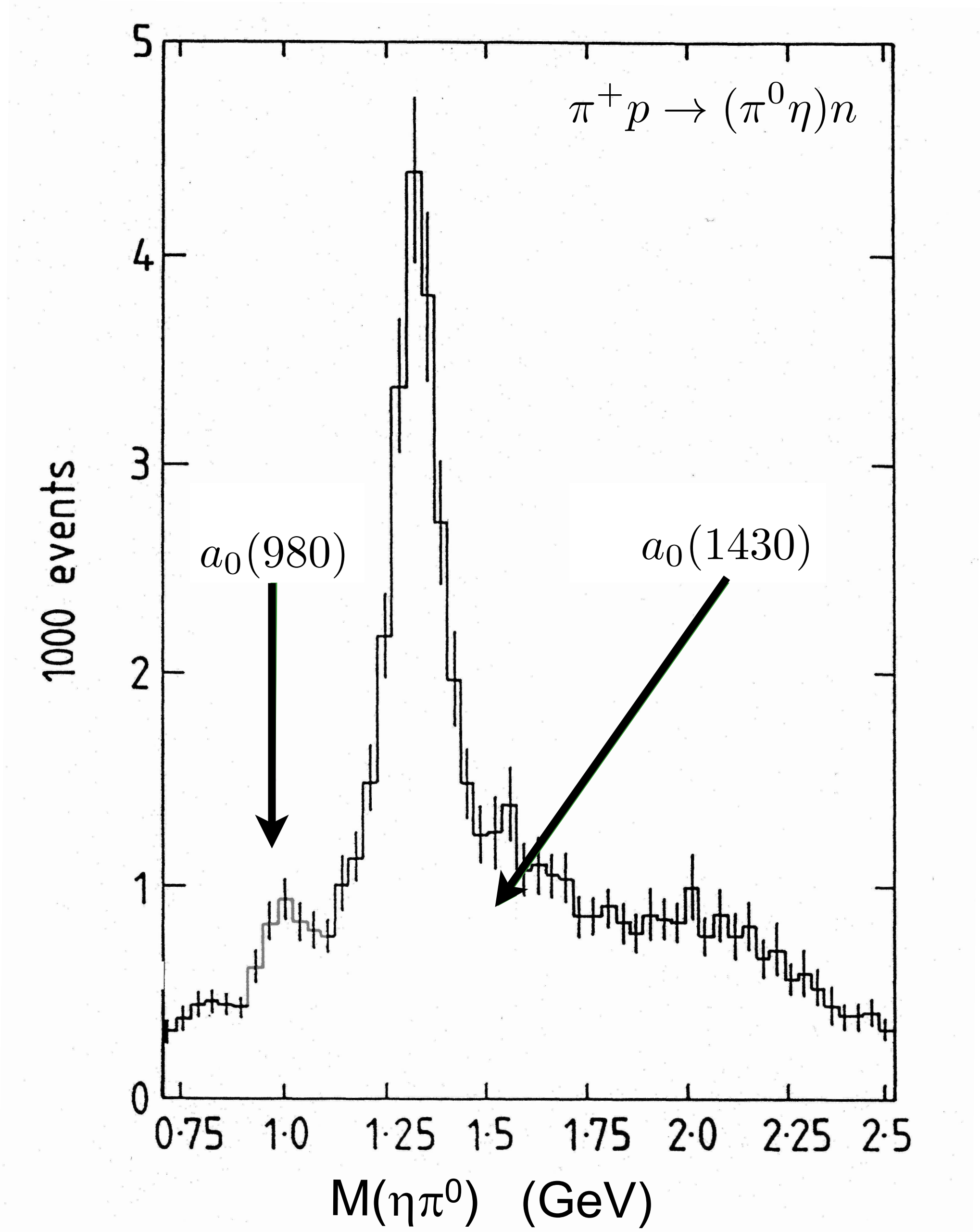
PRD 93 094506, arXiv:1602.05122

$\pi\eta$-$K\bar{K}$-$\pi\eta'$

$I = 1 \;\; J = 0$







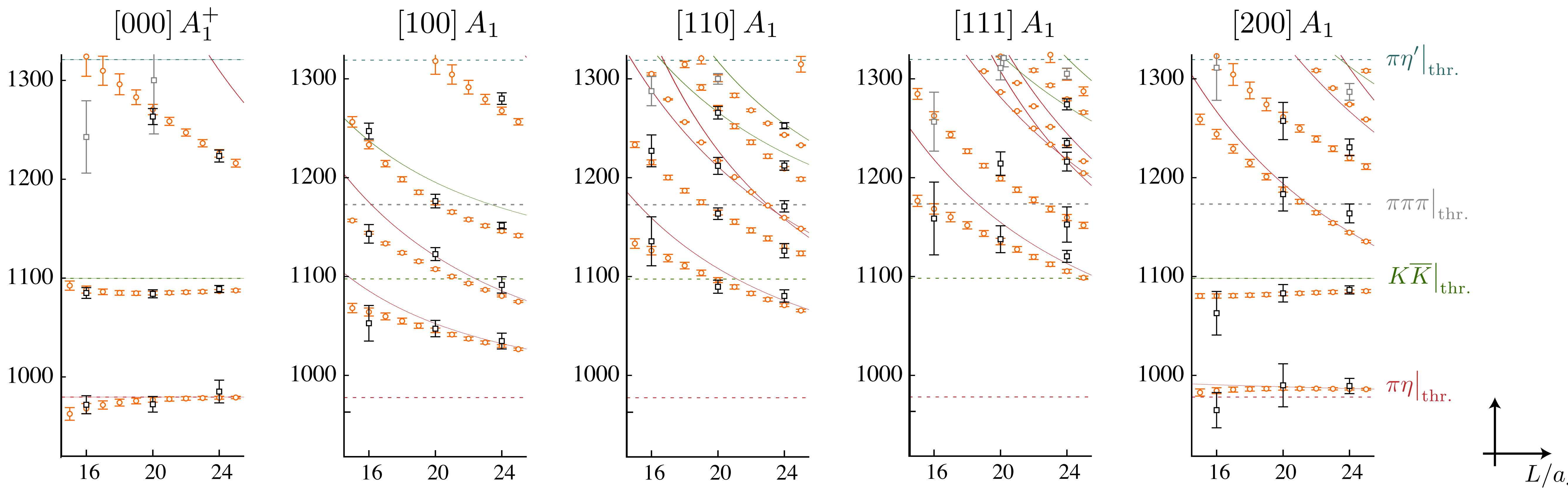
GAMS, Alde *et al* PLB 203 397, 1988.

$m_\pi = 391$ MeV

a_0 resonance - two channel region

$\pi\eta$-$K\bar{K}$

$$K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$



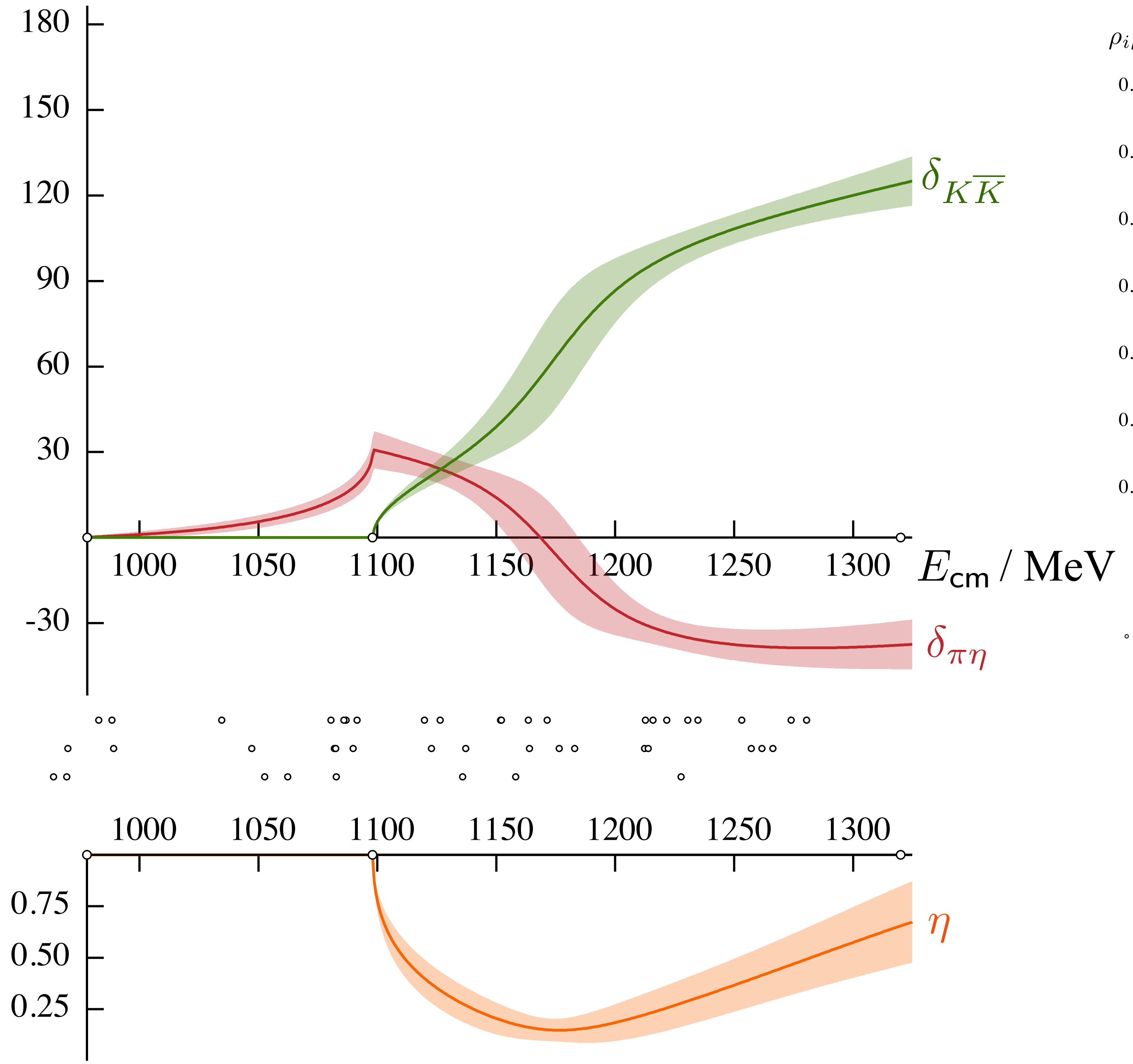
$$
\begin{array}{rl}
m = & (\ 1254 \pm 16) \cdot \mathrm{GeV} \\
g_{\pi\eta} = & (\quad 515 \pm 16) \cdot \mathrm{GeV} \\
g_{K\bar{K}} = & (-730 \pm 85) \cdot \mathrm{GeV} \\
\gamma_{\pi\eta,\,\pi\eta} = & -0.16 \pm 0.24 \\
\gamma_{\pi\eta,\,K\bar{K}} = & -0.56 \pm 0.29 \\
\gamma_{K\bar{K},\,K\bar{K}} = & \ \ 0.12 \pm 0.38
\end{array}
\quad
\begin{bmatrix}
1 & 0.58 & -0.06 & -0.51 & 0.39 & 0.02 \\
 & 1 & -0.63 & -0.87 & 0.84 & -0.49 \\
 & & 1 & 0.52 & -0.68 & 0.83 \\
 & & & 1 & -0.90 & 0.53 \\
 & & & & 1 & -0.78 \\
 & & & & & 1
\end{bmatrix}
$$

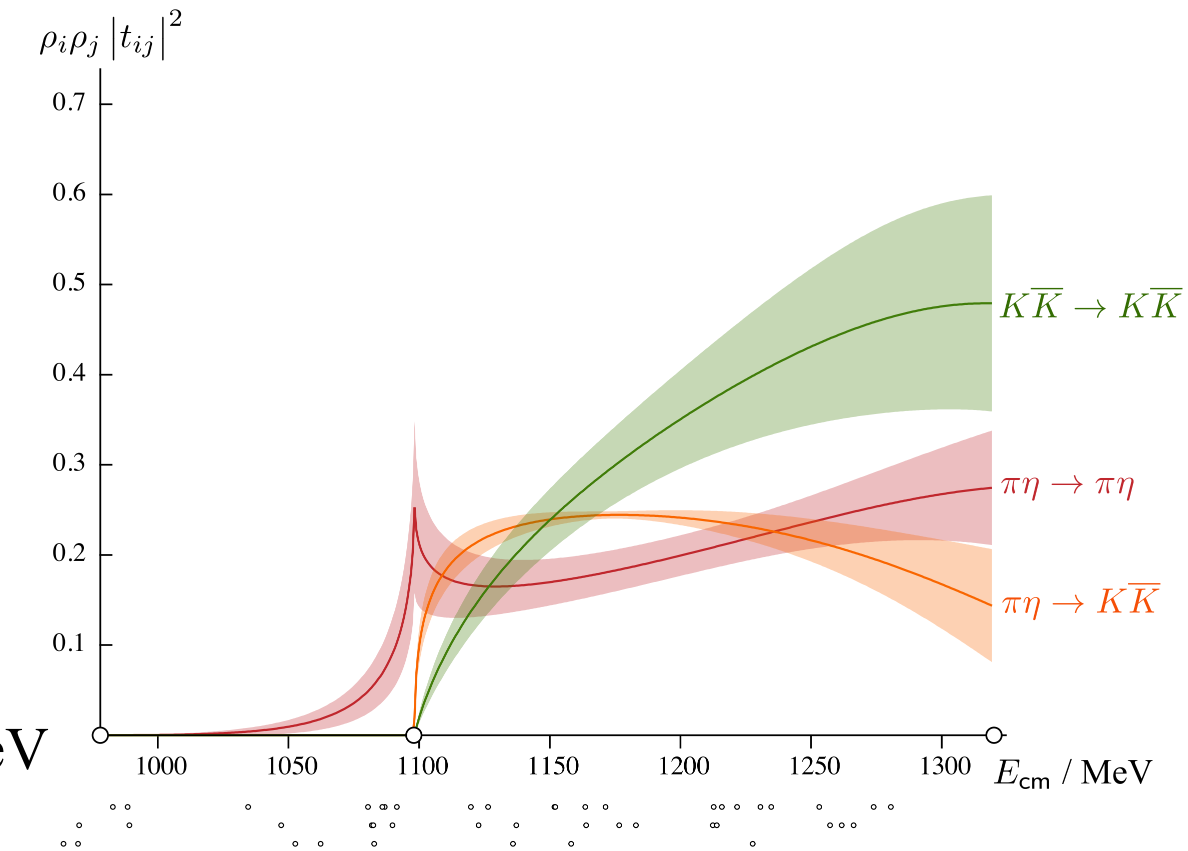
$$\chi^2/N_{\mathrm{dof}} = \frac{58.0}{47-6} = 1.41$$

$m_\pi = 391$ MeV

a_0 resonance - two channel region

S-wave $\pi\eta$-$K\bar{K}$

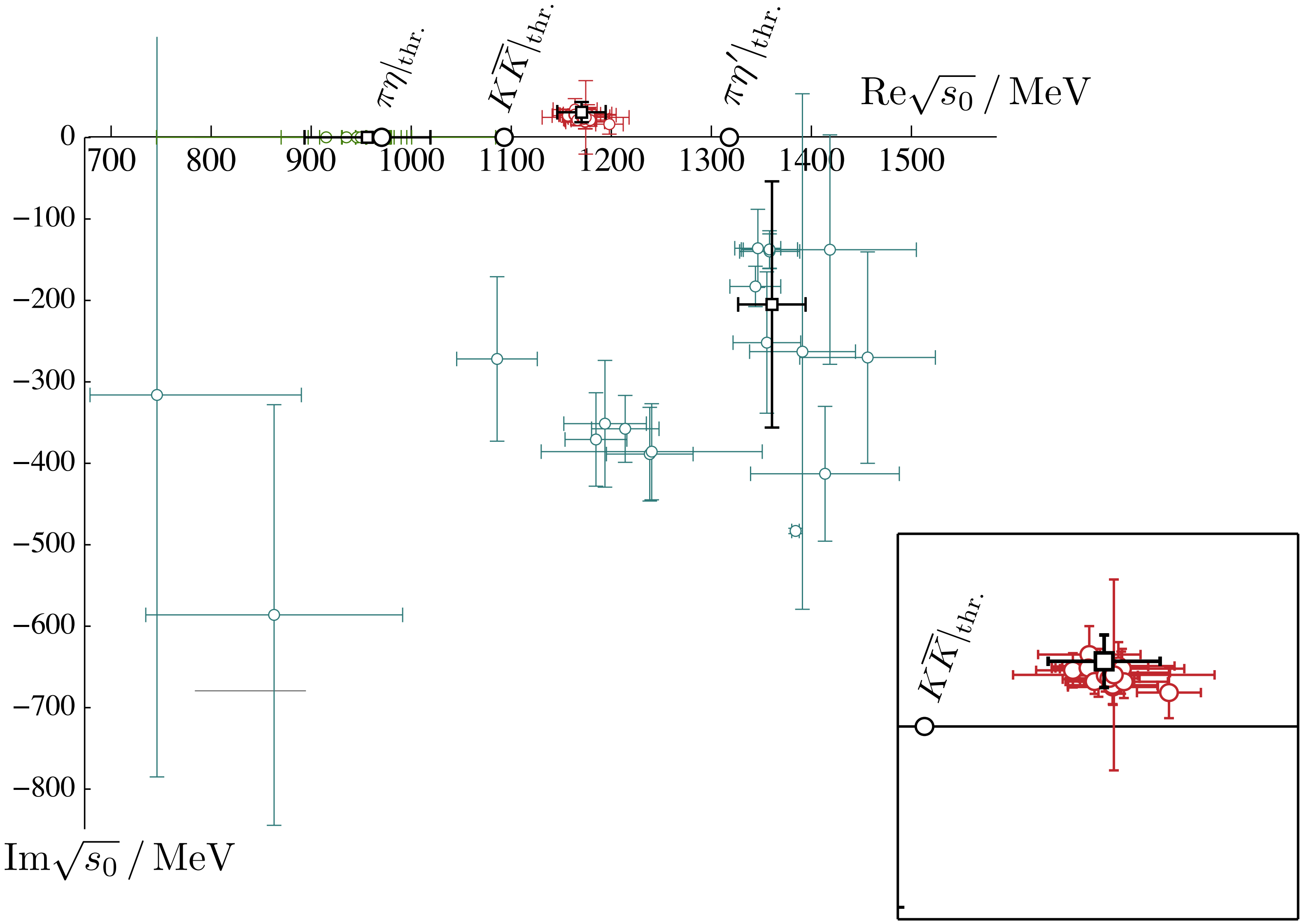


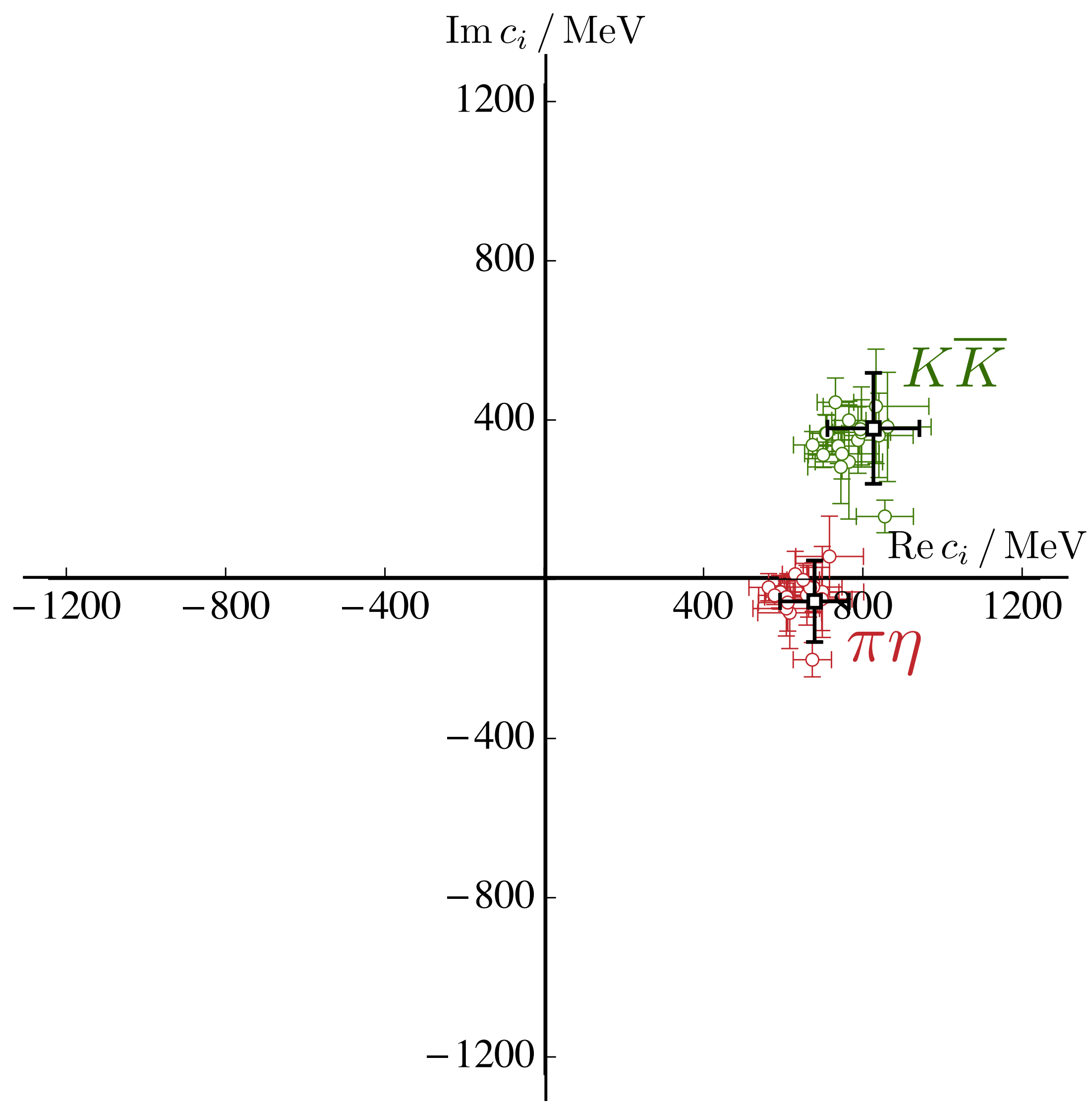


$m_\pi = 391$ MeV

a_0 resonance pole

$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$





$$\left|c_{\pi\eta}\right| = 652(130)\ \mathrm{MeV}$$

$$\left|c_{K\bar{K}}\right| = 844(170)\ \mathrm{MeV}$$

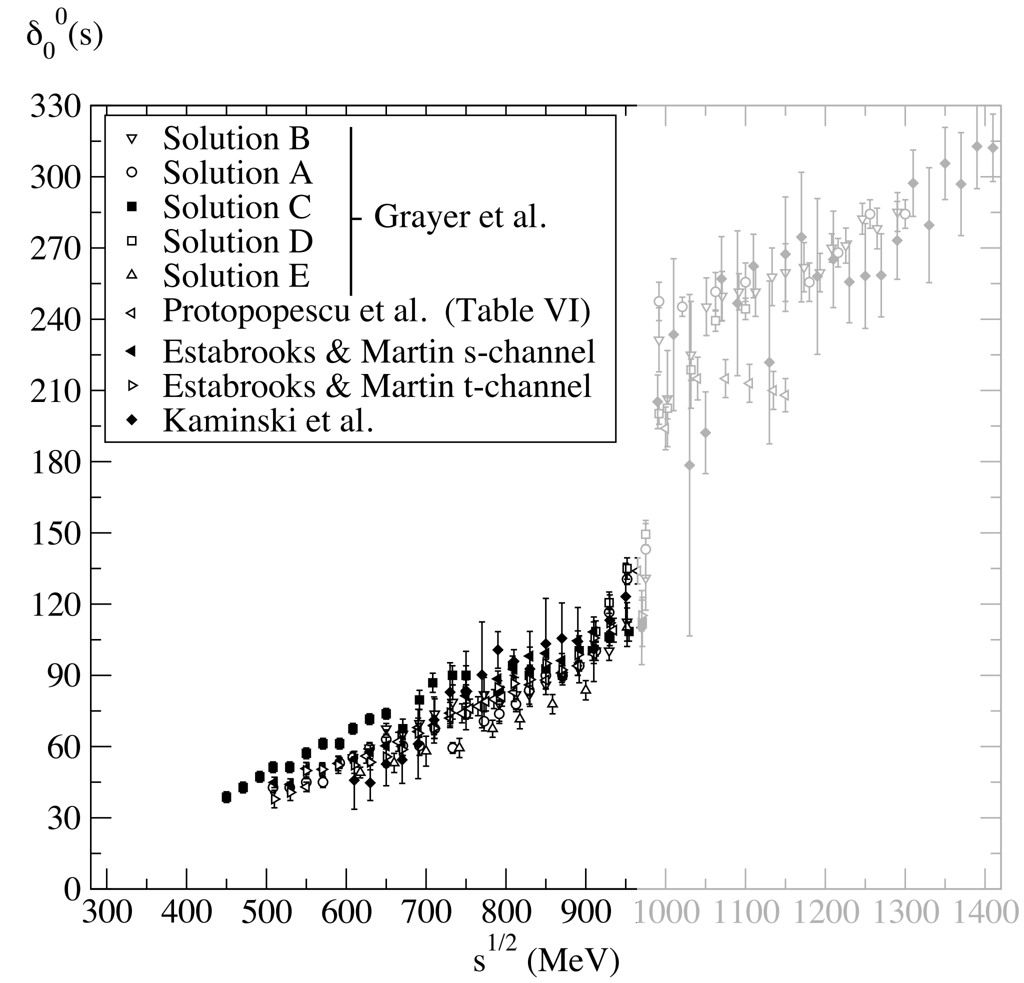
$$\sqrt{s_0} = \left((1177 \pm 27) + \frac{i}{2}(49 \pm 33) \right)\ \mathrm{MeV}$$

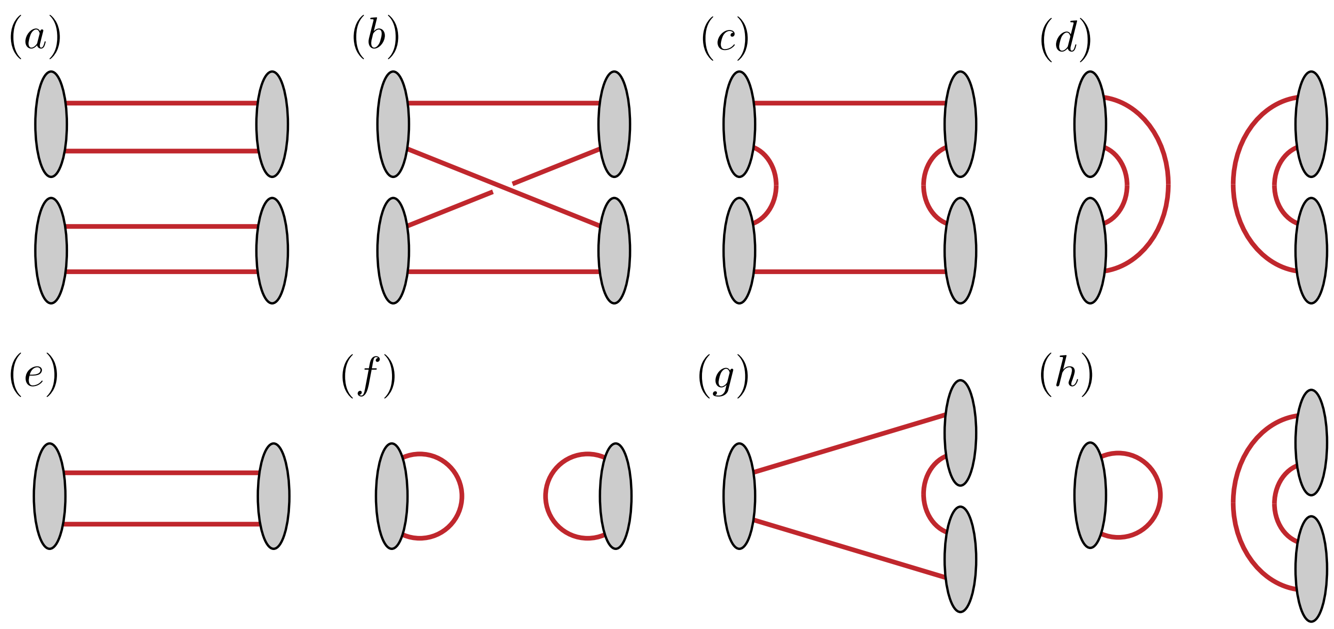
$$m_\pi = 391\ \mathrm{MeV}$$

The $f_0(500)/\sigma$ resonance

arXiv:1607.05900

elastic scattering with
vacuum quantum numbers
$\pi\pi$ in $I = 0$, $J = 0$

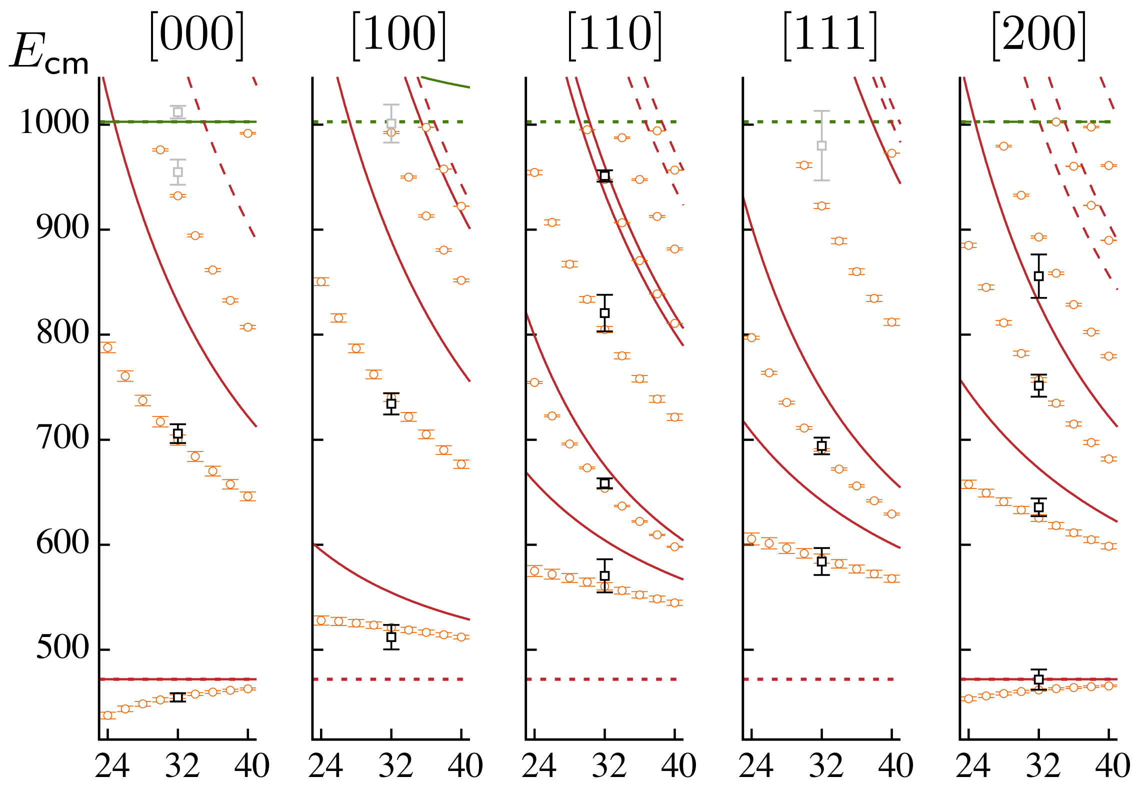




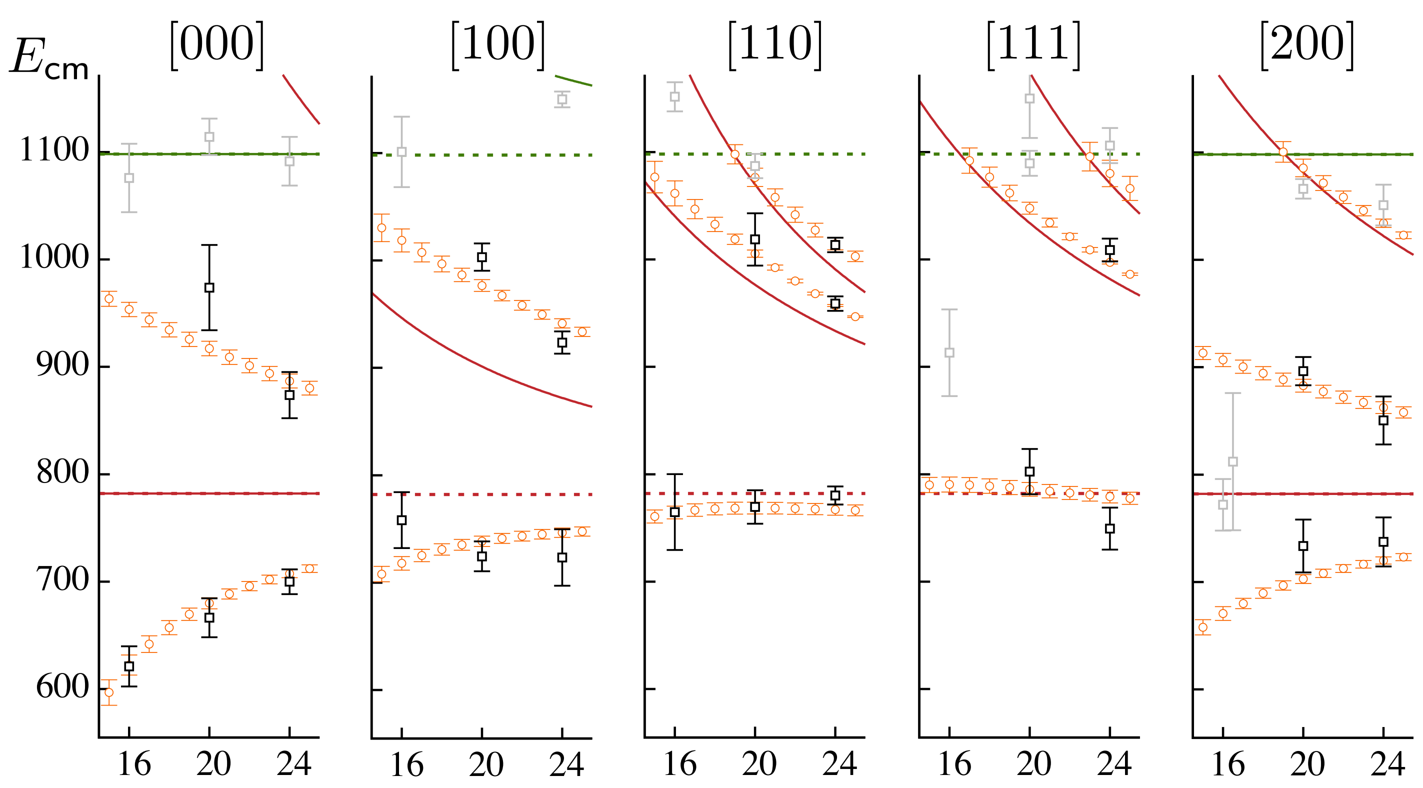
The $f_0(500)/\sigma$ resonance

elastic scattering with vacuum quantum numbers
$\pi\pi$ in $I = 0,\ J = 0$

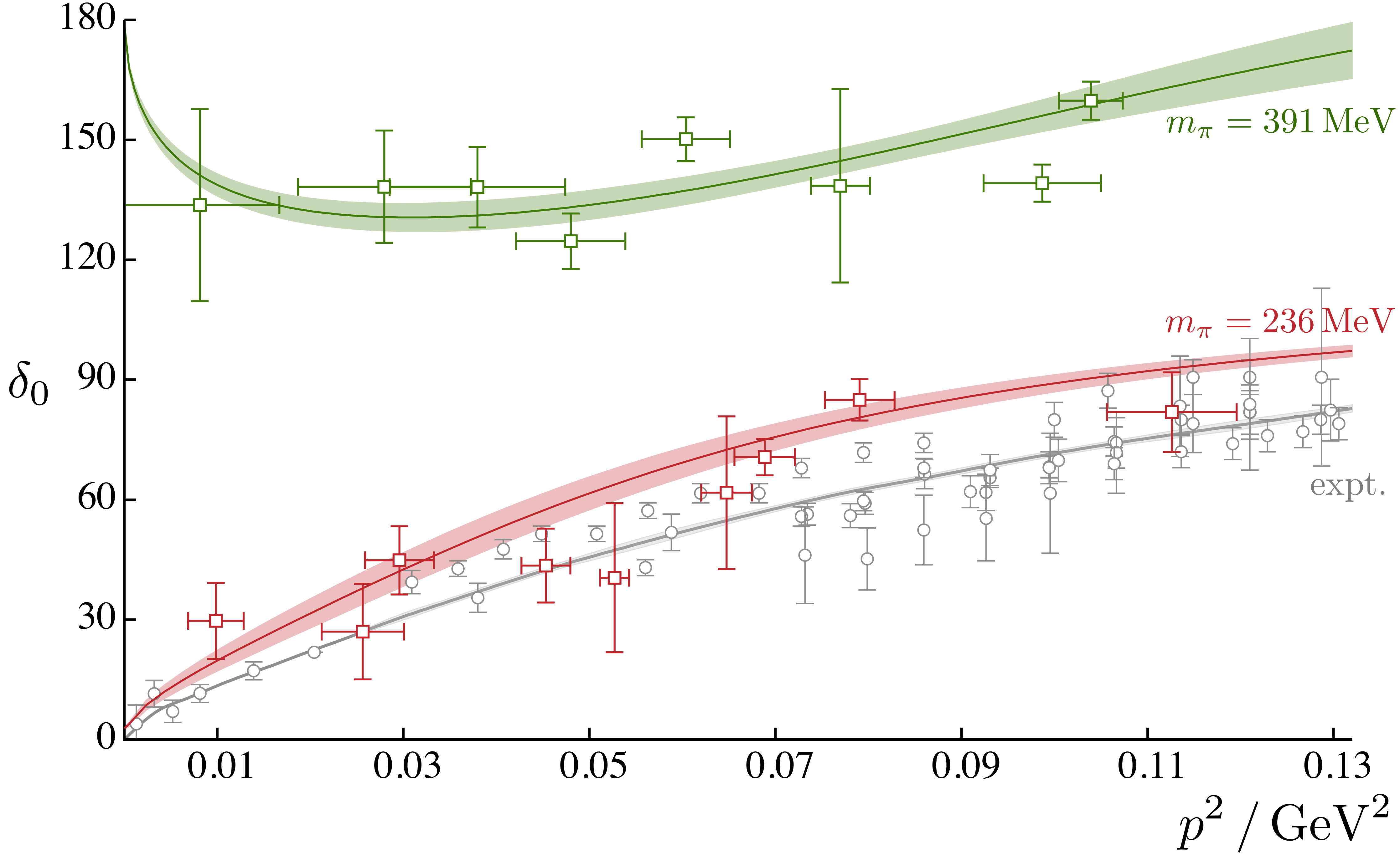
$m_\pi = 236$ MeV



$m_\pi = 391$ MeV

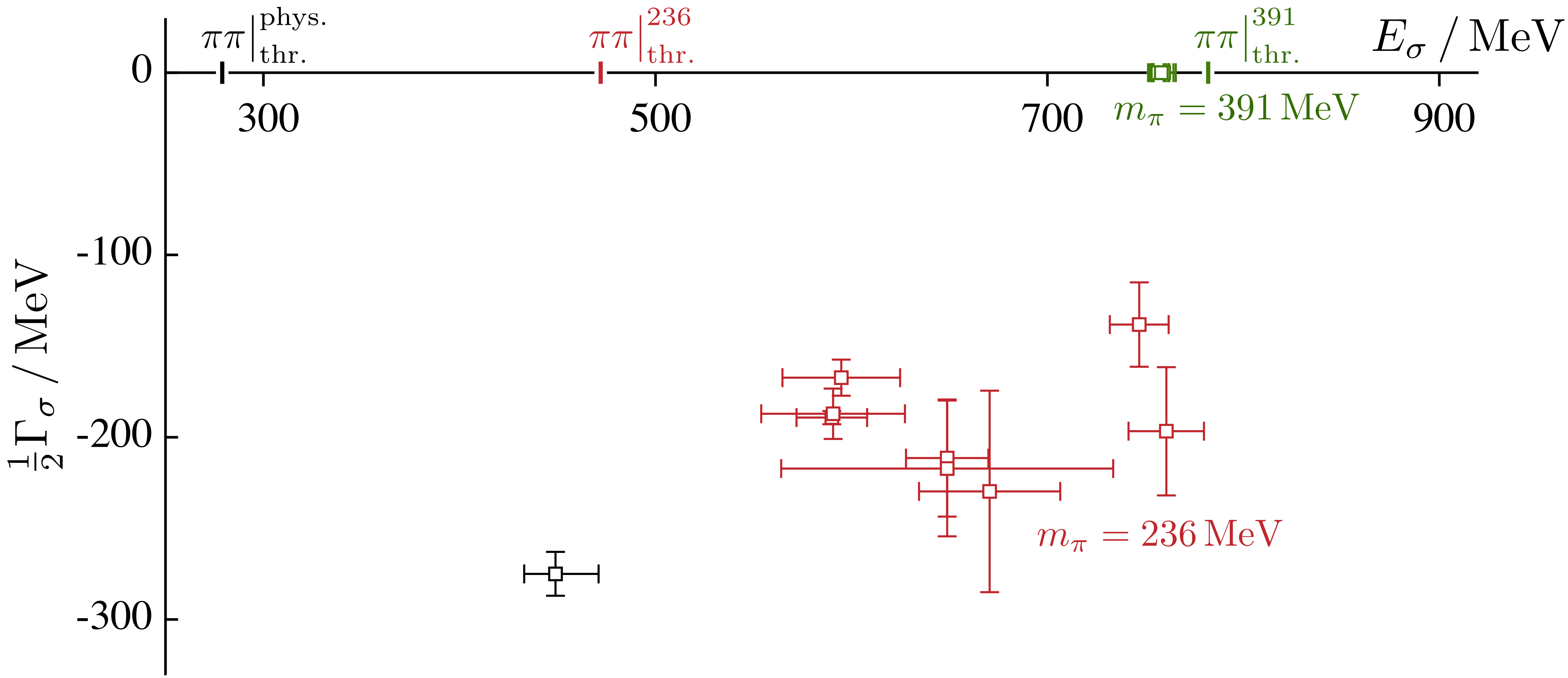


The $f_0(500)/\sigma$ resonance



The $f_0(500)/\sigma$ resonance

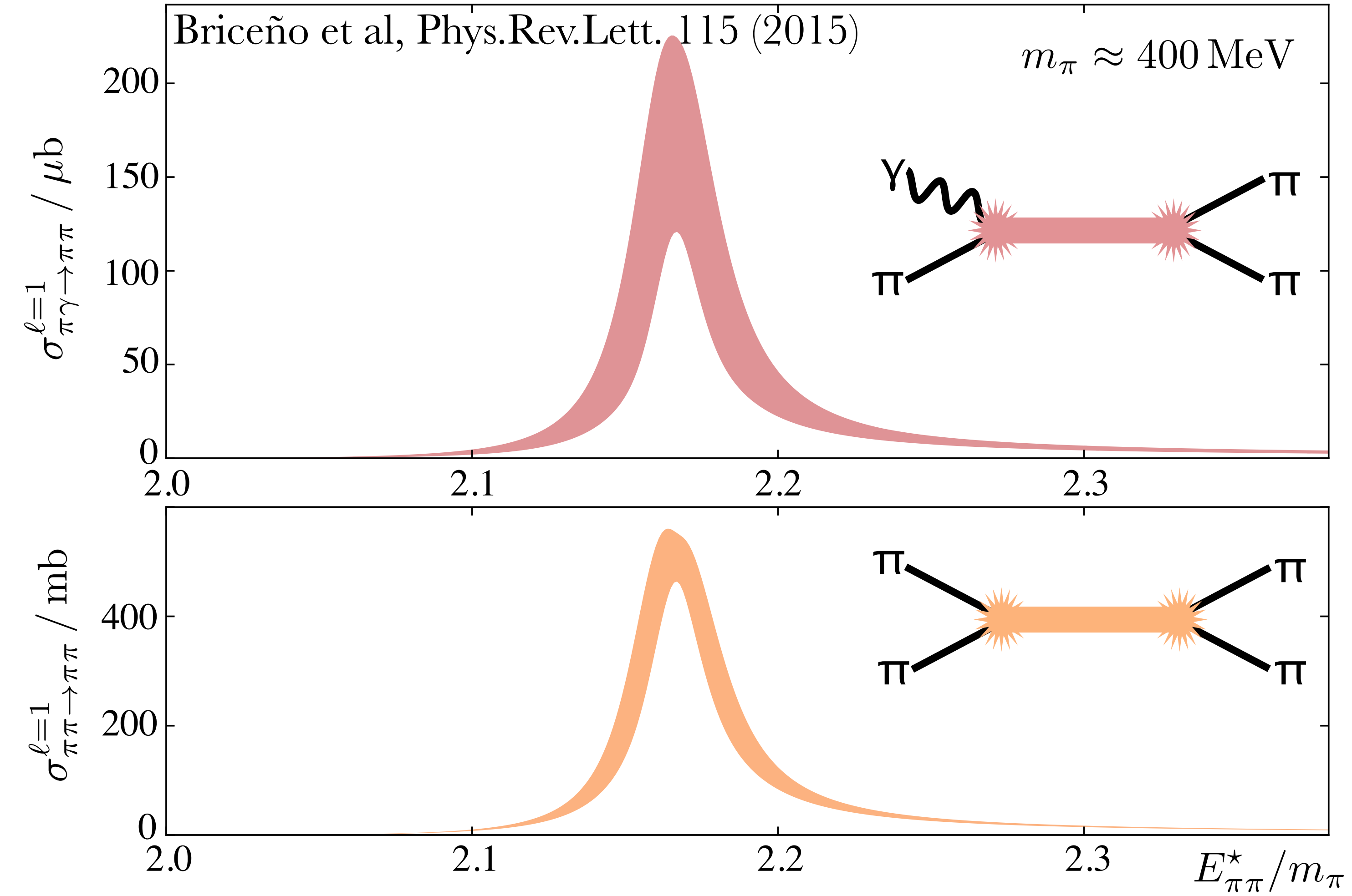
arXiv:1607.05900



Future directions

two-body coupled-channel

$f_0(980)$
$D\bar{D}$
$D\bar{D}^\star$
$N\pi$
$\gamma a \to bc$



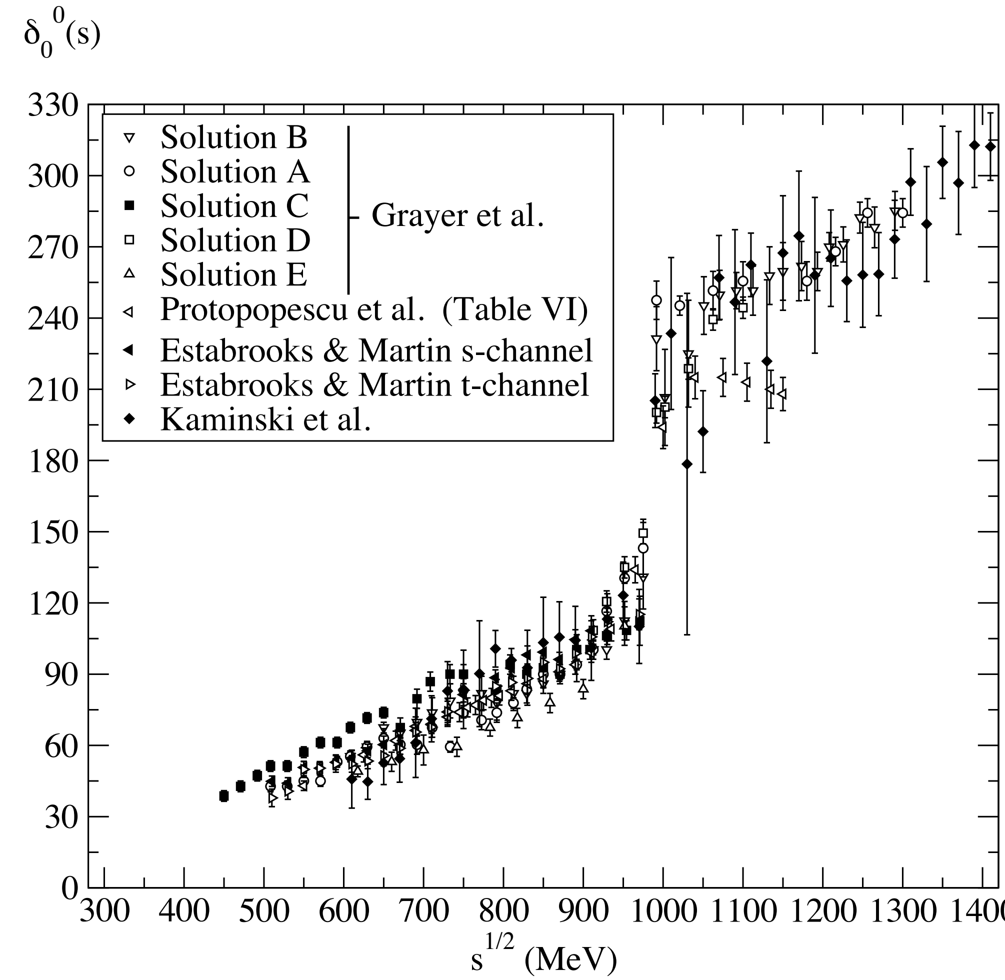
further operator structures - glueball, tetraquark, ...

formalism for three-body and beyond
- needed for higher energies
- needed to get closer to the physical mass

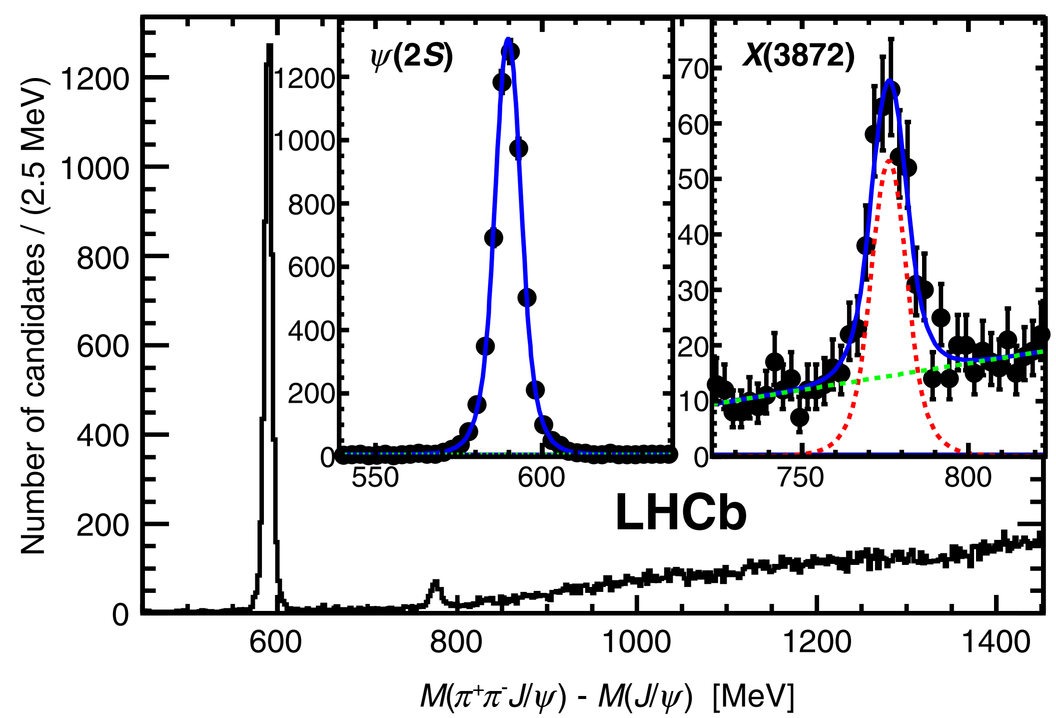
Backup

$m_\pi = 391$ MeV

Coupled-channel scattering



$a_0(980)$, $f_0(980)$
$a_1(1260)$
X(3872), and other XYZ states
$N^\star(1440)$, $\Lambda(1405)$, ...



all decay into multiple final states
all are resonant enhancements in multiple channels
to understand these rigorously, we need coupled-channel analyses

Coupled-channel scattering

Direct extension of the elastic quantization condition derived by Lüscher

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

phase space

infinite volume scattering
t-matrix

known finite-volume
functions

Many derivations, **all in agreement**:

He, Feng, Liu 2005 - two channel QM, strong coupling
Hansen & Sharpe 2012 - field theory, multiple two-body channels
Briceño & Davoudi 2012 - strongly-coupled Bethe-Salpeter amplitudes
Guo et al 2012 - Hamiltonian & Lippmann-Schwinger

Also derivations in specific channels, or for a specific parameterization of the interactions like NREFT, chiral PT, Finite Volume Hamiltonian, etc.

Briceño 2014 - Generalised to scattering of particles with non-zero spin, and spin-½.

Significant steps towards a general 3-body quantization condition have been made

Amplitude parameterization

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E) \cdot \boldsymbol{t}(E) \cdot (\mathbf{1} + i\boldsymbol{\mathcal{M}}(E, L))\right] = 0$$

determinant condition:
- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

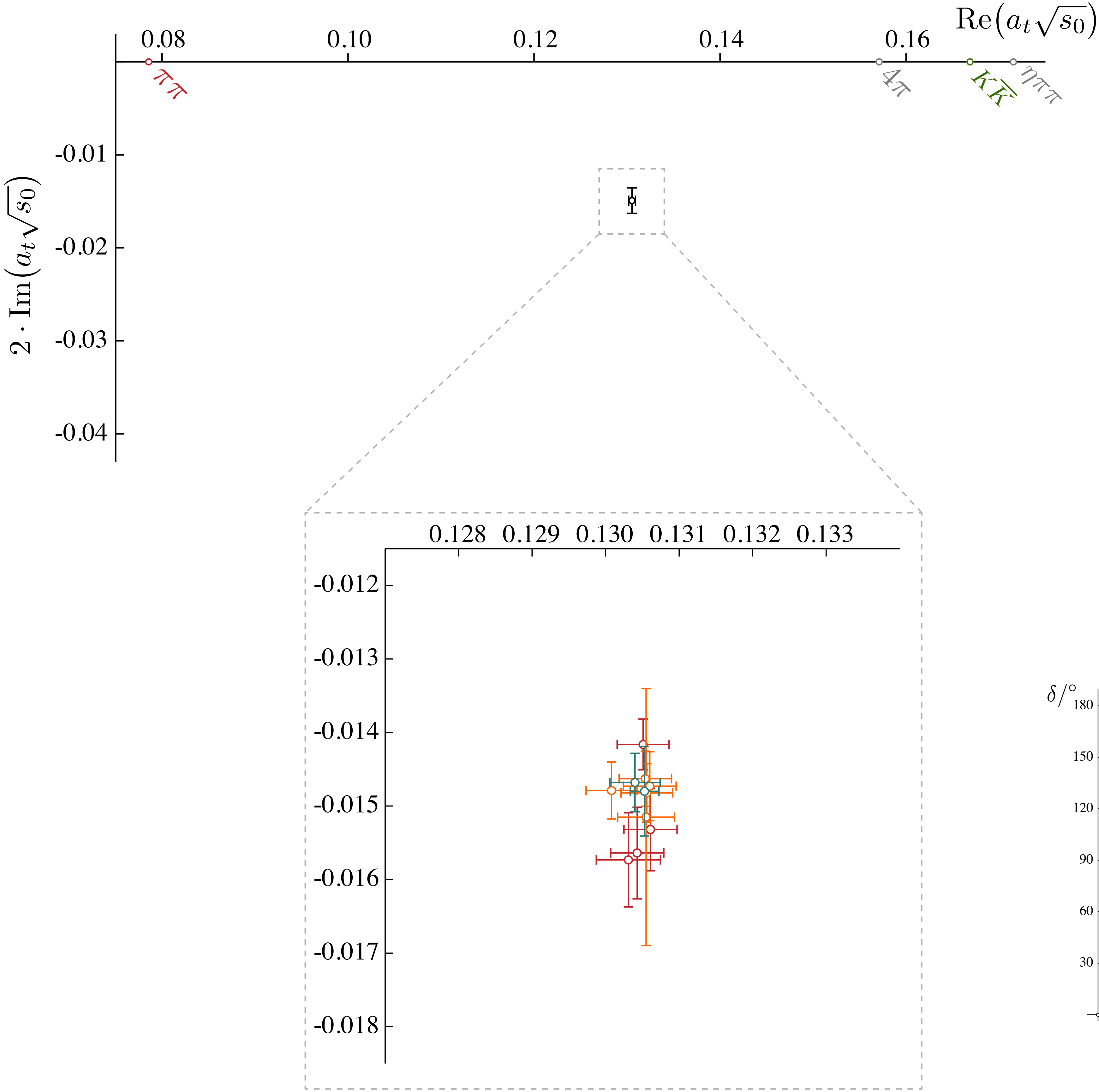
one solution: use energy dependent parameterizations
- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger\mathbf{S} = \mathbf{1} \quad \to \quad \mathrm{Im}\,\mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad \rho_{ij} = \delta_{ij}\frac{2k_i}{E_{\mathsf{cm}}}$$

K-matrix approach:

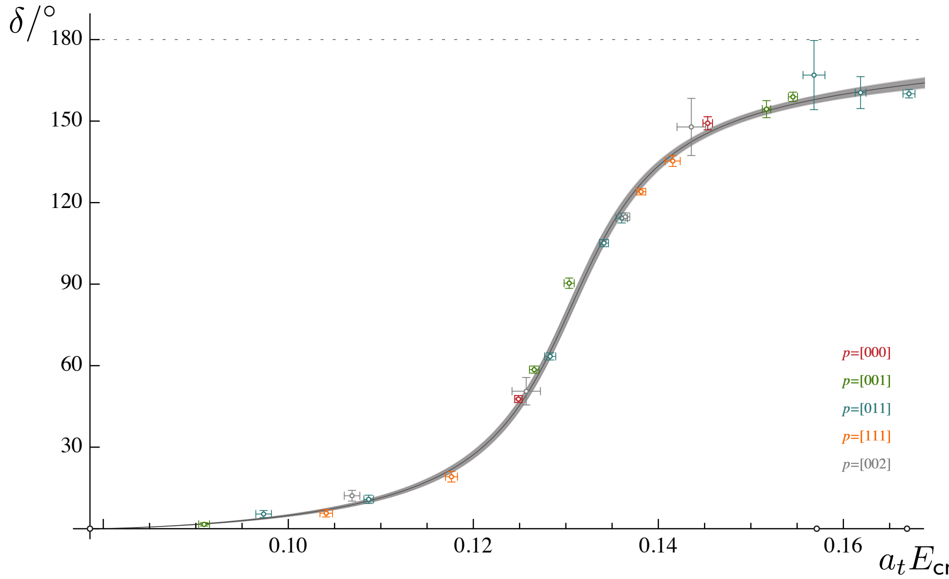
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho} \qquad \text{e.g.: } K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$

ρ resonance pole



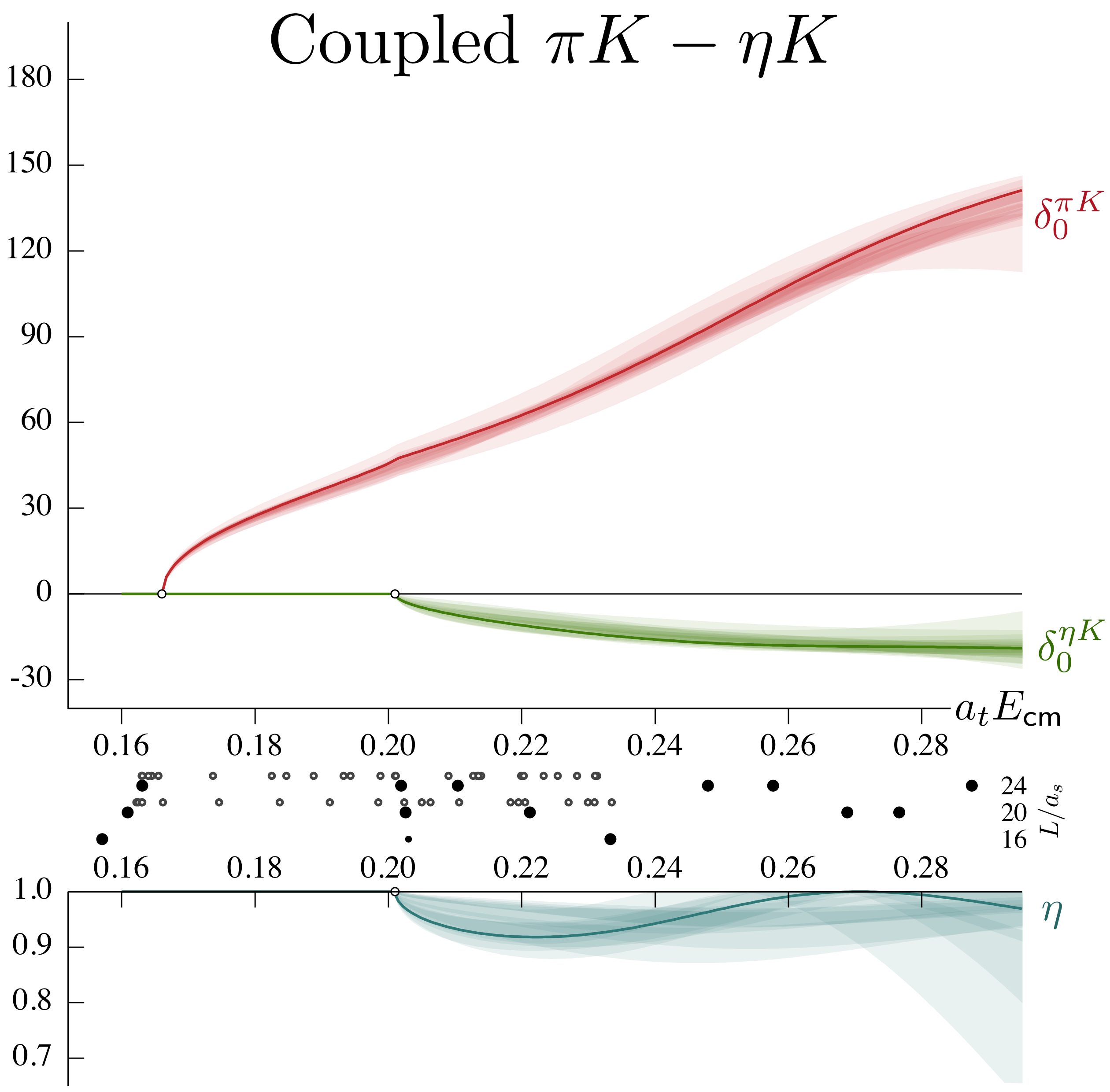
near a pole:

$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

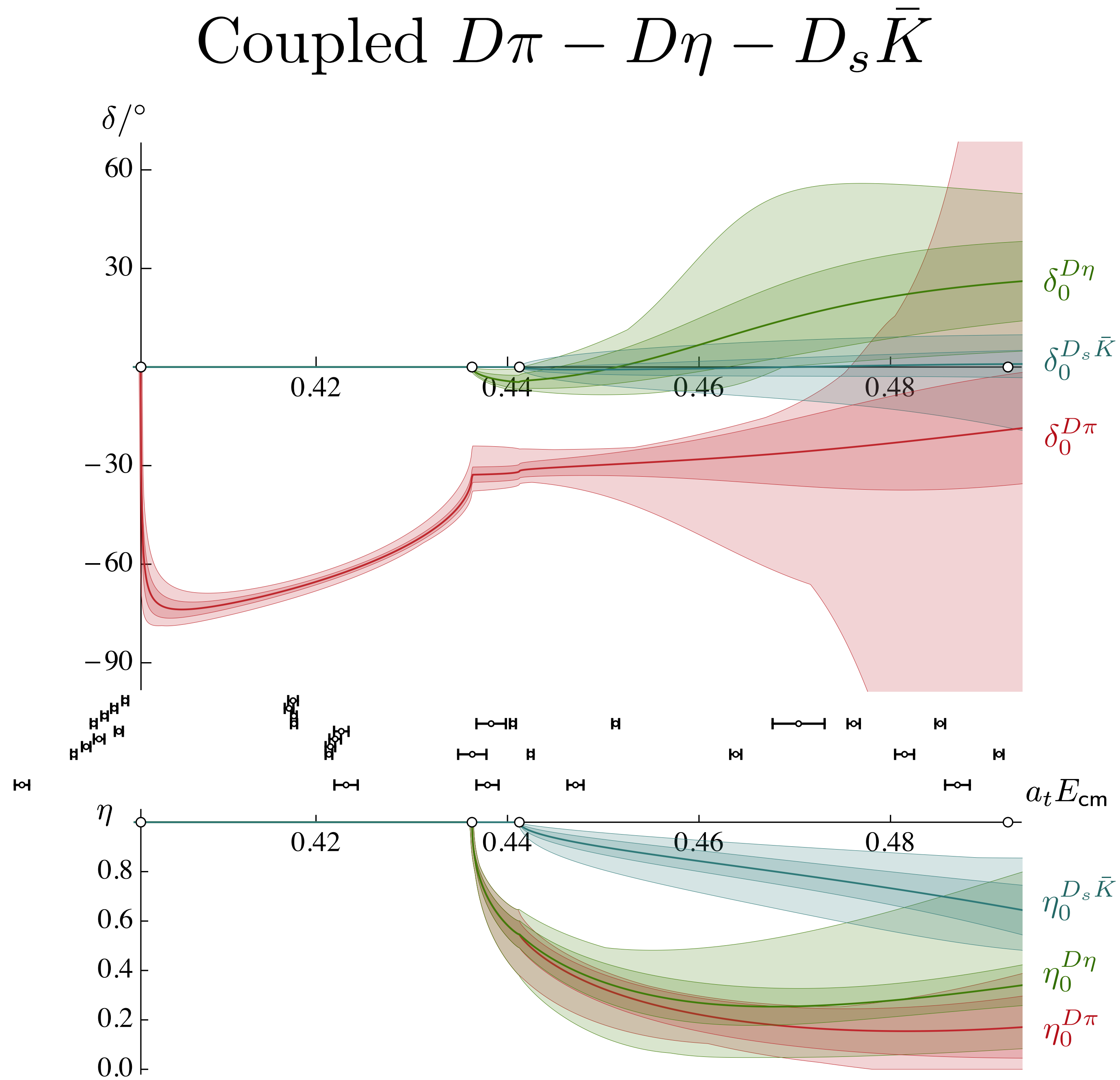


$m_\pi = 236$ MeV

Other calculations



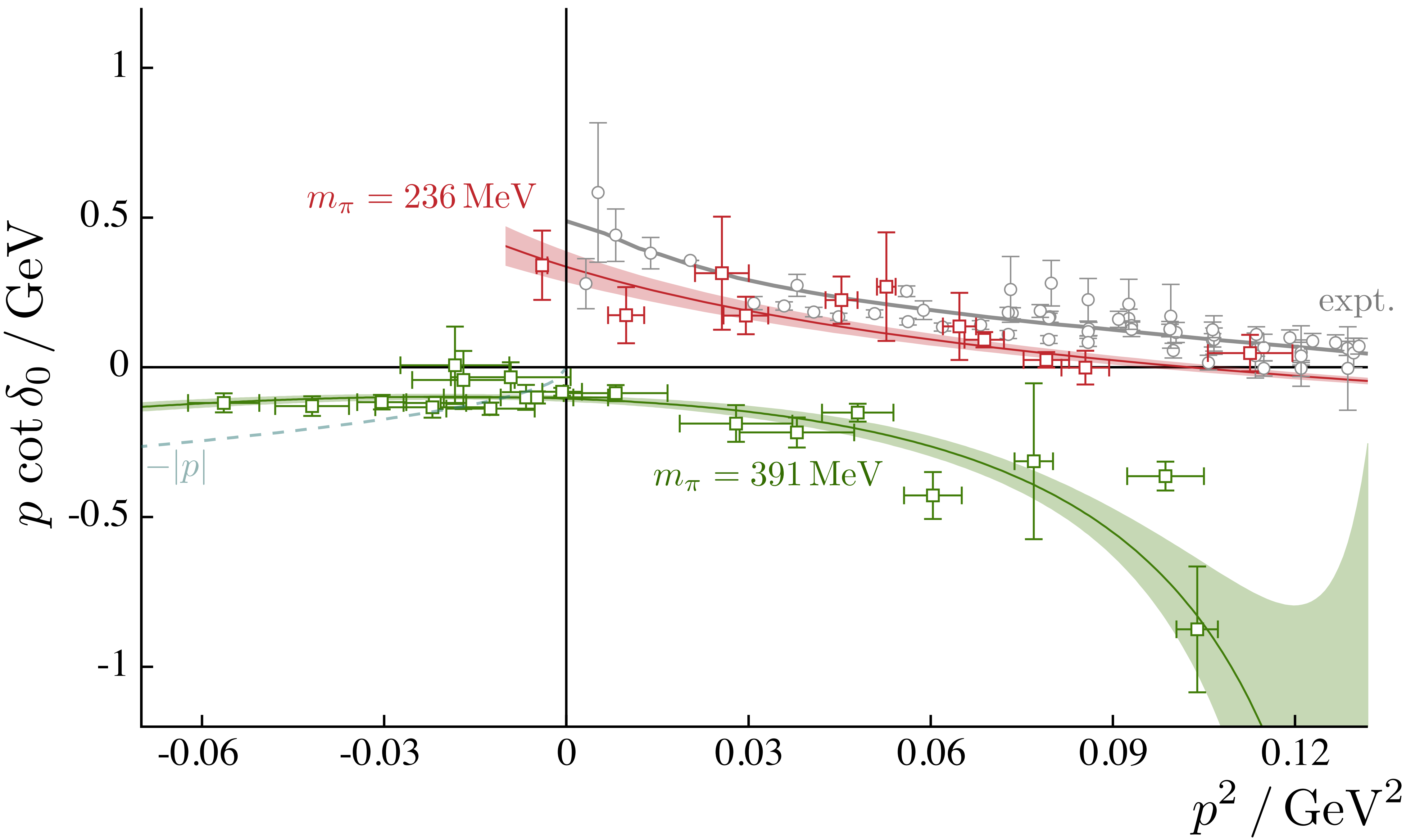
Combined S & P-wave analysis
80 energy levels from 3 volumes
arXiv:1406.4158, PRL 113 (2014) no.18, 182001



Combined S & P-wave analysis
3 coupled channels in S-wave
47 energy levels from 3 volumes
arXiv:1607.07093

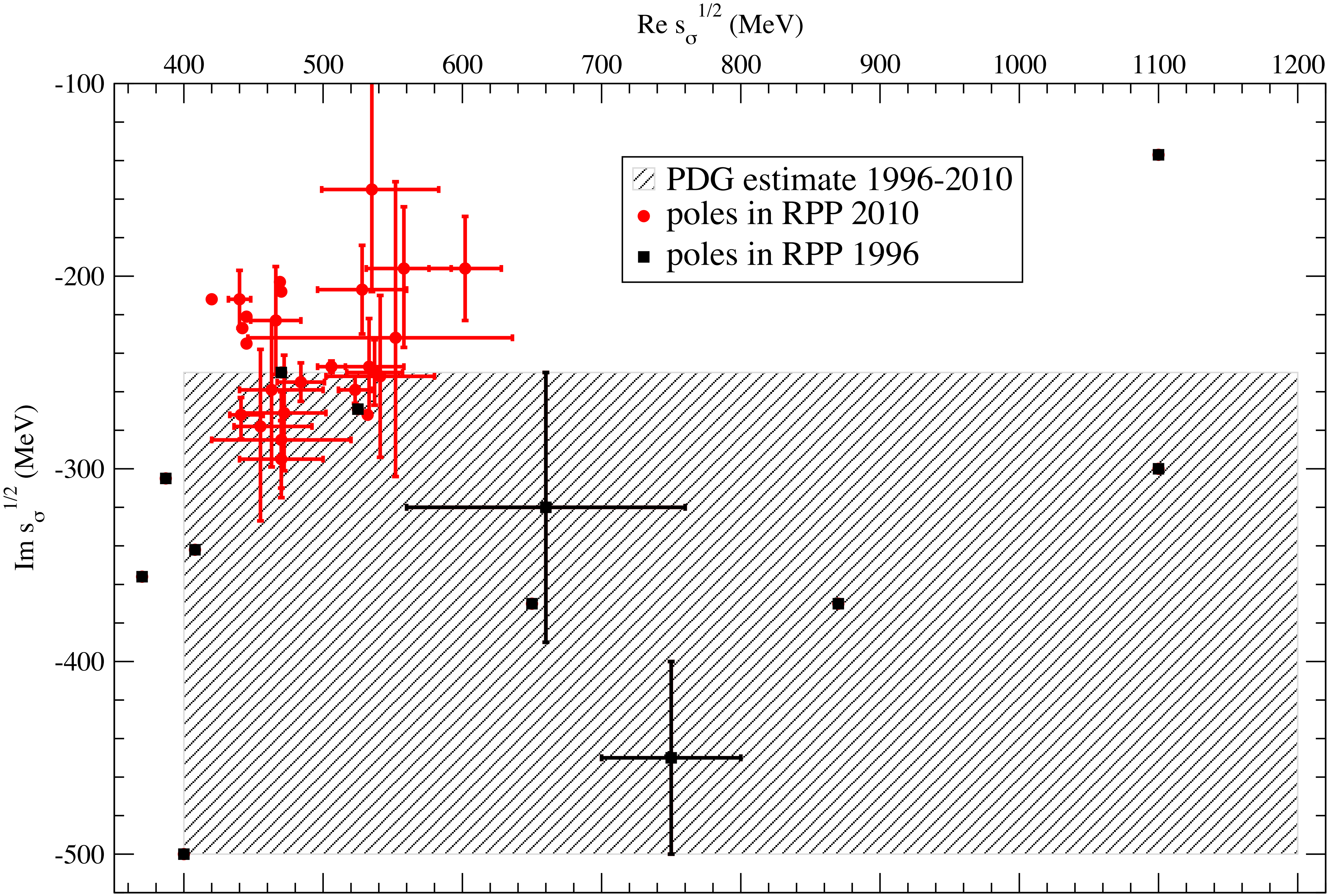
$m_\pi = 391$ MeV

The $f_0(500)/\sigma$ resonance



The $f_0(500)/\sigma$ resonance

from J. R. Pelaez, arXiv:1510.00653



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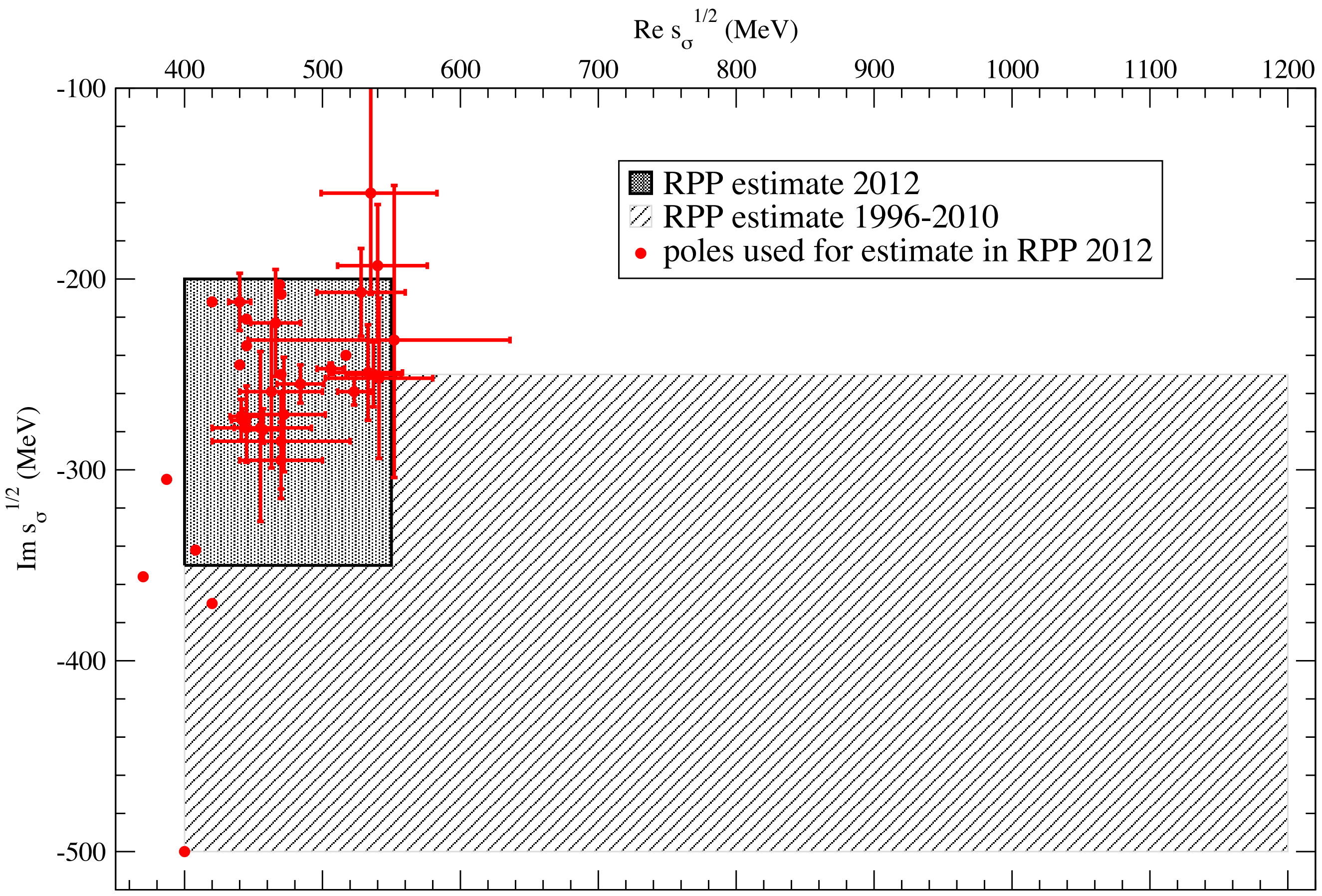
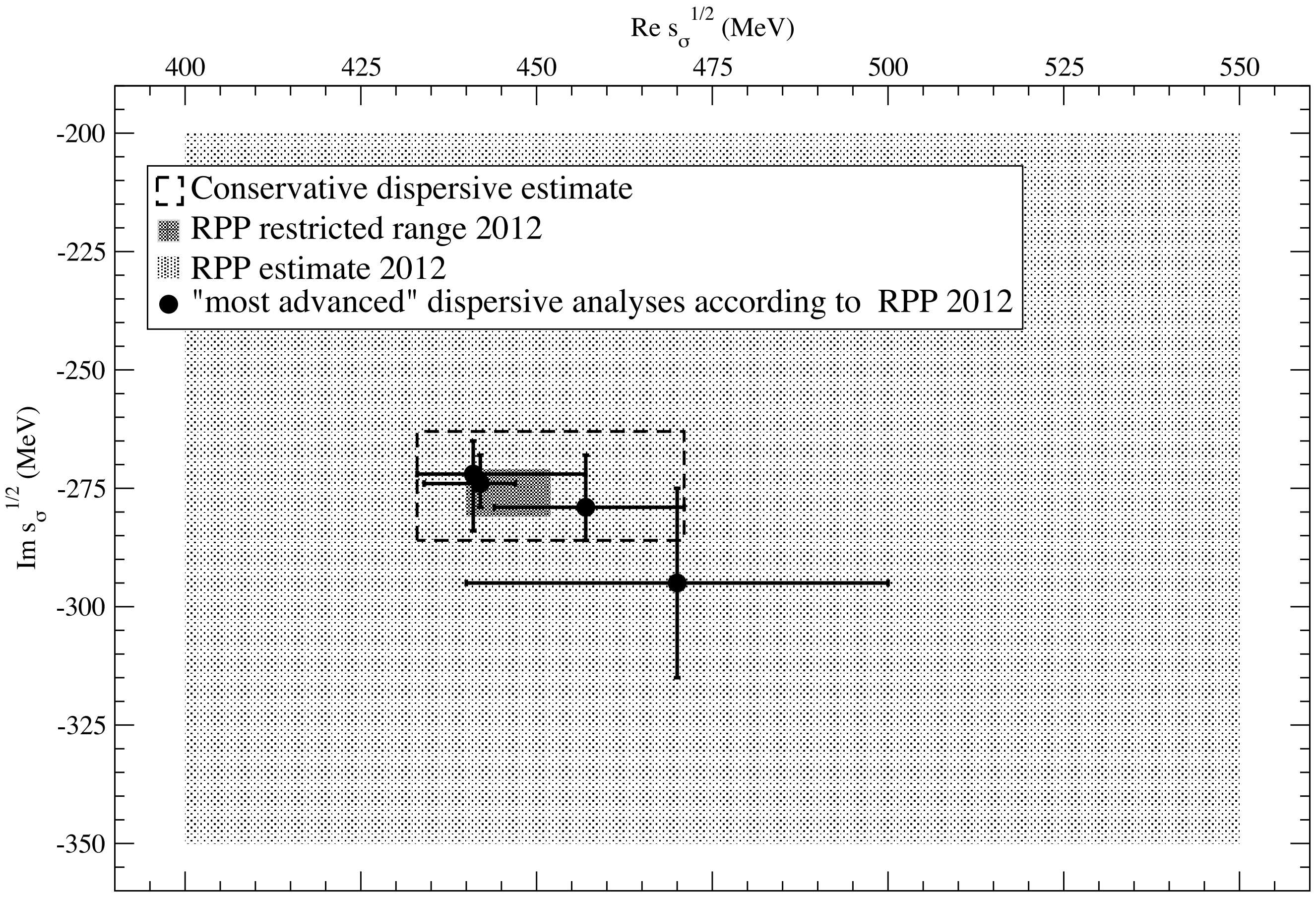
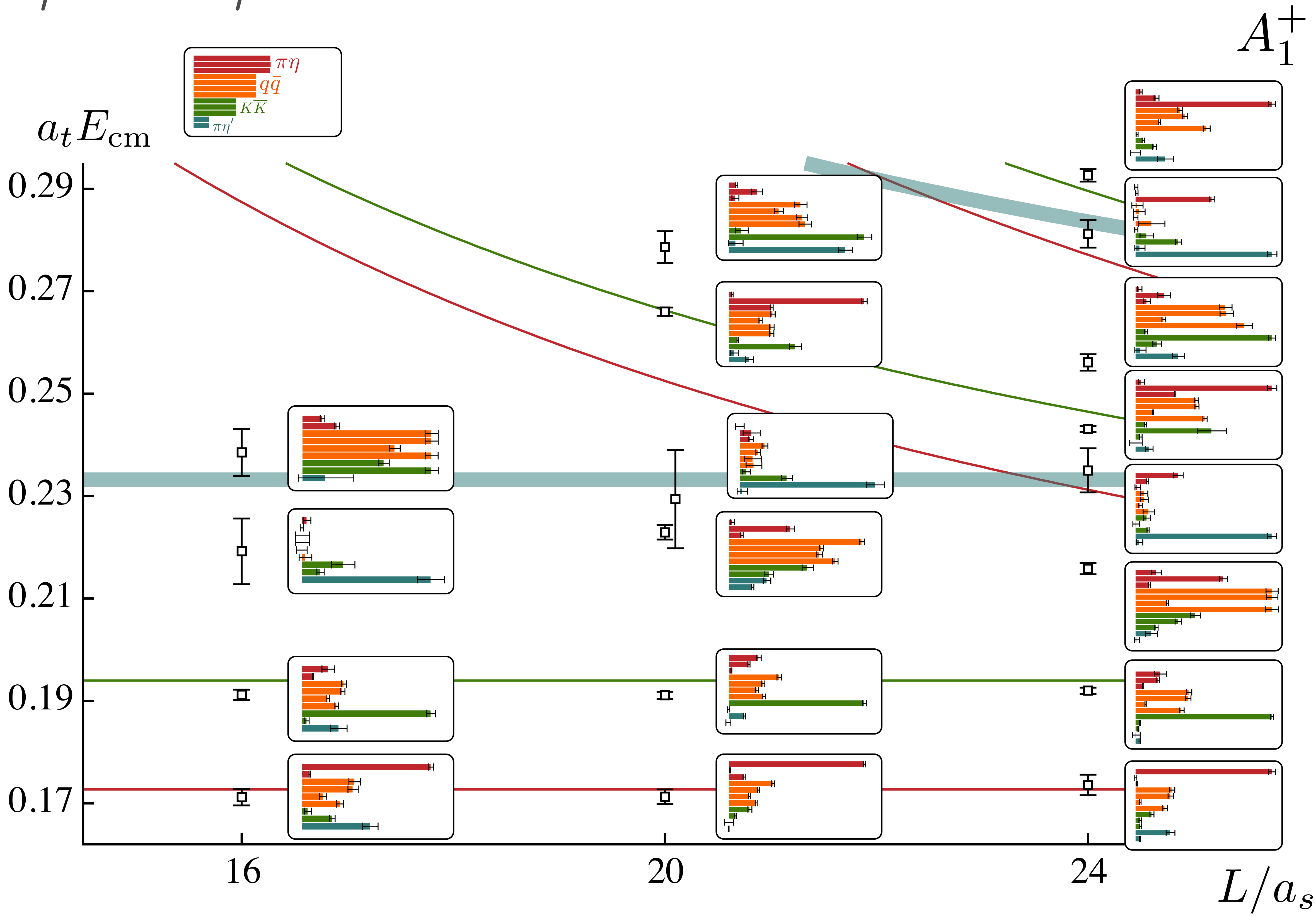




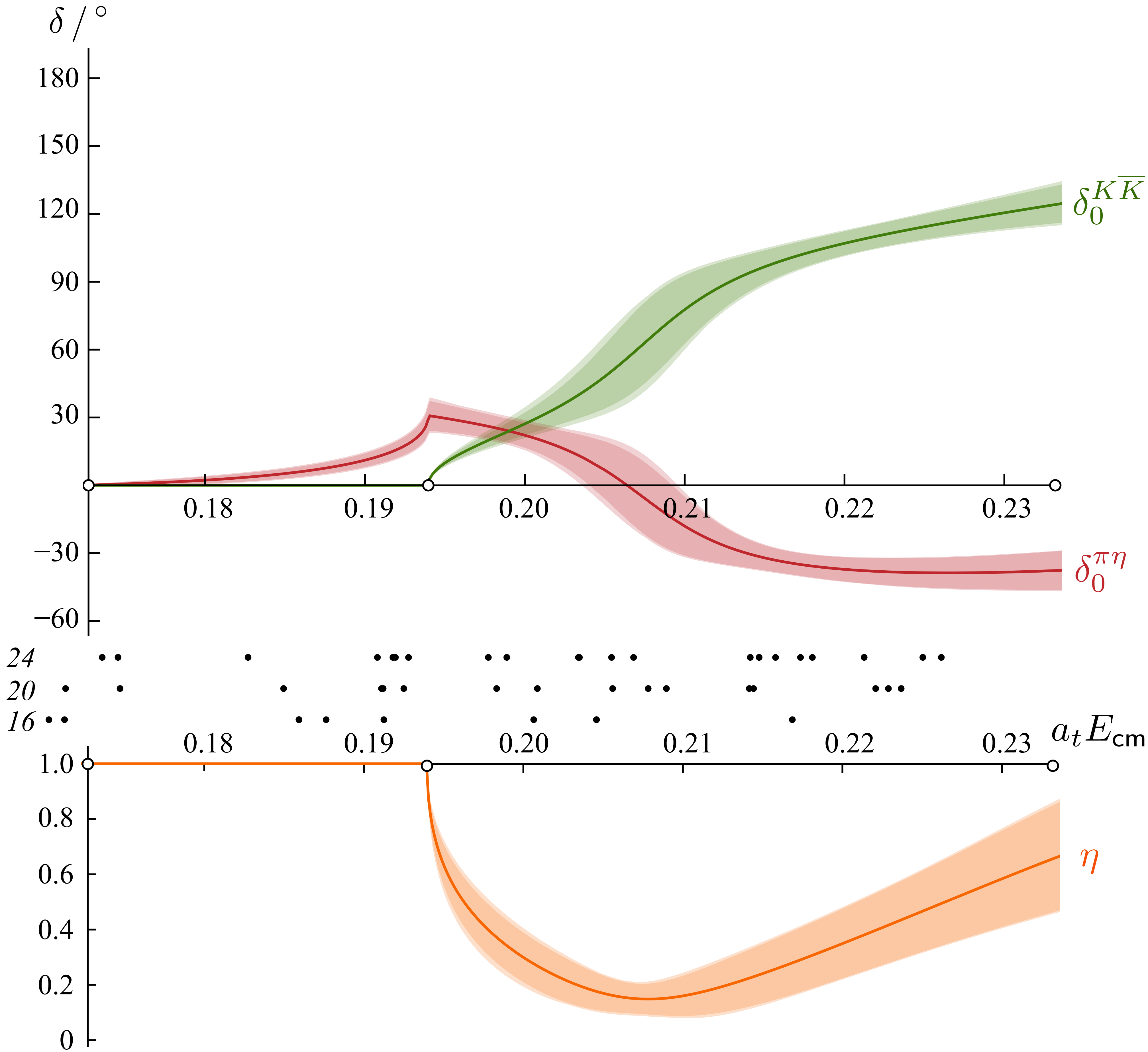
An a_0 resonance

- for more see Jozef Dudek, Tuesday 26 Jul 2016 at 16:50

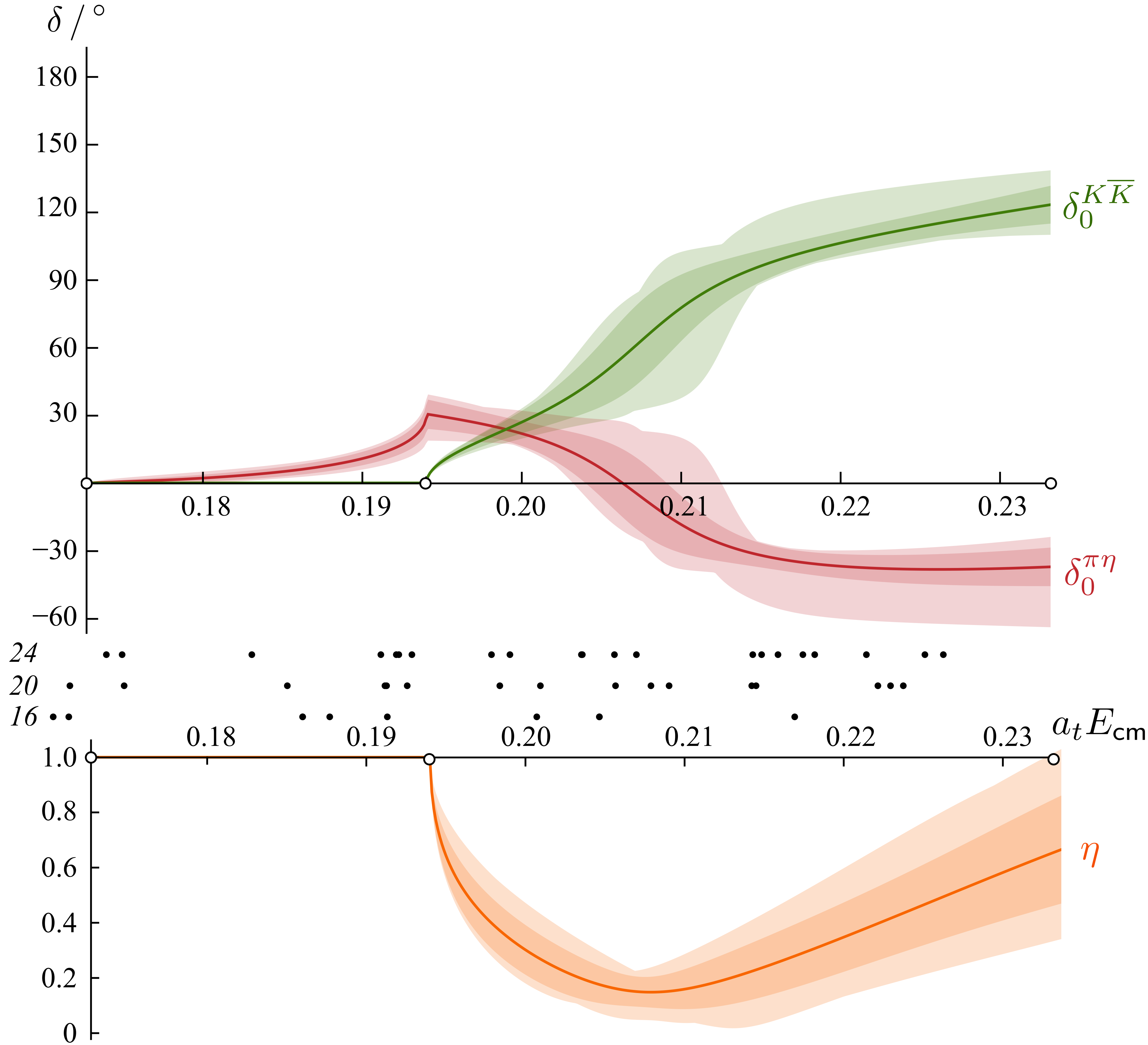
$\pi\eta$-$K\bar{K}$-$\pi\eta'$



$m_\pi = 391$ MeV



$\delta\,/\,^\circ$
180
150
120
90
60
30
−30
−60
0.18
0.19
0.20
0.21
0.22
0.23
$\delta_0^{K\overline{K}}$
$\delta_0^{\pi\eta}$
24
20
16
$a_t E_{\mathsf{cm}}$
1.0
0.8
0.6
0.4
0.2
0
η


$\delta\,/\,^\circ$
180
150
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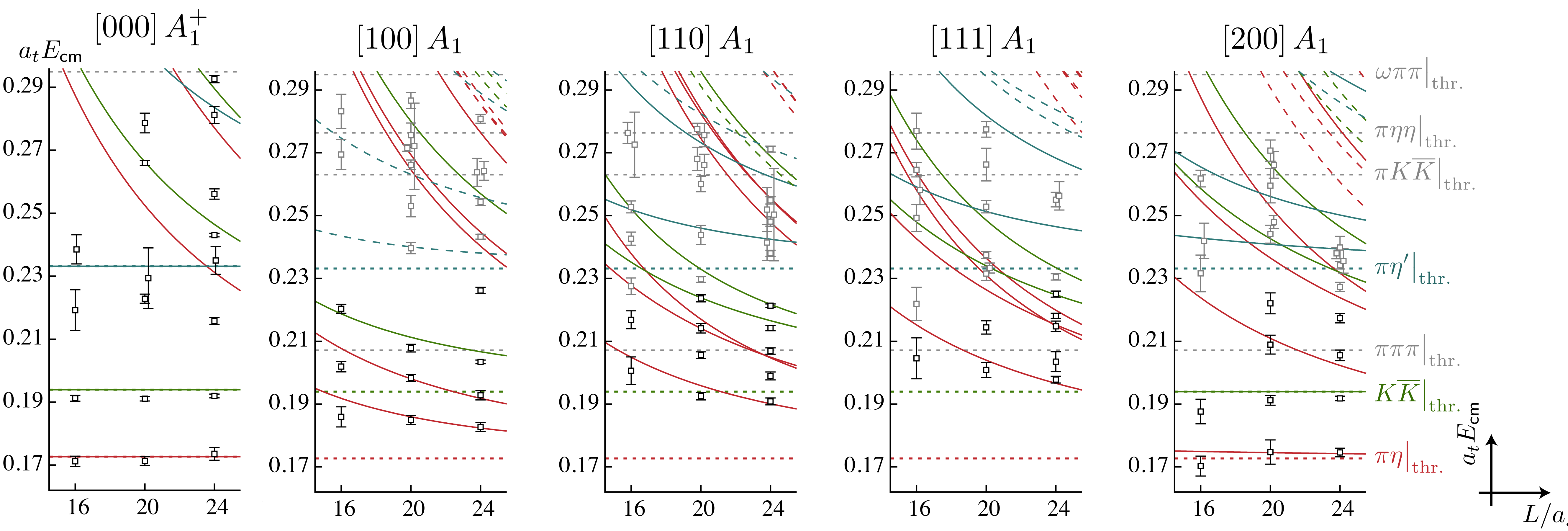
90
60
30
−30
−60
0.18
0.19
0.20
0.21
0.22
0.23
$\delta_0^{K\overline{K}}$
$\delta_0^{\pi\eta}$
24
20
16
$a_t E_{\mathsf{cm}}$
1.0
0.8
0.6
0.4
0.2
0
η

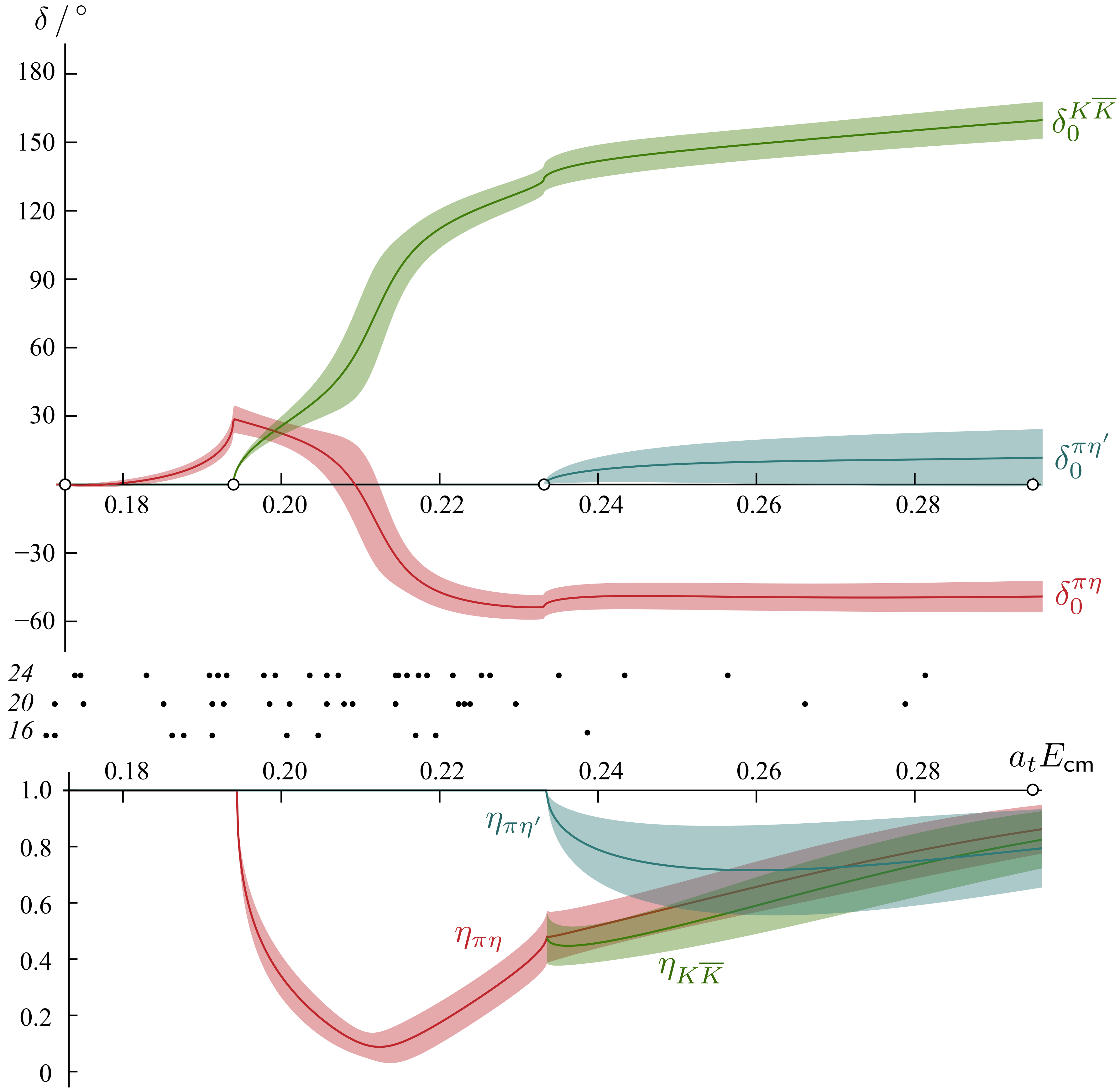
An a_0 resonance

$\pi\eta$-$K\bar{K}$-$\pi\eta'$



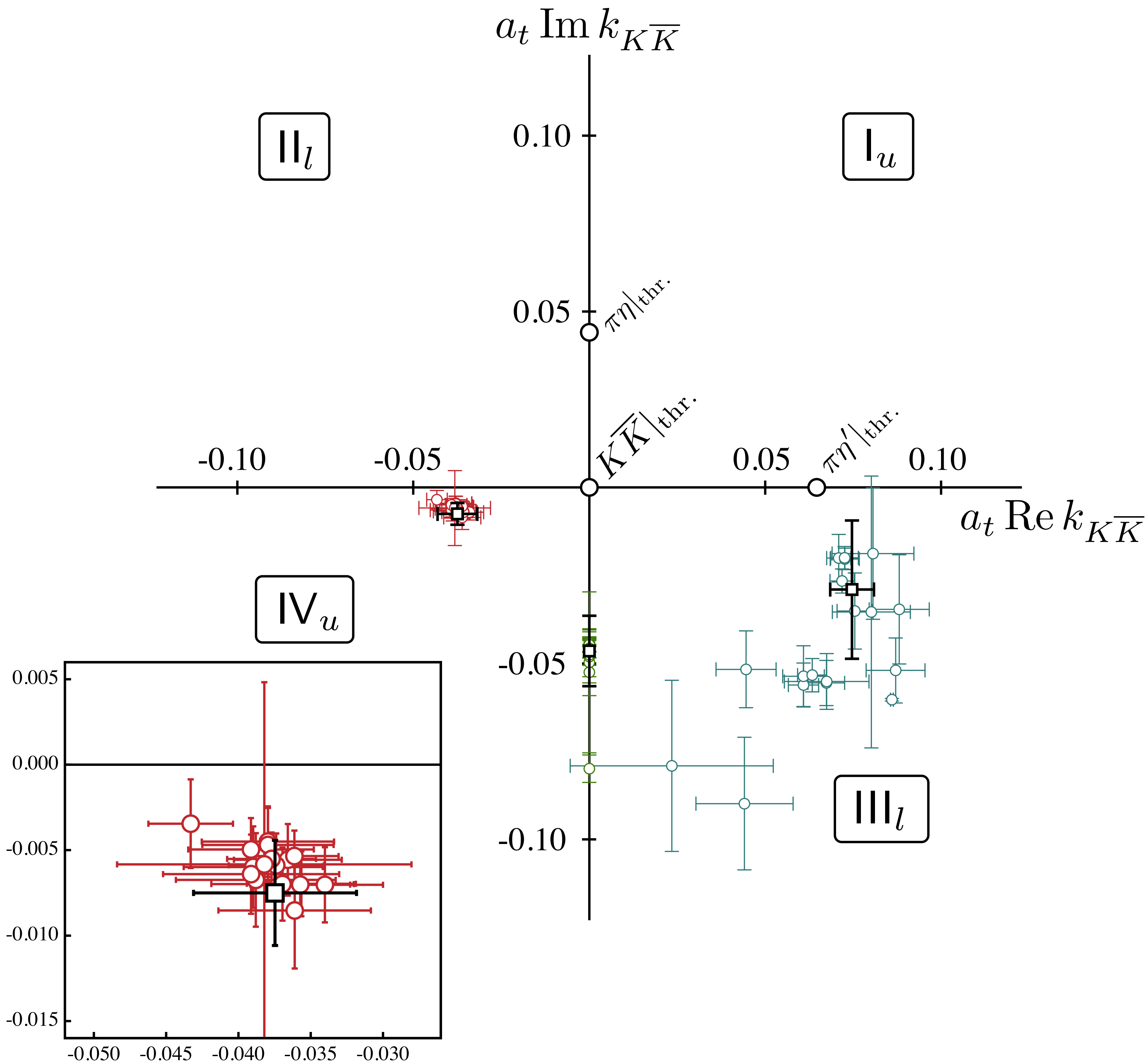
$m_\pi = 391$ MeV

An a_0 resonance - three channel region



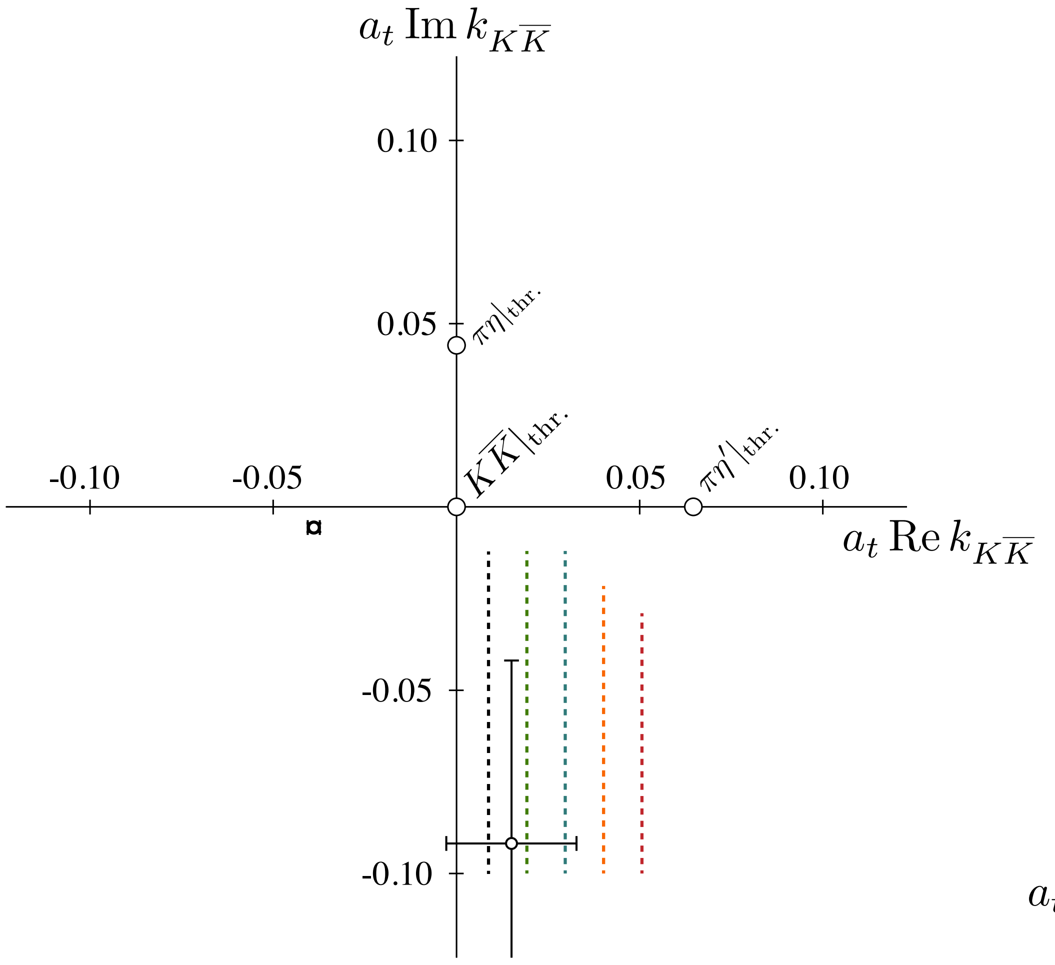
$m_\pi = 391$ MeV

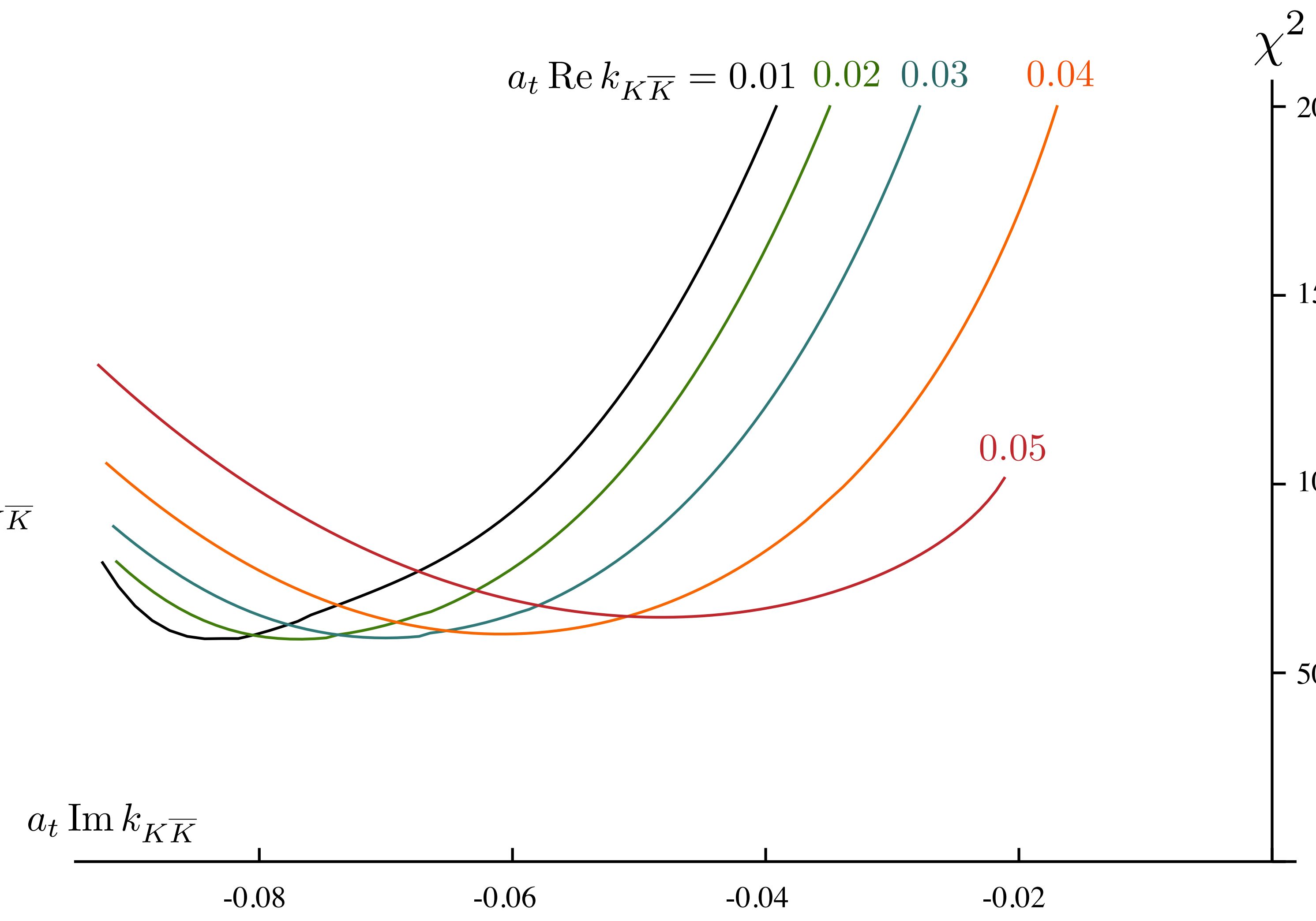
Poles



$m_\pi = 391$ MeV

Poles





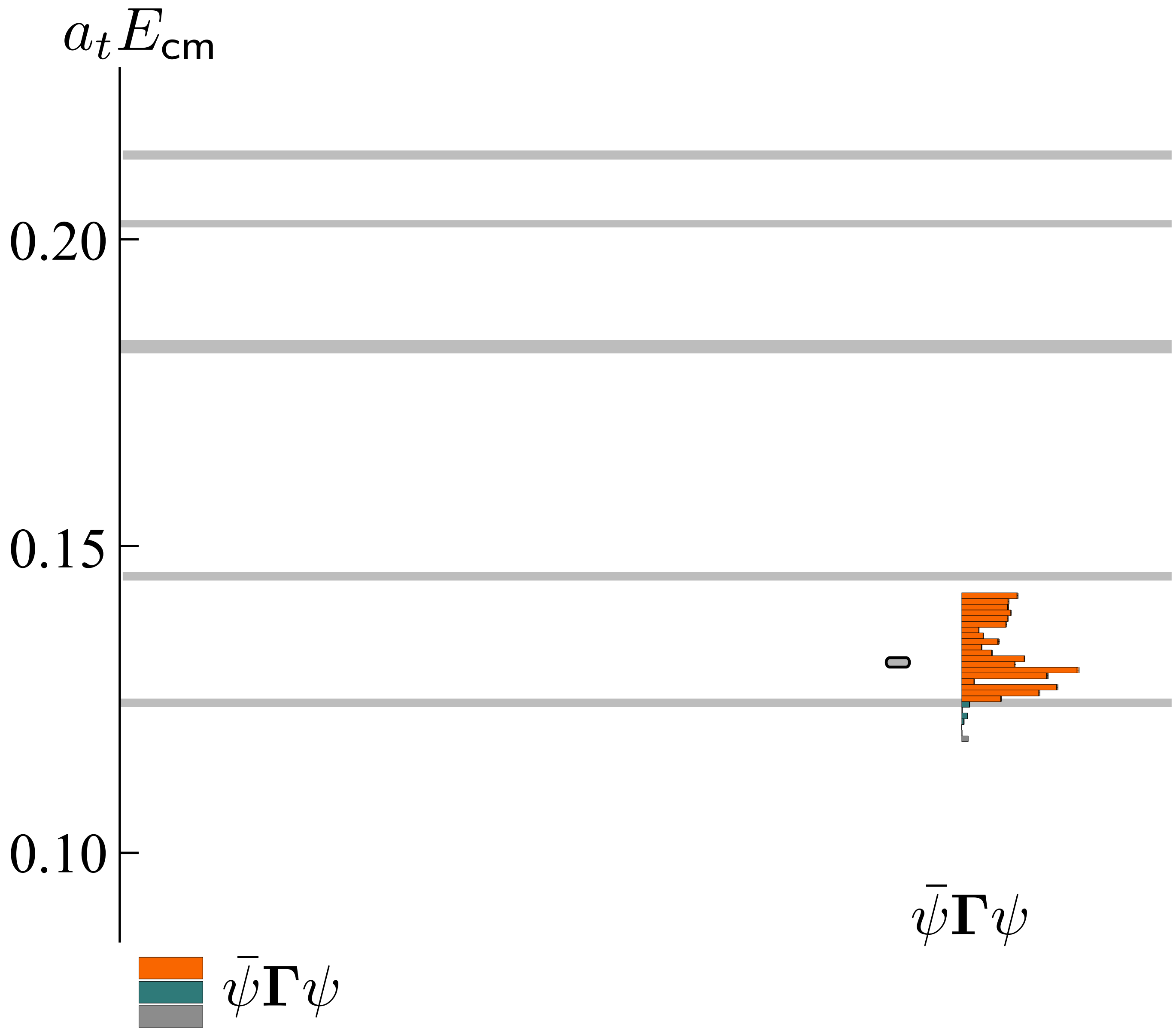
$m_\pi = 391$ MeV

Extracting resonance properties

build a large basis of operators: $\mathcal{O}^\dagger \sim \bar{\psi}\mathbf{\Gamma}\overleftrightarrow{D}\ldots\overleftrightarrow{D}\psi$

compute large correlation matrices: $C_{ij}(t) = \langle 0|\, \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\, |0\rangle$

solve GEVP: $C_{ij}(t)v_j^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t,t_0)C_{ij}(t_0)v_j^{\mathfrak{n}}$



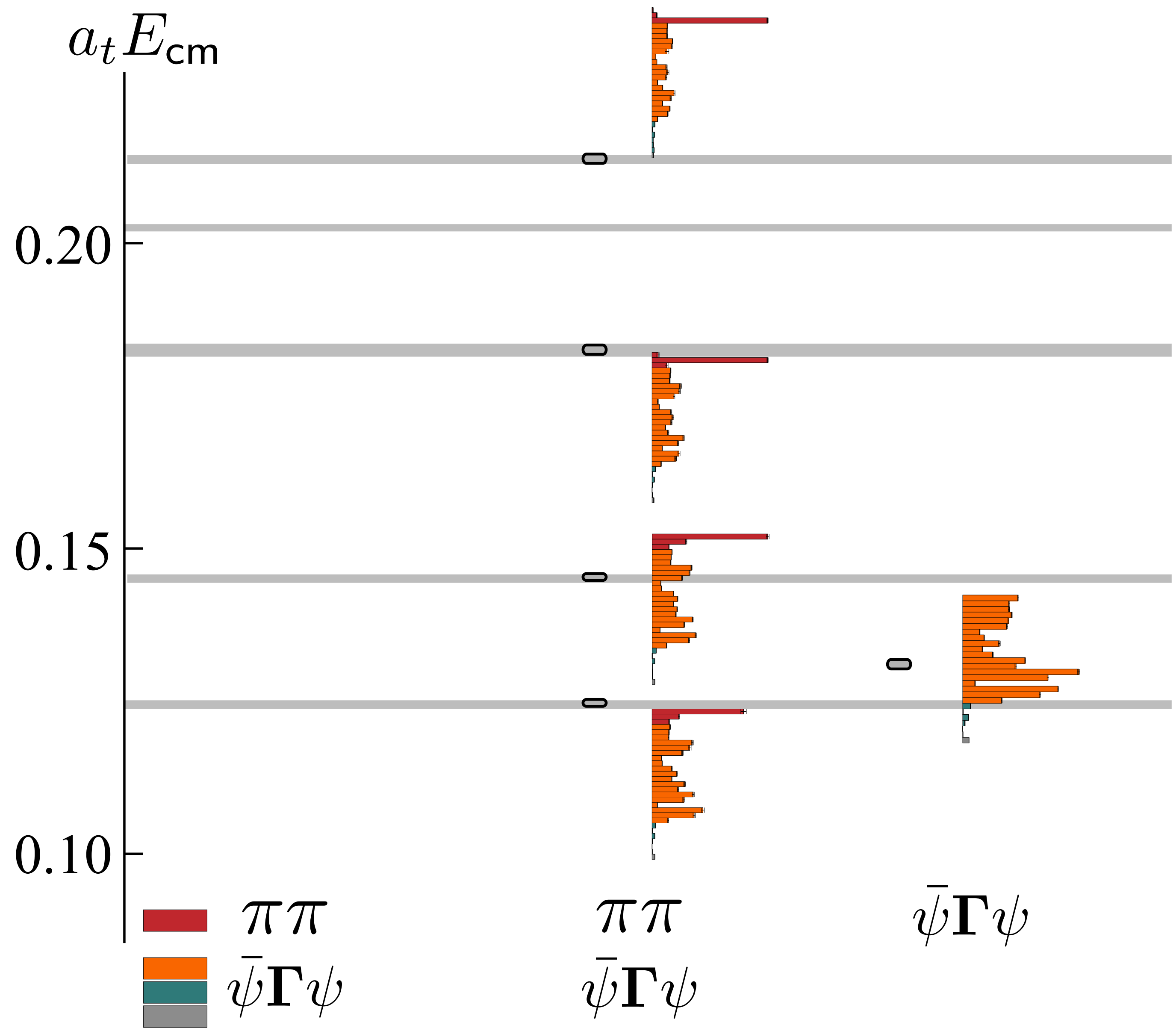
$[000]T_1^-$

$m_\pi = 236$ MeV

Extracting resonance properties

add in $\pi\pi$ operators using a variationally optimal pion $\pi^\dagger = \sum_i v_i^\pi \mathcal{O}_i^\dagger$

combine in pairs $(\pi\pi)^\dagger = \sum_{\vec{p_1}+\vec{p_2}=\vec{P}} \mathcal{C}(\vec{p}_1,\vec{p}_2)\pi^\dagger(\vec{p}_1)\pi^\dagger(\vec{p}_2)$

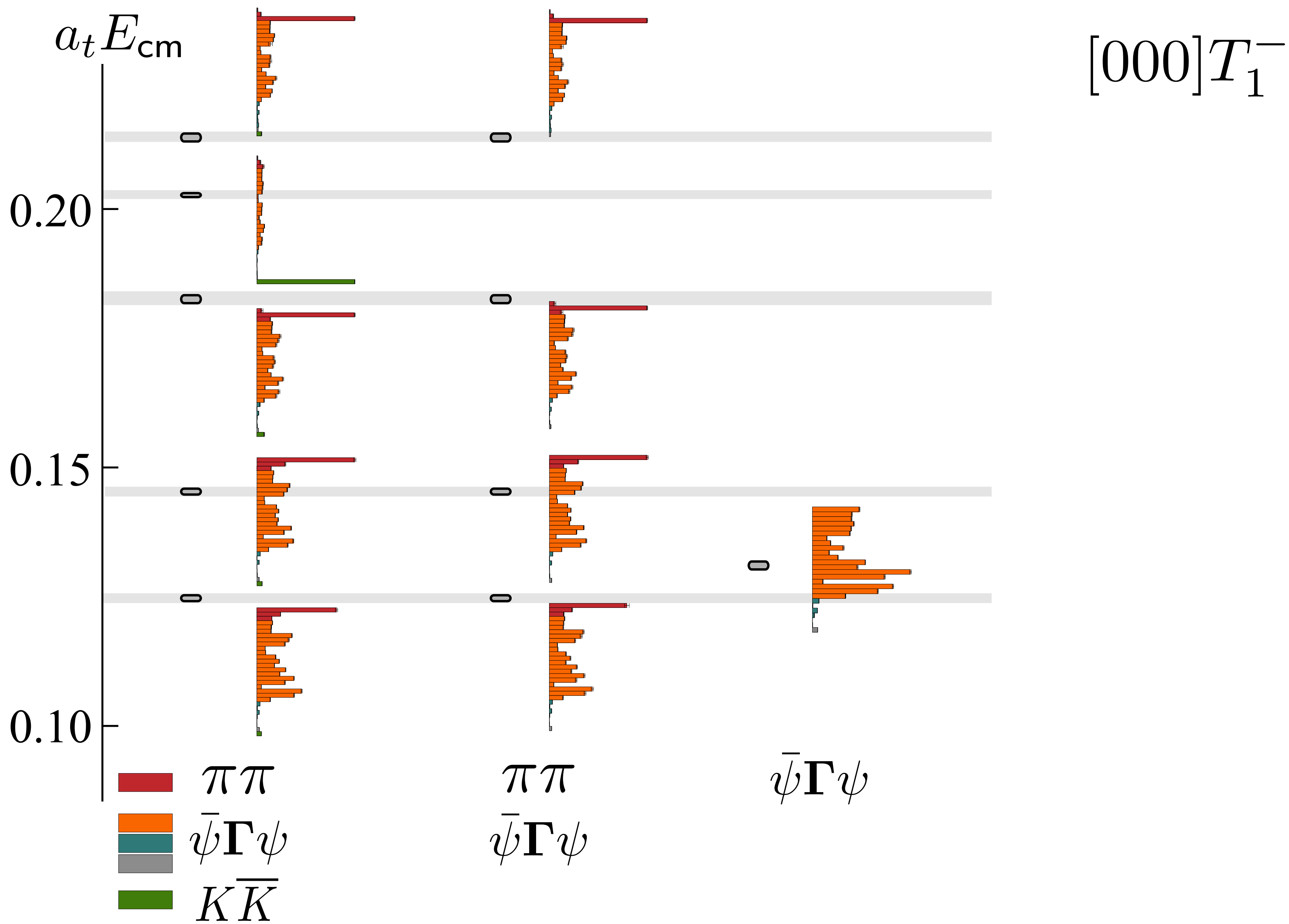


$[000]T_1^-$

$m_\pi = 236$ MeV

Extracting resonance properties

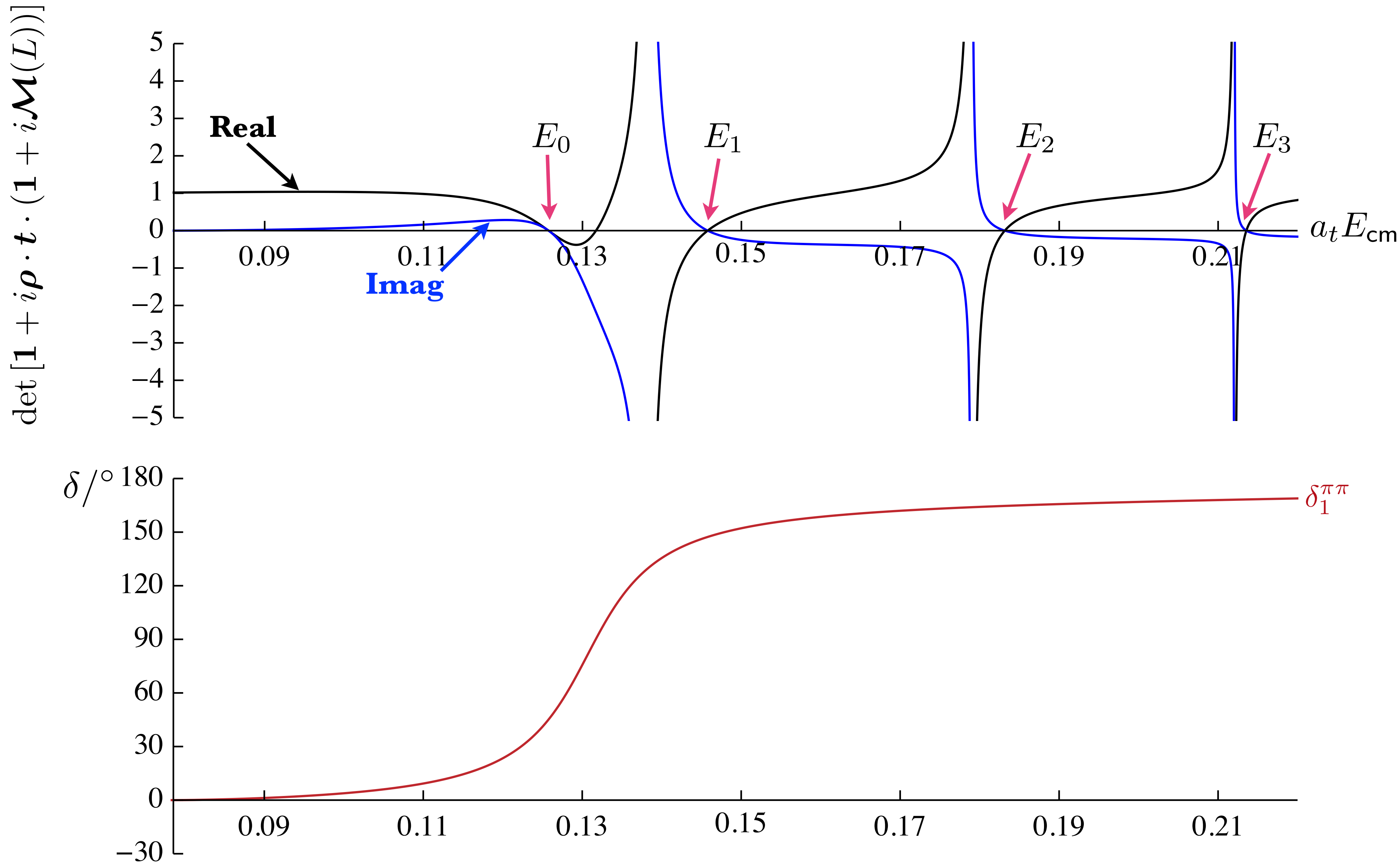
essential to have operators that overlap onto “meson” and “meson-meson” contributions to the physical spectrum



$m_\pi = 236$ MeV

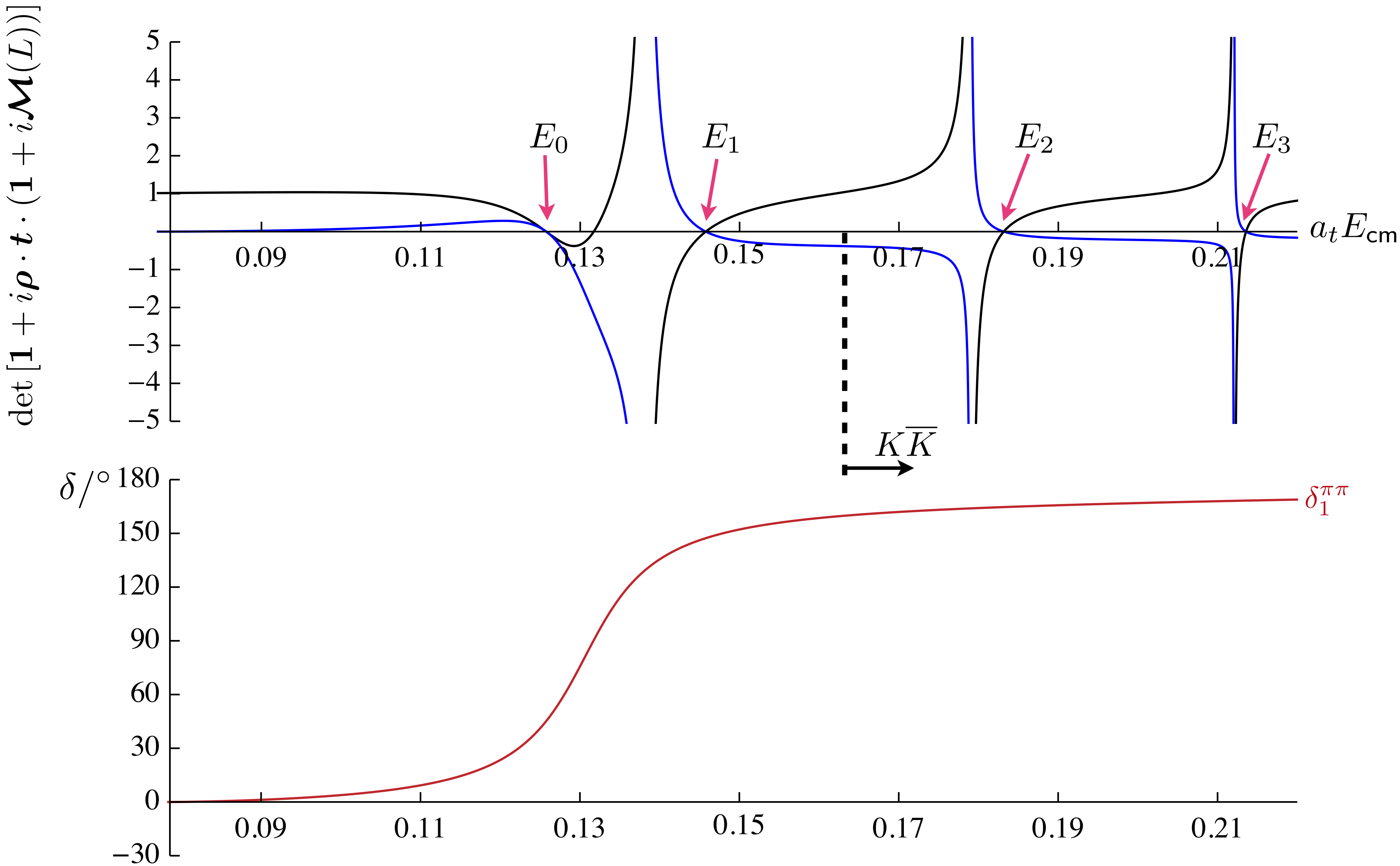
Determinant

$$t = (\pi\pi \to \pi\pi)$$



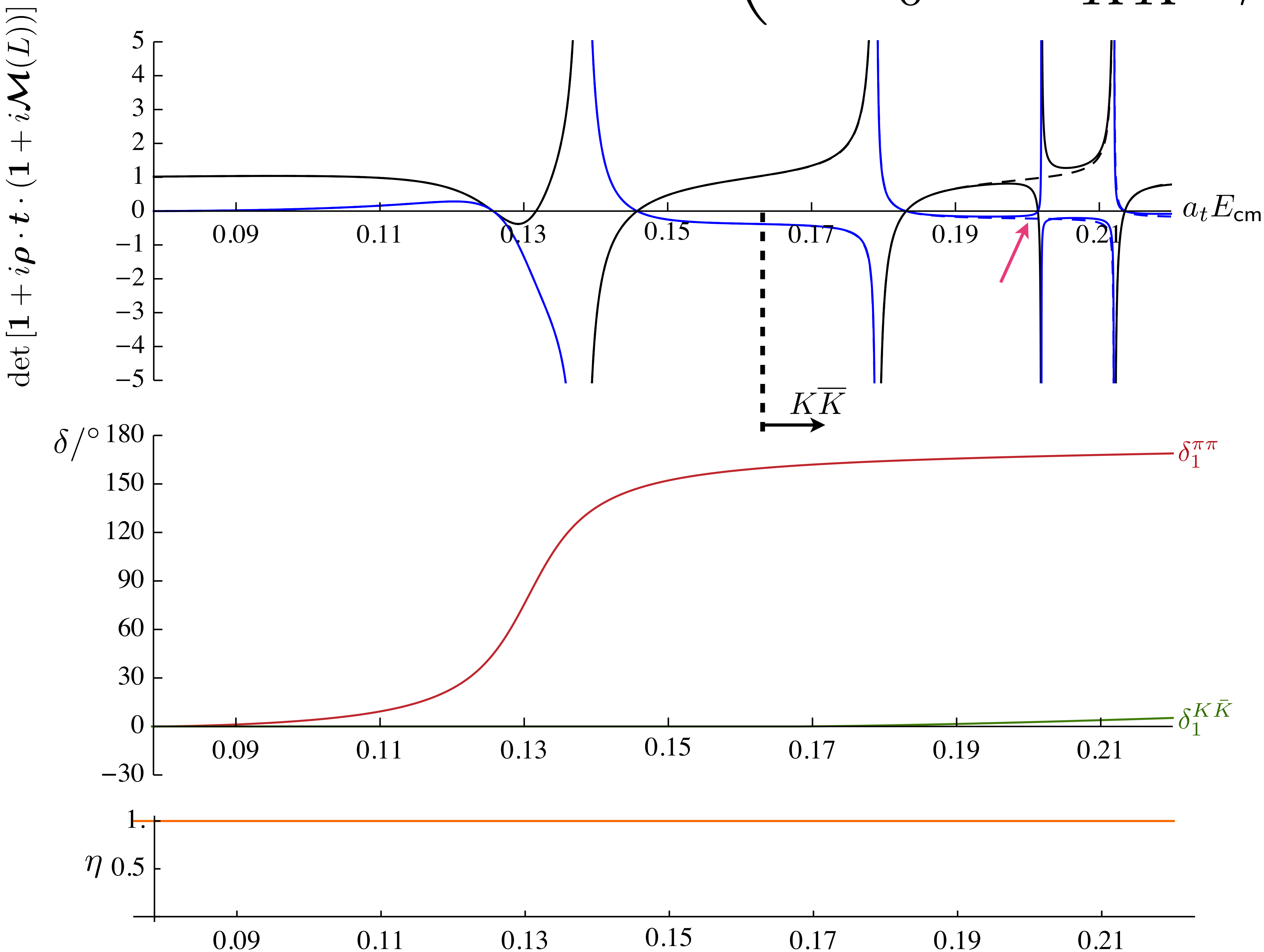
Determinant

$$t = (\pi\pi \to \pi\pi)$$



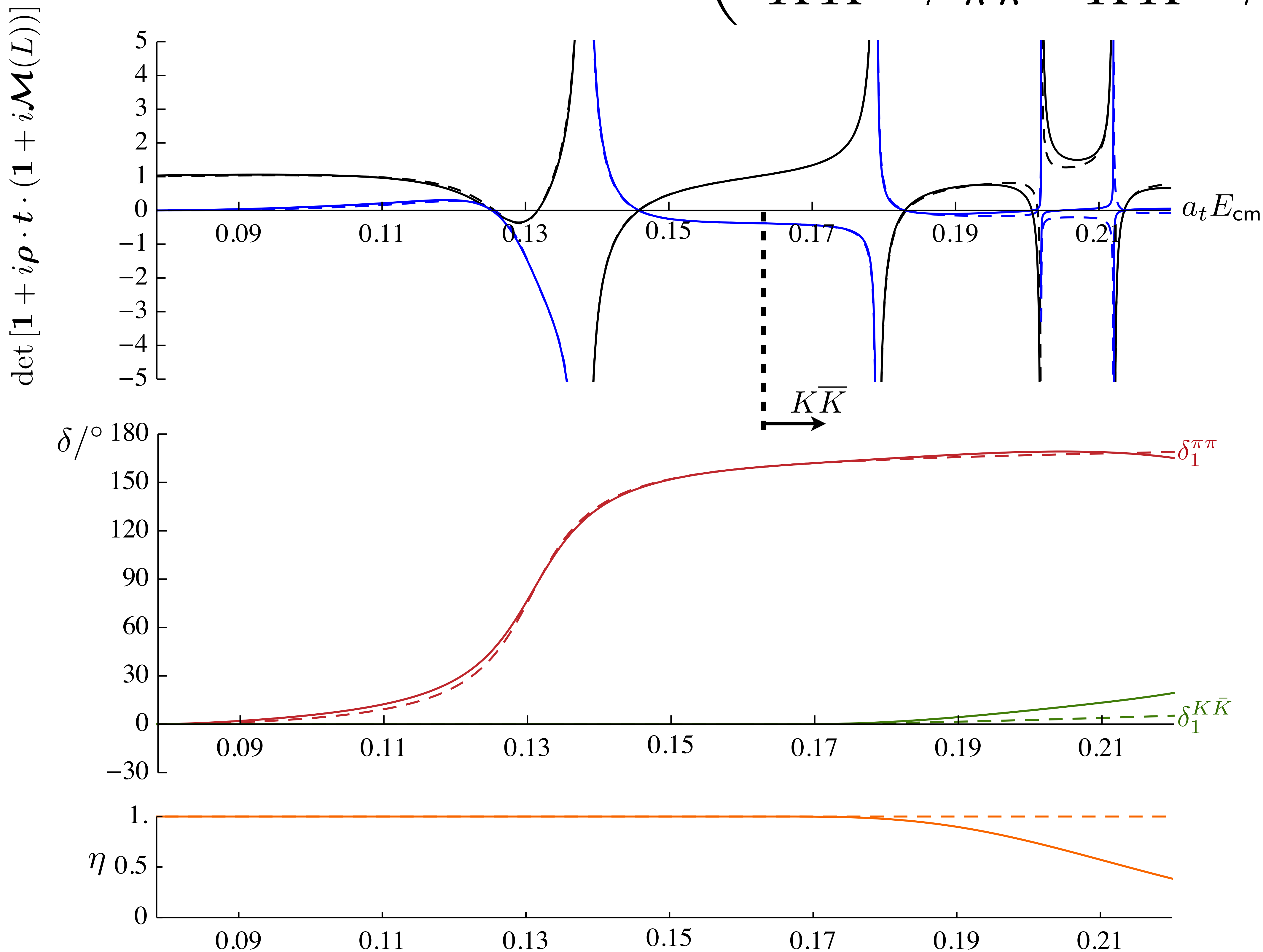
Determinant

$$t = \begin{pmatrix} \pi\pi \to \pi\pi & 0 \\ 0 & K\bar{K} \to K\bar{K} \end{pmatrix}$$



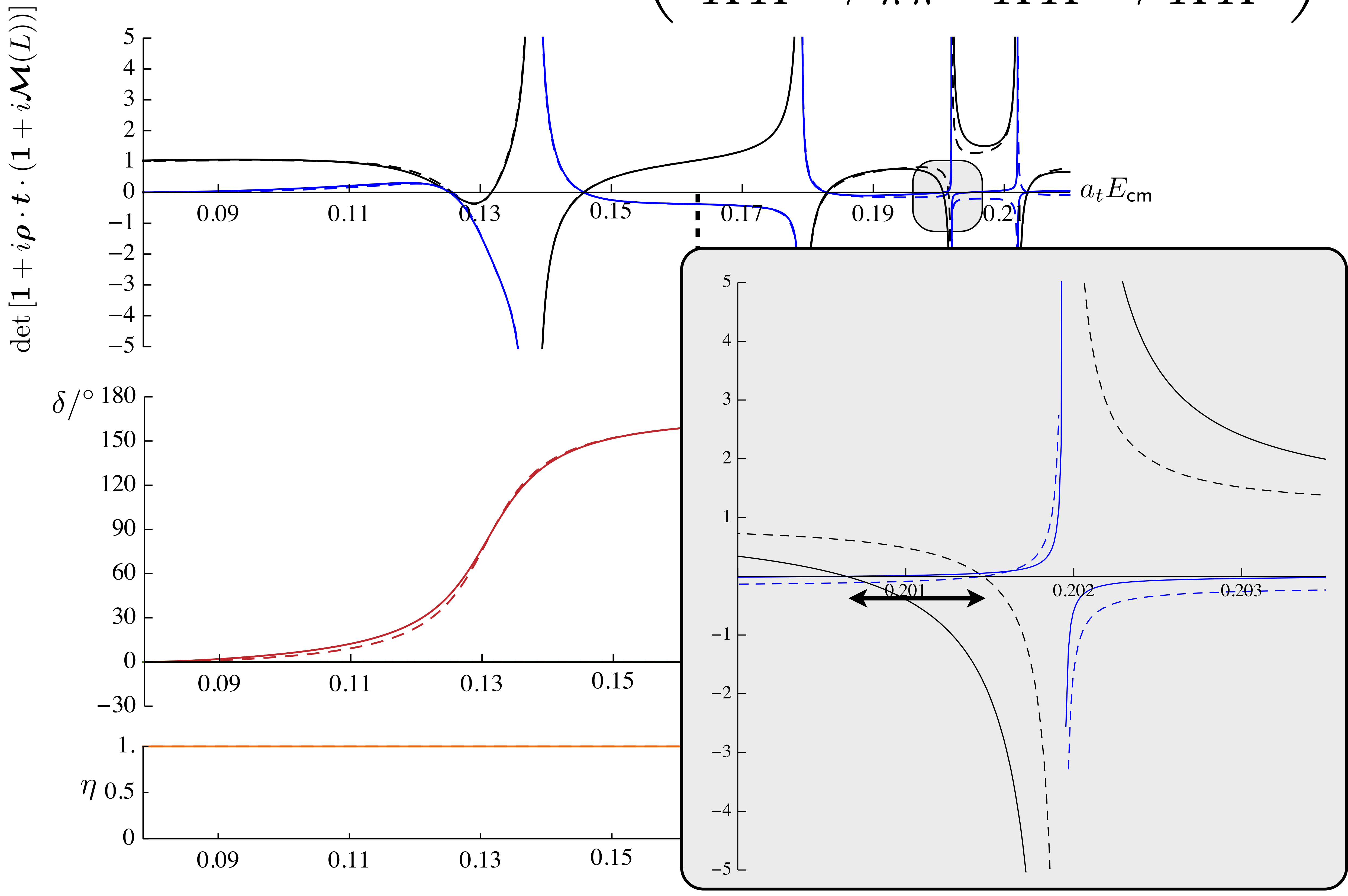
Determinant

$$t = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$



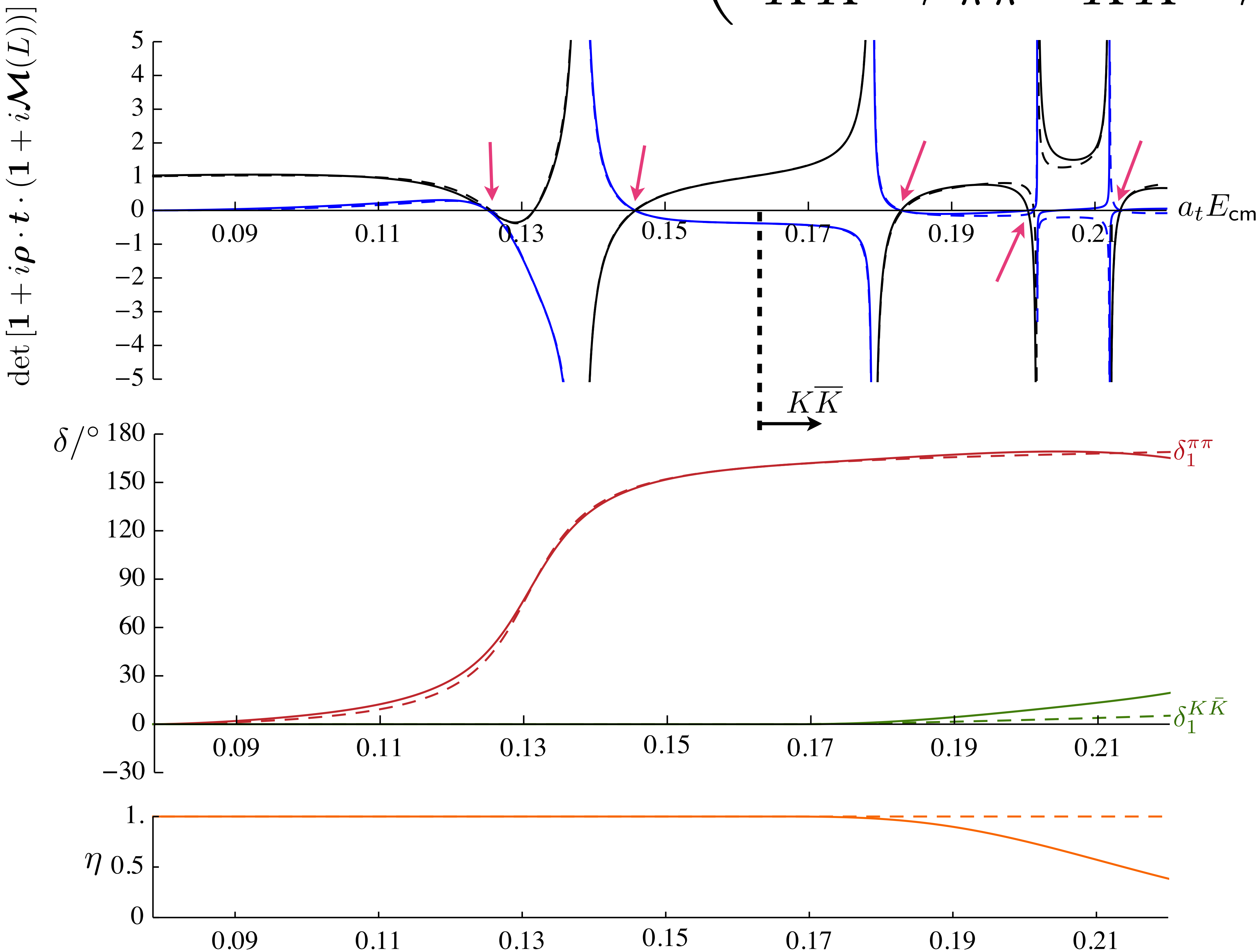
Determinant

$$t = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$



Determinant

$$t = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$



Chiral perturbation theory ρ extrapolation