

# *Production of dijets in forward direction as a probe of dense system of partons*

*Krzysztof Kutak*



*Based on :*

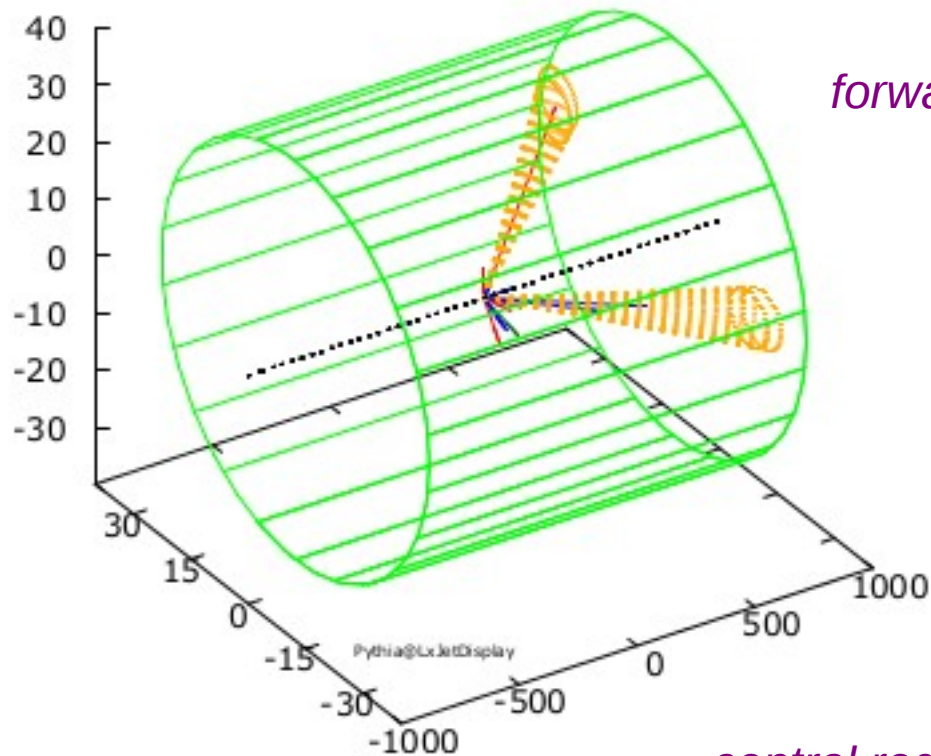
*P. Kotko, K. Kutak, C. Marquet,  
E. Petreska, A. van Hameren,  
S. Sapeta*

*JHEP 1509 (2015) 106*

*arxiv 1611130*

*Bury, Deak, Kutak. Sapeta  
PLB '16*

## *Dilute-dense: forward-forward*



*forward region*

*The collisions we consider:*

*minimal  $p_T$  20 GeV*

*both jets are forward  $\rightarrow y > 3.5$*

*central region*

*From: Piotr Kotko  
LxJet*

*There is certain class of processes where one can assume that partons in one of hadrons are just collinear with hadron and in other are not*

# First attempt: hybrid factorization and dijets

$$\frac{d\sigma_{\text{SPS}}^{P_1 P_2 \rightarrow \text{dijets} + X}}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \frac{p_{1t} p_{2t}}{8\pi^2 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/P_1}(x_1, \mu^2) |\overline{\mathcal{M}}_{ag^* \rightarrow cd}|^2 \mathcal{F}_{g/P_2}(x_2, k_t^2) \frac{1}{1 + \delta_{cd}}$$

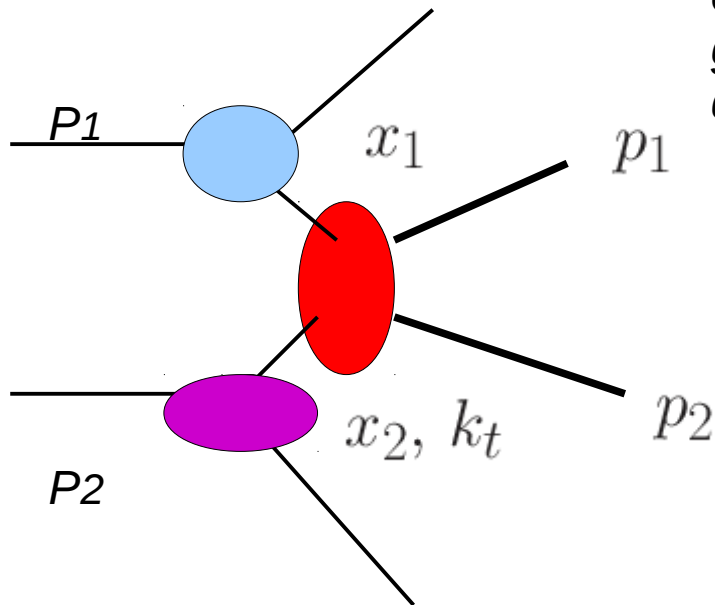
conjecture

Deak, Jung, Kutak, Hautmann '09

obtained from CGC after neglecting all nonlinearities

$g^*g \rightarrow gg$  *Iancu, Laidet*

$qg^* \rightarrow qg$  *Van Hameren, Kotko, Kutak, Marquet, Petreska, Sapeta*



resummation of logs of  $x$

logs of hard scale

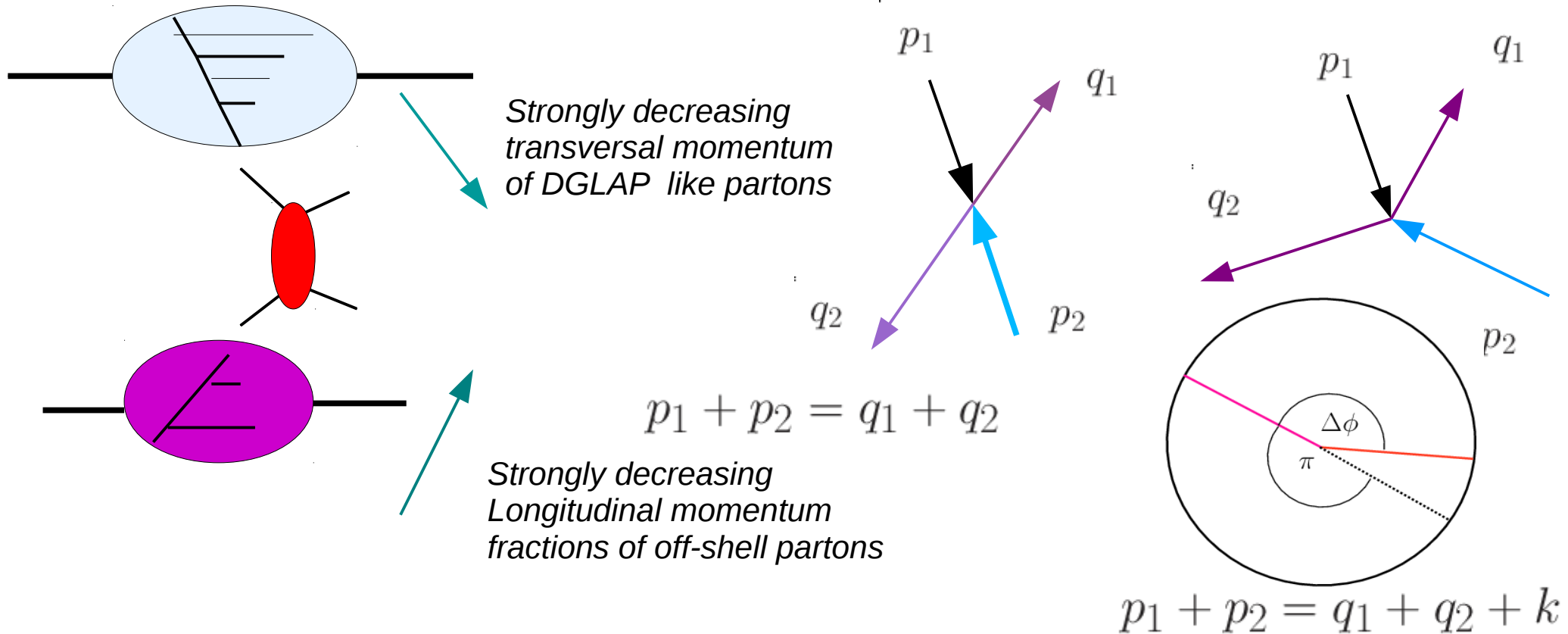
knowing well parton densities at large  $x$  one can get information about low  $x$  physics

$$\begin{aligned} x_1 &= \frac{1}{\sqrt{s}} (|\vec{p}_{1t}| e^{y_1} + |\vec{p}_{2t}| e^{y_2}) \\ x_2 &= \frac{1}{\sqrt{s}} (|\vec{p}_{1t}| e^{-y_1} + |\vec{p}_{2t}| e^{-y_2}) \end{aligned} \quad \xrightarrow{y_1, y_2 \gg 0} \quad \begin{aligned} x_1 &\sim 1 \\ x_2 &\ll 1 \end{aligned}$$

Inbalance momentum:

$$|\vec{k}_t|^2 = |\vec{p}_{1t} + \vec{p}_{2t}|^2 = |\vec{p}_{1t}|^2 + |\vec{p}_{2t}|^2 + 2|\vec{p}_{1t}||\vec{p}_{2t}| \cos \Delta\phi$$

# hybrid High Energy Factorization



# Relevant scales and factorization

$P_t$  average transverse momentum of dijets

$k_t$  target gluon's transverse momentum

$Q_s$  scale at which gluon recombination nonlinear effects at the target start to be relevant

$P_t \sim k_t$  High Energy Factorization  $\rightarrow$  partons carry some  $k_t$

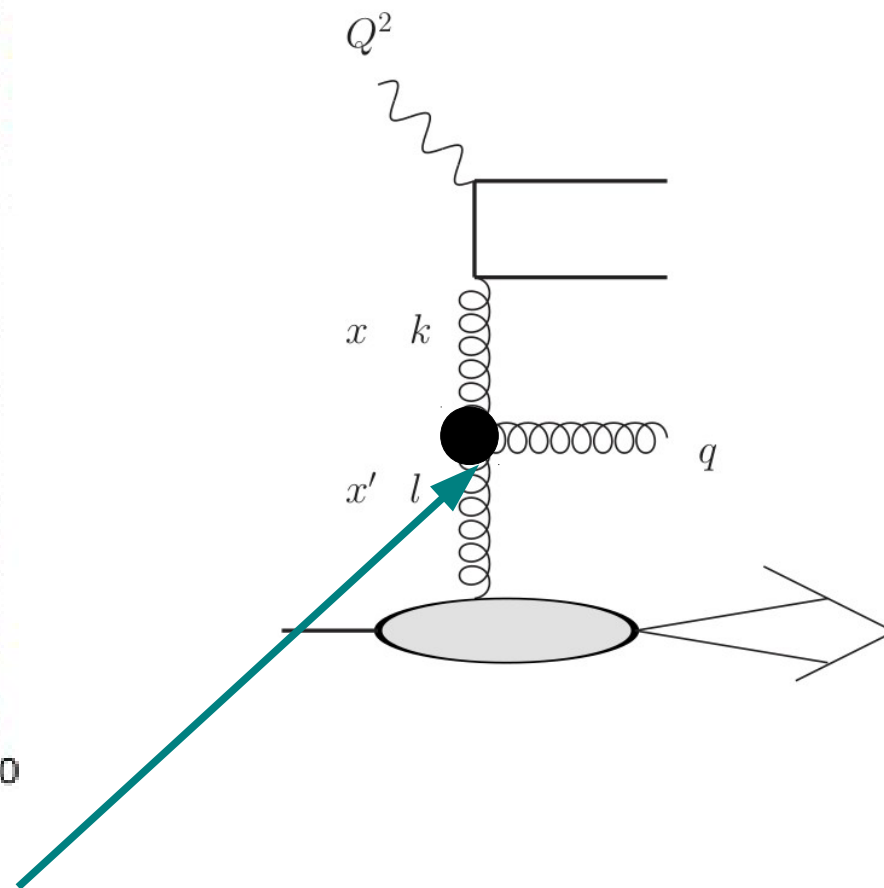
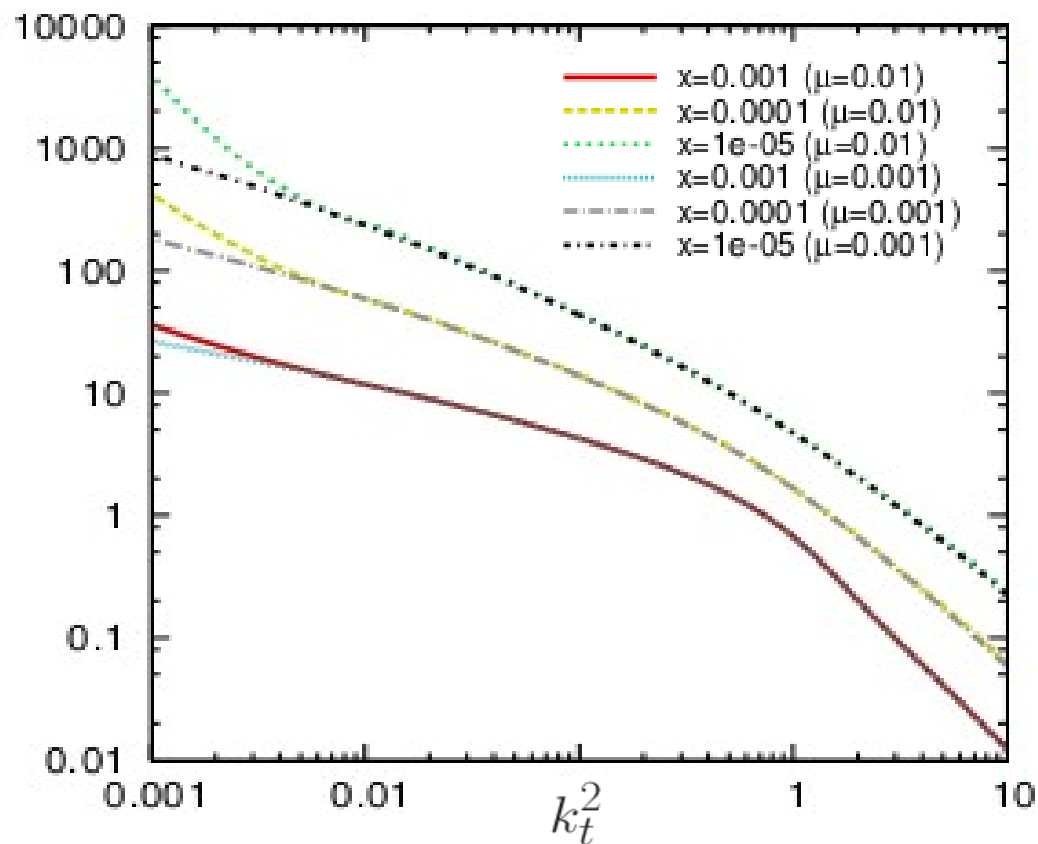
$k_t \ll P_t$  Collinear Factorization  $\rightarrow$  partons in one of hadrons are just collinear with hadron  
 $k_t$  is neglected

$Q_s \sim k_t \ll P_t$  generalized Transverse Momentum Dependent Factorization  $\rightarrow$  rescatterings  
formal treatment of nonlinearities but does not allow for calculation of  
decorrelations

$Q_s, k_t, P_t$  Improved Transverse Momentum Dependent Factorization

# The saturation problem: sensitivity to gluons at small $k_t$

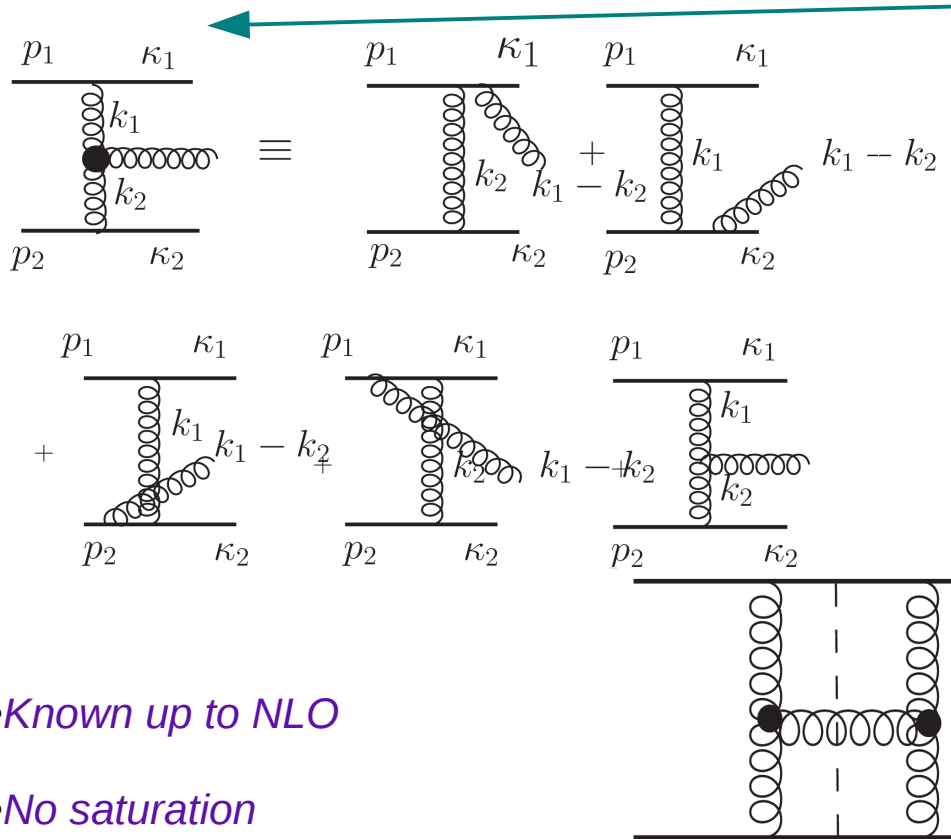
Solution of BFKL equation



$$\mathcal{F} = \mathcal{F}_0 + K \otimes \mathcal{F}$$

# Prototype for TMD evolution - the BFKL evolution

Balitsky, Fadin, Kuraev, Lipatov '77



$$J^\mu = -ig\bar{u}(p_1 + k_1)\gamma^\mu u(p_1) \approx -2igp_1^\mu$$

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1t}$$

$$k_2 = \alpha_2 p_1 + \beta_2 p_2 + k_{2t}$$

$$1 \gg \alpha_1 \gg \alpha_2$$

*strong ordering*

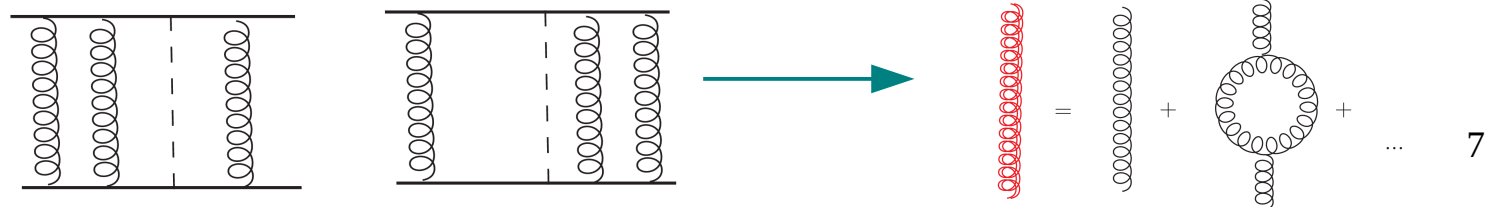
$$1 \gg |\beta_1| \gg |\beta_2|$$

• Known up to NLO

• No saturation

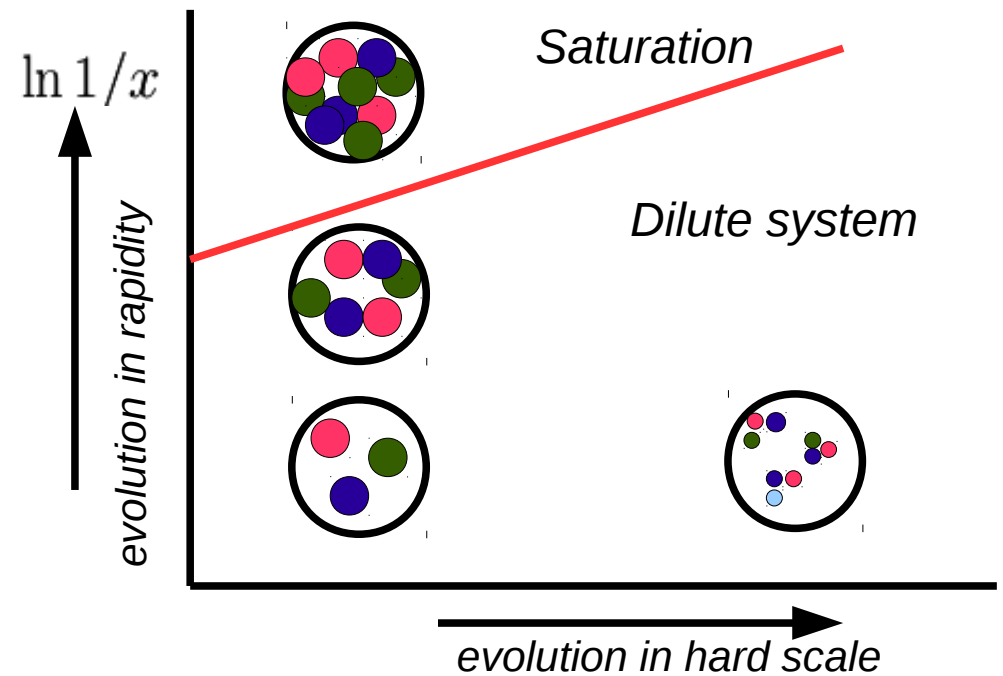
• No applicable to final states: “evolution without observer”

*reggeized gluon*



# High energy factorization and saturation

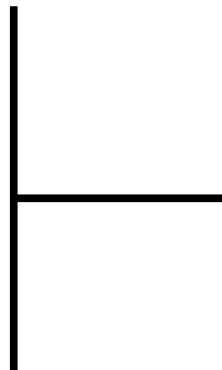
**Saturation** – state where number of gluons stops growing due to high occupation number. Way to fulfill unitarity requirements in high energy limit of QCD.



On microscopic level it means that  
gluon apart splitting recombine

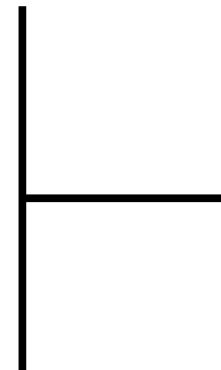
splitting

Linear evolution  
equation

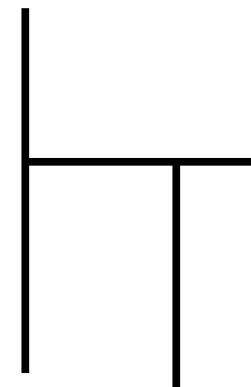


Nonlinear evolution  
equations

splitting

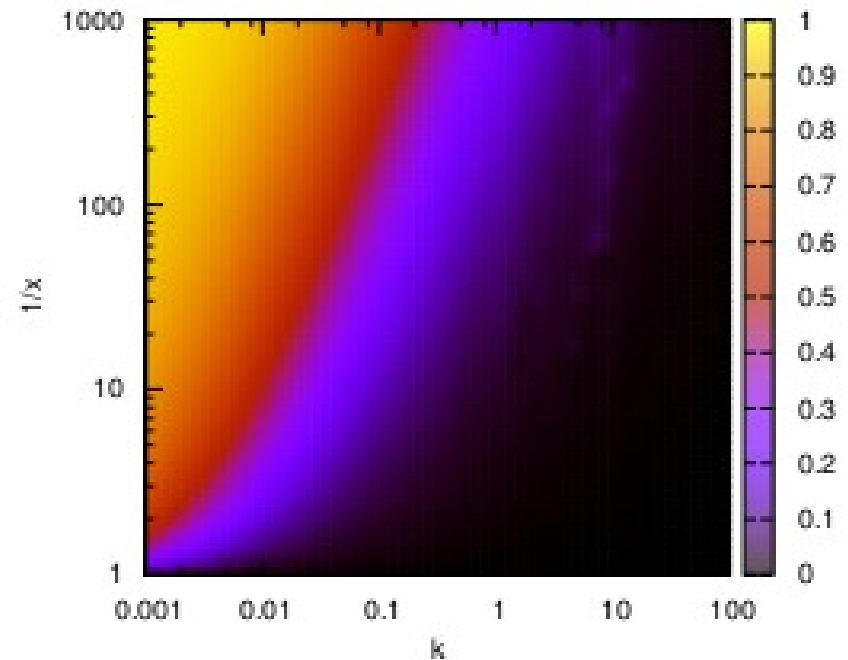


recombination



# High energy factorization and saturation

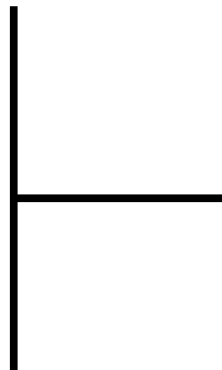
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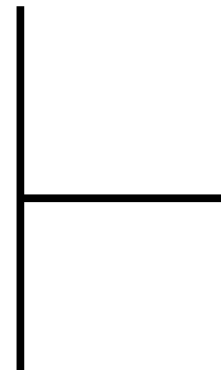
splitting

Linear evolution  
equation

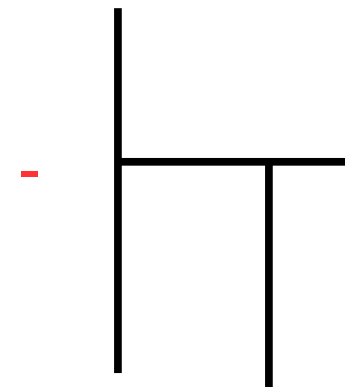


Nonlinear evolution  
equations

splitting



recombination



# The saturation problem: suppressing gluons at small $k_t$

Originally formulated in coordinate space

Balitsky '96, Kovchegov '99

Now at NLO accuracy

Balitsky, Chirilli '07

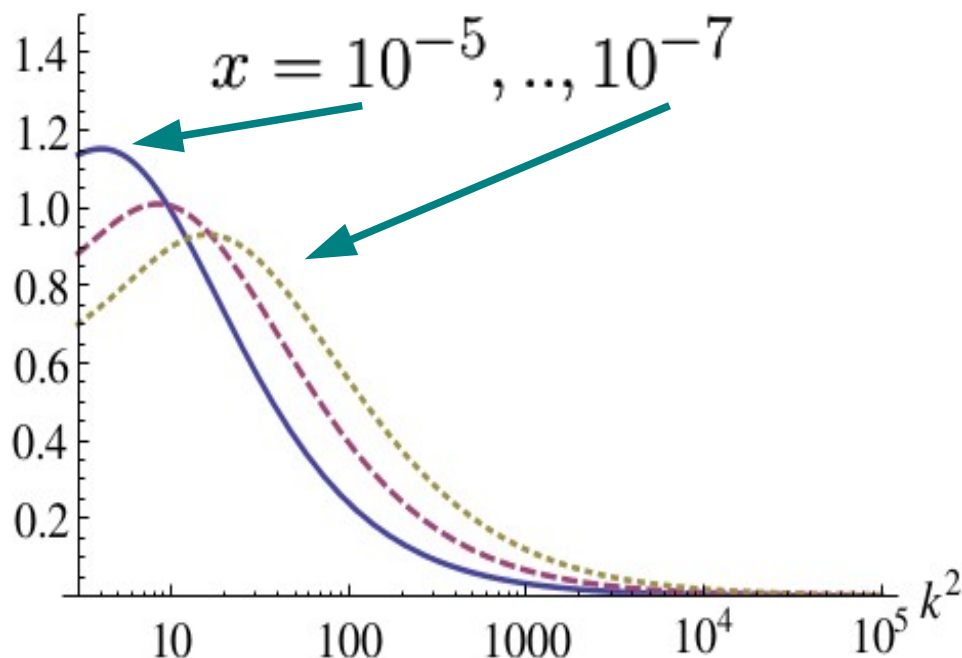
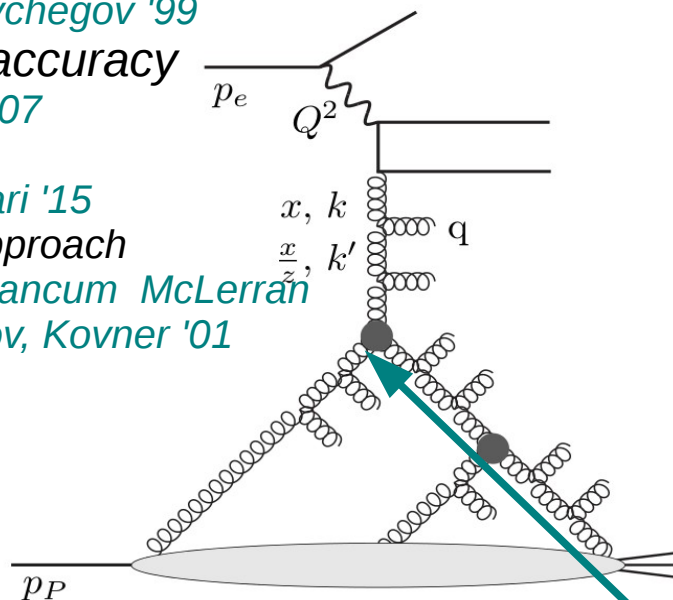
and solved

Lappi, Mantysaari '15

More general approach

Jalilian-Marian, Iancu, McLerran

Weigert, Leonidov, Kovner '01



Solution of the equation

The BK equation for dipole gluon density

$$\mathcal{F} = \mathcal{F}_0 + K \otimes \mathcal{F} - \frac{1}{R^2} V \otimes \mathcal{F}^2$$

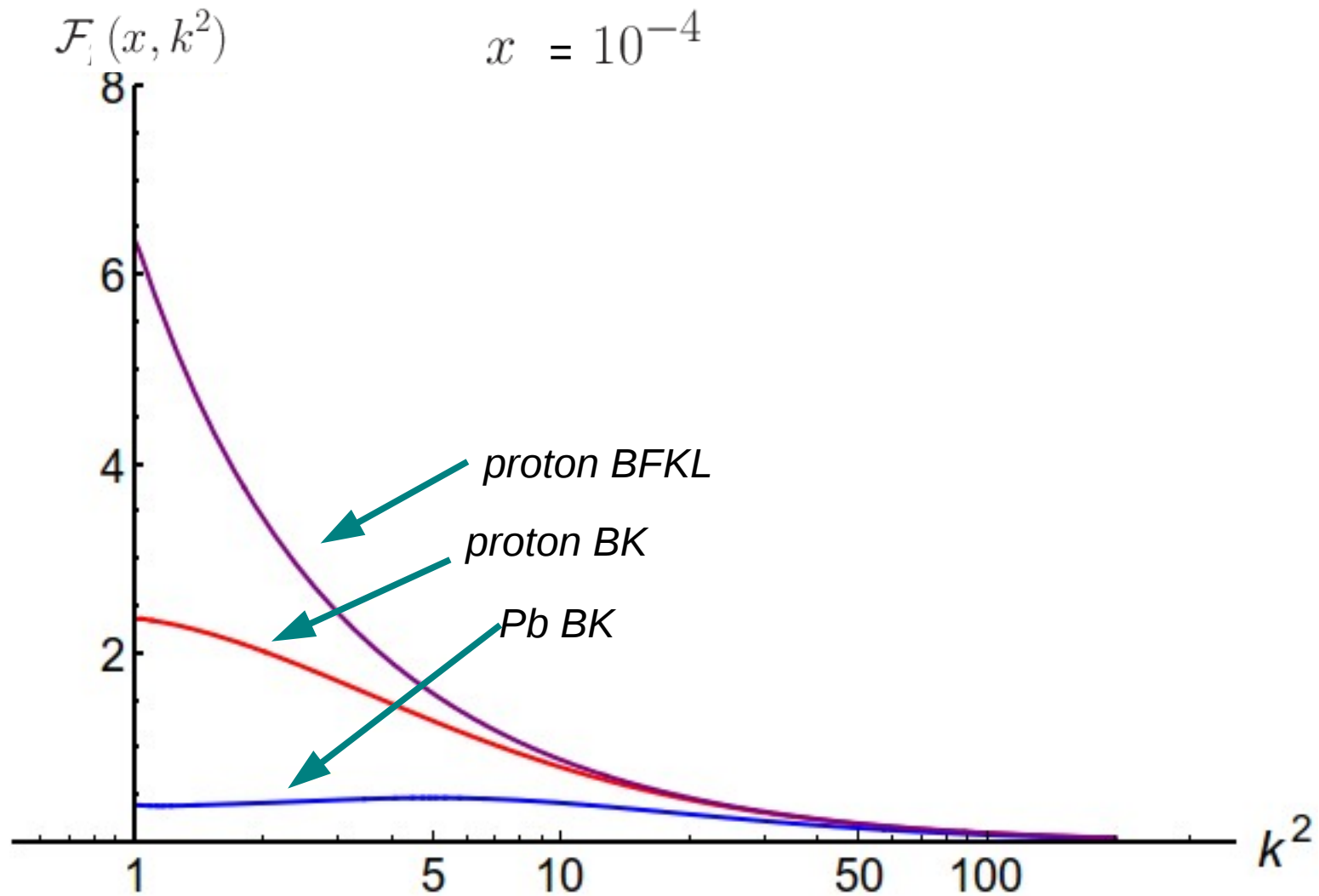
hadron's radius

Kwiecinski, Kutak '02

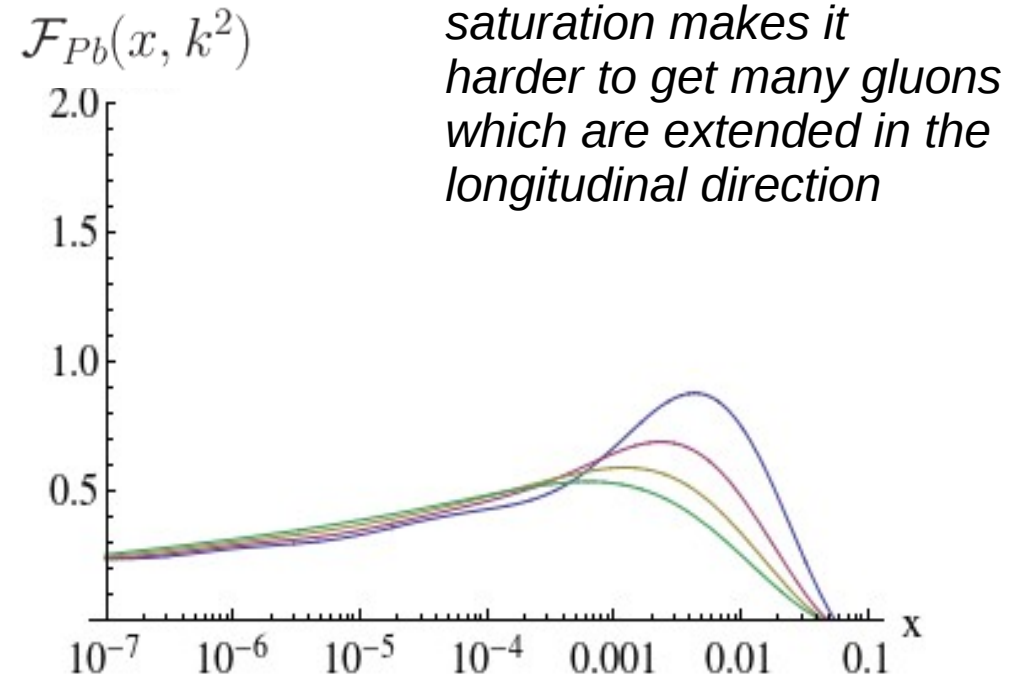
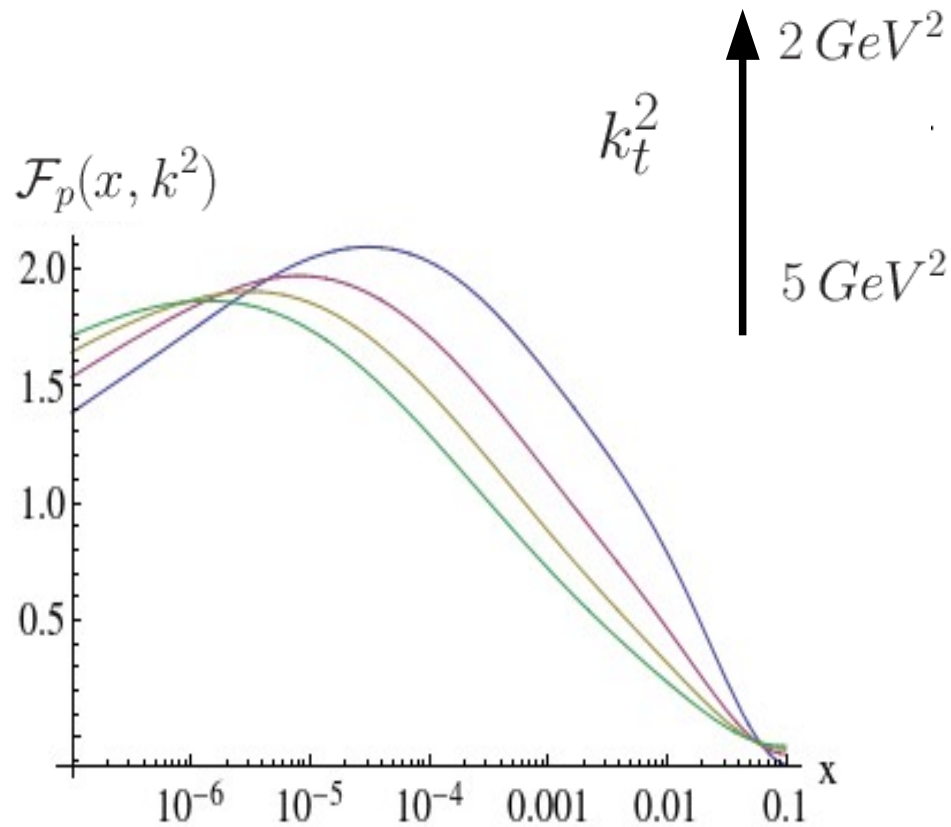
Kutak, Stasto '03

Nikolaev, Schafer '06

## Glue in $p$ vs. glue in $Pb$ vs. linear - $kt$ dependence



## Only nonlinear - glue in $p$ vs. glue in $Pb$



By uncertainty principle this plots show that saturation makes it harder to get many gluons which are extended in the longitudinal direction

Maximum signalize emergence of saturation scale

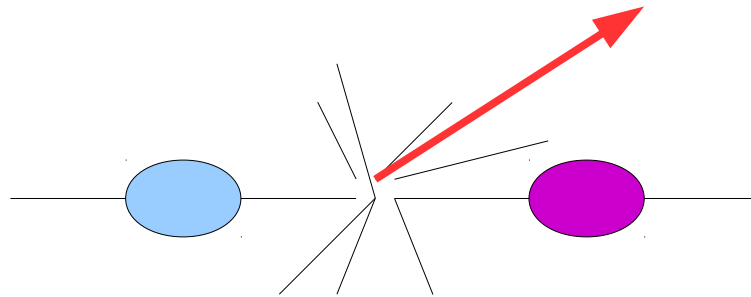
## *PDF we use at present*

*KS (Kutak-Sapeta) nonlinear* → gluon density from extension of momentum space version of BK equation to include:

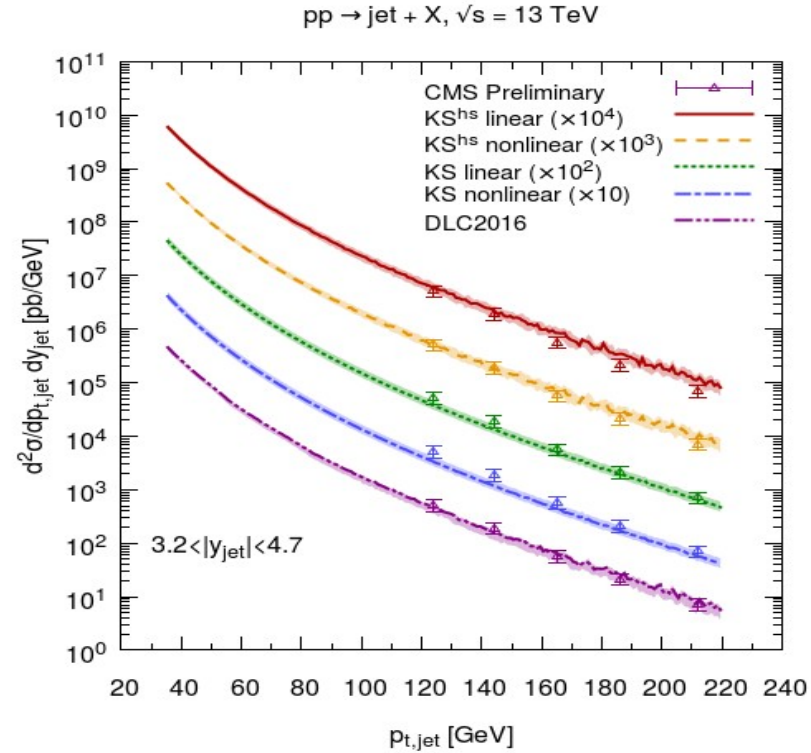
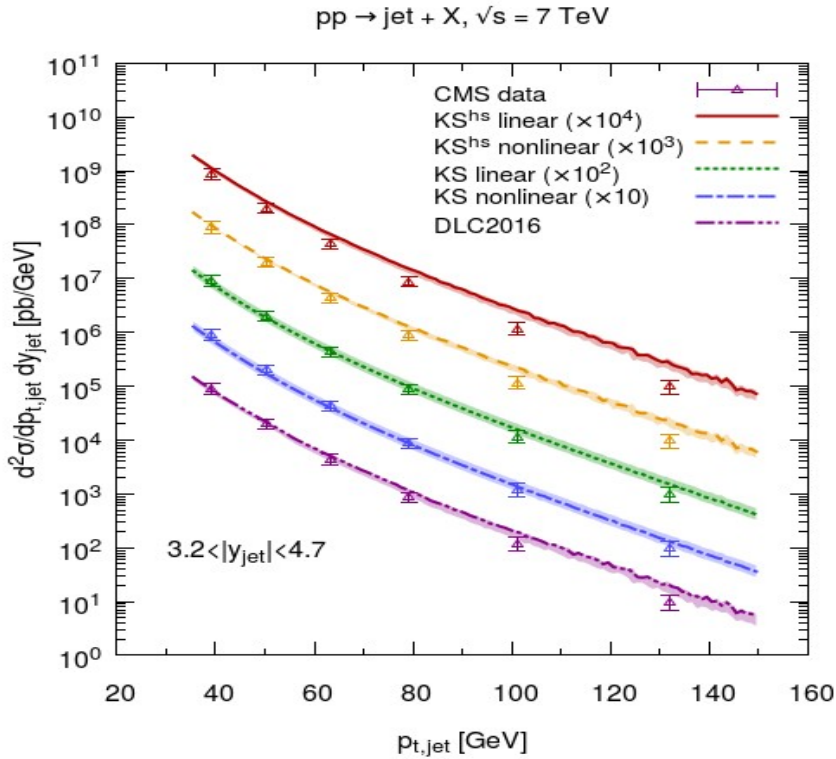
- *kinematical constraint*
- *complete splitting function,*
- *running coupling*
- *quarks*

*KK, Kwiecinski '03 fitted to '10 HERA data KK, Sapeta '12, nonlinear extension of unified BFKL+DGLAP Kwiecinski, Martin, Staśto framework '97.*

## *Inclusive-forward jet*



# Single inclusive $p_t$ jet spectra

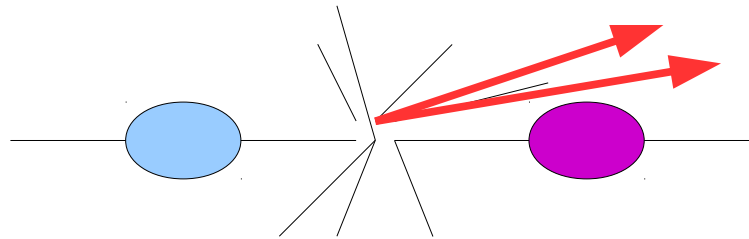


$$|3.2| < y < |4.7|$$

Bury, Deak, Kutak, Sapeta '16

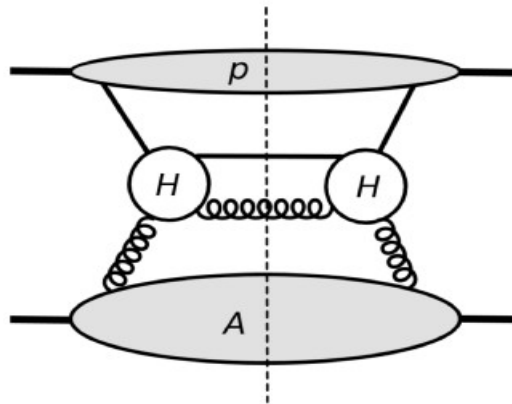
$$\frac{d\sigma}{dy dp_t} = \frac{\pi p_t}{2(x_1 x_2 s)^2} \sum_{a,b,c} \overline{|\mathcal{M}_{ab^* \rightarrow c}|^2} x_1 f_{a/A}(x_1, \mu^2) \mathcal{F}_{b/B}(x_2, p_t^2, \mu^2)$$

# *Forward-forward di-jets*



## Towards TMD for dijets in pA

The used factorization formula for dijets is strictly valid in linear regime and was calculated in a specific gauge. Results for dijets based on it with usage of gluon density coming from nonlinear equation can estimate of strength of saturation. We want to go beyond this



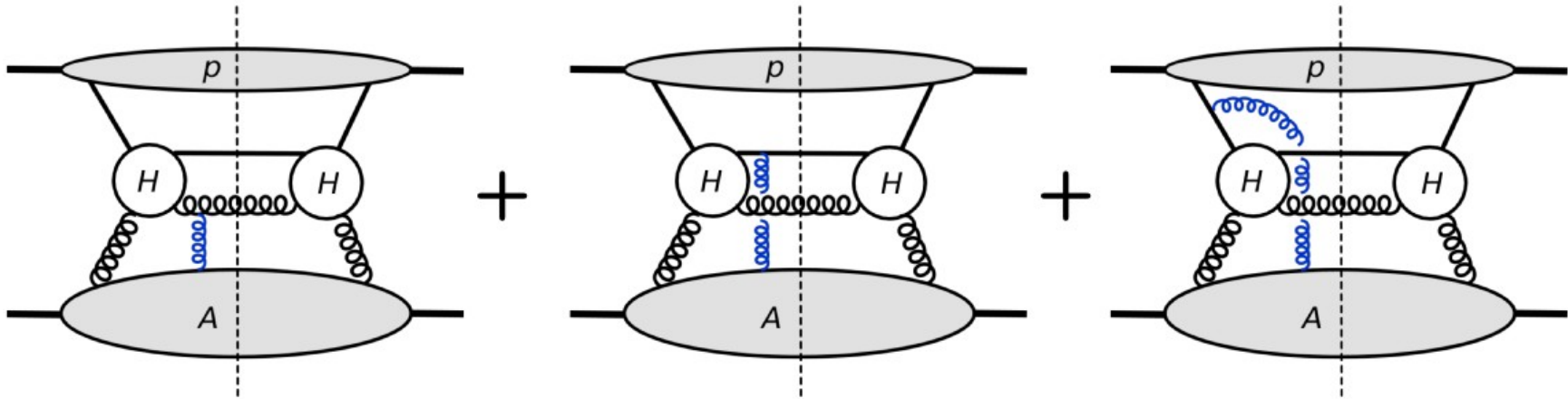
$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{dy_1 dy_2 d^2 p_{1t} d^2 p_{2t}} = \frac{1}{16\pi^3 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1, \mu^2) |\overline{\mathcal{M}_{ag^* \rightarrow cd}}|^2 \mathcal{F}_{g/A}(x_2, k_t) \frac{1}{1 + \delta_{cd}}$$

$$\mathcal{F}_{g/A}(x_2, k_t) \stackrel{\text{naive}}{=} 2 \int \frac{d\xi^+ d^2 \xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) F^{i-}(0)] | A \rangle$$

*Taking all complexity into account leads to following generalization of formula above...*

*Bomhof, Mulders and Pijlman '16.*

## Towards TMD for dijets in pA – gauge link



+ similar diagrams with 2,3,...gluon exchanges.

All this need to be resummed

Bomhof, Mulders, Pijlman 06

This is achieved via gauge link which renders the gluon density gauge invariant

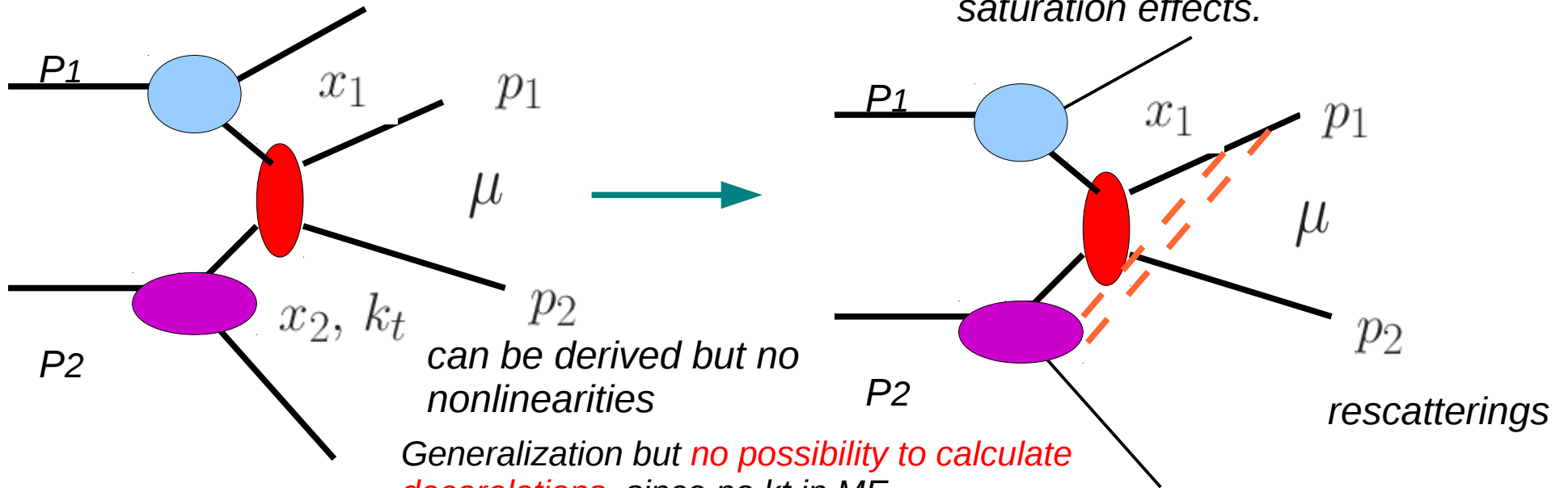
$$\mathcal{P} \exp \left[ -ig \int_{\alpha}^{\beta} d\eta^{\mu} A^a(\eta) T^a \right]$$

$$\mathcal{F}_{g/A}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) U_{[\xi, 0]} F^{i-}(0)] | A \rangle$$

# Improved TMD for dijets

$$\frac{d\sigma_{\text{SPS}}^{P_1 P_2 \rightarrow \text{dijets} + X}}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \frac{p_{1t} p_{2t}}{8\pi^2 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/P_1}(x_1, \mu^2) |\overline{\mathcal{M}}_{ag^* \rightarrow cd}|^2 \mathcal{F}_{g/P_2}(x_2, k_t^2) \frac{1}{1 + \delta_{cd}}$$

can be used for estimates of saturation effects.



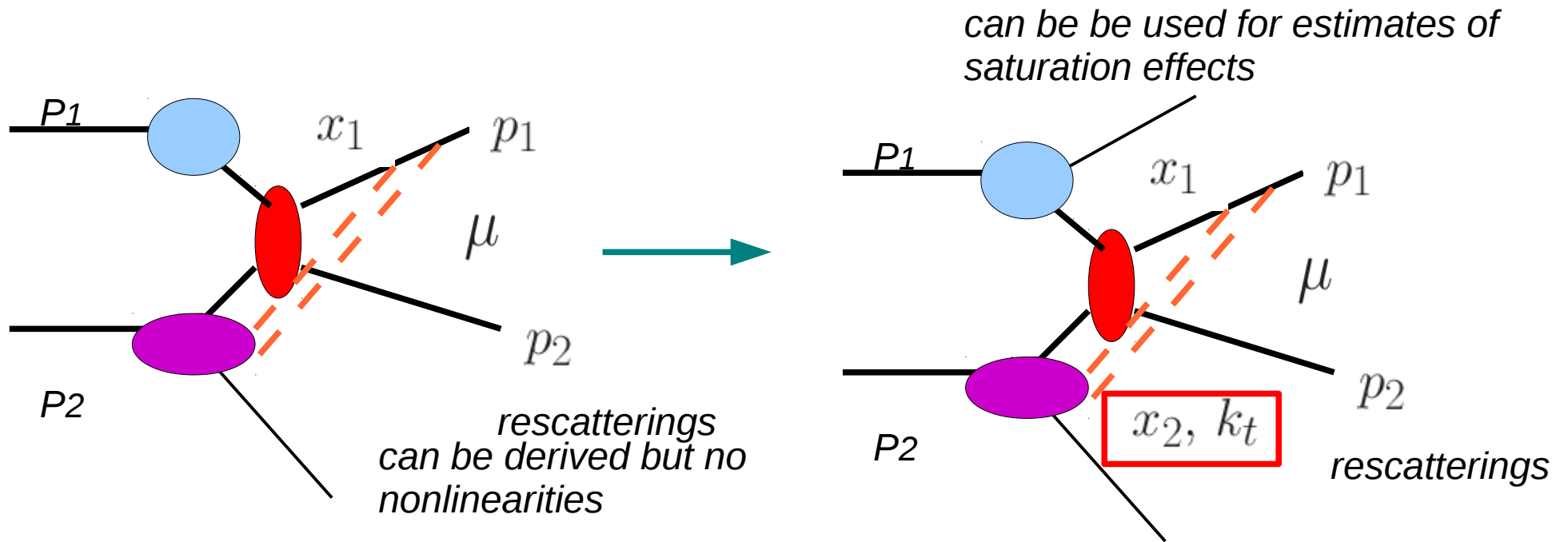
Dominguez, Marquet, Xiao, Yuan '11

Application to differential distributions in d+Au

Stasto, Xiao, Yuan '11

$$\frac{d\sigma^{pA \rightarrow cdX}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} x_1 f_{q/p}(x_1, \mu^2) \sum_{i=1}^n \mathcal{F}_{ag}^{(i)} H_{ag \rightarrow cd}^{(i)} \frac{1}{1 + \delta_{cd}}$$

# Improved TMD for dijets

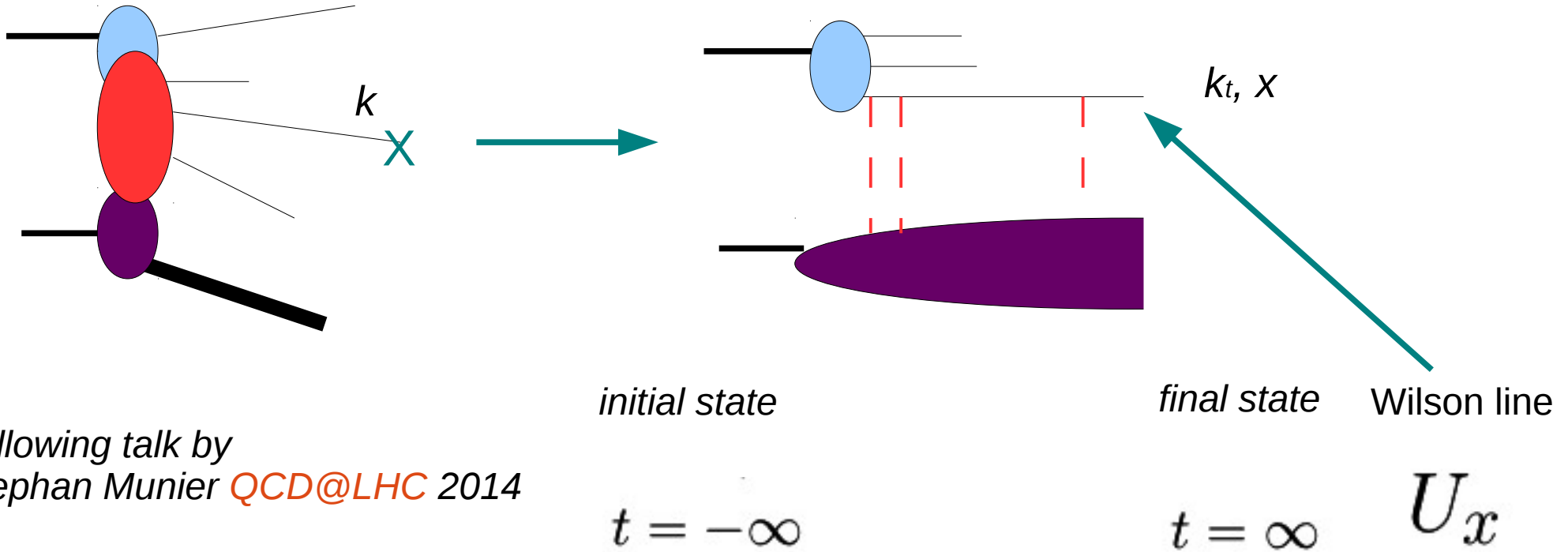


We found a method to include  $k_t$  in ME and express the factorization formula in terms of gauge invariant sub amplitudes  $\rightarrow$  more direct relation to two fundamental gluon densities: **dipole gluon density** and **Weizacker-Williams gluon density**

Kotko, K.K, Marquet, Petreska, Sapeta, van Hameren '15

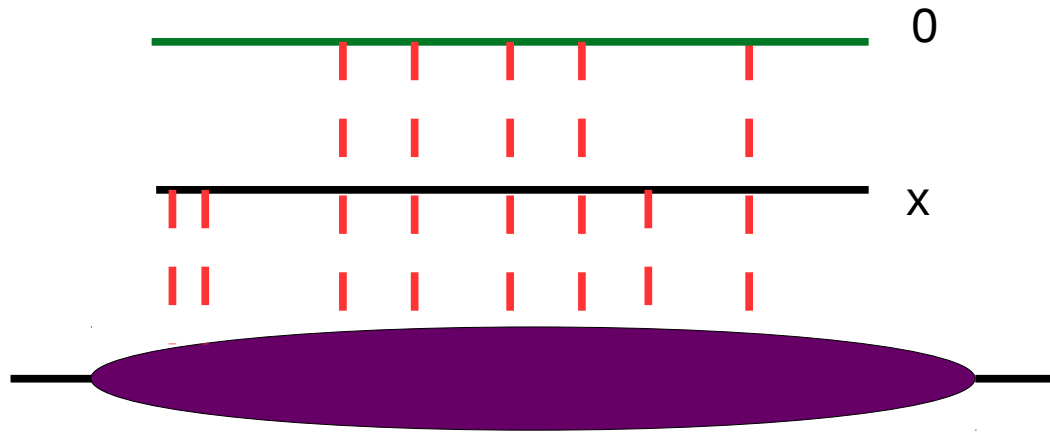
$$\frac{d\sigma^{pA \rightarrow \text{dijets} + X}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1, \mu^2) \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)} \Phi_{ag \rightarrow cd}^{(i)} \frac{1}{1 + \delta_{cd}}^3$$

# Dipole gluon density



Following talk by  
Stephan Munier *QCD@LHC* 2014

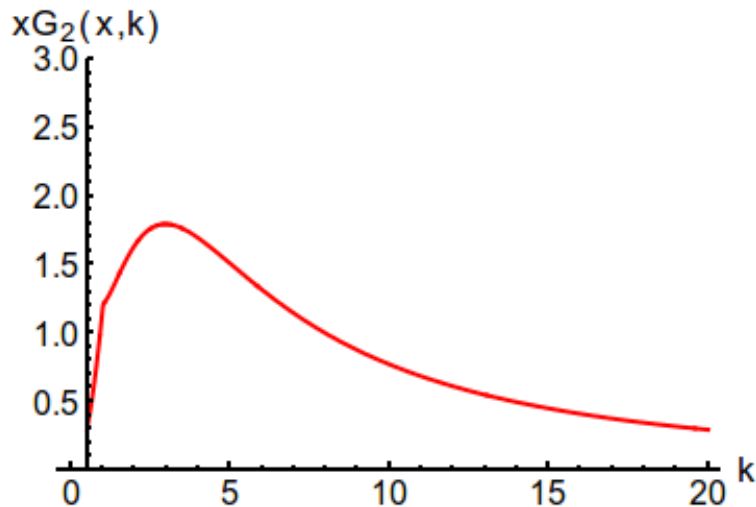
## *Dipole gluon density*



$$\mathcal{F}(x, \mathbf{k}_t^2) \propto \mathbf{k}_t^2 \int \frac{d^2 \mathbf{x}}{(2\pi)^2} e^{-i\mathbf{k}_t \cdot \mathbf{x}} S(x, \mathbf{x})$$

## Dipole gluon density

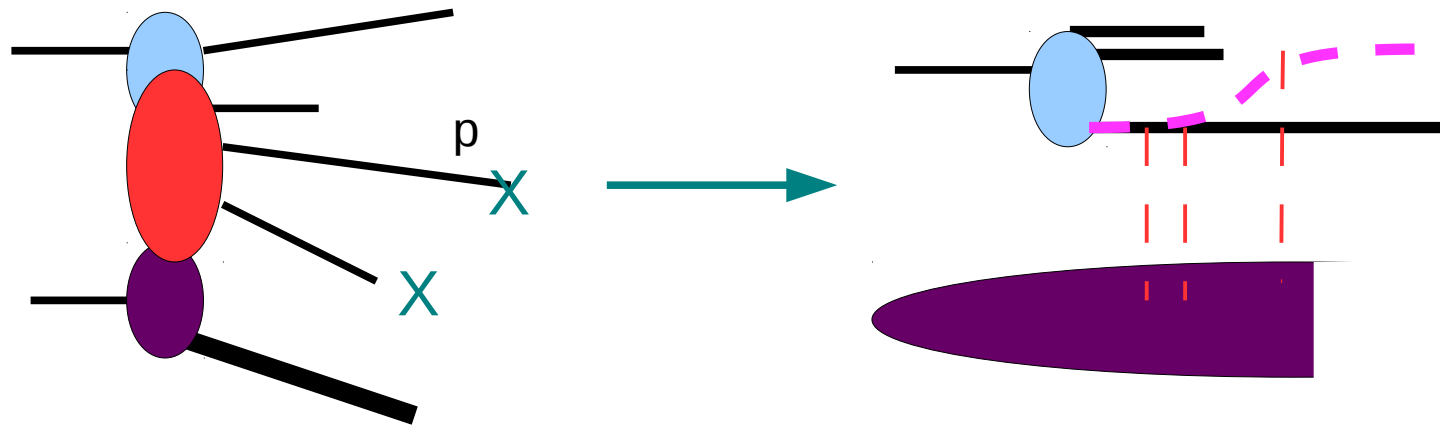
- Enters directly into DIS structure function and DY cross section
- Can be expressed in terms of the expectation value of the  $S$  – matrix for scattering of a  $q\bar{q}$  dipole off a dense target,  $SF$
- One can write BK equation in the momentum space which as a solution gives dipole gluon density



$$\mathcal{F}(x, \mathbf{k}_t^2) \propto \mathbf{k}_t^2 \int \frac{d^2 \mathbf{x}}{(2\pi)^2} e^{-i \mathbf{k}_t \cdot \mathbf{x}} S(x, \mathbf{x})$$

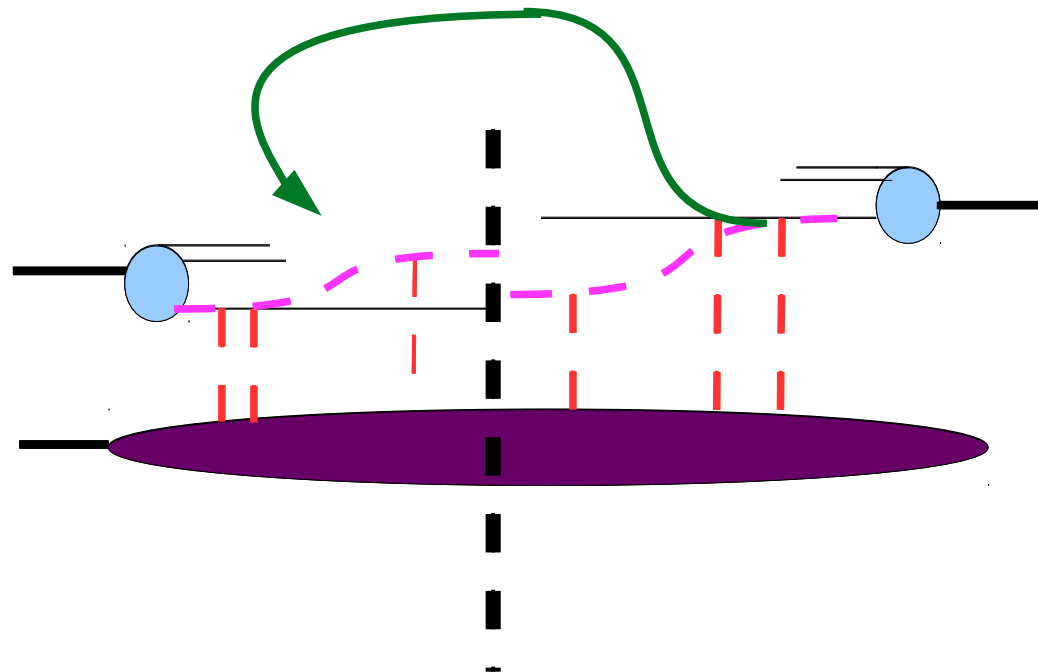
$$xG^{(1)}(x, \mathbf{k}_t^2) \equiv \mathcal{F}(x, \mathbf{k}_t^2)$$

# Weizacker-Williams gluon density

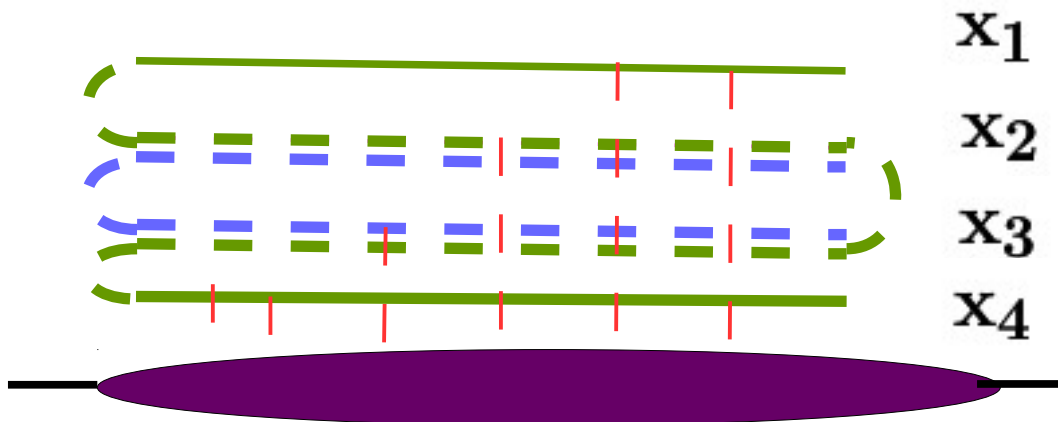


*Double inclusive production*

# Weizacker-Williams gluon density



Large number of  
color limit

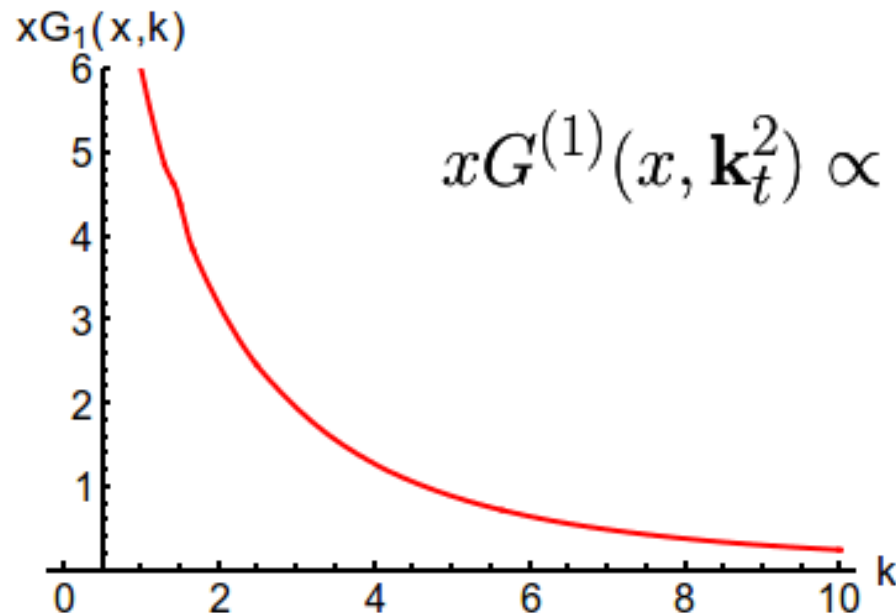


$$S \propto \langle \text{Tr}(U^\dagger(x_2)U(x_3)) \rangle$$

$$Q \propto \langle \text{Tr}(U^\dagger(x_1)U(x_2)U^\dagger(x_3)U(x_4)) \rangle$$

# Weizacker-Williams gluon density

- Can be determined from dijet production in DIS
- In general can be obtained from a quadrupole operator
- For Gaussian distribution of sources one can express it through the expectation value of the S – matrix for scattering of a gg dipole



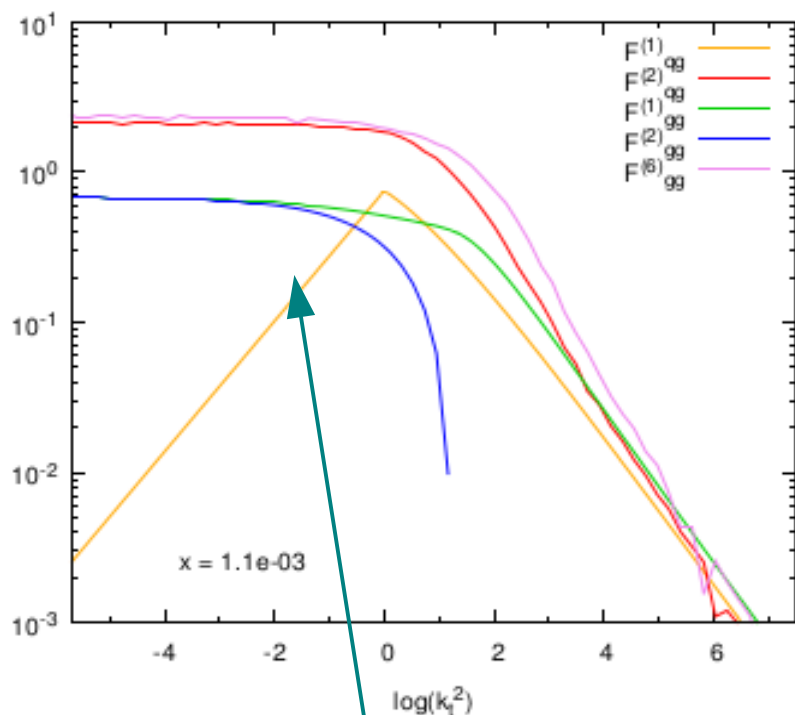
$$xG^{(1)}(x, \mathbf{k}_t^2) \propto \int \frac{d^2\mathbf{x}}{(2\pi)^2} e^{-i\mathbf{k}_t \cdot \mathbf{x}} \frac{(1 - S_A(x, \mathbf{x}))}{\mathbf{x}^2}$$

*In approximation of Gaussian distribution of charges*

$$S_A(x, \mathbf{x}) = [S(x, \mathbf{x})]^2$$

# Glimpse on the first results – HEF vs. ITMD

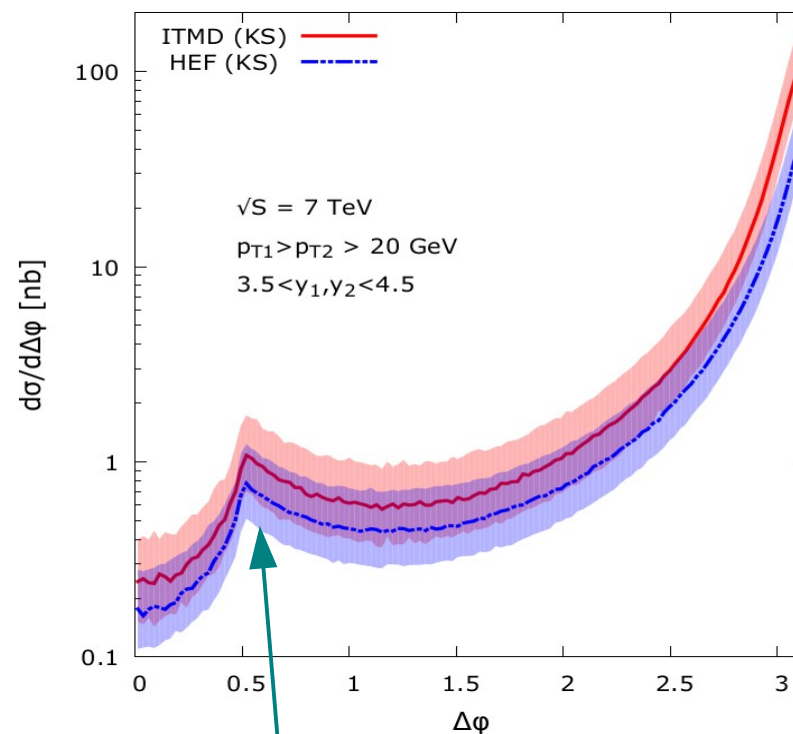
Kotko, K.K, Marquet, Petreska, Sapeta, van Hameren '16



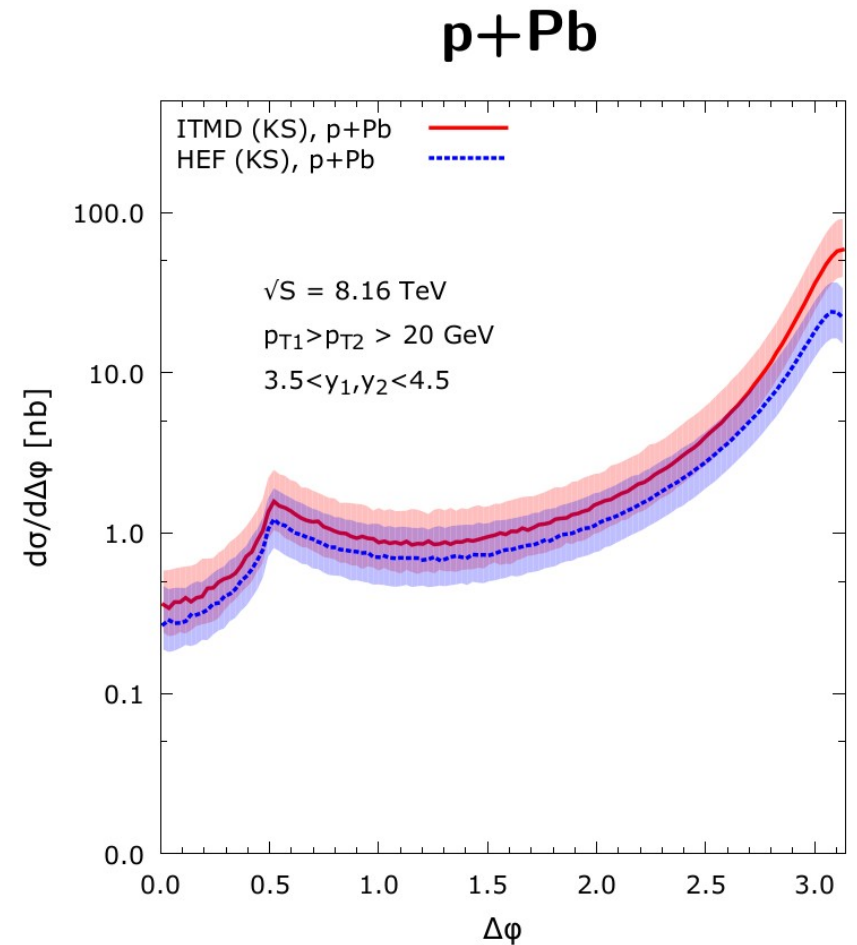
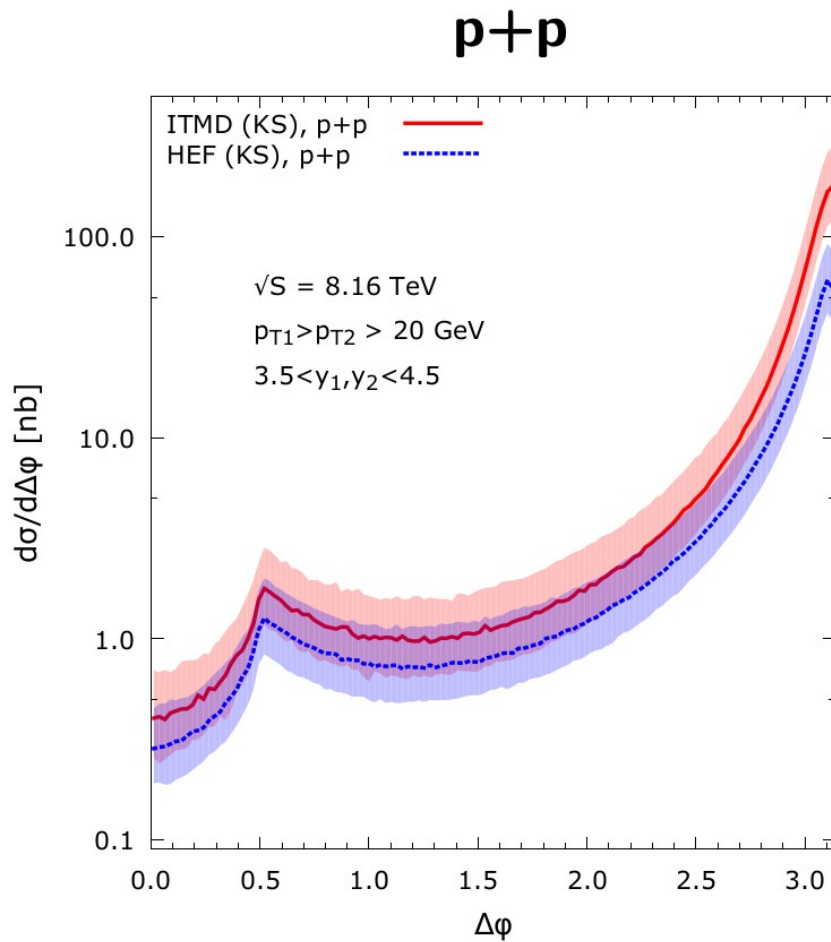
*Standard HEF gluon density*

*The other densities are flat at low  $kt \rightarrow$  less saturation*

*Not negligible differences at large  $kt \rightarrow$  differences at small angles*



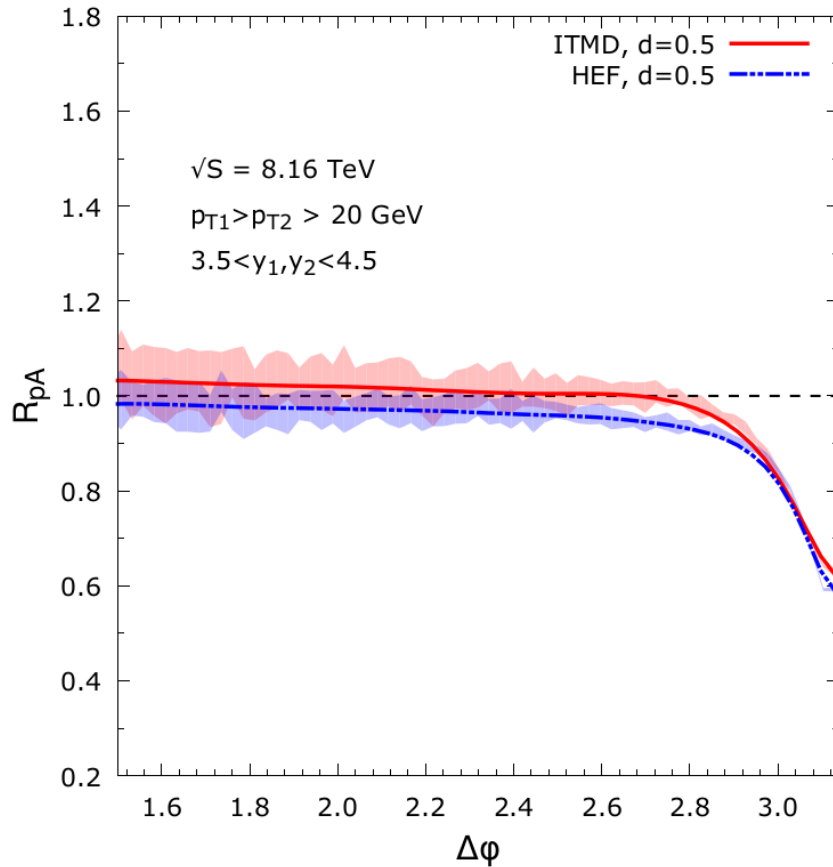
# Azimuthal distance between jets



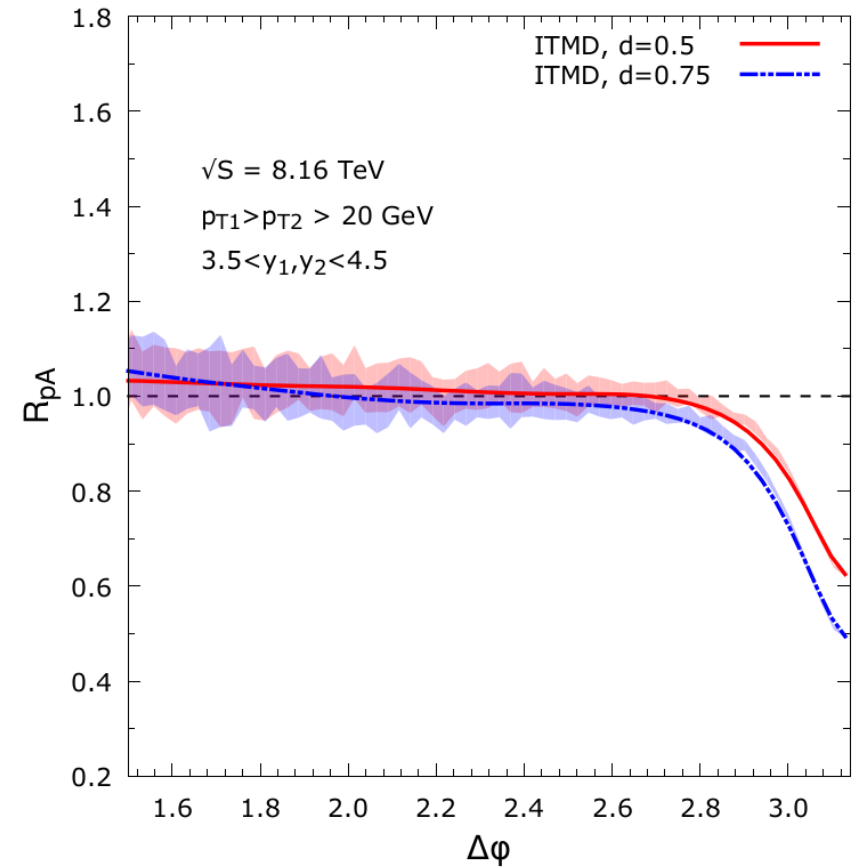
*Differences in normalization but altogether similar behavior when switching from HEF to ITMD*

# $R_{pA}$ : azimuthal distance between jets

## ITMD vs HEF



## ITMD, $d=0.5$ vs $d=0.75$



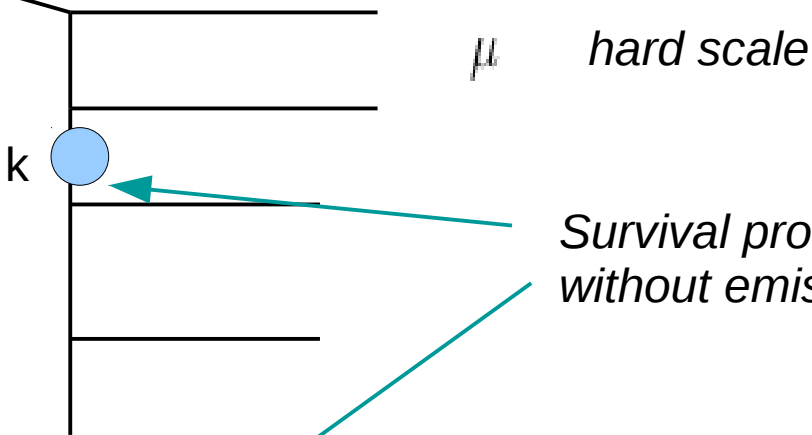
$R_{pA}$  comes very similar since differences in the cross section effectively cancel out

# Other relevant effects – Sudakov form factor in ISR

The relevance in low  $x$  physics  
at linear level recognized by:

Catani, Ciafaloni, Fiorani, Marchesini;  
Kimber, Martin, Ryskin;  
Collins, Jung

Survival probability  
of the gap without  
emissions



Kimber, Martin, Ryskin procedure '01:  $T_s(\mu^2, k^2) = \exp \left( - \int_{k^2}^{\mu^2} \frac{dk'^2}{k'^2} \frac{\alpha_s(k'^2)}{2\pi} \sum_{a'} \int_0^{1-\Delta} dz' P_{a'a}(z') \right)$

$$\mathcal{F}(x, k^2, \mu^2) \sim \partial_{\lambda^2} (T(\lambda^2, \mu^2) x g(x, \lambda^2)) |_{\lambda^2=k^2}$$

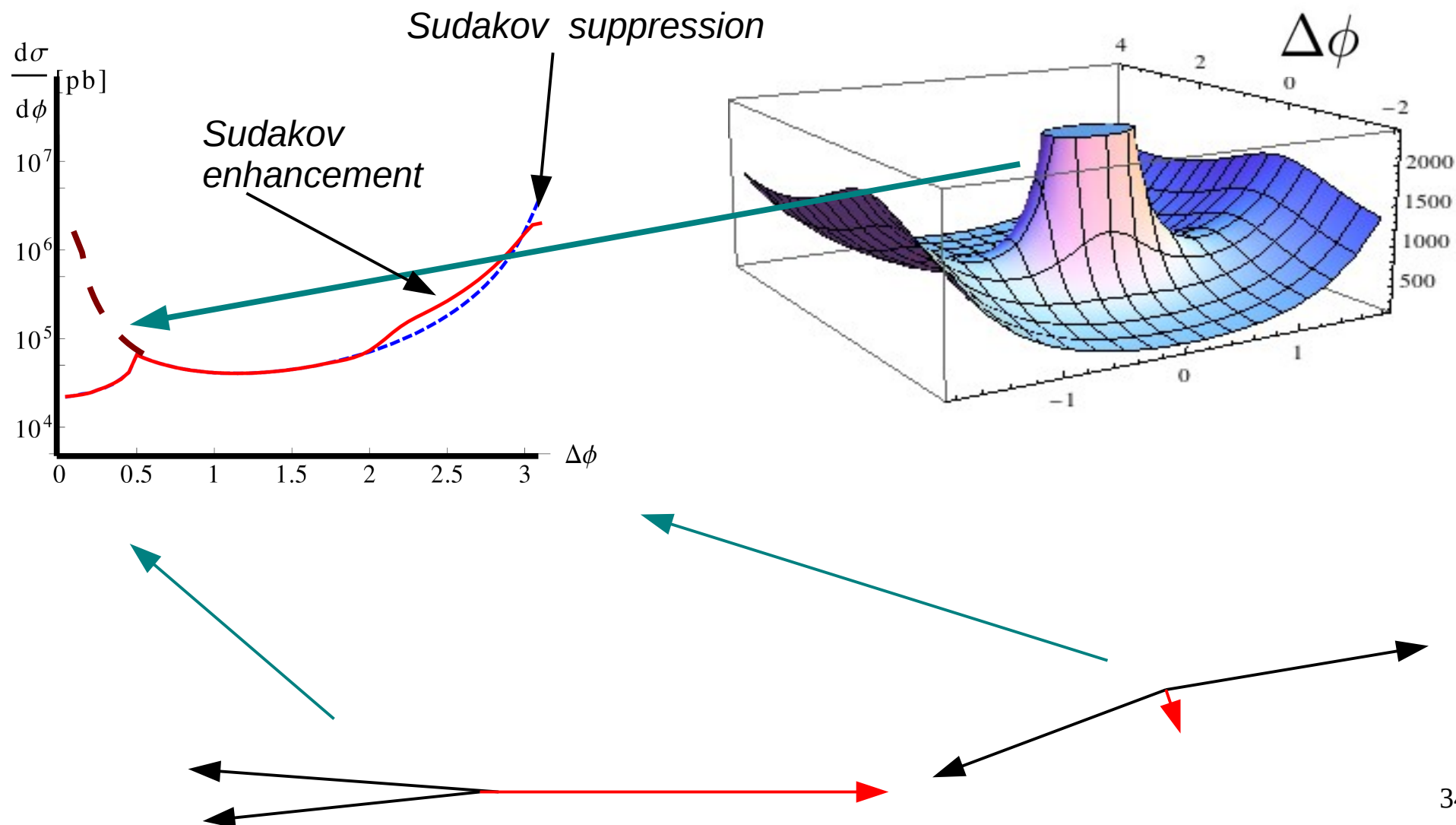
$$\Delta = \frac{\mu}{\mu+k}$$

In context of dijet studied also by  
Mueller, Xiao, Yan '12  
Mueller, Xiao, Yan '13

## Other relevant effects – Sudakov form factor in ISR

Divergence regularized  
by jet algorithm

KK '14

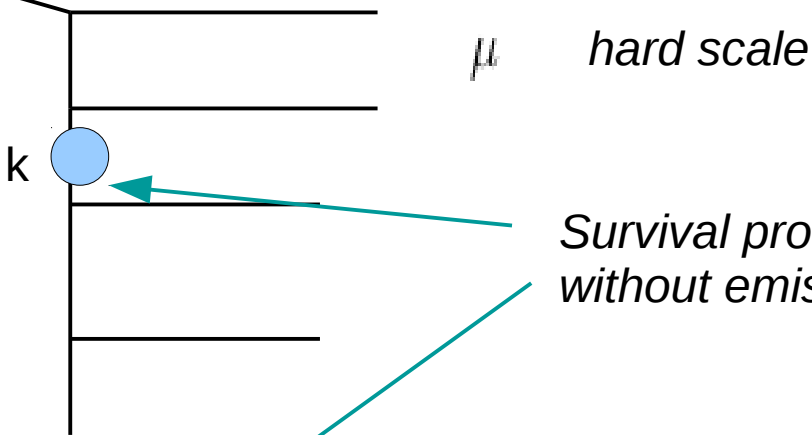


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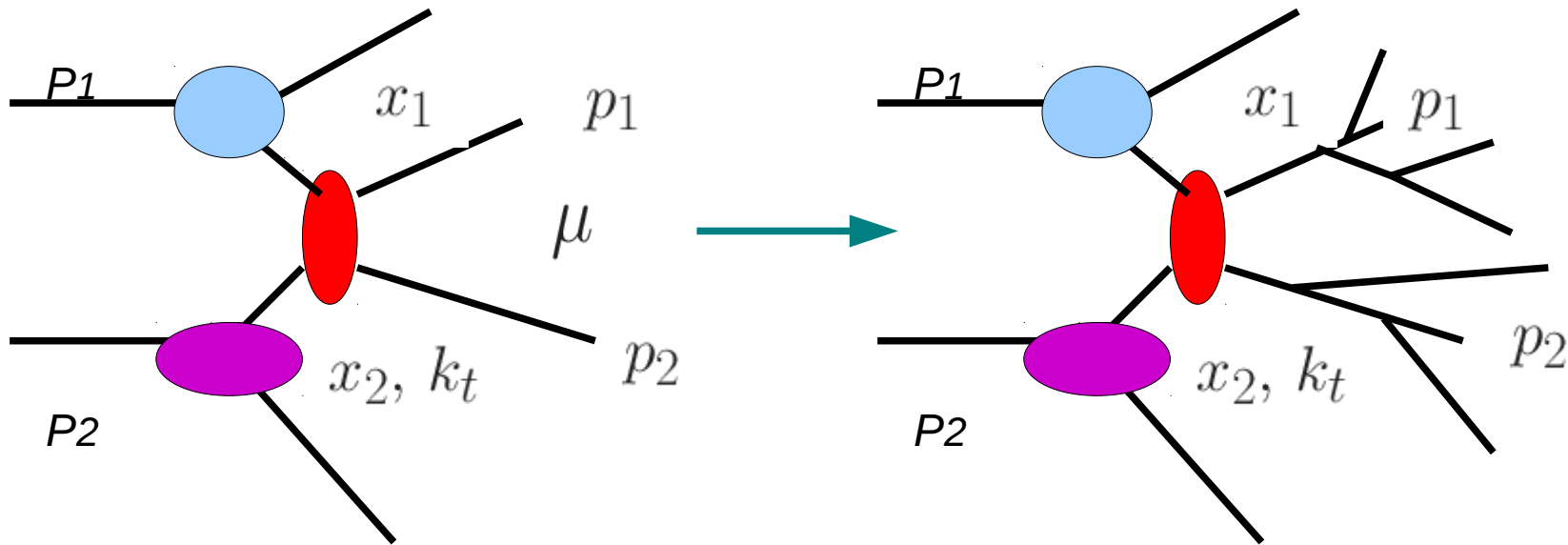
Kimber, Martin, Ryskin procedure '01:  $T_s(\mu^2, k^2) = \exp \left( - \int_{k^2}^{\mu^2} \frac{dk'^2}{k'^2} \frac{\alpha_s(k'^2)}{2\pi} \sum_{a'} \int_0^{1-\Delta} dz' P_{a'a}(z') \right)$

$$\mathcal{F}(x, k^2, \mu^2) \sim \partial_{\lambda^2} (T(\lambda^2, \mu^2) x g(x, \lambda^2)) |_{\lambda^2=k^2}$$

$$\Delta = \frac{\mu}{\mu+k}$$

In context of dijet studied also by  
Mueller, Xiao, Yan '12  
Mueller, Xiao, Yan '13  
Kutak '14

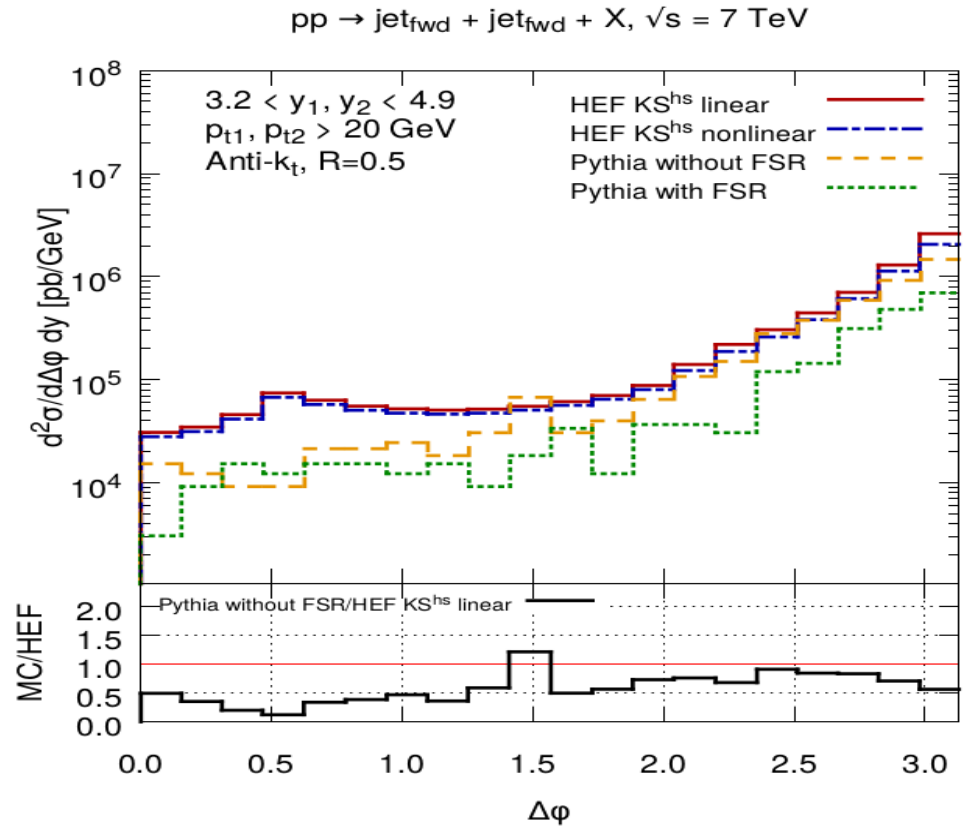
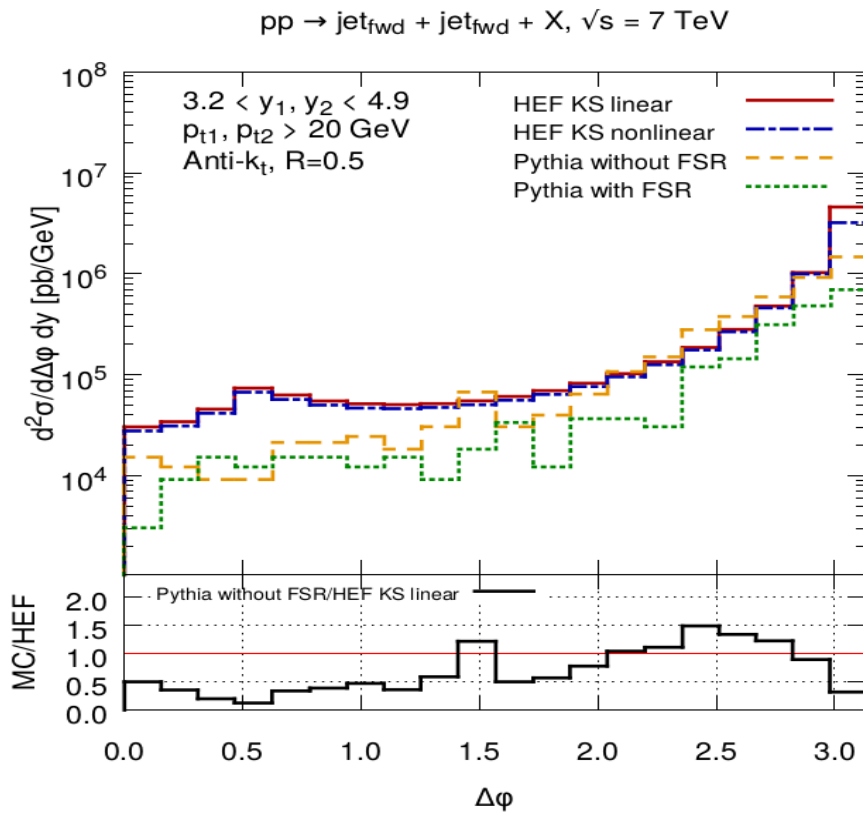
## Other relevant effects – Final State Radiation



Wide angle soft emissions lower cross section for hard jets

# Other relevant effects – final state parton shower with Pythia

Bury, Deak, Kutak, Sapeta '16



HEF framework with KShardsacle or DLC2016 → compatible with ISR in Pythia at moderate and large angles → importance of Sudakov resummation.

# Outlook

*Impact parameter dependence*

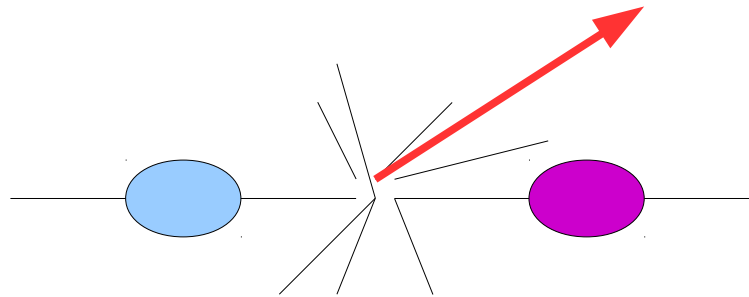
*Use another dipole densities*

*Corrections of higher orders to ME*

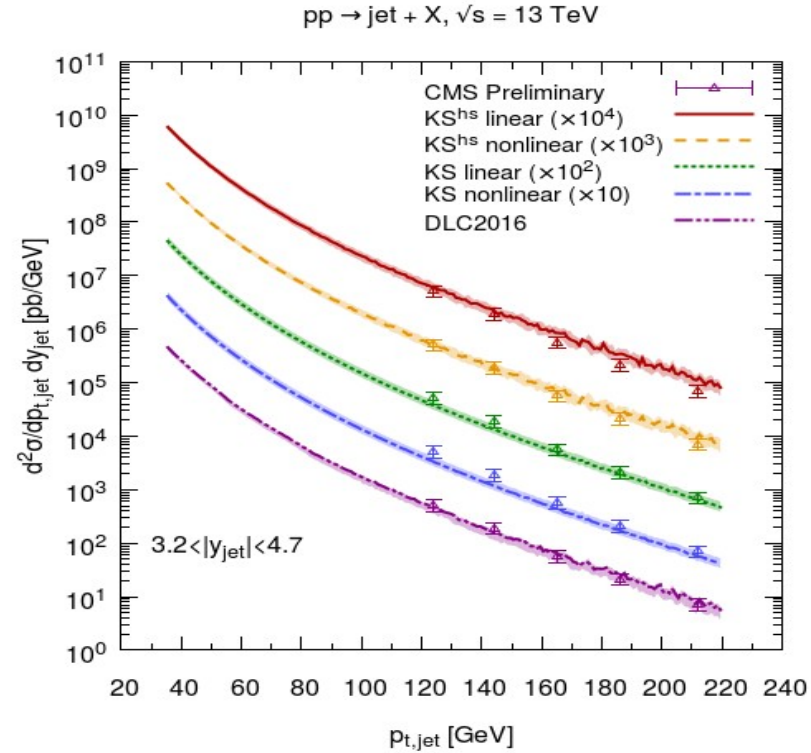
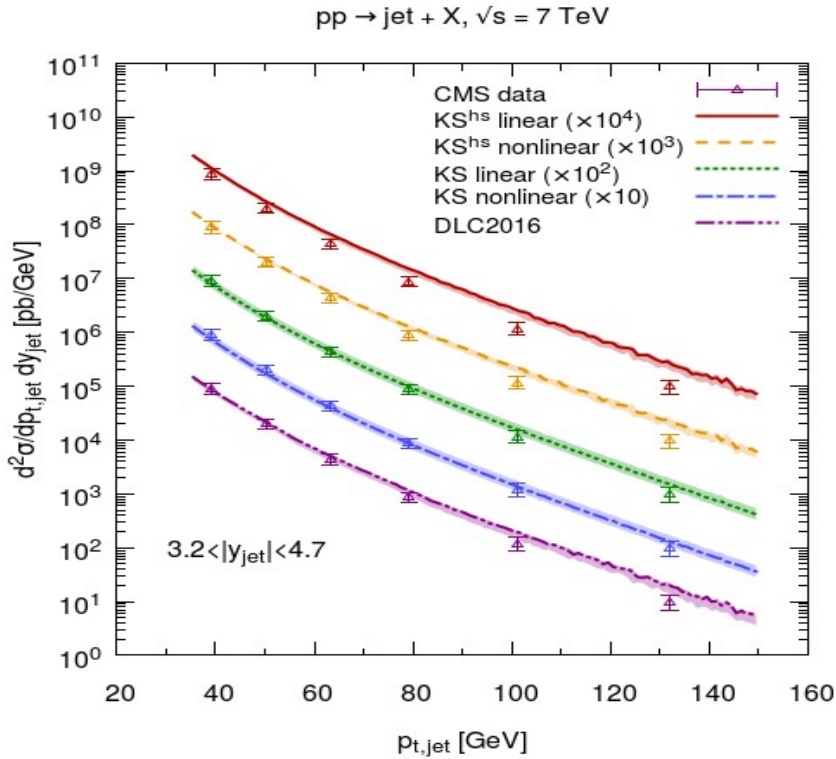
*Include FSR*

*Back up*

# *Inclusive-forward jet*



# Single inclusive $p_t$ jet spectra



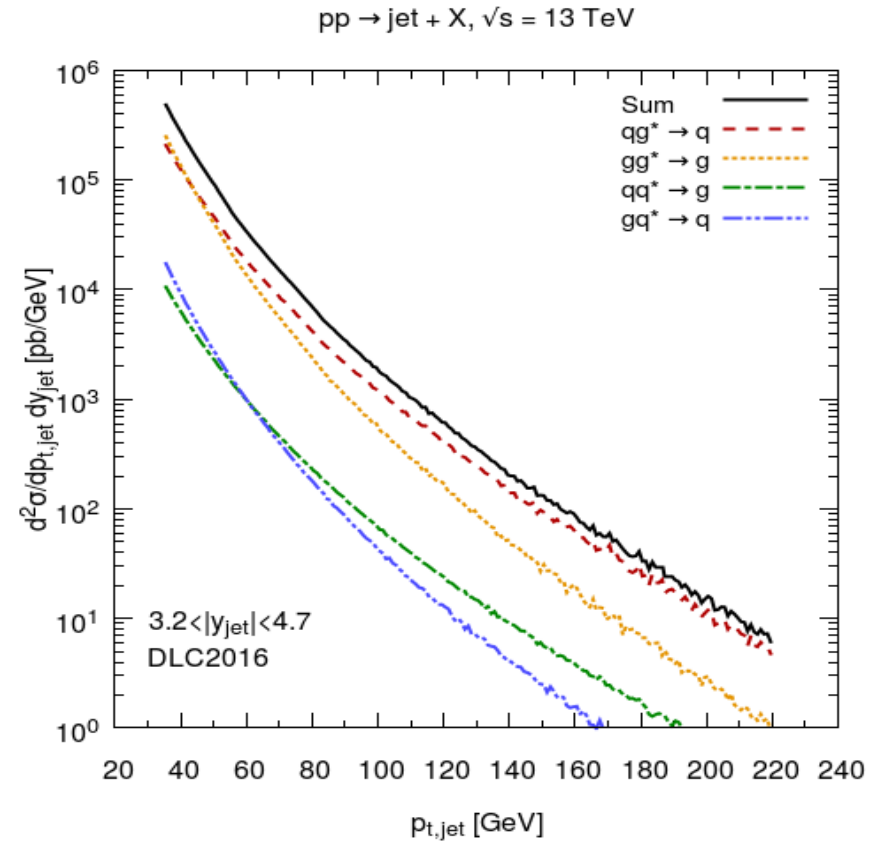
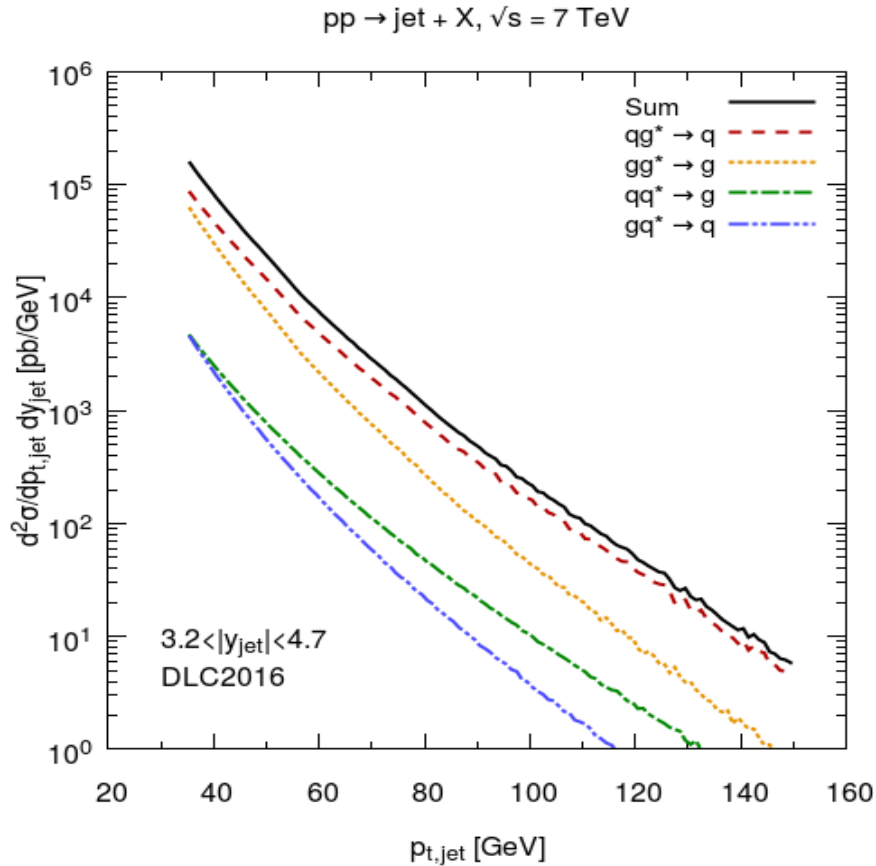
$$|3.2| < y < |4.7|$$

Bury, Deak, Kutak, Sapeta '16

$$\frac{d\sigma}{dy dp_t} = \frac{\pi p_t}{2(x_1 x_2 s)^2} \sum_{a,b,c} \overline{|\mathcal{M}_{ab^* \rightarrow c}|^2} x_1 f_{a/A}(x_1, \mu^2) \mathcal{F}_{b/B}(x_2, p_t^2, \mu^2)$$

# Single inclusive $p_t$ jet spectra

## – process decomposition



The dominant contribution comes from  $qq^* \rightarrow q$ .  
This is due to steeper falling of gluon collinear pdf and sum over quark flavor number since ME  $qq^* \rightarrow q$  and  $gg^* \rightarrow$  differ only by color factor.