HIGHER-ORDERS IN HEAVY QUARK PROCESSES WITHIN THE LTD APPROACH

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arXiv:1608.01584 [hep-ph]

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Based in:

- Catani et al, JHEP 09 (2008) 065; Buchta et al, JHEP 11 (2014) 014
- Hernandez-Pinto, GS and Rodrigo, JHEP 02 (2016) 044
- GS, Driencourt-Mangin, Hernandez-Pinto and Rodrigo, arXiv:1604.06699 [hep-ph]
- Rodrigo, Driencourt-Mangin and GS, arXiv:1608.01584 [hep-ph]

Introduction

Theoretical motivation

- Higher orders in pQCD are needed to increase **theoretical accuracy**
- Deal with **ill-defined expressions** in intermediate steps → **DREG!!!**
  - *Space-time analitically continued from d=4 to d=4-2 ε dimensions.*
  - *Singularities in four-dimensions manifest as poles in ε*
  - *Extend spinor/vector algebra to d-dimensions. It is not unique, and leads to different Regularization Schemes*
- Physical observables are finite since they are IR-safe
  - **KLN** theorem guarantees cancelation of singularities among **real and virtual** contributions
  - In **DREG**, the cancellation manifests **after integration**
- **Idea:** Achieve the **cancellation before integration** and without using DREG in intermediate expressions

→ **Use the Loop-Tree Duality theorem**

Catani et al, JHEP 09 (2008) 065

Introduction

Dual representation of one-loop integrals

Loop Feynman integral

$$L^{(1)}(p_1,\dots,p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i) = \int_\ell \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + i0}$$

Dual integral

$$L^{(1)}(p_1,\dots,p_N) = -\sum_{i=1}^{N} \int_\ell \tilde{\delta}(q_i) \prod_{j=1, j\neq i}^{N} G_D(q_i;q_j)$$

Sum of phase-space integrals!

- **Dual integrals** contain propagators with a **modified prescription.**
- **LTD** is equivalent to **Feynman Tree Theorem (FTT)** but only uses **single-cuts** (multiple cuts codified in the dual prescription)

Dual propagator

$$G_D(q_i,q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta(q_j - q_i)}$$

Modified prescription

(η is space- or light-like)

$$\tilde{\delta}(q_i) = i2\pi\,\theta(q_{i,0})\,\delta(q_i^2 - m_i^2)$$

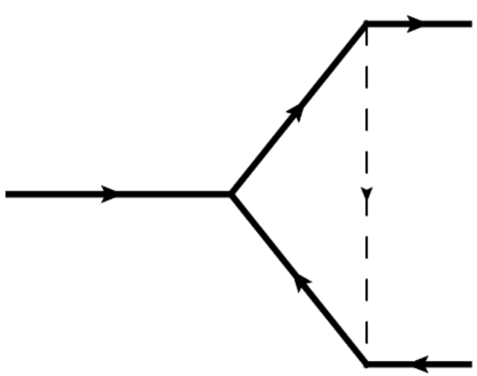
On-shell condition

(loop measure -> PS measure)

Catani et al, JHEP 09 (2008) 065

LTD with massive particles

- **Idea I:** apply LTD directly to **virtual amplitudes** ⇨ **PS integrals**
- **Idea II:** use **dual kinematics** to generate **real-emission on-shell kinematics**
- **Idea III:** write UV counter-terms and perform **integrand-level subtraction.** This will lead to purely **4-dimensional integrable expressions**
- *Reference example:* scalar three-point function with masses
  - Two massive on-shell external particles; one incoming off-shell particle
  - One internal massless state (*gluon, photon,…*)

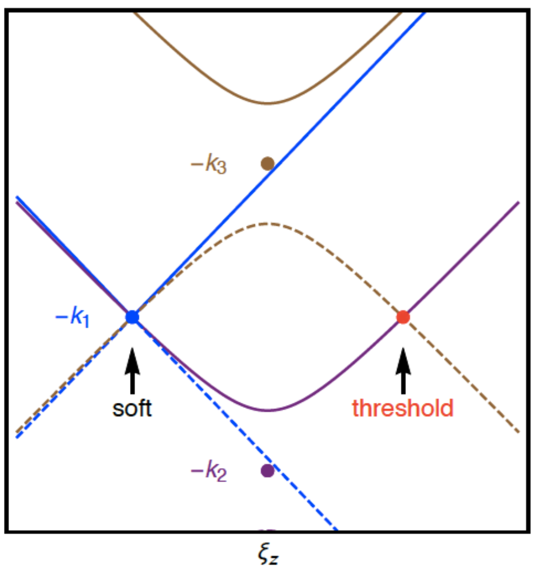


LTD with massive particles

Location of IR singularities

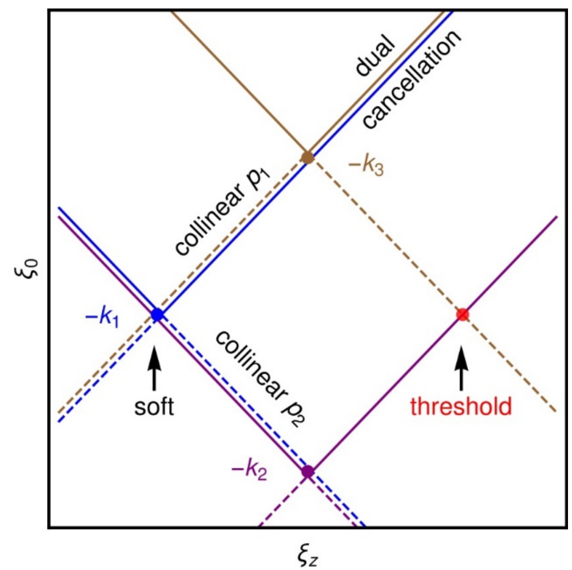
- Analize the dual integration region. It is obtained as the positive energy solution of the on-shell condition;

$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0 \quad \Rightarrow \quad q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$



Massive case: on-shell hyperboloids

- **Forward** (backward) on-shell hyperboloids associated with **positive** (negative) energy mode.
- Degenerate to light-cones for massless propagators.
- *Dual integrands become singular at intersections* (two or more on-shell propagators)



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LTD with massive particles

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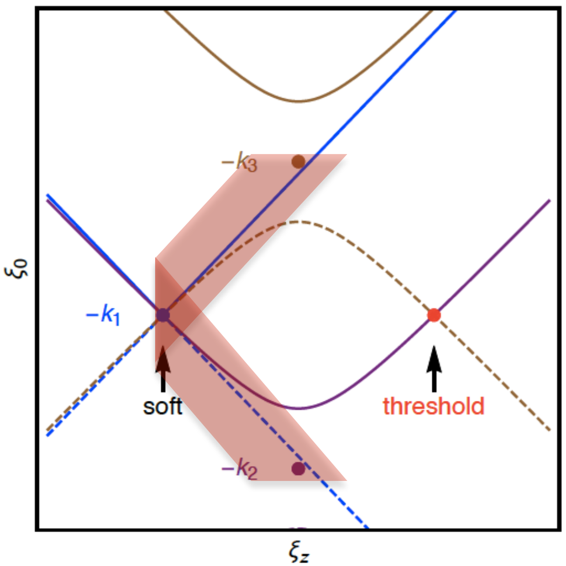

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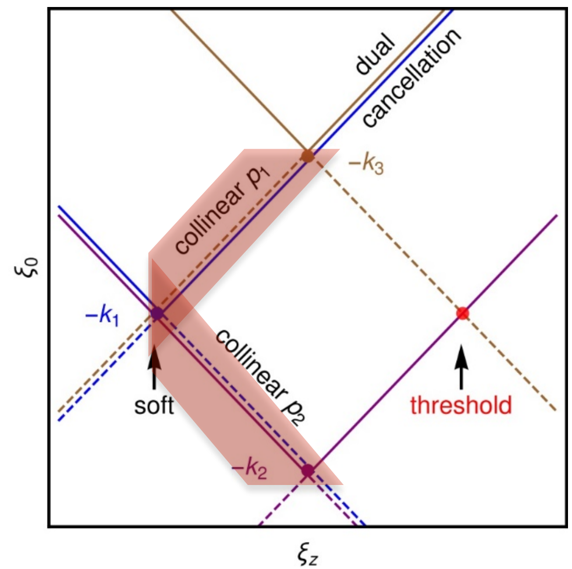
- Degenerate to light-cones for massless propagators.
- *Dual integrands become singular at intersections* (two or more on-shell propagators)
- **Quasi-collinear** configurations lead to **Log(m²)**, which is singular in the massless limit



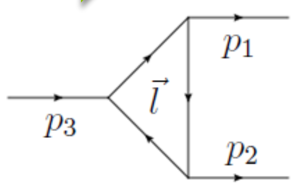
Massless case: light-cones

LTD with massive particles

Real-virtual momentum mapping

- NLO computations require to combine **one-loop** and **real-emission** contributions ➡ **Different kinematics!!!!**

VIRTUAL
(N OS particles + loop momentum)



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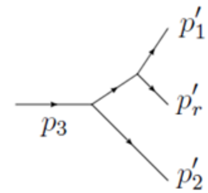


REAL
(N+1 OS particles)

$$p_3 \to p_1 + p_2 + \vec{l} \Rightarrow p_3 \to p'_1 + p'_2 + p'_r$$

- **LTD** express **virtual** amplitudes as **dual** integrals. They depend on **LO kinematics** and the **loop three-momentum** $\vec{l}$ (integration variable)
- **Real contribution** includes **one additional** physical particle in final state. **Split** the phase-space to **isolate IR singularities** (*only one in each region*)

$$\mathcal{R}_i = \{y'_{ir} < \min(y'_{jk})\} , \qquad \sum \mathcal{R}_i = 1$$

- **IDEA:** Use the **loop 3-momentum and N-particle kinematics** to generate **N+1-particle kinematics** ➡ Achieve a **local matching** of singular regions among **real** and **dual** contributions (exploiting the partition)

Rodrigo et al, JHEP02(2016)044; arXiv:1604.06699, 1608.01584 [hep-ph]

LTD with massive particles

Real-virtual momentum mapping

- **Real-virtual momentum mapping with massive particles**
  - Consider **1** the **emitter**, **r** the **radiated particle** and **2** the **spectator**
  - Apply the PS partition and restrict to the only region where **1//r** is allowed (i.e. $\mathcal{R}_1 = \{y'_{1r} < \min y'_{kj}\}$)
  - Propose the following mapping:

$$
\begin{aligned}
p'^{\mu}_r &= q^{\mu}_1 \\
p'^{\mu}_1 &= (1-\alpha_1)\,\hat{p}^{\mu}_1 + (1-\gamma_1)\,\hat{p}^{\mu}_2 - q^{\mu}_1 \\
p'^{\mu}_2 &= \alpha_1\,\hat{p}^{\mu}_1 + \gamma_1\,\hat{p}^{\mu}_2
\end{aligned}
$$

Impose on-shell conditions to determine mapping parameters

  - *Express the loop three-momentum with the same parameterization used for describing the dual contributions!*

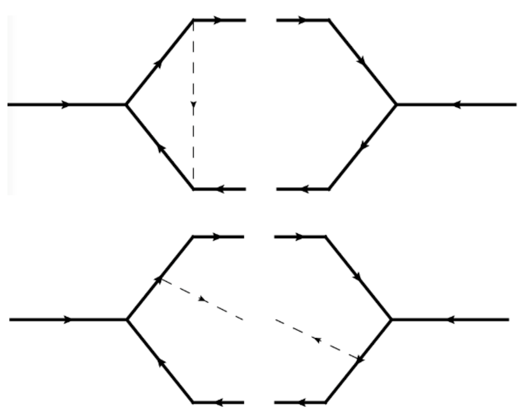
Repeat in each region of the partition…

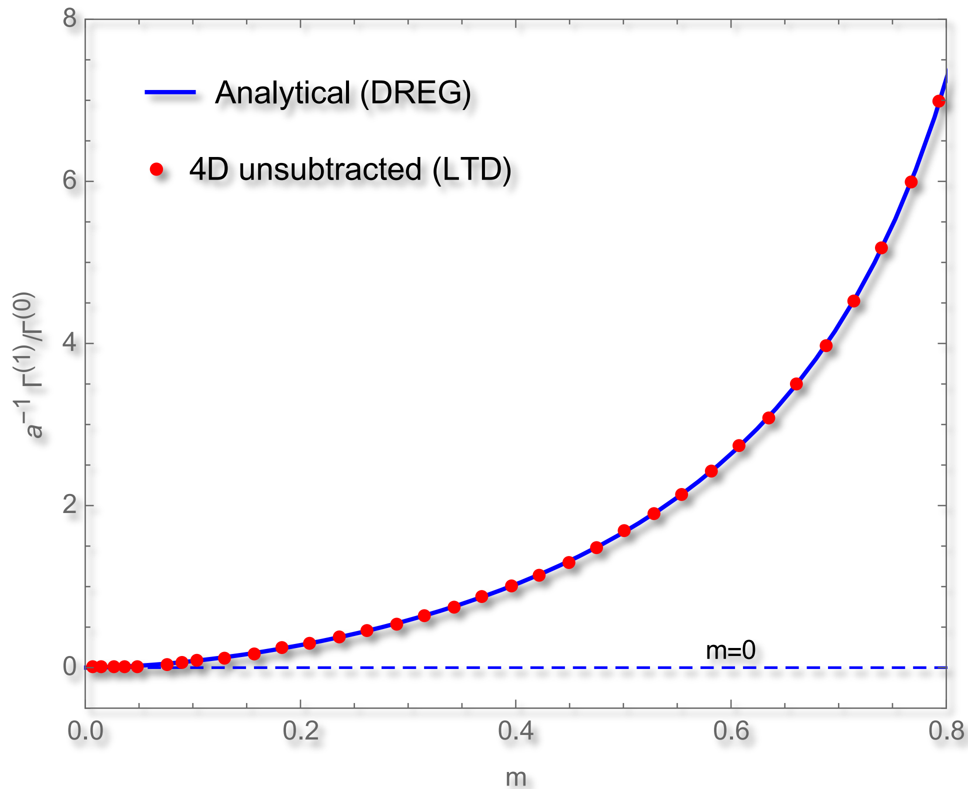
Rodrigo et al, JHEP02(2016)044; arXiv:1604.06699, 1608.01584 [hep-ph]

LTD with massive particles

Example: massive scalar three-point function (DREG vs LTD)

- We combine the dual contributions with the real terms (after applying the proper mapping) to get the total decay rate in the scalar toy-model.
  - The result agrees ***perfectly*** with standard DREG.
  - **Massless limit** is **smoothly** approached due to proper treatment of **quasi-collinear** configurations in the **RV mapping**





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LTD with massive particles

UV counterterms and renormalization

- LTD can also deal with **UV singularities** by building **local** versions of the usual UV counterterms.
- **1: Expand** internal propagators around the *"UV propagator"*

$$
\frac{1}{q_i^2 - m_i^2 + \imath 0} = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} \times \left[1 - \frac{2 q_{\rm UV} \cdot k_{i,{\rm UV}} + k_{i,{\rm UV}}^2 - m_i^2 + \mu_{\rm UV}^2}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} + \frac{(2 q_{\rm UV} \cdot k_{i,{\rm UV}})^2}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0)^2} \right] + \mathcal{O}\left((q_{\rm UV}^2)^{-5/2} \right)
$$

Becker, Reuschle, Weinzierl, JHEP 12 (2010) 013

- **2:** Apply LTD to get the **dual representation** for the expanded UV expression, and **subtract** it from the **dual+real** combined integrand.

$$
I_{\rm UV}^{\rm cnt} = \int_\ell \frac{\tilde\delta(q_{\rm UV})}{2 \left(q_{{\rm UV},0}^{(+)} \right)^2}
$$

$$
q_{{\rm UV},0}^{(+)} = \sqrt{\mathbf{q}_{\rm UV}^2 + \mu_{\rm UV}^2 - i0}
$$

LTD extended to deal with multiple poles (use residue formula to obtain the dual representation)

Bierenbaum *et al.* JHEP 03 (2013) 025

LTD with massive particles

UV counterterms and renormalization

- Requires **unintegrated** wave-function, mass and vertex renormalization constants
- Self-energy corrections with **on-shell renormalization** conditions

$$\Sigma_R(\not{p}_1 = M) = 0 \qquad \left. \frac{d\Sigma_R(\not{p}_1)}{d\not{p}_1} \right|_{\not{p}_1 = M} = 0$$

- Wave function renormalization constant, **both IR and UV poles**

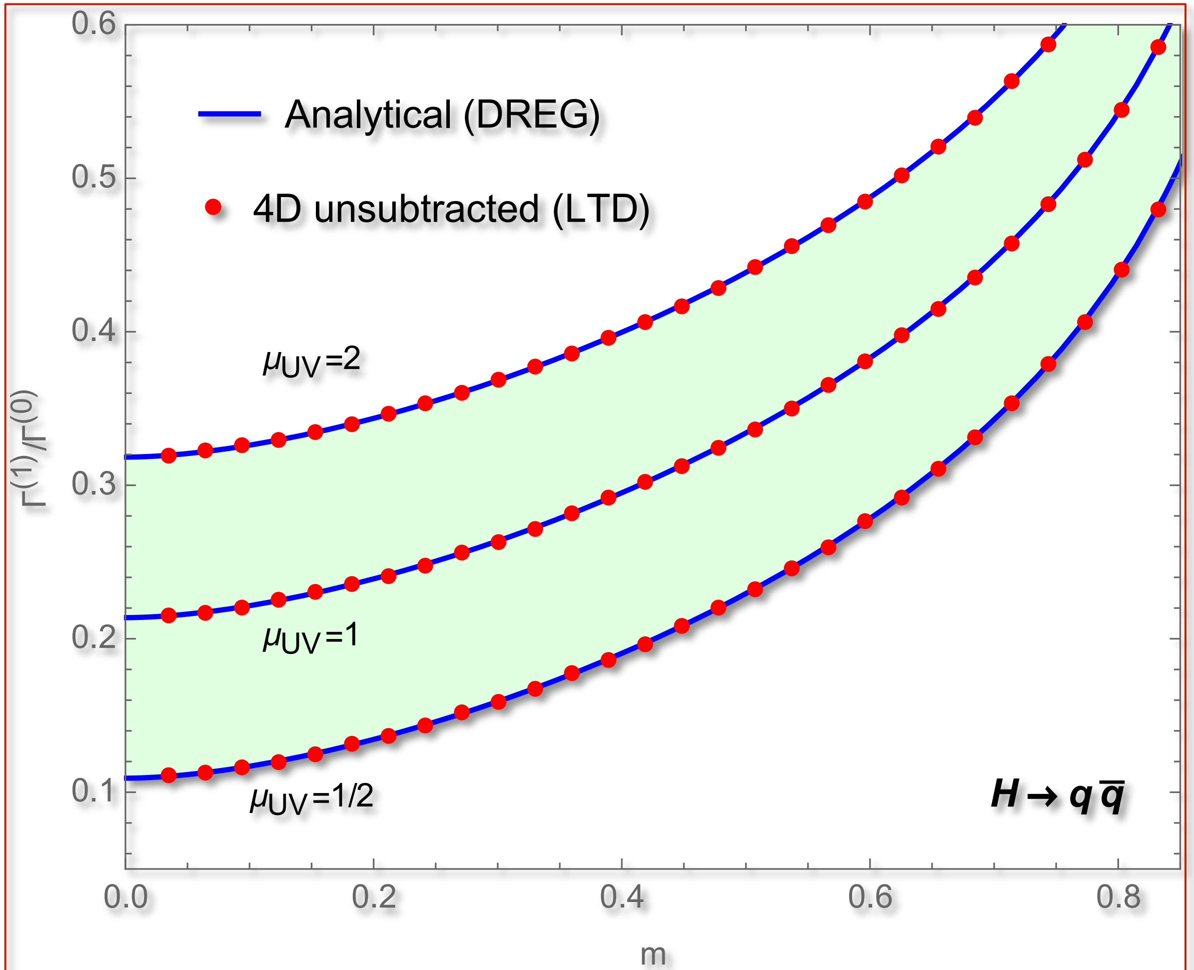
$$\Delta Z_2(p_1) \;=\; -g_{\rm S}^2 C_F \int_\ell G_F(q_1) G_F(q_3) \left((d-2) \frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 4M^2 \left(1 - \frac{q_1 \cdot p_2}{p_1 \cdot p_2} \right) G_F(q_3) \right)$$

- ***Remove UV poles*** *by expanding around the UV-propagator (same for the* ***vertex counterterm****)*
- Integrated form of local counterterms agrees with standard UV counterterms

Rodrigo et al, JHEP02(2016)044; arXiv:1604.06699, 1608.01584 [hep-ph]

Physical example: $A^* \to q\bar{q}(g)$@NLO

Results and comparison with DREG

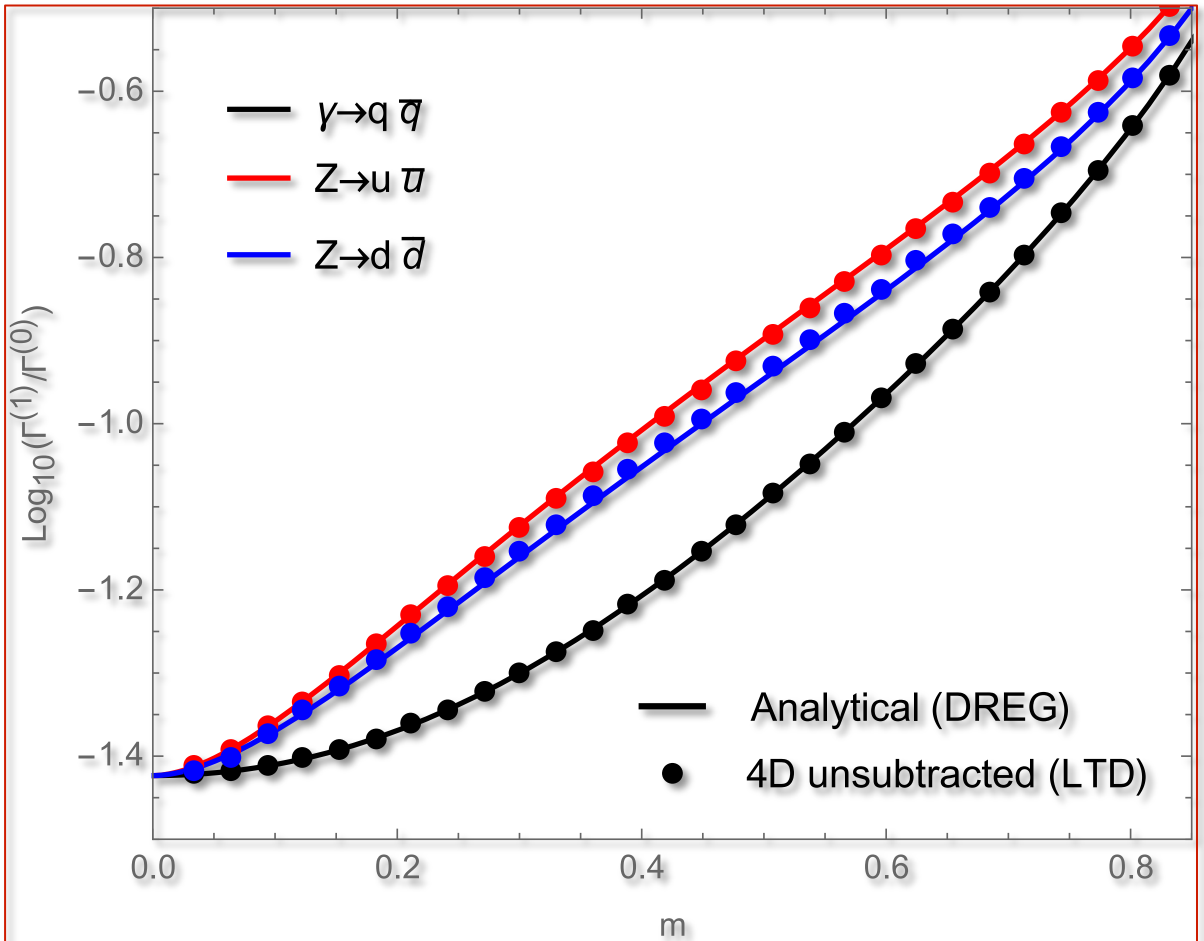


- *Total decay rate for Higgs into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Local version of UV counterterms succesfully reproduces the expected behaviour
  - Efficient numerical implementation

Rodrigo et al, arXiv:1608.01584 [hep-ph]

Physical example: $A^* \to q\bar{q}(g)$@NLO

Results and comparison with DREG



- *Total decay rate for a vector particle into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Efficient numerical implementation

Rodrigo et al, arXiv:1608.01584 [hep-ph]

Physical example: $A^* \to q\bar{q}(g)$@NLO

- The total decay-rate can be expressed using purely **four-dimensional integrands**
- We recover the total NLO correction, **avoiding to deal with DREG**

- **Main advantages:**
  - ✓ Direct **numerical** implementation (integrable functions for ε **=0**)
  - ✓ No need of tensor reduction (**avoids the presence of Gram determinants**, which could introduce numerical instabilities)
  - ✓ **Smooth transition** to the massless limit (due to the efficient treatment of **quasi-collinear** configurations)

Rodrigo et al, arXiv:1608.01584 [hep-ph]

Conclusions and perspectives

- ✓ Physical interpretation of **IR/UV singularities** in loop integrals (intersections of on-shell hyperboloids)
- ✓ **Integrand-level renormalization** (fully local cancellation of singularities)
- ✓ **Combined virtual-real terms are integrable in 4D**
- ✓ **Smooth transition to the massless limit**
- ✓ **First (realistic) physical implementation**
- **Perspectives:**
  - Apply the technique to compute other physical observables (including heavy particles and multi-leg processes)
  - Extend the procedure to higher orders!!!

Thanks!!!