TOWARDS REGULARIZED H.O. COMPUTATIONS IN QFT WITHOUT DREG

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JHEP02(2016)044, arXiv:1604.06699, 1608.01584 [hep-ph]

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  - Motivation and dual representation
  - Threshold and IR singularities
- Real-virtual momentum mapping
- UV and renormalization in LTD
- Physical example: $\gamma^* \to q\bar{q}(g)$ @NLO
- Conclusions

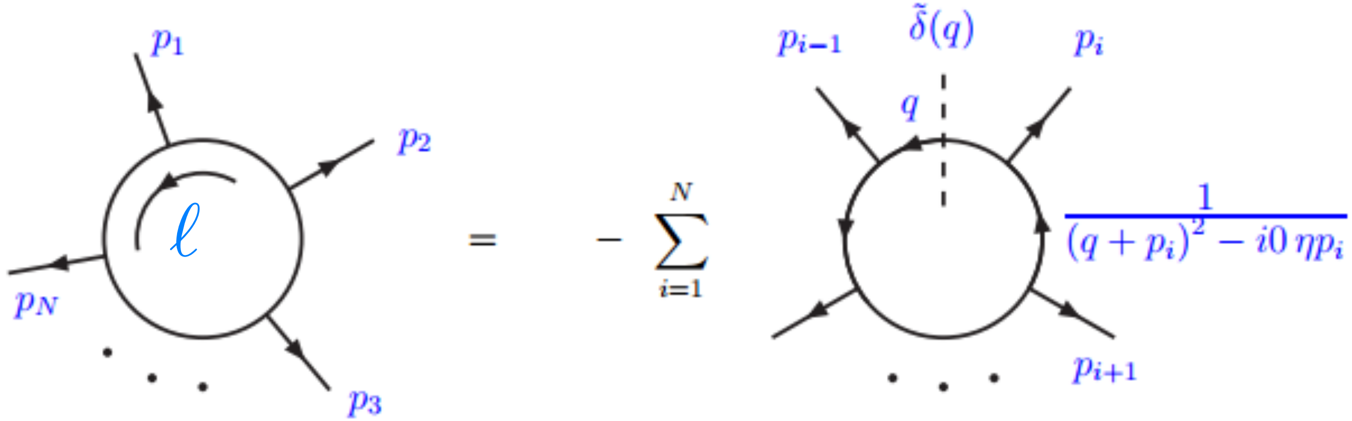
Based in:

- Catani et al, JHEP 09 (2008) 065; Buchta et al, JHEP 11 (2014) 014
- Hernández-Pinto, GS and Rodrigo, JHEP 02 (2016) 044
- GS, Driencourt-Mangin, Hernández-Pinto and Rodrigo, arXiv:1604.06699 [hep-ph]
- Rodrigo, Driencourt-Mangin and GS, arXiv:1608.01584 [hep-ph]

Loop-tree duality (LTD)

Motivation

- KLN theorem suggests that **virtual and real** contributions have the **same IR divergent structure** (because they cancel in IR-safe observables)
- **Cut contributions** are similar to **tree-level** scattering amplitudes, if all the loops are cut. At one-loop, **1-cuts are tree-level objects** (higher-cuts are products of unconnected graphs)
- **MOTIVATION:** Combine real and virtual contributions at **integrand level** and perform the **computation in 4-dimensions** (take $\epsilon \to 0$)
- We will use **Loop-Tree Duality** to express **loop integrals** in terms of dual contributions, which resemble **phase-space integrals**



Catani et al, JHEP 09 (2008) 065

Loop-tree duality (LTD)

Dual representation of one-loop integrals

Loop Feynman integral

$$L^{(1)}(p_1,\ldots,p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i) = \int_\ell \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + i0}$$

Dual integral

$$L^{(1)}(p_1,\ldots,p_N) = -\sum_{i=1}^{N} \int_\ell \tilde{\delta}(q_i) \prod_{j=1, j\neq i}^{N} G_D(q_i; q_j)$$

Sum of phase-space integrals!

- **Dual integrals** contain propagators with a **modified prescription**
- **LTD** is equivalent to **Feynman Tree Theorem (FTT)** but only uses **single-cuts** (multiple cuts codified in the dual prescription)

Dual propagator

$$G_D(q_i, q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta(q_j - q_i)}$$

Modified prescription

(η is space- or light-like)

$$\tilde{\delta}(q_i) = i2\pi\,\theta(q_{i,0})\,\delta(q_i^2 - m_i^2)$$

On-shell condition

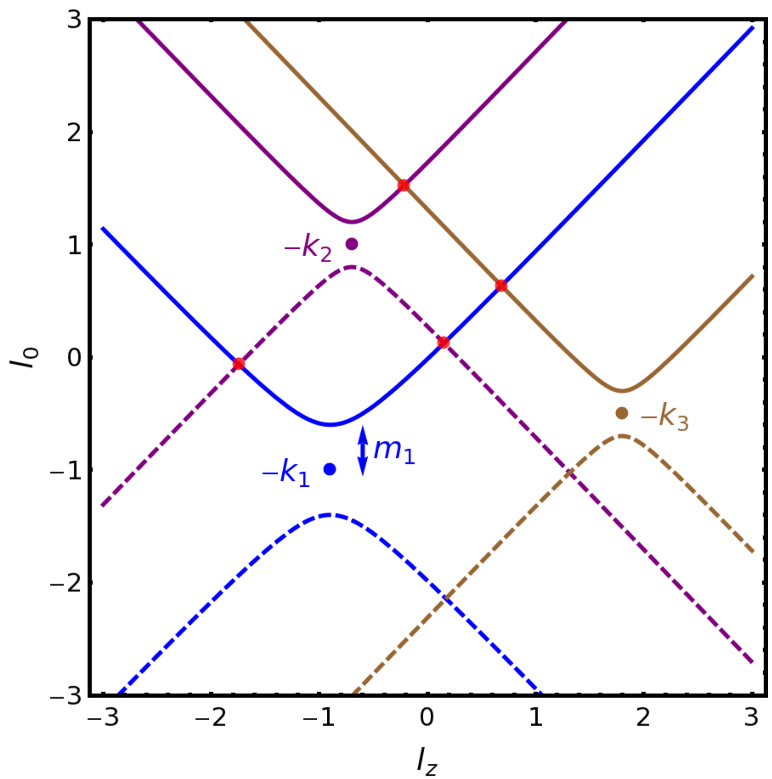
(loop measure -> PS measure)

Catani et al, JHEP 09 (2008) 065

Loop-tree duality (LTD)

Threshold and IR singularities

- Feynman integrands develop singularities when propagators go on-shell. LTD allows to understand it as soft/collinear divergences of real radiation



Massive case: on-shell hyperboloids

- Forward (backward) on-shell hyperboloids associated with positive (negative) energy solutions

$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0$$

$$q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$

- LTD is equivalent to integrate along the **forward on-shell hyperboloids**

DISCUSSED IN A PREVIOUS TALK!!!
More details available in
arXiv:1608.01584[hep-ph]

Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, arXiv:1604.06699 [hep-ph]

Loop-tree duality (LTD)

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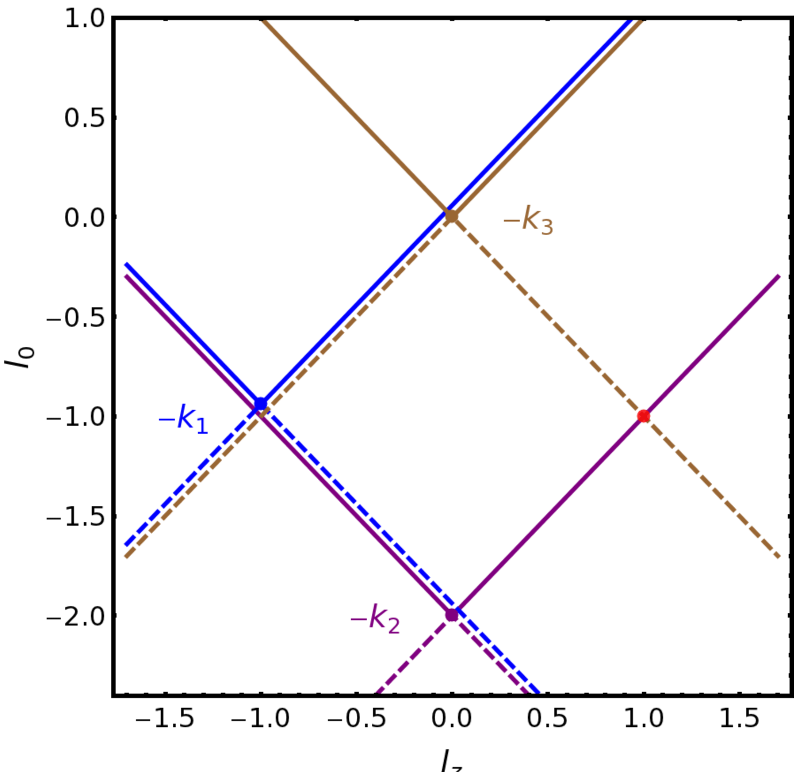


Massless case: light-cones

- Forward (backward) on-shell hyperboloids associated with positive (negative) energy solutions

$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0$$

$$q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$

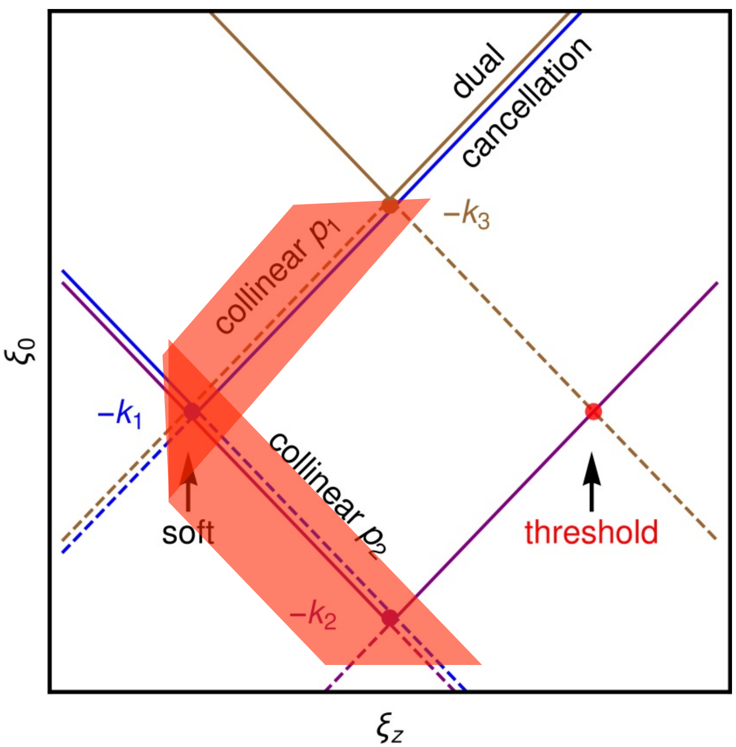
- LTD is equivalent to integrate along the forward on-shell hyperboloids
- Degenerate to light-cones for massless propagators

Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, arXiv:1604.06699 [hep-ph]

Loop-tree duality (LTD)

Threshold and IR singularities

- Feynman integrands develop singularities when propagators go on-shell. LTD allows to understand it as soft/collinear divergences of real radiation.



- Forward (backward) on-shell hyperboloids associated with positive (negative) energy solutions.

$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0$$

$$q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$

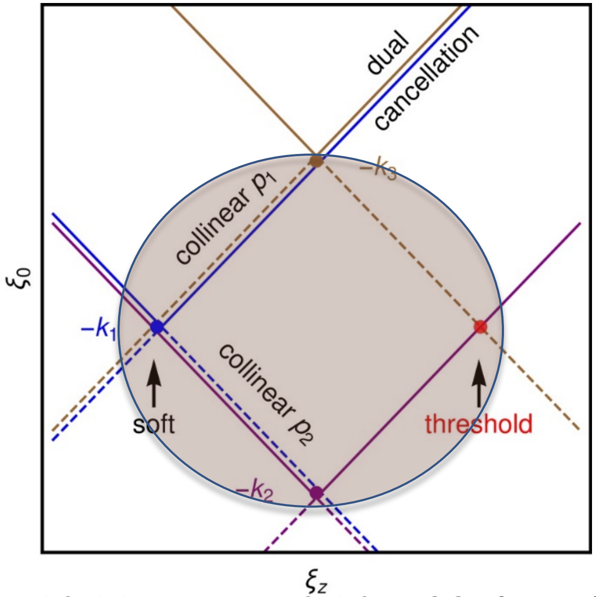
- LTD is equivalent to integrate along the forward on-shell hyperboloids.
- Degenerate to light-cones for massless propagators.
- Dual integrands become singular at intersections (two or more on-shell propagators)

Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, arXiv:1604.06699 [hep-ph]

Loop-tree duality (LTD)

Threshold and IR singularities: Remarks

- **Forward on-shell hyperboloids** (light-cones) corresponds to the **integration domain** of dual contributions
- **FB intersections** associated with **IR/Threshold singularities** (i.e. **soft/ collinear poles** in DREG)
- **FF** intersections compensate among dual contributions (**cancelled in the sum**).
- *Threshold singularities are integrable* (but numerically unstable…)
- IR singularities should **cancel** with the **real radiation** contribution (**KLN theorem**).



- **Main property of LTD: IR singularities** of the virtual amplitudes are located in a **compact region of the loop three-momentum**

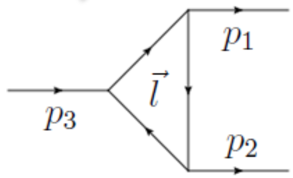
Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, arXiv:1604.06699 [hep-ph]

Real-virtual momentum mapping

Basic idea and formulae

- NLO computations require to combine **one-loop** and **real-emission** contributions ➡ **Different kinematics!!!!**

VIRTUAL
(N OS particles + loop momentum)



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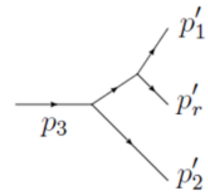


REAL
(N+1 OS particles)

$$p_3 \to p_1 + p_2 + \vec{l} \Rightarrow p_3 \to p'_1 + p'_2 + p'_r$$

- **LTD** express **virtual** amplitudes as **dual** integrals. They depend on **LO kinematics** and the **loop three-momentum** $\vec{l}$ (integration variable)
- **Real contribution** includes **one additional** physical particle in final state. **Split** the phase-space to **isolate IR singularities** (*only one in each region*)

$$\mathcal{R}_i = \{y'_{ir} < \min(y'_{jk})\}, \qquad \sum \mathcal{R}_i = 1$$

- **IDEA:** Use the **loop 3-momentum and N-particle kinematics** to generate **N+1-particle kinematics**! ➡ Achieve a **local matching** of singular regions among **real** and **dual** contributions (exploiting the partition)

Rodrigo et al, JHEP02(2016)044; arXiv:1604.06699, 1608.01584 [hep-ph]

Real-virtual momentum mapping

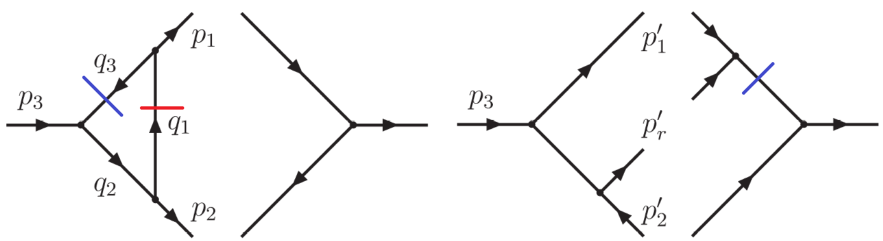
Basic idea and formulae

Massive case: See Heavy Quark's talk!

- **Real-virtual momentum mapping (massless particles)**
  - Consider **1** the **emitter**, **r** the **radiated particle** and **2** the **spectator**
  - Apply the PS partition and restrict to the only region where **1//r** is allowed (i.e. $\mathcal{R}_1 = \{y'_{1r} < \min y'_{kj}\}$)
  - Propose the following mapping:

$$
\boxed{
\begin{aligned}
p'^{\mu}_r &= q^{\mu}_1 \,, & p'^{\mu}_1 &= p^{\mu}_1 - q^{\mu}_1 + \alpha_1\, p^{\mu}_2 \,, \\
p'^{\mu}_2 &= (1-\alpha_1)\, p^{\mu}_2 \,, & \alpha_1 &= \frac{q_3^2}{2q_3 \cdot p_2} \,,
\end{aligned}
}
$$

Dual-real correspondence



  - *Express the loop three-momentum with the same parameterization used for describing the dual contributions*

Repeat in each region of the partition…

Rodrigo et al, JHEP02(2016)044; arXiv:1604.06699, 1608.01584 [hep-ph]

Real-virtual momentum mapping

Real-virtual combination and local regularization

- Once **dual** and **real** contributions are expressed with the **same variables**, ***add them at integrand level***. The integrand is **free of singularities**, and, thus, **integrable in four-dimensions**

- ***Technical points:***
  - Compare the **topologies** of dual (**cut**) contributions and **real diagrams**; group those that have the same behaviour in the **IR limits** (*use the LC plots to make a proper assignment*)
  - Express all the dual momenta $\mathbf{q_i}$ in the **same coordinate system.** Then, apply the corresponding change of variables in each partition.

Rodrigo et al, JHEP02(2016)044; arXiv:1604.06699, 1608.01584 [hep-ph]

UV and renormalization in LTD

General ideas

- LTD can also deal with **UV singularities** by building **local** versions of the usual UV counterterms.
- **1: Expand** internal propagators around the *"UV propagator"*

$$\frac{1}{q_i^2 - m_i^2 + \imath 0} = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} \times \left[1 - \frac{2q_{\rm UV}\cdot k_{i,\rm UV} + k_{i,\rm UV}^2 - m_i^2 + \mu_{\rm UV}^2}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} + \frac{(2q_{\rm UV}\cdot k_{i,\rm UV})^2}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0)^2}\right] + \mathcal{O}\left((q_{\rm UV}^2)^{-5/2}\right)$$

Becker, Reuschle, Weinzierl, JHEP 12 (2010) 013

- **2:** Apply LTD to get the **dual representation** for the expanded UV expression, and **subtract** it from the **dual+real** combined integrand.

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{\tilde{\delta}(q_{\rm UV})}{2\left(q_{{\rm UV},0}^{(+)}\right)^2}$$

$$q_{{\rm UV},0}^{(+)} = \sqrt{\mathbf{q}_{\rm UV}^2 + \mu_{\rm UV}^2 - \imath 0}$$

LTD extended to deal with multiple poles
(use residue formula to obtain the dual representation)

Bierenbaum *et al.* JHEP 03 (2013) 025

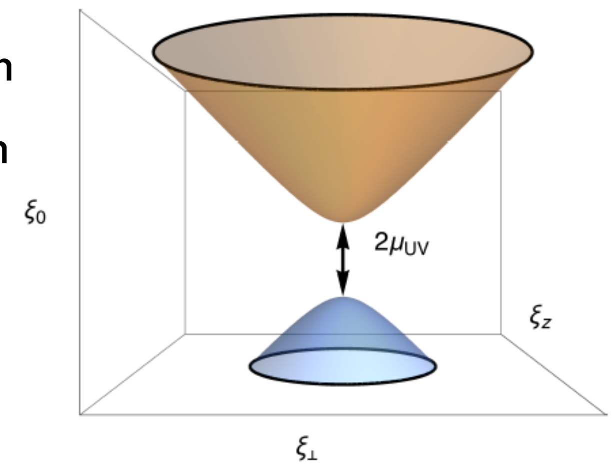
Rodrigo et al, JHEP02(2016)044; arXiv:1604.06699, 1608.01584 [hep-ph]

UV and renormalization in LTD

Some important remarks

- Divergences arise from the high-energy region (UV poles), which implies loop integration for loop energies larger than μ_{UV}: the proposed counter-terms do not alter the IR cancellations!
- A local subtraction takes place, thus allowing to take the limit $\epsilon \to 0$ at integrand level!
- **Self-energies and vertex corrections** must be taken into account even if they vanish in DREG (IR/UV non trivial behaviour)
- Finite pieces are fixed by subleading terms in the UV limit. It allows to **fix** the renormalization **scheme** (defined at ***integral level***)

More details about renormalization in the Heavy Quark's talk



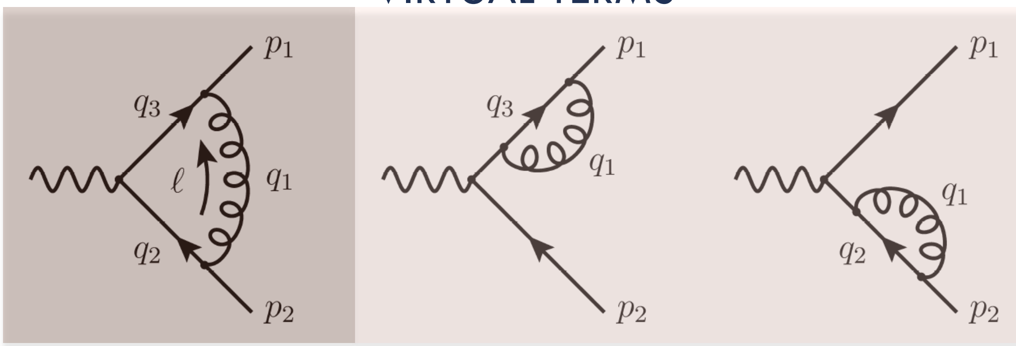
On-shell hyperboloids for UV counter-terms

Rodrigo et al, JHEP02(2016)044; arXiv:1604.06699, 1608.01584 [hep-ph]

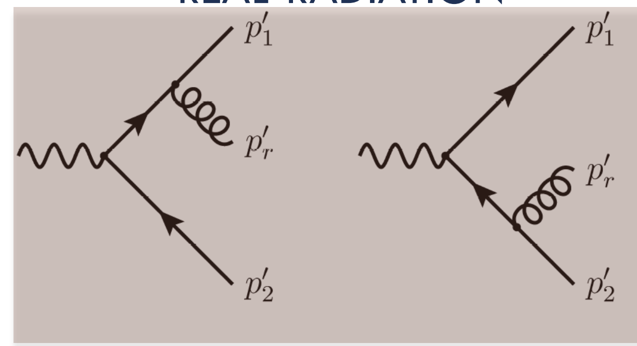
Physical example: $\gamma^* \to q\bar{q}(g)$ @NLO

Setup and dual contributions

VIRTUAL TERMS



REAL-RADIATION



- LTD representation of the vertex corrections:

$$
\begin{aligned}
\langle \mathcal{M}^{(0)}_{q\bar{q}} | \mathcal{M}^{(1)}_{q\bar{q},1} \rangle &= -2\, g_S^2\, C_F\, |\mathcal{M}^{(0)}_{q\bar{q}}|^2 \int d[\xi_{1,0}]\, d[v_1]\, \left(\xi_{1,0}^{-1}\, v_1^{-1} + 1\right) \left((1-v_1)^{-1} - \xi_{1,0}\right) \,, \\
\langle \mathcal{M}^{(0)}_{q\bar{q}} | \mathcal{M}^{(1)}_{q\bar{q},2} \rangle &= -2\, g_S^2\, C_F\, |\mathcal{M}^{(0)}_{q\bar{q}}|^2 \int d[\xi_{2,0}]\, d[v_2]\, \frac{\xi_{2,0}}{1-\xi_{2,0}+\imath 0} \left(v_2 \left((1-v_2)^{-1} - \xi_{2,0}\right) - \epsilon\right) \\
\langle \mathcal{M}^{(0)}_{q\bar{q}} | \mathcal{M}^{(1)}_{q\bar{q},3} \rangle &= -2\, g_S^2\, C_F\, |\mathcal{M}^{(0)}_{q\bar{q}}|^2 \int d[\xi_{3,0}]\, d[v_3]\, \frac{\xi_{3,0}}{1+\xi_{3,0}} \left((1-v_3)\left(v_3^{-1} + \xi_{3,0}\right) - \epsilon\right) \,,
\end{aligned}
$$

Rodrigo et al, JHEP02(2016)044; arXiv:1601.04634, 1604.06699 [hep-ph]

Physical example: $\gamma^* \to q\bar{q}(g)$ @NLO

Self-energies and other subtleties

- Self-energy corrections are vanishing at integral level, but they ARE crucial to cancel both IR and UV singularities. They contribute to the wave function corrections (and are usually neglected for massless particles)

Self-energy contribution

$$\begin{aligned}\langle \mathcal{M}^{(0)}_{q\bar{q}} | \Sigma(p_1) \rangle &= -2(1-\epsilon)\, g_S^2\, C_F\, |\mathcal{M}^{(0)}_{q\bar{q}}|^2 \left[\int d[\xi_{1,0}]\, d[v_1]\, v_1^{-1}\, (1-v_1)\, \xi_{1,0} \right. \\ &\left. - \int d[\xi_{3,0}]\, d[v_3]\, v_3^{-1}\, (1+(1-v_3)\, \xi_{3,0}) \right],\end{aligned}$$

Self-energy UV contribution

$$\begin{aligned}\langle \mathcal{M}^{(0)}_{q\bar{q}} | \Sigma_{\rm UV}(p_1) \rangle &= -2(1-\epsilon)\, g_S^2\, C_F\, |\mathcal{M}^{(0)}_{q\bar{q}}|^2 \int d[\xi_{\rm UV}]\, d[v_{\rm UV}]\, \frac{\xi_{\rm UV}^2}{(\xi_{\rm UV}^2 + m_{\rm UV}^2)^{3/2}} \\ &\times \left[\left(2 + \xi_{\rm UV}\, (1-2v_{\rm UV})\right) \left(1 - \frac{3\, (2\,\xi_{\rm UV}\, (1-2v_{\rm UV}) - m_{\rm UV}^2)}{4\, (\xi_{\rm UV}^2 + m_{\rm UV}^2)}\right) - \frac{1}{2}\right]\end{aligned}$$

- It cancels the IR singularities of the squared real amplitudes, whilst the UV contribution is subtracted

$$\sigma_V^{(1)} = \frac{1}{2s_{12}} \int d\Phi_{1\to 2} \left[2{\rm Re}\langle \mathcal{M}^{(0)} | \overline{\mathcal{M}}^{(1)} \rangle - (\Delta Z_2(p_1) + \Delta Z_2(p_2))\, |\mathcal{M}^{(0)}|^2 \right]$$

Complete *renormalized* virtual contribution

Rodrigo et al, JHEP02(2016)044; arXiv:1601.04634, 1604.06699 [hep-ph]

Physical example: $\gamma^* \to q\bar{q}(g)$ @NLO

 Self-energies and other subtleties

- Each component can be expressed using purely **four-dimensional integrands**

$$\bar{\sigma}_{\mathrm{V}}^{(1)} = \left(\sum_{i=1,2} \bar{\sigma}_{\mathrm{V},i}^{(1)}\right) + \sigma_{\mathrm{V},3}^{(1)} - \sigma_{\mathrm{V}}^{(1,\mathrm{UV})}$$ Renormalized virtual remnant

$$\widetilde{\sigma}_i^{(1)} = \widetilde{\sigma}_{\mathrm{V},i}^{(1)} + \widetilde{\sigma}_{\mathrm{R},i}^{(1)}$$ Real+dual IR-regulated terms

$$\begin{aligned}
\widetilde{\sigma}_1^{(1)} &= \sigma^{(0)} \frac{\alpha_{\mathrm{S}}}{4\pi} C_F \left(19 - 32\log(2)\right) , \\
\widetilde{\sigma}_2^{(1)} &= \sigma^{(0)} \frac{\alpha_{\mathrm{S}}}{4\pi} C_F \left(-\frac{11}{2} + 8\log(2) - \frac{\pi^2}{3}\right) , \\
\bar{\sigma}_{\mathrm{V}}^{(1)} &= \sigma^{(0)} \frac{\alpha_{\mathrm{S}}}{4\pi} C_F \left(-\frac{21}{2} + 24\log(2) + \frac{\pi^2}{3}\right) ,
\end{aligned}$$

$$\Rightarrow \quad \widetilde{\sigma}_1^{(1)} + \widetilde{\sigma}_2^{(1)} + \bar{\sigma}_{\mathrm{V}}^{(1)} = \sigma^{(0)}\, 3\, C_F \frac{\alpha_{\mathrm{S}}}{4\pi}$$

- We recover the total NLO correction, **avoiding to deal with DREG**
- **Main advantages:**
  - ✓ Direct **numerical** implementation (integrable functions for $\varepsilon = 0$)
  - ✓ No need of tensorial reduction (**avoids the presence of Gram determinants**, which could introduce numerical instabilities)

Rodrigo et al, JHEP02(2016)044; arXiv:1601.04634, 1604.06699 [hep-ph]

Conclusions and perspectives

- ✓ Loop-tree duality allows to treat virtual and real contributions in the same way (simplification of implementation)
- ✓ Physical interpretation of **IR/UV singularities** in loop integrals (light-cone diagrams)
- ✓ Expression of *loop integrals* as *phase-space integrals*
- ✓ **Combined virtual-real terms are integrable in 4D**

- **Perspectives:**
  - Apply the technique to compute other physical observables (including heavy particles and multi-leg processes)
  - Extend the procedure to higher orders

Thanks!!!

$\gamma^* \to q\bar{q}(g)$ @NLO: 4D formulae

- Integration regions:

$$\mathcal{R}_1(\xi_0, v) = \theta(1-2v_1)\,\theta\left(\frac{1-2v_1}{1-v_1} - \xi_{1,0}\right)\Big|_{\{\xi_{1,0},v_1\}\to\{\xi_{3,0},v_3\}=\{\xi_0,v\}}$$

$$\mathcal{R}_2(\xi_0, v) = \theta\left(\frac{1}{1+\sqrt{1-v}} - \xi_0\right)$$

- Four-dimensional cross-sections:

$$\widetilde{\sigma}_1^{(1)} = \sigma^{(0)}\,\frac{\alpha_S}{4\pi}\,C_F \int_0^1 d\xi_{1,0} \int_0^{1/2} dv_1\, 4\,\mathcal{R}_1(\xi_{1,0}, v_1) \left[2\left(\xi_{1,0} - (1-v_1)^{-1}\right) - \frac{\xi_{1,0}(1-\xi_{1,0})}{\left(1-(1-v_1)\,\xi_{1,0}\right)^2}\right]$$

$$\widetilde{\sigma}_2^{(1)} = \sigma^{(0)}\,\frac{\alpha_S}{4\pi}\,C_F \int_0^1 d\xi_{2,0} \int_0^1 dv_2\, 2\,\mathcal{R}_2(\xi_{2,0}, v_2)\,(1-v_2)^{-1} \left[\frac{2\,v_2\,\xi_{2,0}\left(\xi_{2,0}(1-v_2)-1\right)}{1-\xi_{2,0}}\right]$$

$$\begin{aligned}
\overline{\sigma}_V^{(1)} = {} & \sigma^{(0)}\,\frac{\alpha_S}{4\pi}\,C_F \int_0^\infty d\xi \int_0^1 dv \left\{ -2\,\left(1-\mathcal{R}_1(\xi,v)\right)\,v^{-1}(1-v)^{-1}\,\frac{\xi^2(1-2v)^2+1}{\sqrt{(1+\xi)^2-4v\,\xi}} \right. \\
& + 2\,\left(1-\mathcal{R}_2(\xi,v)\right)\,(1-v)^{-1}\left[2\,v\,\xi\,\left(\xi(1-v)-1\right)\left(\frac{1}{1-\xi+\imath 0} + \imath\pi\delta(1-\xi)\right) - 1 + v\,\xi\right] \\
& + 2\,v^{-1}\left(\frac{\xi(1-v)(\xi(1-2v)-1)}{1+\xi} + 1\right) - \frac{(1-2v)\,\xi^3\,(12-7m_{\rm UV}^2-4\xi^2)}{(\xi^2+m_{\rm UV}^2)^{5/2}} \\
& \left. - \frac{2\,\xi^2(m_{\rm UV}^2+4\xi^2(1-6v(1-v)))}{(\xi^2+m_{\rm UV}^2)^{5/2}} \right\}
\end{aligned}$$