Gauge Coupling Unification in Radiative Neutrino Mass Models

Stella Riad,
KTH Royal Institute of Technology
ICHEP, Chicago, August 5, 2016

In collaboration with: C. Hagedorn, T. Ohlsson, M. Schmidt
Outline

• Motivation
• RG running in radiative neutrino mass models
  1. Minimal UV completion models with $d=7$
  2. Models with DM
  3. Models with adjoint representation of SU(3)
• Summary and conclusions
Motivation

Address two of the problems with the SM:
• Neutrino mass
• Grand unification

Bonus: dark matter

Idea: RG running in radiative neutrino mass models. Can there be unification?
Choice of models

Three classes of models:
I. Minimal UV completions of dimension-7 operators + DM
II. Models with dark matter + DM
III. Models with particles in the adjoint representations of SU(3)

Assumptions:
• New particle masses: 1 TeV
• Only gauge coupling running
• Only top yukawa
• Non-perturbative models

Error margin on unification
Running in radiative models

\[
\frac{dg_i}{dt} = \beta_i(g) = b_i \frac{g_i^3}{16\pi^2} + \sum_k \left( b_{ik} \frac{g_k^2 g_i^3}{(4\pi)^4} + \frac{g_i^3}{(4\pi)^4} \text{Tr}(C_k^{u} Y_u^\dagger Y_u + C_k^{d} Y_d^\dagger Y_d + C_k^{e} Y_e^\dagger Y_e) \right)
\]

[Mačacek, Vaughn, 1983]

Determine contribution to gauge coupling running and solve RGE analytically (one-loop) and numerically using PyR@TE (two-loop) [Lyonnet, Schienbein, Staub, Wingerter, 2014]

**Model categories:**

1. Those unifying
2. Those which doesn’t unify but also doesn’t diverge
3. Those which doesn’t unify and contain a Landau pole
I. Minimal UV completions of d=7 operators

[Cai, Clarke, Schmidt, Volkas, 2015]

- 15 models with d=7 operators
  - $\Delta L = 2$
  - Three topologies
- Two new particles. Two scalars or one scalar + one fermion
- We assume 1 to 6 generations of each new particle.

Models SI-J

<table>
<thead>
<tr>
<th>Scalar</th>
<th>Scalar</th>
<th>Operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1, 2, \frac{1}{2})$</td>
<td>$(1, 1, 1)$</td>
<td>$\mathcal{O}_{2,3,4}$ [28]</td>
</tr>
<tr>
<td>$(3, 2, \frac{1}{6})$</td>
<td>$(3, 1, -\frac{1}{3})$</td>
<td>$\mathcal{O}_{3,8}$ [38, 44]</td>
</tr>
<tr>
<td>$(3, 2, \frac{1}{6})$</td>
<td>$(3, 3, -\frac{1}{3})$</td>
<td>$\mathcal{O}_3$</td>
</tr>
</tbody>
</table>

$$
\mathcal{O}_2 = LLL\bar{e}H, \quad \mathcal{O}_3 = LLQ\bar{d}H,
\mathcal{O}_4 = LLQ^\dagger \bar{u}^\dagger H, \quad \mathcal{O}_8 = L\bar{d}\bar{e}^\dagger \bar{u}^\dagger H,
$$
I. Minimal UV completions of d=7 operators

Analytical study

\[ n_1 = \frac{2\pi}{L} \frac{B_{23}^2 \alpha_{1,SM}^{-1}(\Lambda) + B_{31}^2 \alpha_{2,SM}^{-1}(\Lambda) + B_{12}^2 \alpha_{3,SM}^{-1}(\Lambda)}{B_{23}^1 B_{31}^2 - B_{23}^2 B_{31}^1} \],

\[ n_2 = \frac{2\pi}{L} \frac{B_{23}^1 \alpha_{1,SM}^{-1}(\Lambda) + B_{31}^1 \alpha_{2,SM}^{-1}(\Lambda) + B_{12}^1 \alpha_{3,SM}^{-1}(\Lambda)}{B_{23}^2 B_{31}^1 - B_{23}^1 B_{31}^2} \].

\[ B_{kl}^i = b_k^i - b_l^i \quad , \quad L = \ln \left( \frac{\Lambda}{\Lambda_{NP}} \right) \]

- Two new particles in \( 1 \leq n_1 \leq 6 \) and \( 1 \leq n_2 \leq 6 \) generations
- Models unify if range in \( \Lambda \) overlap

\[ \rightarrow \text{S1-2, S2-4, S2-6 and S2-11.} \]
I. Minimal UV completions of d=7 operators

Numerical analysis

- Unification in 4 models
- Unification scale $10^{14}$-$10^{16}$ GeV
- One "basic" model

<table>
<thead>
<tr>
<th>Model</th>
<th>P1</th>
<th>P2</th>
<th>$\Lambda$ (GeV)</th>
<th>$\alpha^{-1}(\Lambda)$</th>
<th>$\frac{\Delta \log_{10}(\Lambda)}{\log_{10}(\Lambda)}$ (%)</th>
<th>$\frac{\Delta \alpha^{-1}}{\alpha^{-1}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1-2</td>
<td>$3(3, 2, \frac{1}{6})_S$</td>
<td>$(3, 1, -\frac{1}{3})_S$</td>
<td>$2.4 \cdot 10^{15}$</td>
<td>37.5</td>
<td>1.0</td>
<td>0.52</td>
</tr>
<tr>
<td>S1-2</td>
<td>$4(3, 2, \frac{1}{6})_S$</td>
<td>$4(3, 1, -\frac{1}{3})_S$</td>
<td>$1.8 \cdot 10^{16}$</td>
<td>35.1</td>
<td>1.6</td>
<td>0.80</td>
</tr>
<tr>
<td>S2-4</td>
<td>$(3, 1, \frac{2}{3})_F$</td>
<td>$5(3, 2, \frac{1}{6})_S$</td>
<td>$4.3 \cdot 10^{15}$</td>
<td>32.3</td>
<td>1.8</td>
<td>0.87</td>
</tr>
<tr>
<td>S2-11</td>
<td>$(1, 2, -\frac{1}{2})_F$</td>
<td>$(3, 2, \frac{1}{6})_S$</td>
<td>$1.2 \cdot 10^{14}$</td>
<td>38.4</td>
<td>1.2</td>
<td>0.61</td>
</tr>
</tbody>
</table>
Adding dark matter

Add 1 to 6 generations of DM to “basic model”

<table>
<thead>
<tr>
<th>Scalar</th>
<th>Fermion</th>
<th>Scalar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1, 3, 0)_S$</td>
<td>$(1, 3, 0)_F$</td>
<td>$(1, 2, \frac{1}{2})_S$</td>
</tr>
<tr>
<td>$(1, 5, 0)_S$</td>
<td>$(1, 5, 0)_F$</td>
<td>$(1, 4, \frac{1}{2})_S$</td>
</tr>
<tr>
<td>$(1, 7, 0)_S$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Unification in 15 models
- Lower unification scale
- Gauge coupling same ball park

[Goodman, Witten 1985],
[Cirelli, Fornengo, Strumia, 2006],
[Cirelli, Strumia 2009]
II. Models with dark matter

• One-loop realization of Weinberg operator
• 4 topologies
• 2-4 new particles
• Color neutral

<table>
<thead>
<tr>
<th>Model</th>
<th>$m$</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>$\Lambda$ (GeV)</th>
<th>$\alpha^{-1}(\Lambda)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-1-D</td>
<td>1</td>
<td>$1, 2, \frac{1}{2} S$</td>
<td>$1, 1, 0 S$</td>
<td>$1, 2, \frac{1}{2} F$</td>
<td>$1, 3, 1 S$</td>
<td>$1.3 \cdot 10^{13}$</td>
<td>38.4</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>$(1, 2, -\frac{1}{2}) S$</td>
<td>$(1, 1, -1) S$</td>
<td>$(1, 2, -\frac{1}{2}) F$</td>
<td>$(1, 3, 0) S$</td>
<td>$3.1 \cdot 10^{13}$</td>
<td>38.2</td>
</tr>
<tr>
<td>T1-2-A</td>
<td>0</td>
<td>$(1, 1, 0) F$</td>
<td>$(1, 2, \frac{1}{2}) S$</td>
<td>$(1, 1, 0) S$</td>
<td>$(1, 2, \frac{1}{2}) F$</td>
<td>$5.3 \cdot 10^{13}$</td>
<td>39.4</td>
</tr>
<tr>
<td>T1-2-B</td>
<td>0</td>
<td>$(1, 1, 0) F$</td>
<td>$(1, 2, \frac{1}{2}) S$</td>
<td>$(1, 3, 0) S$</td>
<td>$(1, 2, \frac{1}{2}) F$</td>
<td>$4.6 \cdot 10^{13}$</td>
<td>38.4</td>
</tr>
<tr>
<td></td>
<td>-2</td>
<td>$(1, 1, -1) F$</td>
<td>$(1, 2, -\frac{1}{2}) S$</td>
<td>$(1, 3, -1) S$</td>
<td>$(1, 2, -\frac{1}{2}) F$</td>
<td>$3.2 \cdot 10^{12}$</td>
<td>35.9</td>
</tr>
<tr>
<td>T1-3-A</td>
<td>0</td>
<td>$(1, 1, 0) F$</td>
<td>$(1, 2, \frac{1}{2}) F$</td>
<td>$(1, 1, 0) S$</td>
<td>$(1, 2, -\frac{1}{2}) F$</td>
<td>$2.8 \cdot 10^{13}$</td>
<td>37.7</td>
</tr>
<tr>
<td>T3-A</td>
<td>0</td>
<td>$(1, 1, 0) S$</td>
<td>$(1, 3, 1) S$</td>
<td>$(1, 2, \frac{1}{2}) F$</td>
<td>-</td>
<td>$1.6 \cdot 10^{13}$</td>
<td>37.3</td>
</tr>
<tr>
<td></td>
<td>-2</td>
<td>$(1, 1, -1) S$</td>
<td>$(1, 3, 0) S$</td>
<td>$(1, 2, -\frac{1}{2}) F$</td>
<td>-</td>
<td>$4.0 \cdot 10^{13}$</td>
<td>38.7</td>
</tr>
<tr>
<td>T1-3-A</td>
<td>0</td>
<td>$(1, 1, 0) F$</td>
<td>$(1, 2, \frac{1}{2}) F$</td>
<td>$2 (1, 1, 0) S$</td>
<td>-</td>
<td>$6.9 \cdot 10^{13}$</td>
<td>39.8</td>
</tr>
<tr>
<td>T1-3-B</td>
<td>0</td>
<td>$(1, 1, 0) F$</td>
<td>$(1, 2, \frac{1}{2}) F$</td>
<td>$2 (1, 3, 0) S$</td>
<td>-</td>
<td>$5.7 \cdot 10^{13}$</td>
<td>38.9</td>
</tr>
</tbody>
</table>

[Restrepo, Zapata, Yaguna, 2013]
III. Models with particles in adjoint representation of SU(3)

[Angel, Cai, Rodd, Schmidt, Volkas, 2013], [Fileviez Pérez, Wise, 2009]

<table>
<thead>
<tr>
<th>Model</th>
<th>Scalar</th>
<th>Fermion</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>$(\bar{3}, 1, \frac{1}{3})_S$</td>
<td>$(8, 1, 0)_F$</td>
</tr>
<tr>
<td>U2</td>
<td>$(8, 2, \frac{1}{2})_S$</td>
<td>$(8, 1, 0)_F$</td>
</tr>
<tr>
<td>U3</td>
<td>$(8, 2, \frac{1}{2})_S$</td>
<td>$(8, 3, 0)_F$</td>
</tr>
</tbody>
</table>

- Large representations = large contribution to the RG running
- Only one model, U1, without LP
- No model with unification (even if DM added)
Sources of uncertainty

Uncertainty at Z-mass scale

New particle mass

One-loop vs two-loop effect

Effects of the order of 1-10 %

Stella Riad, KTH, RG running in radiative models, ICHEP, 2016
Summary and conclusions

- Connect low-energy radiative neutrino mass models with high energy GUTs.
- Gauge unification is possible
- Unification scale in general low
  - Can be pushed up by colored particles
- Further investigation necessary considering other couplings and other models
- arXiv:1605.03986
THANK YOU!