

m_c , m_b , and α_s
Lattice status and prospects

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Parametric uncertainties in BSM searches

Channel	M_H [GeV]	Γ [MeV]	$\Delta\alpha_s$	Δm_b	Δm_c	Δm_t	THU
$H \rightarrow b\bar{b}$	124	2.37	-1.4%	+1.7%	+0.0%	+0.0%	+0.5%
			+1.4%	-1.7%	-0.0%	-0.0%	-0.5%
	125	2.38	-1.4%	+1.7%	+0.0%	+0.0%	+0.5%
			+1.4%	-1.7%	-0.0%	-0.0%	-0.5%
	126	2.40	-1.4%	+1.7%	+0.0%	+0.0%	+0.5%
			+1.4%	-1.7%	-0.0%	-0.0%	-0.5%
$H \rightarrow \tau^+\tau^-$	124	$2.54 \cdot 10^{-1}$	+0.0%	+0.0%	+0.0%	+0.0%	+0.5%
			-0.0%	-0.0%	-0.0%	-0.0%	-0.5%
	125	$2.56 \cdot 10^{-1}$	+0.0%	+0.0%	+0.0%	+0.0%	+0.5%
			-0.0%	-0.0%	-0.0%	-0.0%	-0.5%
	126	$2.58 \cdot 10^{-1}$	+0.0%	+0.0%	+0.0%	+0.0%	+0.5%
			-0.0%	-0.0%	-0.0%	-0.0%	-0.5%
$H \rightarrow \mu^+\mu^-$	124	$8.83 \cdot 10^{-4}$	+0.0%	+0.0%	+0.0%	+0.0%	+0.5%
			-0.0%	-0.0%	-0.0%	-0.0%	-0.5%
	125	$8.90 \cdot 10^{-4}$	+0.0%	+0.0%	+0.0%	+0.0%	+0.5%
			-0.0%	-0.0%	-0.0%	-0.0%	-0.5%
	126	$8.96 \cdot 10^{-4}$	+0.0%	+0.0%	+0.0%	+0.0%	+0.5%
			-0.0%	-0.0%	-0.0%	-0.0%	-0.5%
$H \rightarrow c\bar{c}$	124	$1.17 \cdot 10^{-1}$	-1.9%	-0.0%	+5.3%	+0.0%	+0.5%
			+1.9%	-0.0%	-5.2%	-0.0%	-0.5%
	125	$1.18 \cdot 10^{-1}$	-1.9%	-0.0%	+5.3%	+0.0%	+0.5%
			+1.9%	-0.0%	-5.2%	-0.0%	-0.5%
	126	$1.19 \cdot 10^{-1}$	-1.9%	-0.0%	+5.3%	+0.0%	+0.5%
			+1.9%	-0.0%	-5.2%	-0.0%	-0.5%
$H \rightarrow gg$	124	$3.27 \cdot 10^{-1}$	+3.0%	-0.2%	+0.0%	-0.1%	+3.2%
			-3.0%	+0.1%	-0.0%	+0.1%	-3.2%
	125	$3.35 \cdot 10^{-1}$	+3.0%	-0.1%	+0.0%	-0.1%	+3.2%
			-3.0%	+0.1%	-0.0%	+0.1%	-3.2%
	126	$3.43 \cdot 10^{-1}$	+3.1%	-0.1%	+0.0%	-0.1%	+3.2%
			-3.0%	+0.2%	-0.0%	+0.1%	-3.2%
$H \rightarrow \gamma\gamma$	124	$8.97 \cdot 10^{-3}$	+0.0%	+0.0%	+0.0%	+0.0%	+1.0%
			-0.0%	-0.0%	-0.0%	-0.0%	-1.0%
	125	$9.28 \cdot 10^{-3}$	+0.0%	+0.0%	+0.0%	+0.0%	+1.0%
			-0.0%	-0.0%	-0.0%	-0.0%	-1.0%
	126	$9.60 \cdot 10^{-3}$	+0.0%	+0.0%	+0.0%	+0.0%	+1.0%
			-0.0%	-0.0%	-0.0%	-0.0%	-1.0%
$H \rightarrow Z\gamma$	124	$5.72 \cdot 10^{-3}$	+0.0%	+0.0%	+0.0%	+0.0%	+5.0%
			-0.0%	-0.0%	-0.0%	-0.0%	-5.0%
	125	$6.27 \cdot 10^{-3}$	+0.0%	+0.0%	+0.0%	+0.0%	+5.0%
			-0.0%	-0.0%	-0.0%	-0.0%	-5.0%
	126	$6.84 \cdot 10^{-3}$	+0.0%	+0.0%	+0.0%	+0.0%	+5.0%
			-0.0%	-0.0%	-0.0%	-0.0%	-5.0%
$H \rightarrow WW$	124	$7.82 \cdot 10^{-1}$	+0.0%	+0.0%	+0.0%	+0.0%	+0.5%
			-0.0%	-0.0%	-0.0%	-0.0%	-0.5%
	125	$8.74 \cdot 10^{-1}$	+0.0%	+0.0%	+0.0%	+0.0%	+0.5%
			-0.0%	-0.0%	-0.0%	-0.0%	-0.5%
	126	$9.74 \cdot 10^{-1}$	+0.0%	+0.0%	+0.0%	+0.0%	+0.5%
			-0.0%	-0.0%	-0.0%	-0.0%	-0.5%
$H \rightarrow ZZ$	124	$9.43 \cdot 10^{-2}$	+0.1%	+1.0%	+0.2%	+0.0%	+0.5%
			-0.1%	-1.0%	-0.2%	-0.0%	-0.5%
	125	$1.07 \cdot 10^{-1}$	+0.1%	+1.0%	+0.2%	+0.0%	+0.5%
			-0.1%	-1.0%	-0.2%	-0.0%	-0.5%
	126	$1.21 \cdot 10^{-1}$	+0.1%	+1.0%	+0.2%	+0.0%	+0.5%
			-0.1%	-1.0%	-0.2%	-0.0%	-0.5%

Partial Higgs widths

Uncertainties in standard model parameters limit possible precision in searches for new physics.

Partial widths into $b\bar{b}$, $c\bar{c}$, and gg are more dependent on parametric uncertainties than on other theory.

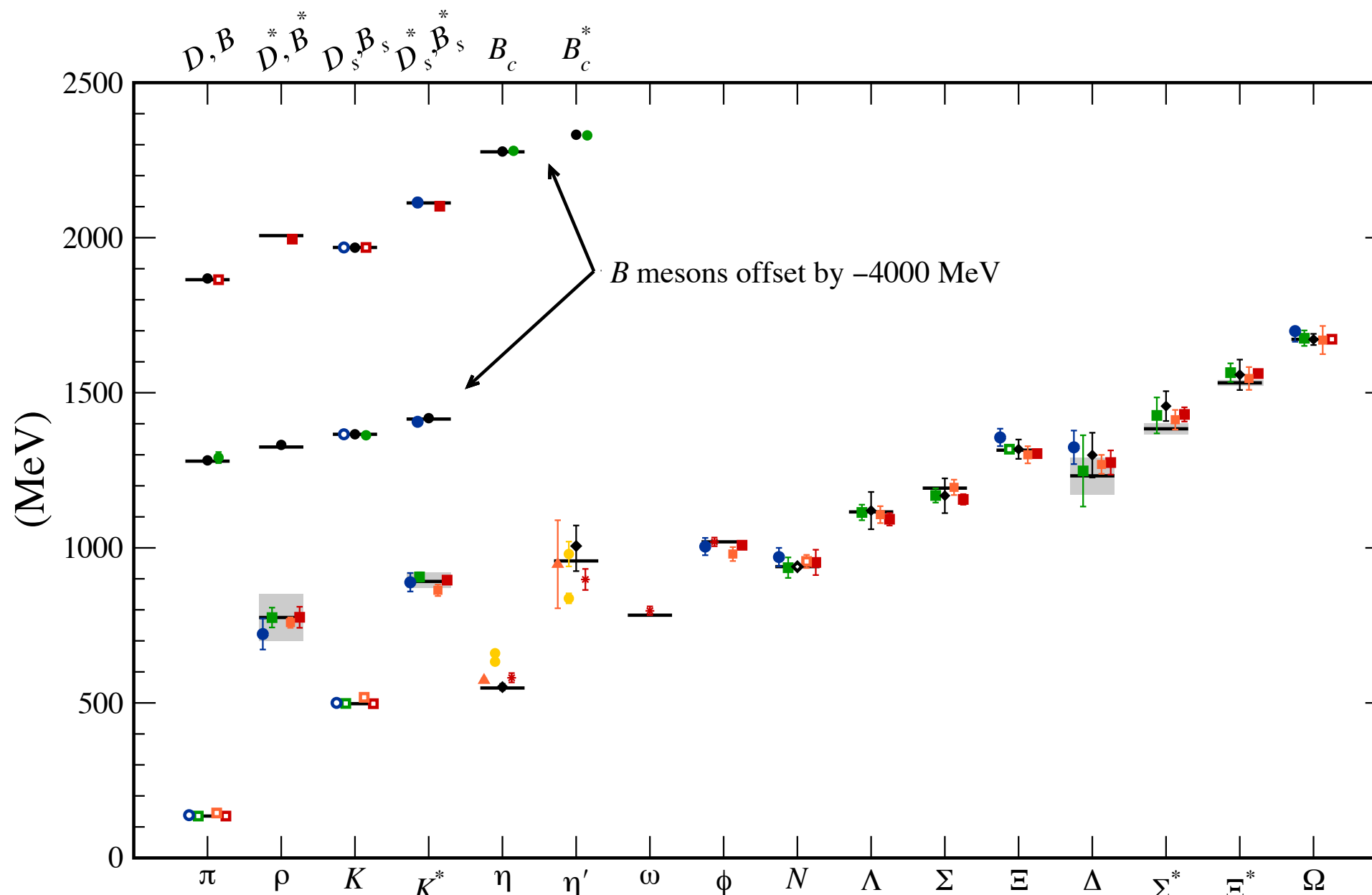
Lattice QCD can provide the most precise determinations of the parameters α_s , m_c , and m_b .

LHC Higgs Cross Section Working Group
M. Grazzini, R. Harlander, B. Mellado, P. Musella (Eds.)
2016 (Draft)

Table 3.11: SM Higgs partial widths and their relative parametric (PU) and theoretical (THU) uncertainties for a selection of Higgs masses. For PU, all the single contributions are shown. For these four columns, the upper percentage value (with its sign) refers to the positive variation of the parameter, while the lower one refers to the negative variation of the parameter.

Lattice in the 21st century

For the past ~ten years, it has been possible to use lattice QCD Monte Carlo methods to calculate simple quantities with understood, complete error budgets.



Andreas Kronfeld

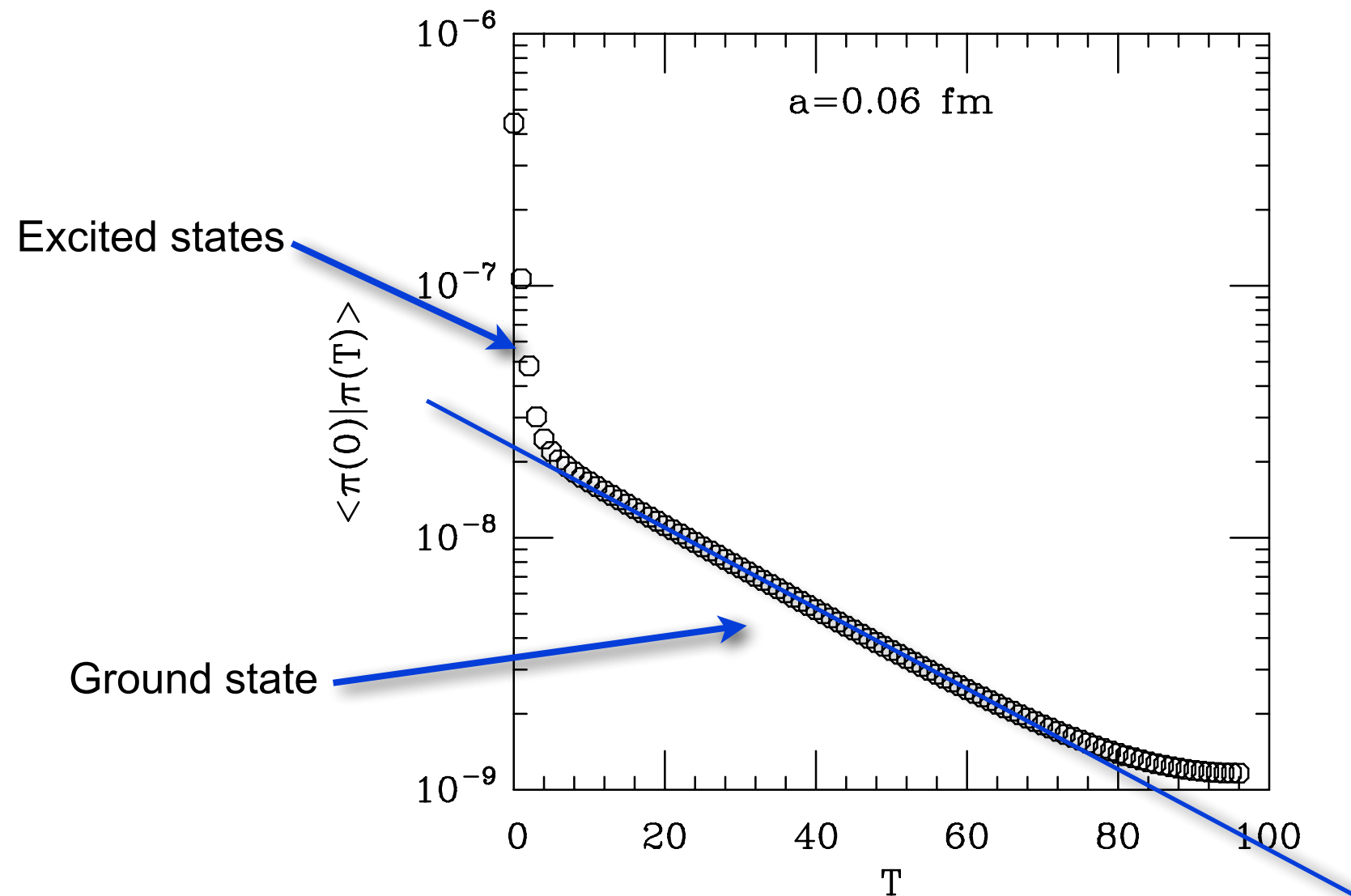
What is “simple”?

- Simplest: stable mesons.
- Over the **last ten years**, many key quantities. Hadronically stable mesons, especially:
 - Heavy and light meson **decay constants**,
 - **Semileptonic decays**,
 - **Meson-antimeson mixing**.
- Make possible important determinations of 8 CKM matrix elements, 5 quark masses, the strong coupling constant.
- **Now**: **$\pi\pi\pi$** systems, **nucleons**

Coming US experimental program

- **Next five years:** lattice calculations are needed *throughout* the entire future US experimental program.
 - $g-2$
 - LHCb, Belle-2: continued improvement of CKM results
 - mu2e, LBNE, Nova: nucleon matrix elements.
 - Underground LBNE: proton decay matrix elements.
 - LHC, Higgs decays: lattice provides the most accurate α_s and m_c now, and m_b in the future

How?



$$\langle \bar{\psi} \gamma_5 \psi(t=0) | \bar{\psi} \gamma_5 \psi(t) \rangle = C \exp(-Mt) + \text{excited states}.$$

If the two quarks were a u and a \bar{u} , the slope would give M_π , C would be proportional to F_π^2 .

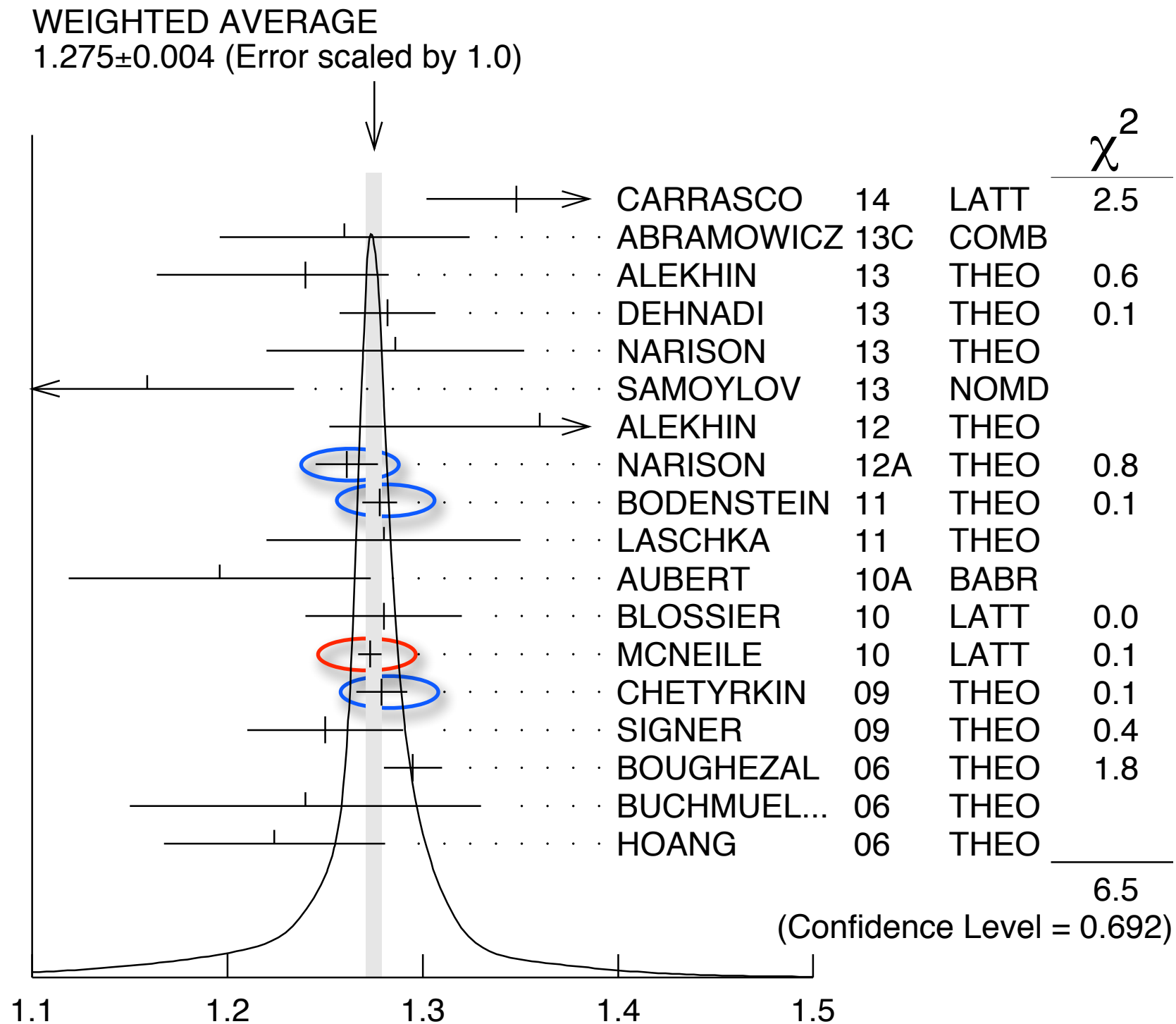
To obtain α_s , m_c , or m_b via the lattice

- In principle,
 - can get $m_{\overline{\text{MS}}}$ from m_{latt} by equating Green's functions calculated in perturbation theory in the two regulators:

The diagram shows a horizontal black line with an arrow pointing to the right, representing a quark. Above this line is a red, wavy loop representing a gluon. The diagram is part of a series, indicated by a plus sign and three dots to its right. This is followed by an equals sign, then another identical diagram (black line with arrow and red gluon loop), also followed by a plus sign and three dots.

- In practice,
 - Calculating short-distance quantities to third order perturbation theory is hard and messy.
 - Calculating some short-distance quantities nonperturbatively is easy and clean.
- The art of determining α_s or m_q via the lattice is finding a quantity as easy to calculate as possible
 - with continuum perturbation theory, *and*
 - nonperturbatively with the lattice.

m_c

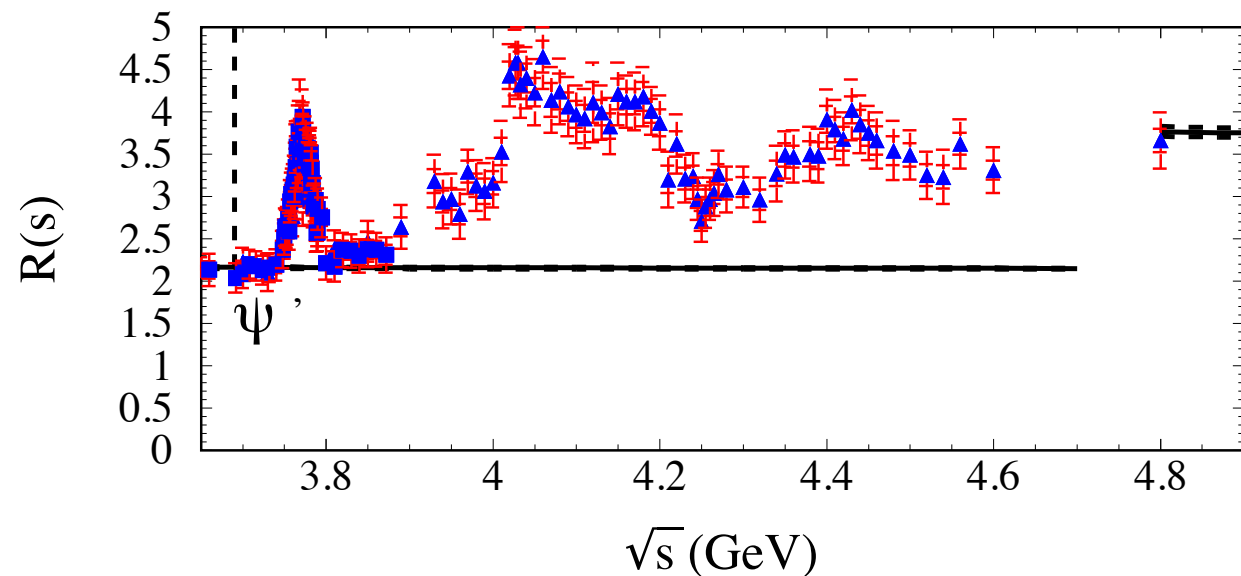


The most precise non-lattice determinations of m_c use e^+e^- annihilation data and ITEP sum rules. (Karlsruhe group, Chertyrkin et al.)

Recent lattice determination of HPQCD uses the same type of perturbation theory, but lattice QCD to supply the correlation functions rather than experiment.

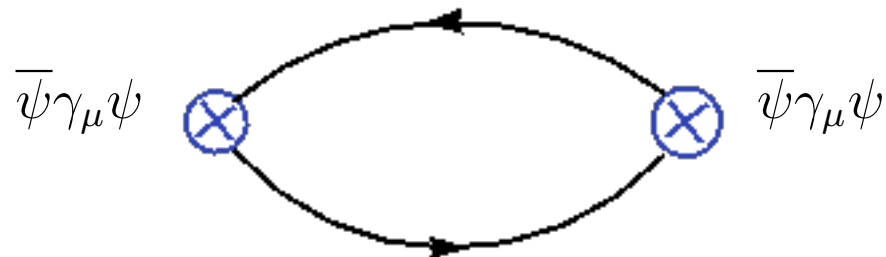
K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014) and 2015 update

$$e^+e^- \rightarrow m_c$$



$$\mathcal{M}_n \equiv \int \frac{ds}{s^{n+1}} R_Q(s)$$

Moments of the heavy quark production cross section in e^+e^- annihilation can be related to the derivatives of the vacuum polarization at $q^2=0$.



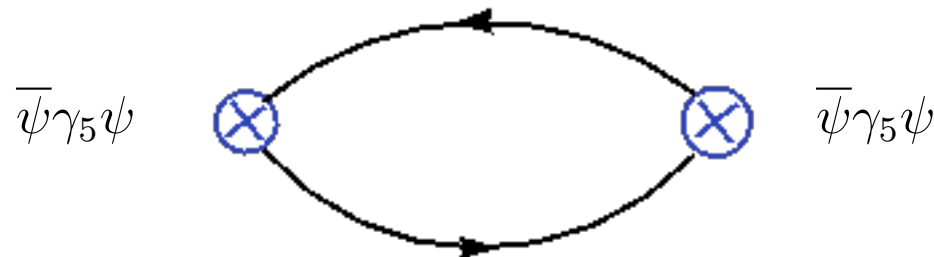
$$\mathcal{M}_n = \frac{12\pi^2}{n!} \left(\frac{d}{dq^2} \right)^n \Pi_Q(q^2) |_{q^2=0}$$

Can be calculated in perturbation theory.
Known to $O(\alpha_s^3)$ (Chetyrkin et. al.)

Lattice QCD

can also compute such correlation functions with high accuracy.

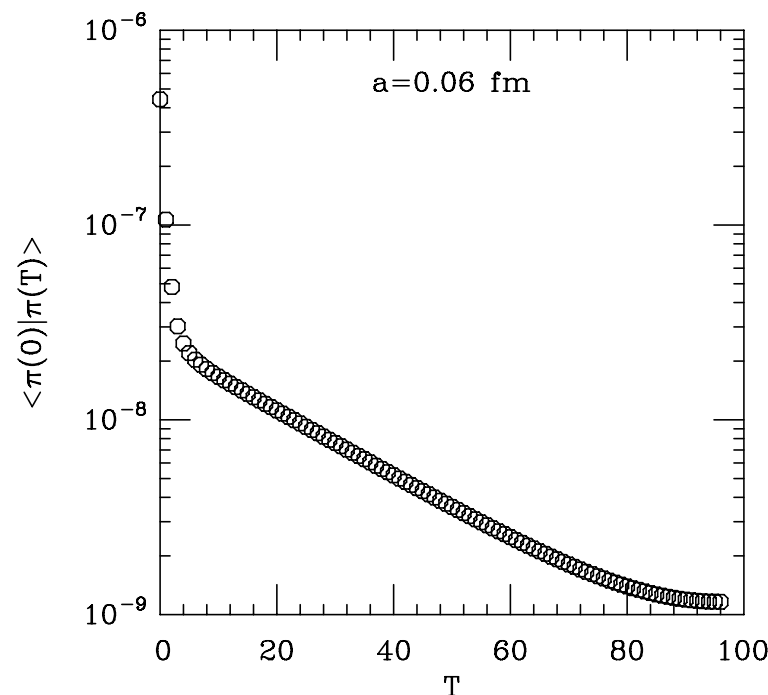
Correlation functions of all currents can be calculated in perturbation theory (and with the lattice). The most precise m_c can be obtained by choosing the one that is most precise on the lattice: the pseudoscalar correlator.



$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0, 0) | 0 \rangle,$$

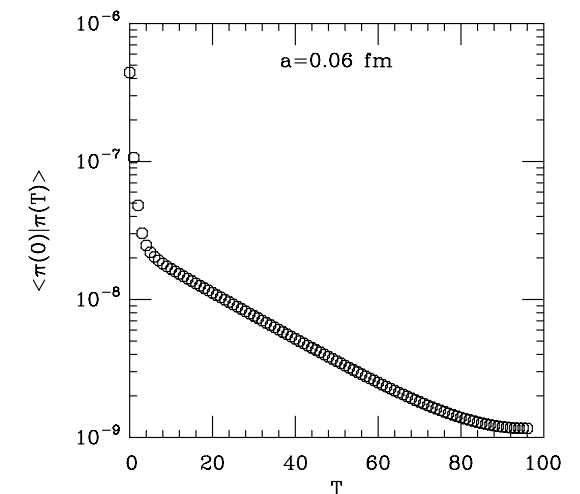
$$G_n \equiv \sum_t (t/a)^n G(t),$$

Perturbation theory to α_s^3 from the Karlsruhe group.



Technical tricks to make the lattice calculation more precise

Choose pseudoscalar (easiest) current correlator.
(Easier to calculate than a pion or charmonium mass.)

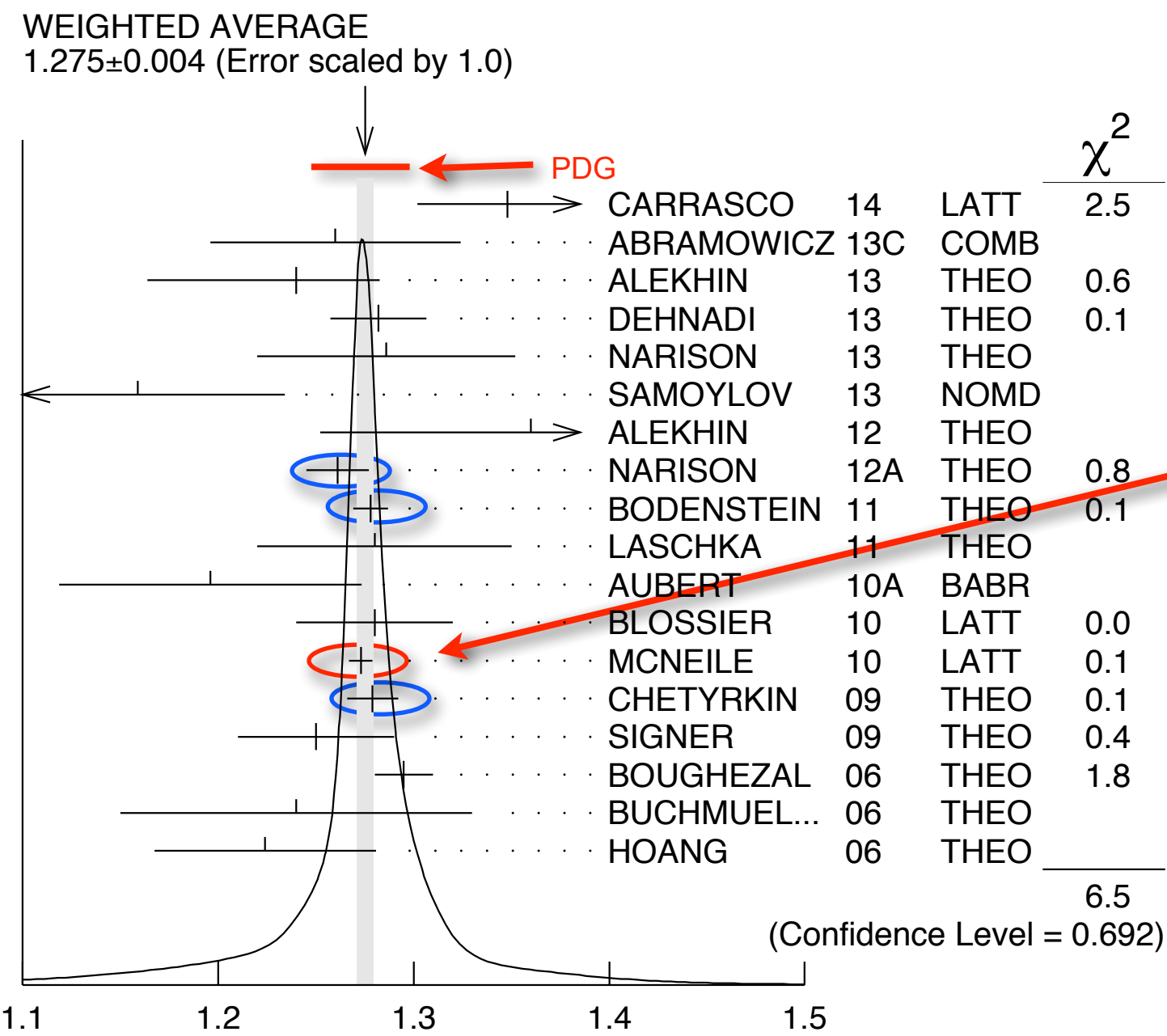


In matching perturbative and nonperturbative results, divide both by the tree level correlator. (Removes leading discretization errors.)

In the lattice calculation of, for example, the charm correlator, use $M\eta_c$ as experimental input to set the energy scale. (Reduces sensitivity to the tuning of the lattice mass used.)

$$R_n \equiv \begin{cases} G_4/G_4^{(0)} & \text{for } n = 4 \\ \frac{am_{\eta_h}}{2am_{0h}} (G_n/G_n^{(0)})^{1/(n-4)} & \text{for } n \geq 6 \end{cases}$$

m_c results



	$m_c(3)$
a^2 extrapolation	0.2%
Perturbation theory	0.5
Statistical errors	0.1
m_h extrapolation	0.1
Errors in r_1	0.2
Errors in r_1/a	0.1
Errors in m_{η_c}, m_{η_b}	0.2
α_0 prior	0.1
Gluon condensate	0.0
Total	0.6%

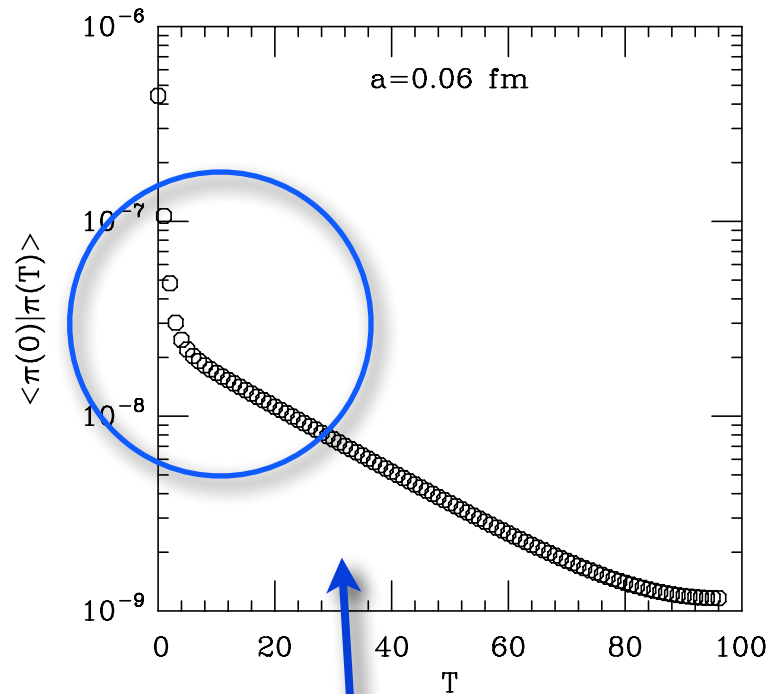
$$m_c(m_c, n_f = 4) = 1.273(6) \text{ GeV.}$$

HPQCD, McNeile et al.

Uncertainty is dominated by the same perturbation theory used in all of the most precise results.



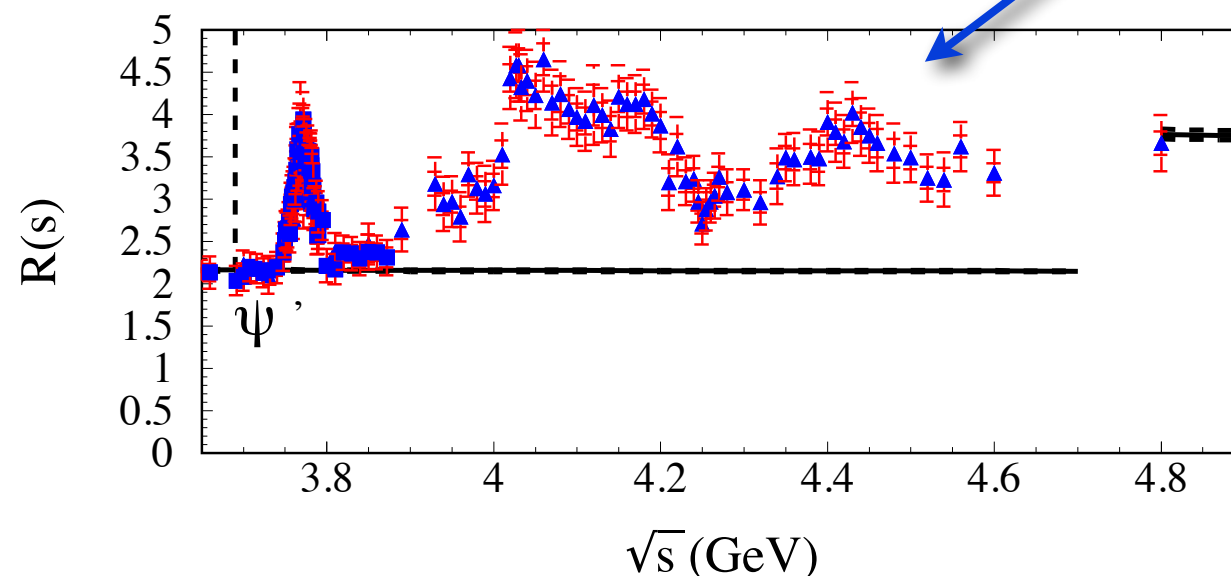
Why can lattice determinations of m_c from correlation functions be more precise than those from e^+e^- ?



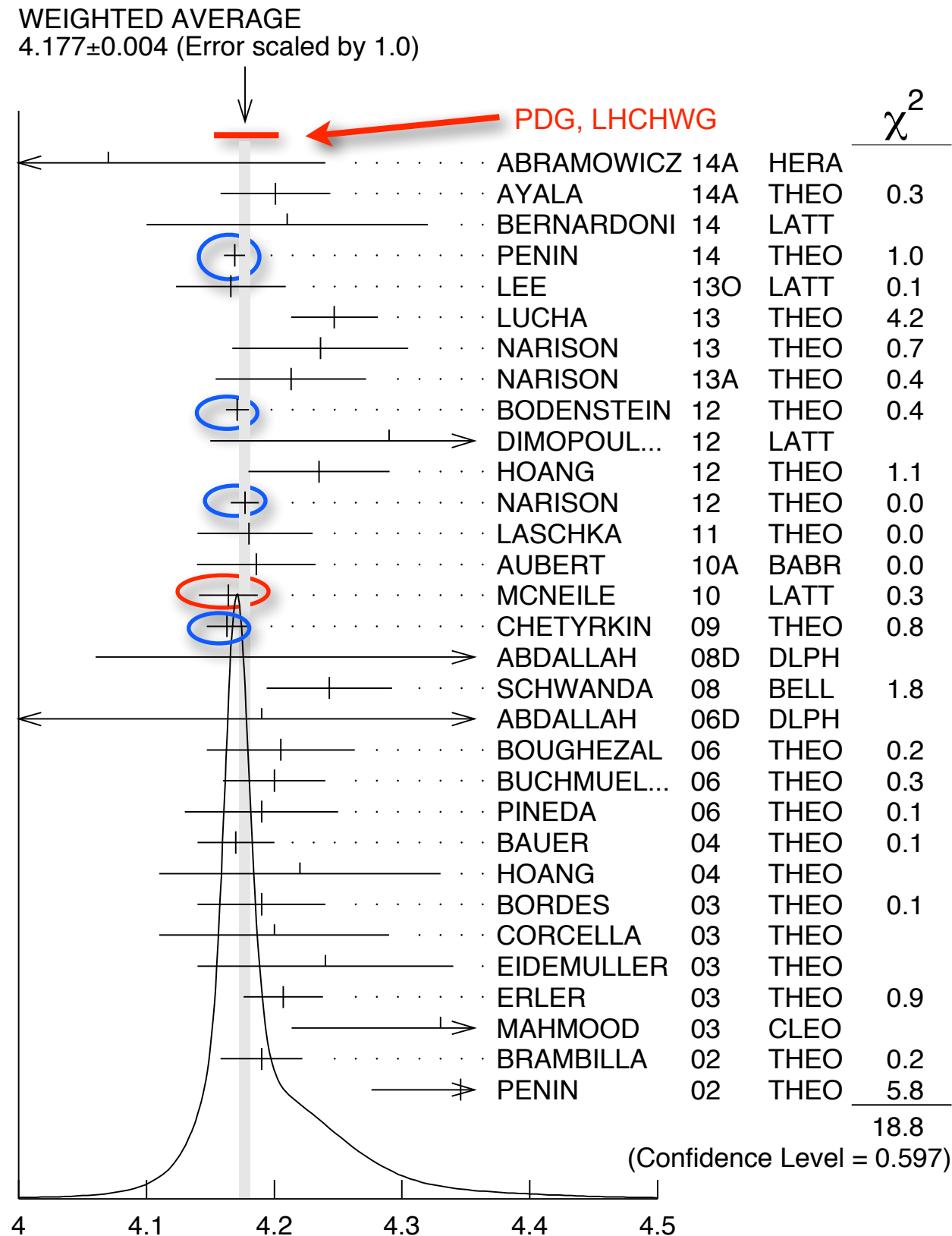
Moments of correlation functions are even easier than what I earlier told you have been considered the easiest quantities for the last ten years.

We need the correlation functions at finite T , and not their asymptotic form at large T .

Because **this** is cleaner data than **this**.



m_b



The most precise non-lattice determinations of m_c use e^+e^- annihilation data and ITEP sum rules. (Karlsruhe group, Chertyrkin et al.)

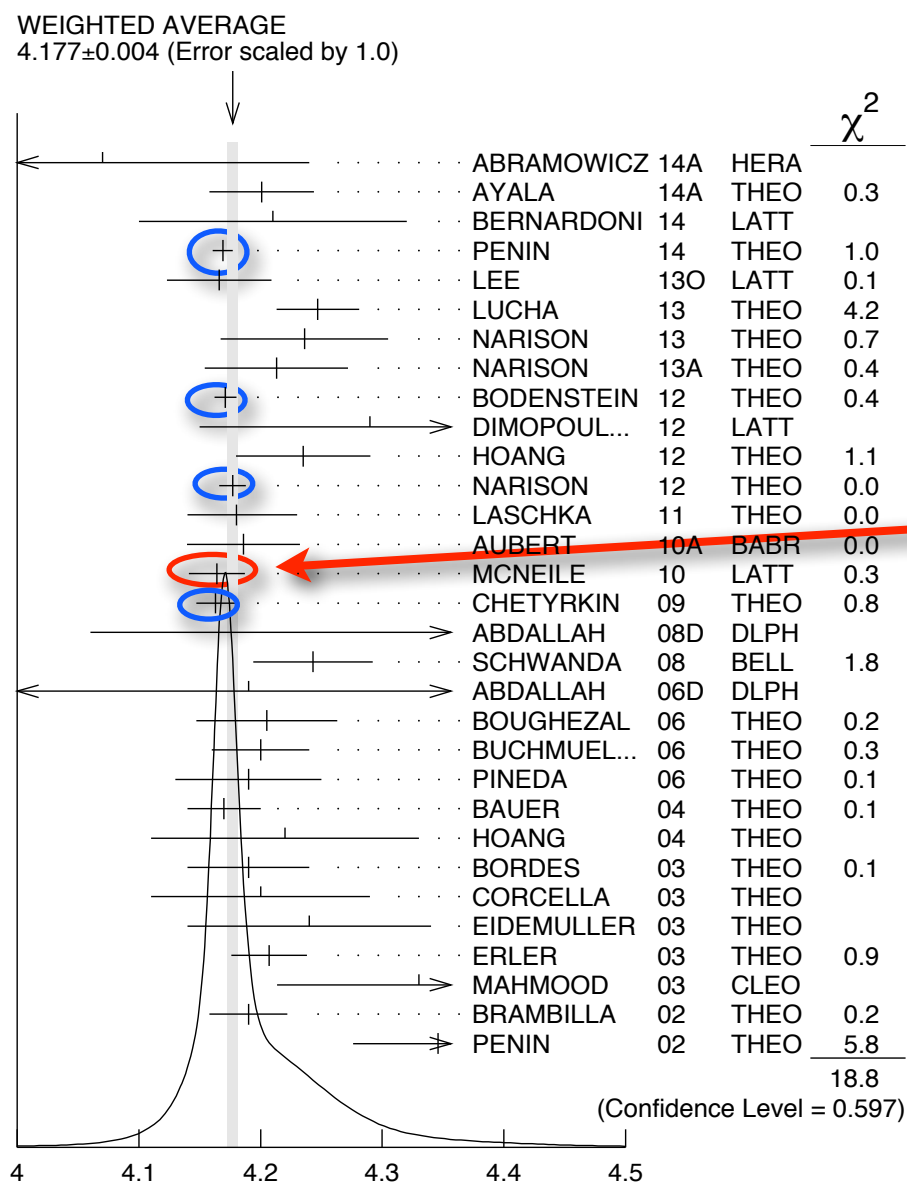
Recent lattice determination uses the same type of perturbation theory, but lattice QCD to supply the correlation functions rather than experiment.

For m_b , perturbative errors are tiny. ($\alpha(m_b)^4 \ll \alpha(m_c)^4$.)

K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014) and 2015 update

m_b results

For m_b , these lattice correlator methods are just barely working at $a=0.045$ fm.
(They treat the b as light compared with $1/a$.)
Need $a=0.03$ fm to be comfortable.
Discretization errors and statistics dominate current uncertainties. Both can be attacked with brute force computing power.
Needed configurations are projected to be generated in the next few years.



	$m_b(10)$
a^2 extrapolation	0.6%
Perturbation theory	0.1
Statistical errors	0.3
m_h extrapolation	0.1
Errors in r_1	0.1
Errors in r_1/a	0.3
Errors in m_{η_c}, m_{η_b}	0.1
α_0 prior	0.1
Gluon condensate	0.0
Total	0.7%

$$m_b(m_b, n_f = 5) = 4.164(23) \text{ GeV}$$



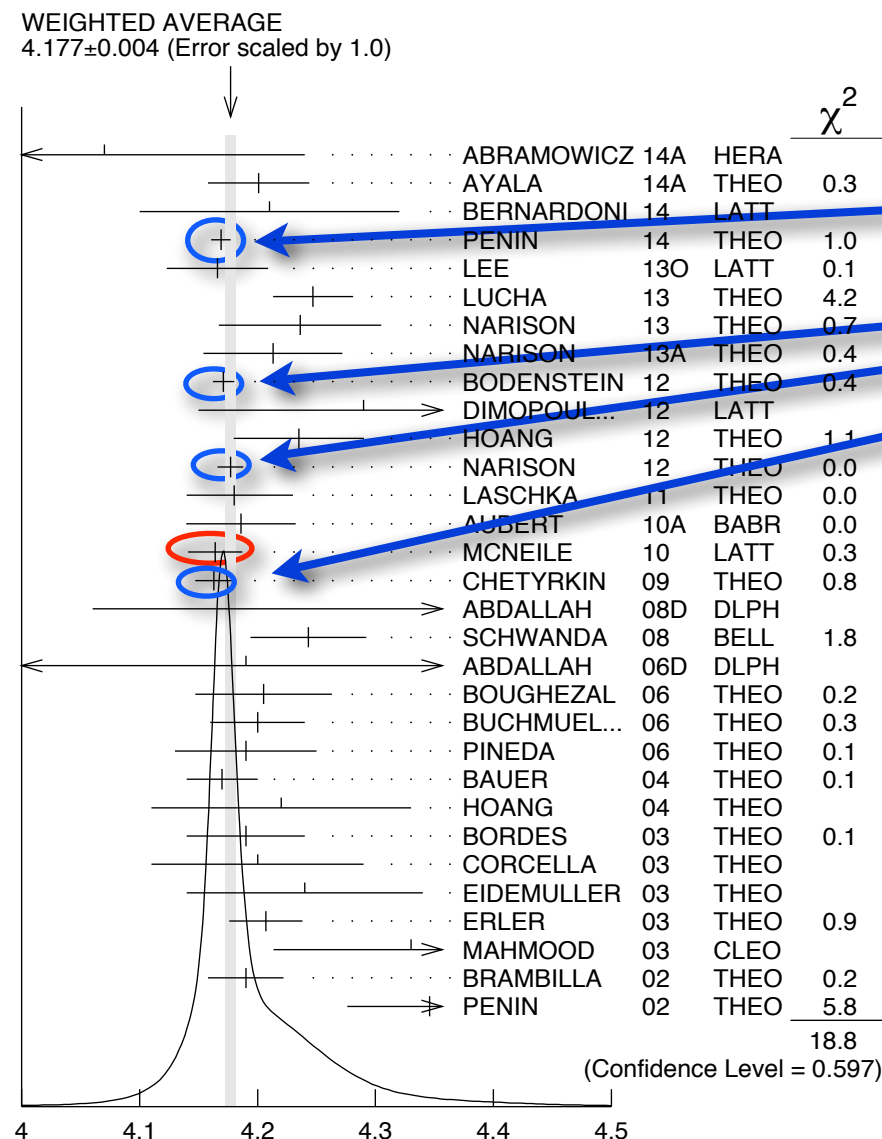
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The most precise determinations of m_b using moments of e^+e^- data arrive at different estimates of the precision from the same data and the same perturbation theory.

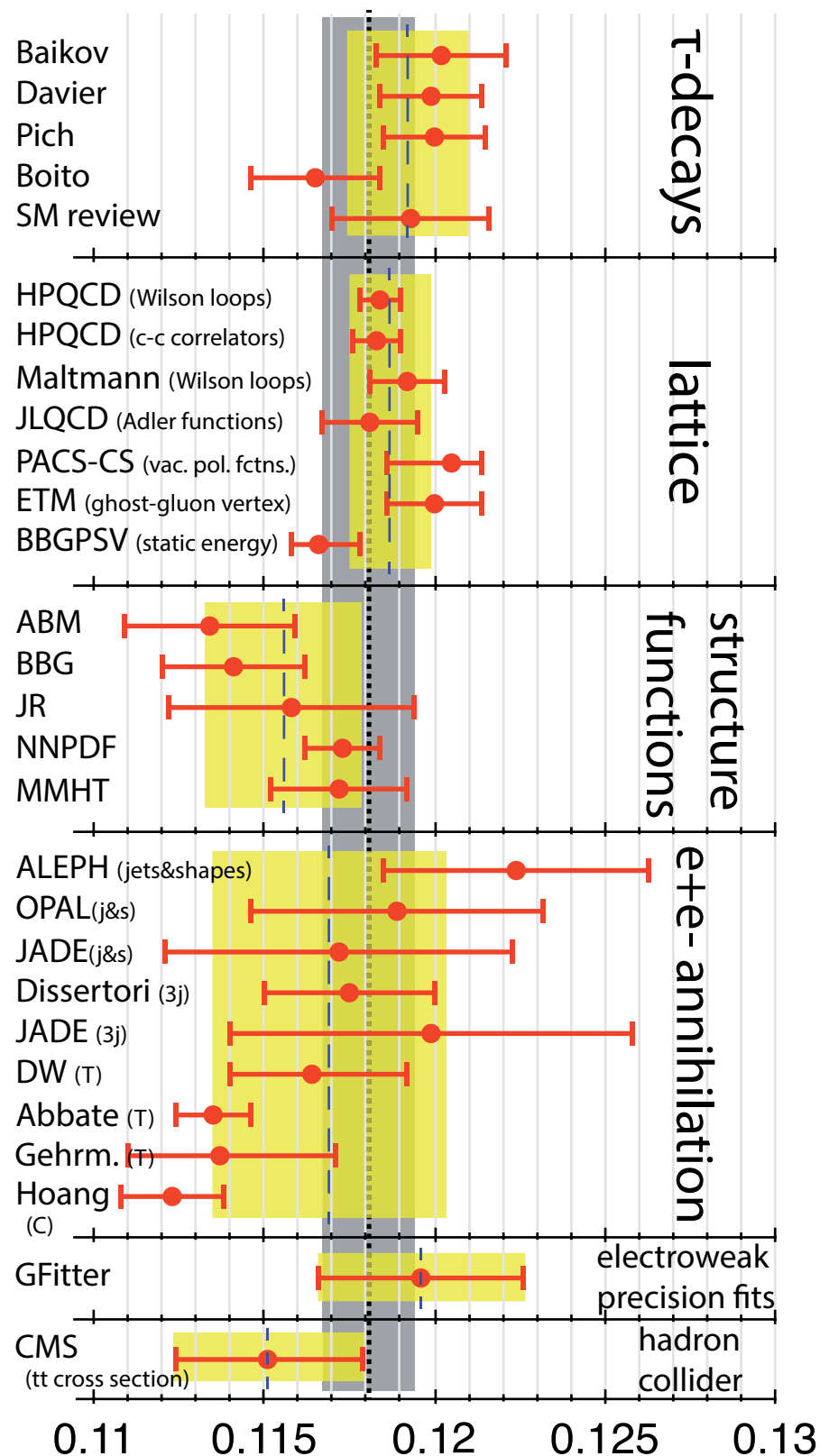
Coming lattice calculations should be able to confirm (or not) the more more precise claims.

Unlike m_c , where the lattice and e^+e^- determinations share the same perturbation theory, perturbative uncertainties are negligible and the lattice and e^+e^- determinations will have totally independent uncertainties.

α_s

There are multiple ways of determining α_s , both with and without the lattice.

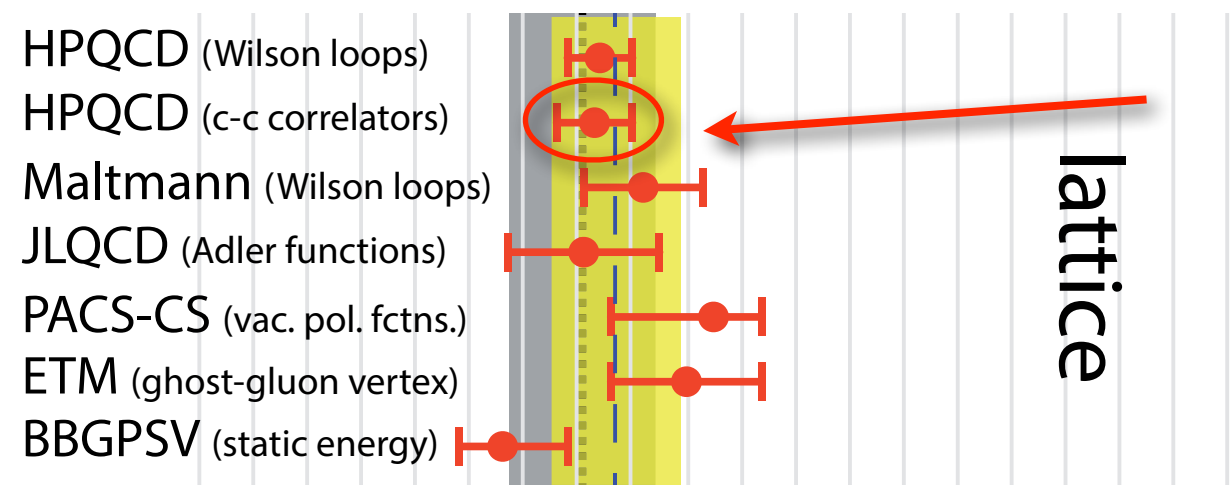
There are several lattice determinations equal to or more precise than all the non-lattice determinations together.



Particle Data Group, revised September 2015 by
S. Bethke, G. Dissertori, and G.P. Salam.



α_s results: correlator method

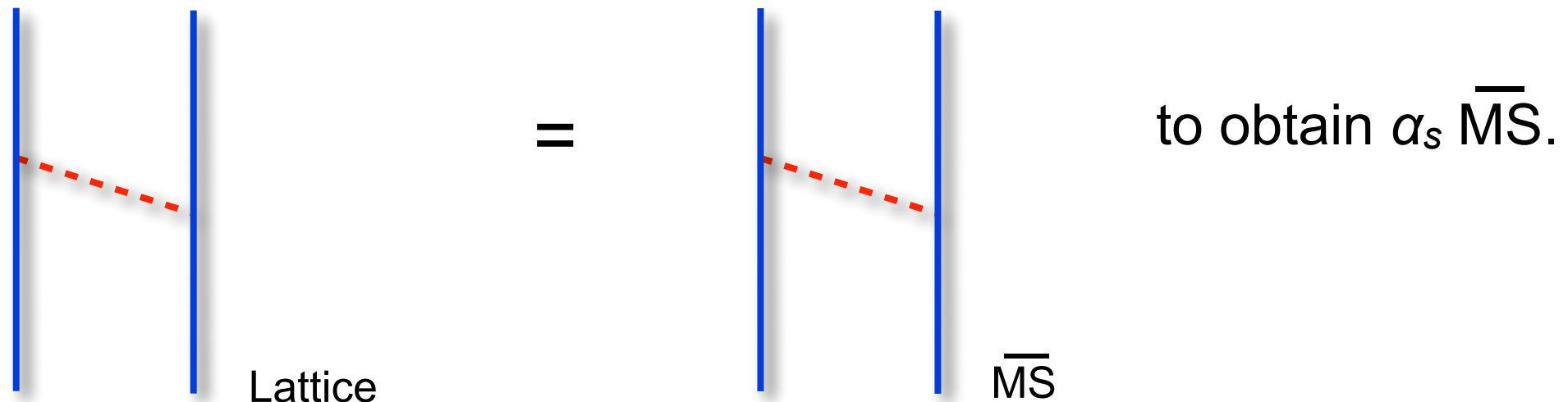


	$\alpha_{\overline{\text{MS}}}(M_Z)$
a^2 extrapolation	0.2%
Perturbation theory	0.4
Statistical errors	0.2
m_h extrapolation	0.0
Errors in r_1	0.1
Errors in r_1/a	0.1
Errors in m_{η_c}, m_{η_b}	0.0
α_0 prior	0.1
Gluon condensate	0.2
Total	0.6%

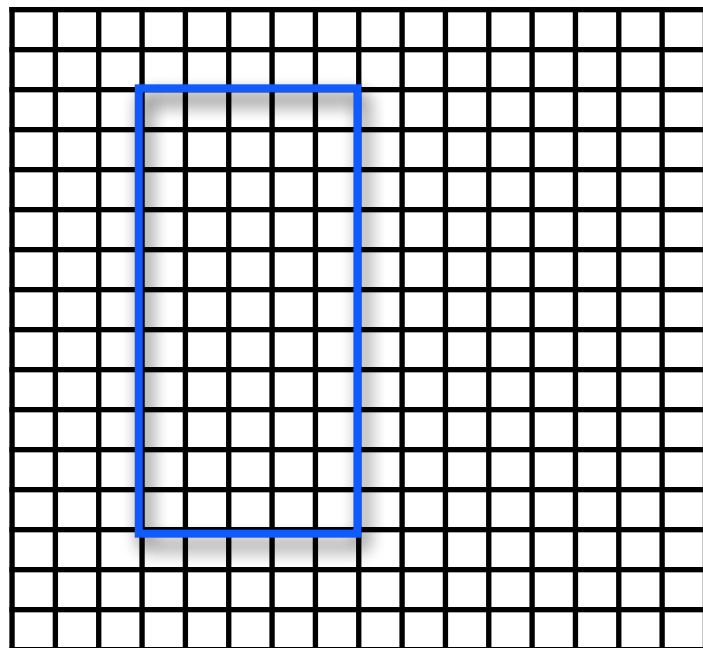
Results are dominated by perturbation theory. Better numerical data will help bound perturbative coefficients.

α_s results: Wilson loops

α_s can be determined with lattice calculations of many other quantities, e.g., the heavy quark potential.



Lattice calculates the heavy quark potential from Wilson loops.



HPQCD has determined α_s directly from Wilson loops.

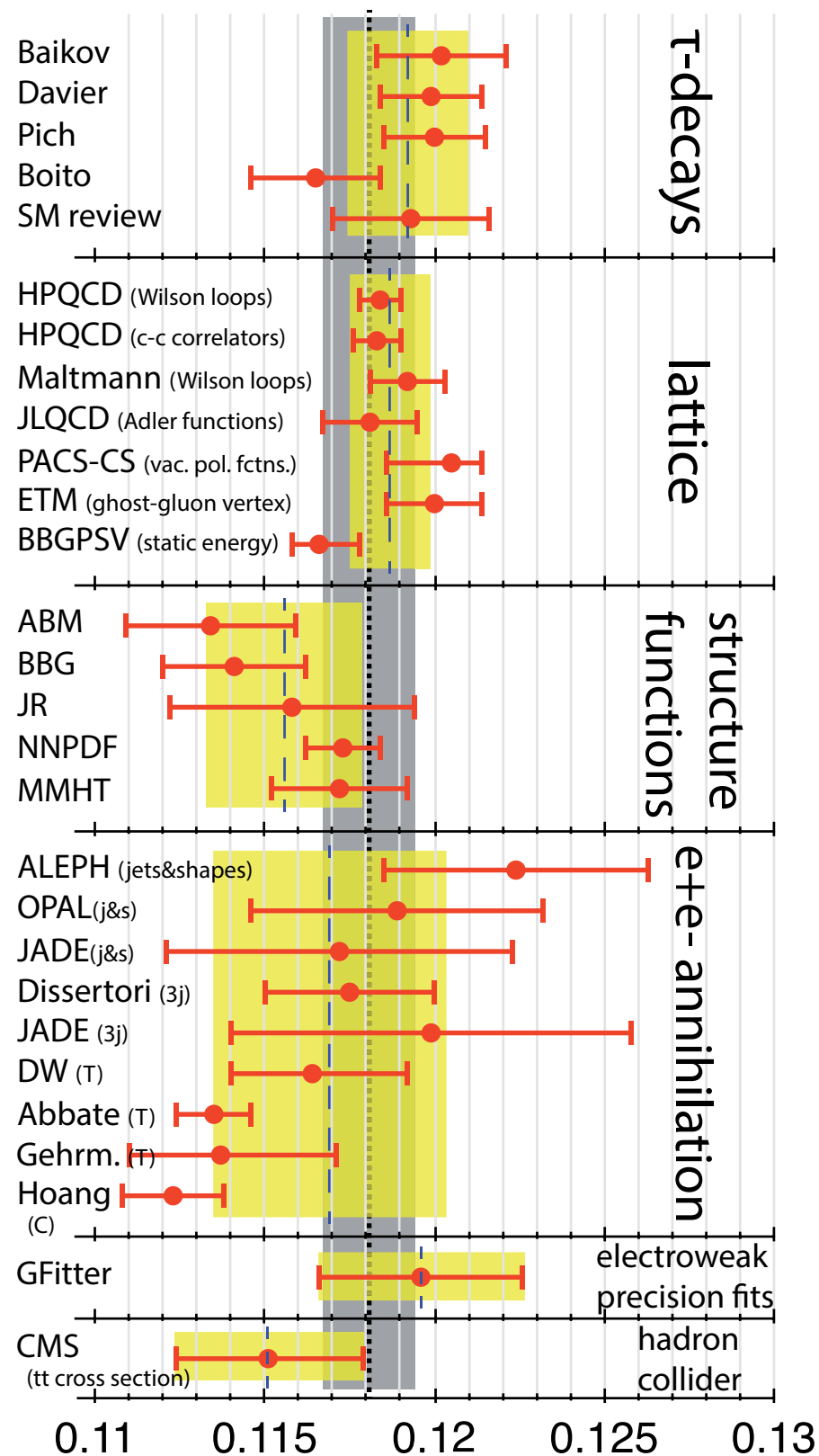
Result compatible with their correlator result, similar precision: $\alpha_s = 0.1184(6)$, but totally different uncertainties, heavy use of lattice perturbation theory.

A. Bazavov et al. use the potential and obtain $\alpha_s = 0.1166 + 0.0012 - 0.0008$.

α_s , other lattice results

There are numerous good ways of determining α_s using lattice QCD.

- The Adler function, JLQCD. Phys.Rev. D82 (2010) 074505.
 - $\alpha_s = 0.1181 \pm 0.0003 + 0.0014 - 0.0012$
- The Schrödinger functional, PACS-CS. JHEP 0910:053,2009.
 - $\alpha_s = 0.1205(8)(5)(+0/-17)$
- The ghost-gluon vertex, European Twisted Mass Collaboration (ETM). Phys.Rev.Lett. 108 (2012) 262002.
 - $\alpha_s = 0.1200(14)$



The lattice results are dominated by the two most precise results from HPQCD, but there are several other lattice results from Europe and Japan, all of which agree with each other and each which is more precise than any non-lattice result.

Best lattice results (HPQCD)	0.1184(06)
Average of other lattice results omitting the best two	0.1188(09)
World average omitting lattice results (PDG)	0.1175(17)
Complete world average (PDG)	0.1185(14)

Prospects: m_c and m_b

- Correlator methods are currently the most precise, both with e^+e^- and with lattice methods.
- For m_c , correlator moments are simple to calculate on the lattice
 - Should be checkable by many lattice groups.
 - Results should be of comparable precision to determinations from e^+e^- .
 - Uncertainty will be dominated by perturbation theory.
- For m_b , most precise lattice determination relies on treating b quark as light compared to $1/a$.
 - Possible with HISQ fermions, may be hard for other lattice methods.
 - The lattice result should catch up to the most precise of the e^+e^- results with more CPU power.
 - The resulting uncertainties in the e^+e^- determinations and the lattice determinations will be totally independent of each other (unlike the case for m_c); perturbative uncertainty is negligible.

Prospects: α_s

- The uncertainties of the Wilson loop and correlator determinations of α_s are dominated by perturbation theory and will improve somewhat.
- α_s can be determined well from lattice calculations of many different quantities. There is likely to be continued improvement in the apparent robustness of the lattice results as more quantities are calculated with increasing precision.
- As of now there are results from
 - six different quantities,
 - five different groups on three continents,
 - four different fermion discretizations.
- Results are completely independent and consistent, and each is more precise than the most precise non-lattice determination.

Prospects for improvement

- Future Higgs factories like the high-luminosity ILC aim to measure branching fractions to sub-per cent accuracy.
- Since parameter uncertainties will limit the predictive power of theory, some have questioned value of such measurements.
- Lattice determinations will have no problem improving to the needed accuracy.

	$\delta m_b(10)$	$\delta \alpha_s(m_Z)$	$\delta m_c(3)$	δ_b	δ_c	δ_g	
current errors [10]	0.70	0.63	0.61	0.77	0.89	0.78	Lepage, Mackenzie, and Peskin estimated improvements to parameters and partial width predictions arising from improved perturbation theory, reduced lattice spacing, and increased statistics.
+ PT	0.69	0.40	0.34	0.74	0.57	0.49	
+ LS	0.30	0.53	0.53	0.38	0.74	0.65	
+ LS ²	0.14	0.35	0.53	0.20	0.65	0.43	
+ PT + LS	0.28	0.17	0.21	0.30	0.27	0.21	
+ PT + LS ²	0.12	0.14	0.20	0.13	0.24	0.17	
+ PT + LS ² + ST	0.09	0.08	0.20	0.10	0.22	0.09	
ILC goal				0.30	0.70	0.60	

Table 1: Projected fractional errors, in percent, for the $\overline{\text{MS}}$ QCD coupling and heavy quark masses under different scenarios for improved analyses. The improvements considered are: PT - addition of 4th order QCD perturbation theory, LS, LS² - reduction of the lattice spacing to 0.03 fm and to 0.023 fm; ST - increasing the statistics of the simulation by a factor of 100. The last three columns convert the errors in input parameters into errors on Higgs couplings, taking account of correlations. The bottom line gives the target values of these errors suggested by the projections for the ILC measurement accuracies.

Lepage, Mackenzie, Peskin;
arXiv:1404.0319v2.

Conclusions

- Lattice calculations now provide the most precise determinations of α_s and m_c . They soon will also provide the most precise determination of m_b .
- Now that lattice methods are well established, they are starting to play a role in unexpected places throughout the HEP program, such as Higgs physics.

Backup slides



Perturbative coefficients for moments

TABLE III. Perturbation theory coefficients ($n_f = 3$) for r_n [2–6]. Coefficients are defined by $r_n = 1 + \sum_{j=1} r_{nj} \alpha_{\overline{\text{MS}}}^j(\mu)$ for $\mu = m_h(\mu)$. The third-order coefficients are exact for $4 \leq n \leq 10$. The other coefficients are based upon estimates; we assign conservative errors to these.

n	r_{n1}	r_{n2}	r_{n3}
4	0.7427	−0.0577	0.0591
6	0.6160	0.4767	−0.0527
8	0.3164	0.3446	0.0634
10	0.1861	0.2696	0.1238
12	0.1081	0.2130	0.1(3)
14	0.0544	0.1674	0.1(3)
16	0.0146	0.1293	0.1(3)
18	−0.0165	0.0965	0.1(3)

HPQCD take uncalculated coefficients in series for moments $r_{nj} \sim \mathcal{O}(0.5 \alpha_s(m_q)^j)$; further constrain the possible sizes for coefficients by comparing nonperturbative results for many quark masses with perturbation theory using Bayesian priors for higher order terms.