Nuclear Matrix Elements for Neutrinoless Double-Beta Decay

J. Engel

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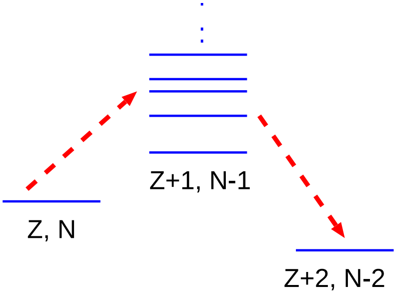
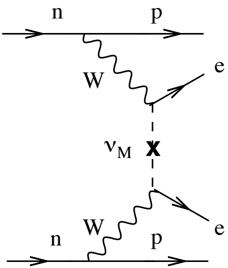
$0\nu\ \beta\beta$ Decay

If energetics are right (ordinary β decay forbidden)…

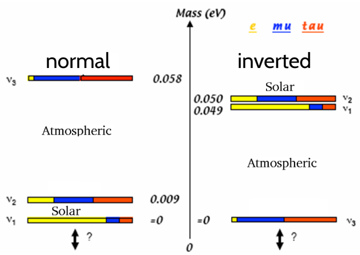


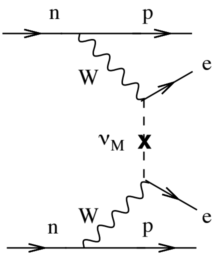
and neutrinos are their own antiparticles…

can observe two neutrons turning into protons, emitting two electrons and nothing else, e.g. via



Considerations



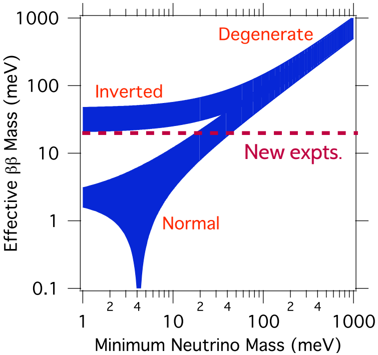


In usual scenario, rate depends on effective neutrino mass:

$$m_{\rm eff} \equiv \sum_i m_i U_{ei}^2$$

If lightest neutrino is light:

- $m_{\rm eff} \propto \sim \sqrt{\Delta m^2_{\rm sol}}$ normal
- $m_{\rm eff} \propto \sim \sqrt{\Delta m^2_{\rm atm}}$ inverted



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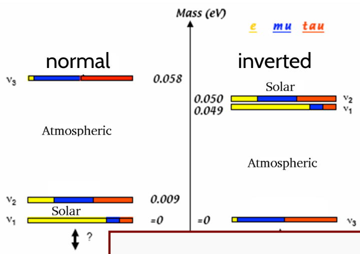

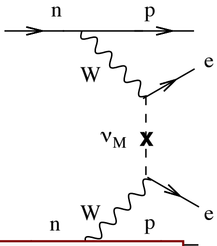

But rate also depends on a nuclear matrix element.

In usua
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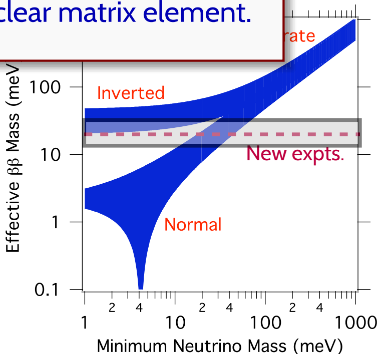
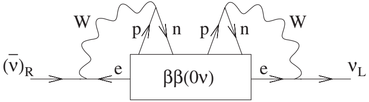
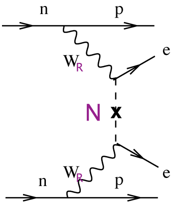


Other Mechanisms Can Contribute

If neutrinoless decay occurs then ν's are Majorana, no matter what:



but light neutrinos may not drive the decay:



Exchange of heavy right-handed neutrino in left-right symmetric model.

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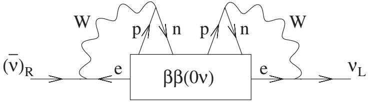

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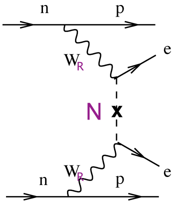

Exchange of heavy right-handed neutrino
in left-right symmetric model.

Amplitude of exotic mechanism:

$$\frac{Z_{0\nu}^{\text{heavy}}}{Z_{0\nu}^{\text{light}}} \approx \left(\frac{M_{W_L}}{M_{W_R}}\right)^4 \left(\frac{\langle q^2\rangle}{m_{\text{eff}}\, m_N}\right)$$

$\langle q^2\rangle \approx 10^4$ MeV^2

$$\approx \mathbf{1} \quad \text{if} \quad m_N \approx 1\,\text{TeV} \quad \text{and} \quad m_{\text{eff}} \approx \sqrt{\Delta m^2_{\text{atm}}}$$

So exotic exchange can occur with roughly the same rate as light-ν exchange. Untangling would seem to require several expts and accurate nuclear matrix elements for all processes.

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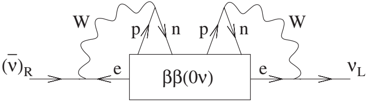

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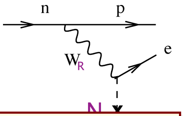

Exchange of heavy right-handed neutrino

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But apparently, LHC should either see many such things or rule them out as competition to light-ν exchange.

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Light-ν-Exchange Matrix Element

$$M_{0\nu} = M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^{F} + \dots$$

with

$$M_{0\nu}^{GT} = \langle F| \, | \sum_{i,j} H(r_{ij}) \, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \; \tau_i^+ \tau_j^+ \; |I\rangle + \dots$$

$$M_{0\nu}^{F} = \langle F| \sum_{i,j} H(r_{ij}) \; \tau_i^+ \tau_j^+ \; |I\rangle + \dots$$

$$H(r) \approx \frac{2R}{\pi r} \int_0^\infty dq \frac{\sin qr}{q + \bar{E} - (E_i + E_f)/2} \quad \text{roughly} \propto 1/r$$

Contribution to integral peaks at $q \approx 200$ MeV inside nucleus. Corrections are from "forbidden" terms, weak nucleon form factors, many-body currents …

Nuclear-Structure Methods in One Slide

- **Density Functional Theory & Related Techniques:** Mean-field-like theory plus relatively simple corrections in very large single-particle space with phenomenological interaction.
- **Shell Model:** Partly phenomenological interaction in a small single-particle space – a few orbitals near nuclear Fermi surface – but with arbitrarily complex correlations.
- **Ab Initio Calculations:** Start from a well justified two-nucleon + three-nucleon Hamiltonian, then solve full many-body Schröd equation to good accuracy in space large enough to include all important correlations. At present, works pretty well in systems near closed shells up to $A \approx 50$.

New!

- **Interacting Boson Model:** Model for collective states (as bosonic excitations).

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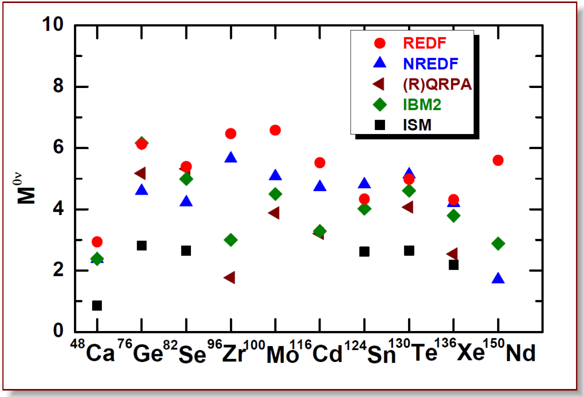
Has potential to combine and ground virtues of shell model and density functional theory.

Level of Agreement So Far

Significant spread. And all the models could be missing important physics.

Uncertainty hard to quantify.



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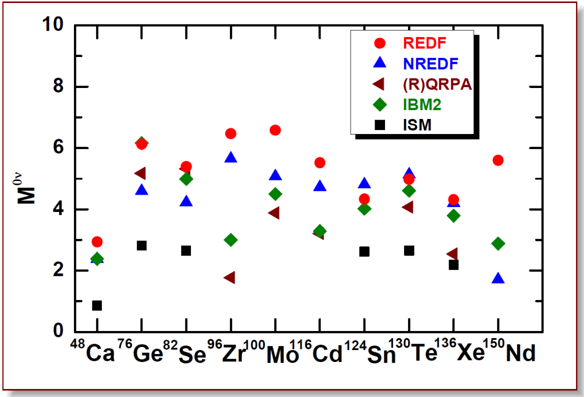
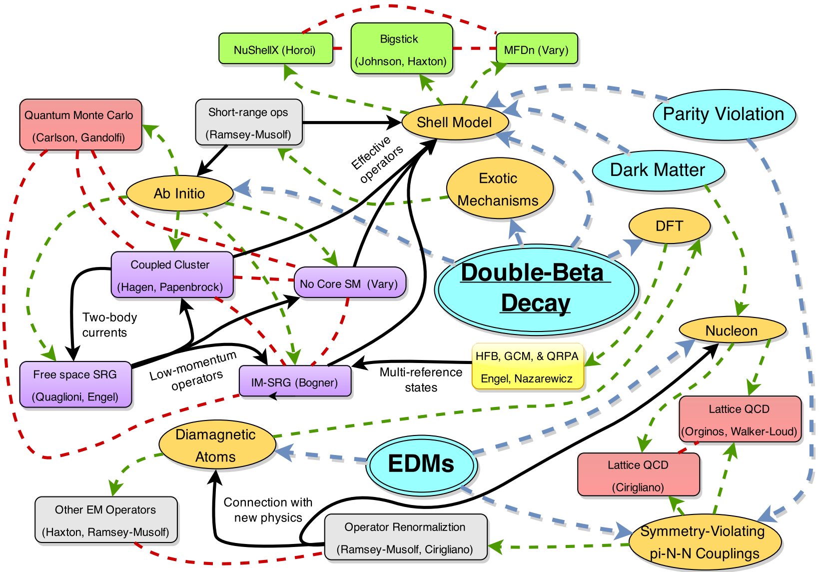


More computing power and new many-body methods responsible for major recent progress in ab initio theory.

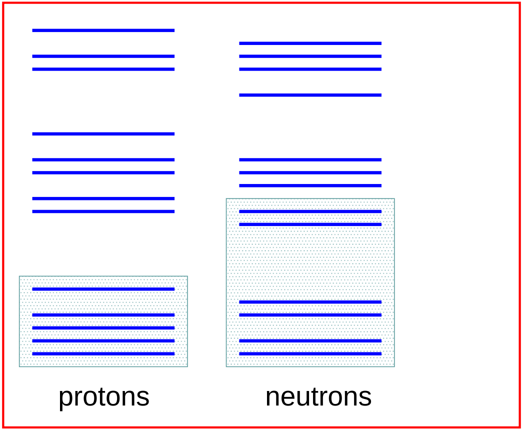
Theorists are organizing; should be able to do better now.

ββ and Fund. Symmetries Topical DOE Collaboration



Traditional Shell Model

Starting point: set of single-particle orbitals in an average potential.



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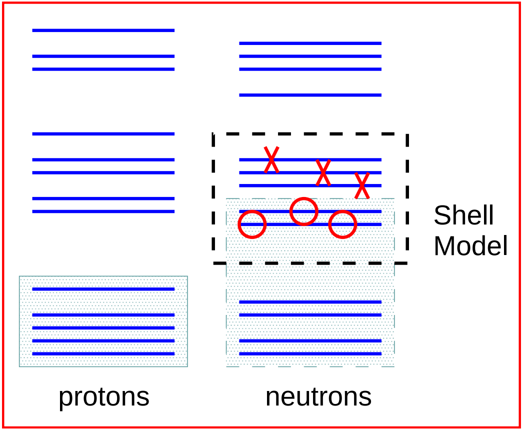
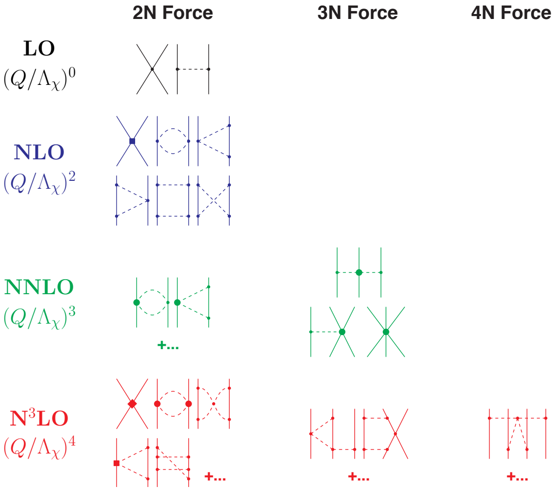


Shell model neglects all but a few orbitals around the Fermi surface, uses phenomenological Hamiltonian.

Ab Initio Nuclear Structure in Heavy Nuclei

Typically starts with chiral effective field theory; degrees of freedom are nucleons and pions below the chiral-symmetry breaking scale.



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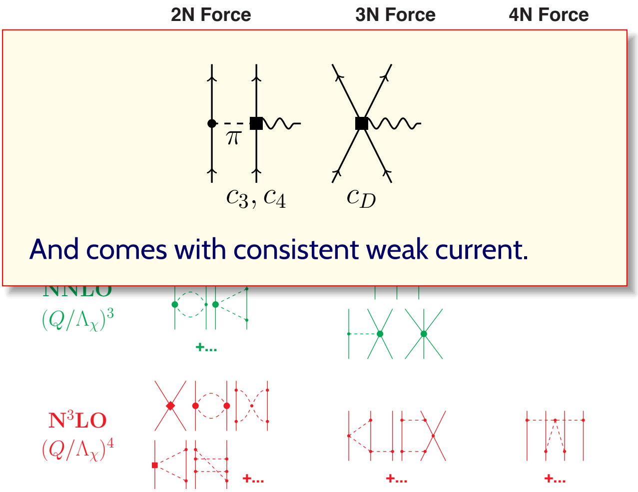
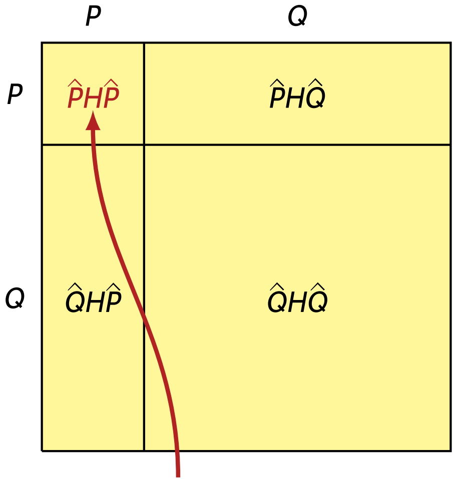


Ab Initio Shell Model

Because un-doctored ab initio calculations far from closed shells still difficult

Partition of Full Hilbert Space



Shell model done here.

P = valence space
Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with H_{eff} in P reproducing most important eigenvalues.

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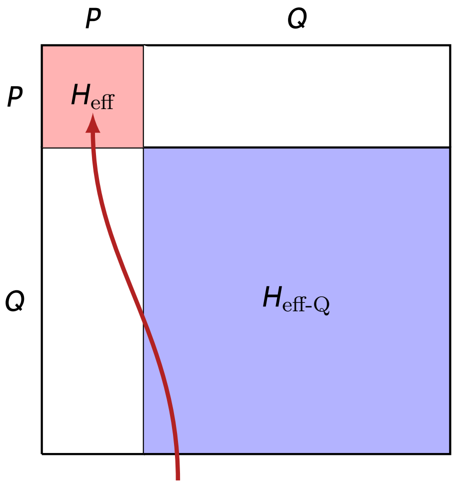

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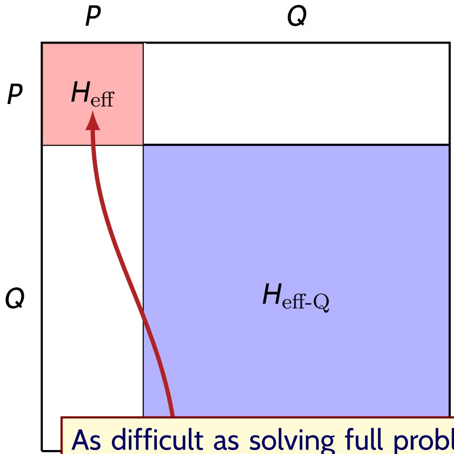

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As difficult as solving full problem. But idea is that N-body effective operators should not be important for $N > 2$ or 3.

Method 1: Coupled-Cluster Theory

Ground state in closed-shell nucleus:

$$|\Psi_0\rangle = e^T |\varphi_0\rangle \qquad T = \sum_{i,m} t_i^m a_m^\dagger a_i + \sum_{ij,mn} \frac{1}{4} t_{ij}^{mn} a_m^\dagger a_n^\dagger a_i a_j + \dots$$

Slater determinant

$m,n>F \quad i,j<F$

States in closed-shell + a few constructed in similar way.

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<u>Construction of Unitary Transformation to Shell Model:</u>

1. Complete calculation of low-lying states in nuclei with 1, 2, and 3 nucleons outside closed shell (where calculations are feasible).
2. Lee-Suzuki mapping of lowest eigenstates onto shell-model space, determine effective Hamiltonian and decay operator.

   Lee-Suzuki maps lowest eigenvectors to orthogonal vectors in shell model space in way that minimizes difference between mapped and original vectors.
3. Use these operators in shell-model for $\beta\beta$-decaying nucleus.

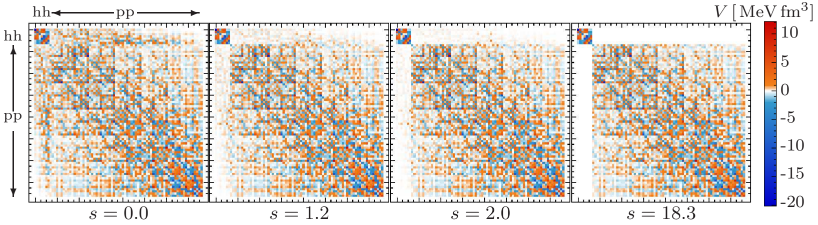
Method 2: In-Medium Similarity Renormalization Group

Flow equation for effective Hamiltonian.

Shell-model space asymptotically decoupled.

$$\frac{d}{ds}H(s) = [\eta(s), H(s)], \qquad \eta(s) = [H_d(s), H_{od}(s)], \quad H(\infty) = H_{\mathrm{eff}}$$

d = diagonal $\qquad$ od = off diagonal

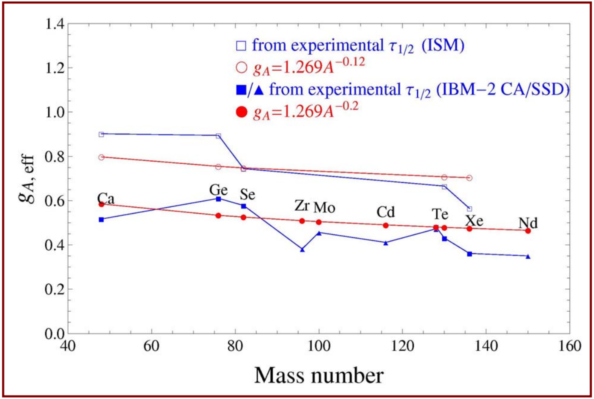


Hergert et al.

Development about as far along as coupled clusters.

Related Issue Facing All Calculations: "g_A"

40-Year-Old Problem Particularly Important in ββ Decay:
Effective g_A needed for two-neutrino decay in shell model and IBM



F. Iachello, MEDEX'13 meeting

If 0ν matrix elements quenched by same amount, experiments will be less sensitive; rates go like fourth power of g_A.

We Should Resolve the Issue Soon

Problem must be due to some combination of:

1. Truncation of model space.

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2. Many-body weak currents.

   Size still not clear, particularly for $0\nu\beta\beta$ decay, where current is needed at finite momentum transfer q.

   Leading terms in chiral EFT for finite q only recently worked out. Careful determination and use in decay computations will happen in next year or two.

Finally…

Existence of topical collaboration will speed progress in next few years on this and other fronts:

- Uncertainty quantification
- Other mechanisms for $\beta\beta$ decay, short-range physics

$\vdots$

Goal is accurate matrix elements with quantified uncertainty by end of collaboration (5 years from now).

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That's all.