Lattice QCD
Properties and Interactions of Light Nuclei

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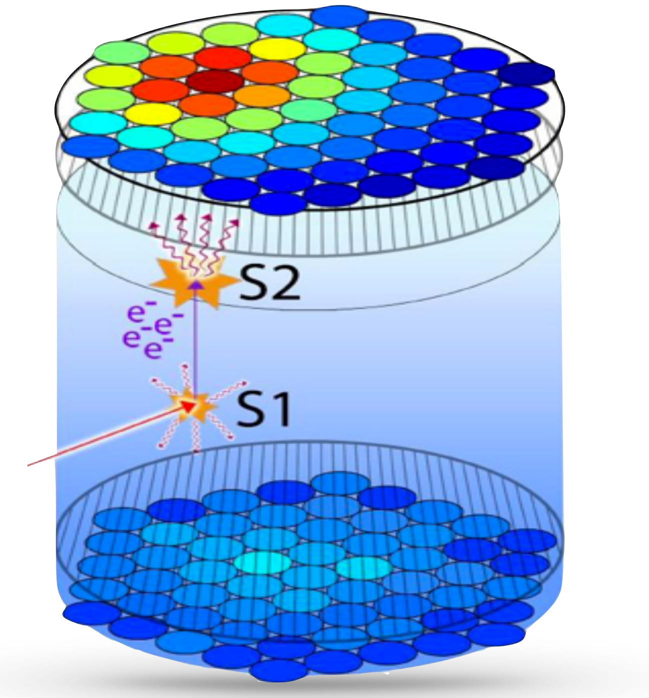
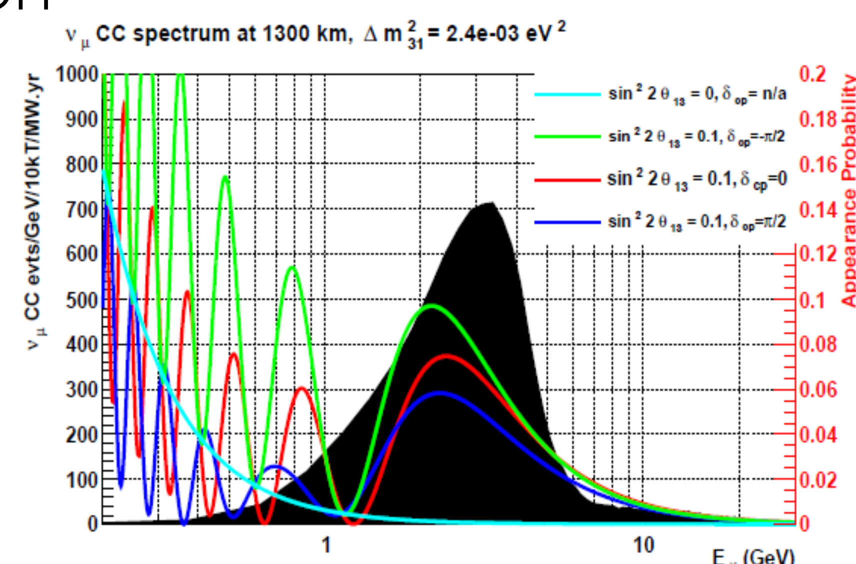
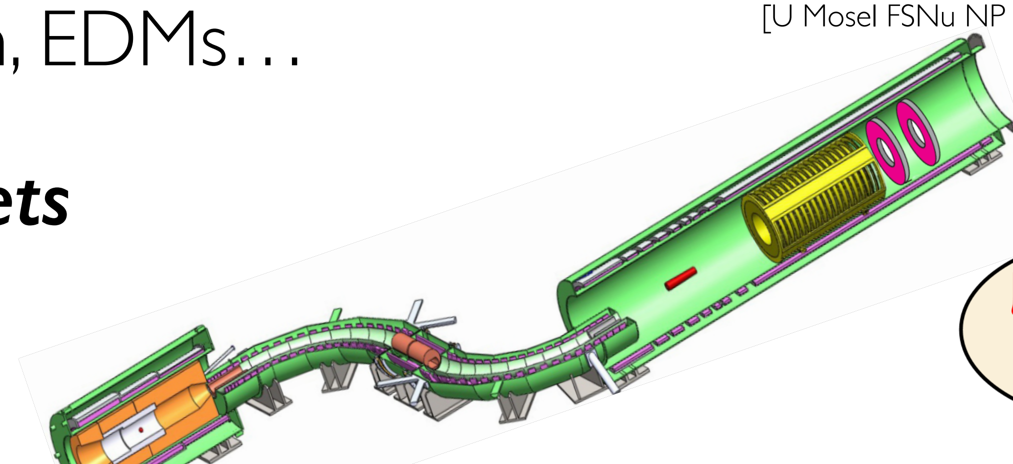
- Seek new physics through quantum effects
- Precise experiments
  - Sensitivity to probe the rarest interactions of the SM
  - Look for effects where there is no SM contribution
- Important focus of HEP(NP) experimental program
  - Dark matter direct detection
  - Neutrino physics
  - Charged lepton flavour violation, EDMs…
- ***Major component is nuclear targets***



http://www.hep.ucl.ac.uk/darkMatter/

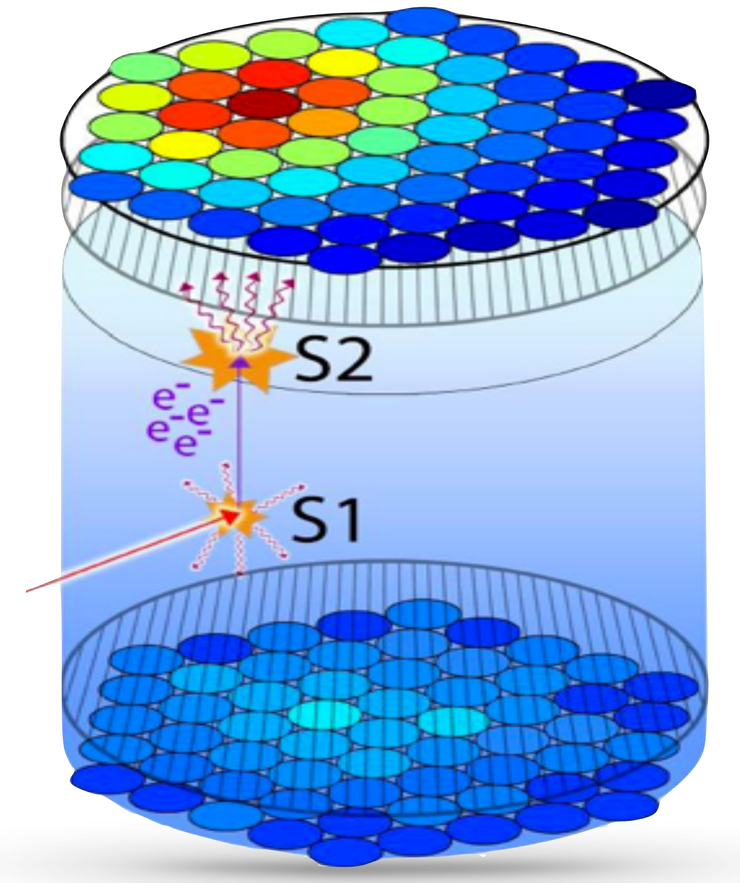


[U Mosel FSNu NP town meeting]



The intensity frontier

- Dark matter direct detection: nuclear recoils in large bucket of nuclei as signal
  - Detection rate/bounds depends on dark matter properties/dynamics and x-sec on nucleus
- 😍 Positive signals would be unambiguous
- 😟 Post-detection: precise nuclear x-sec (with quantified uncertainties) to discern underlying dynamics
- Potentially understand seemingly conflicting positive and negative signals
- Inform experimental design and backgrounds



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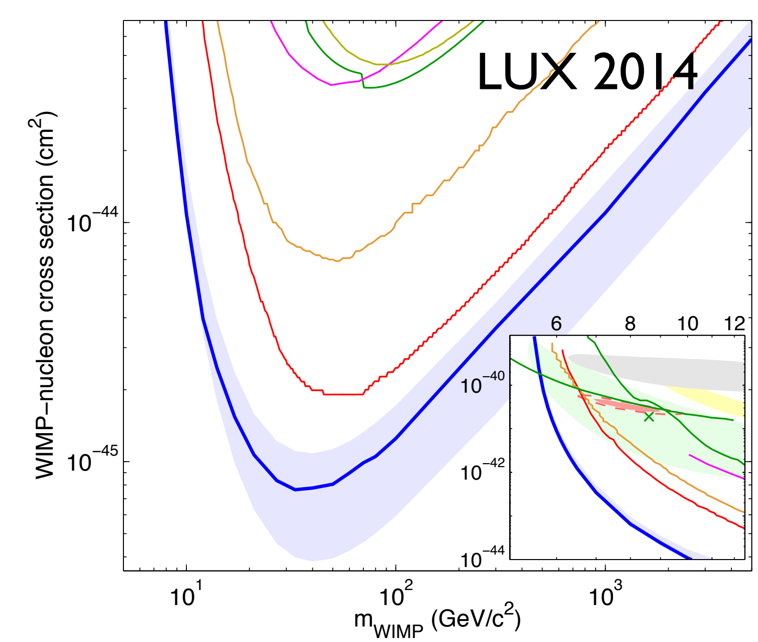


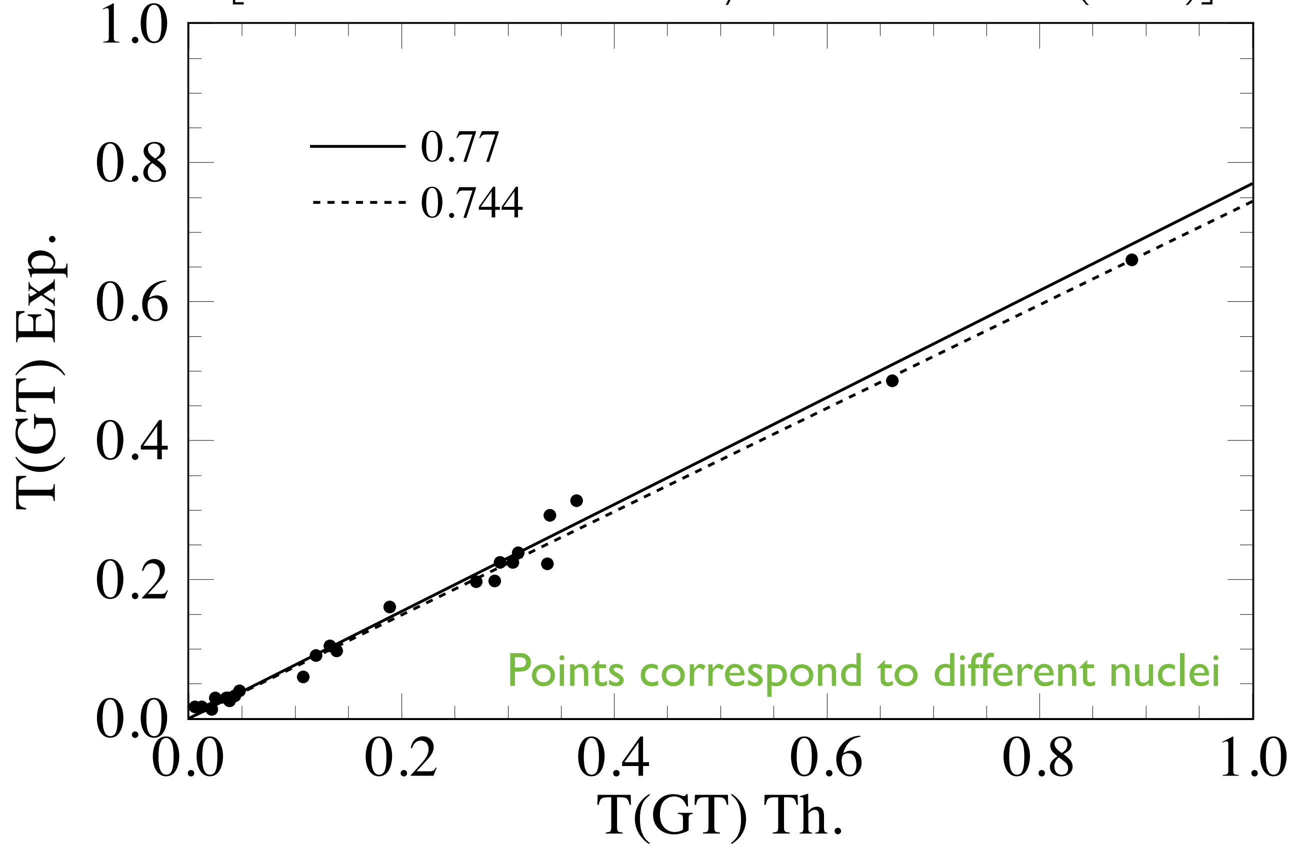
Nuclear uncertainties

- How well do we know nuclear matrix elements?

😢 Stark example of problems: Gamow-Teller transitions in nuclei

- Well measured for large range of nuclei (30<A<60)
- Many nuclear structure calcs (QRPA, shell-model,…) – spectrum well described
- Matrix elements systematically off by 20–30%
- "Correct" by "quenching" axial charge in nuclei ...

[Martinez-Pinedo et al., Phys. Rev. C53, 2602 (1996)]

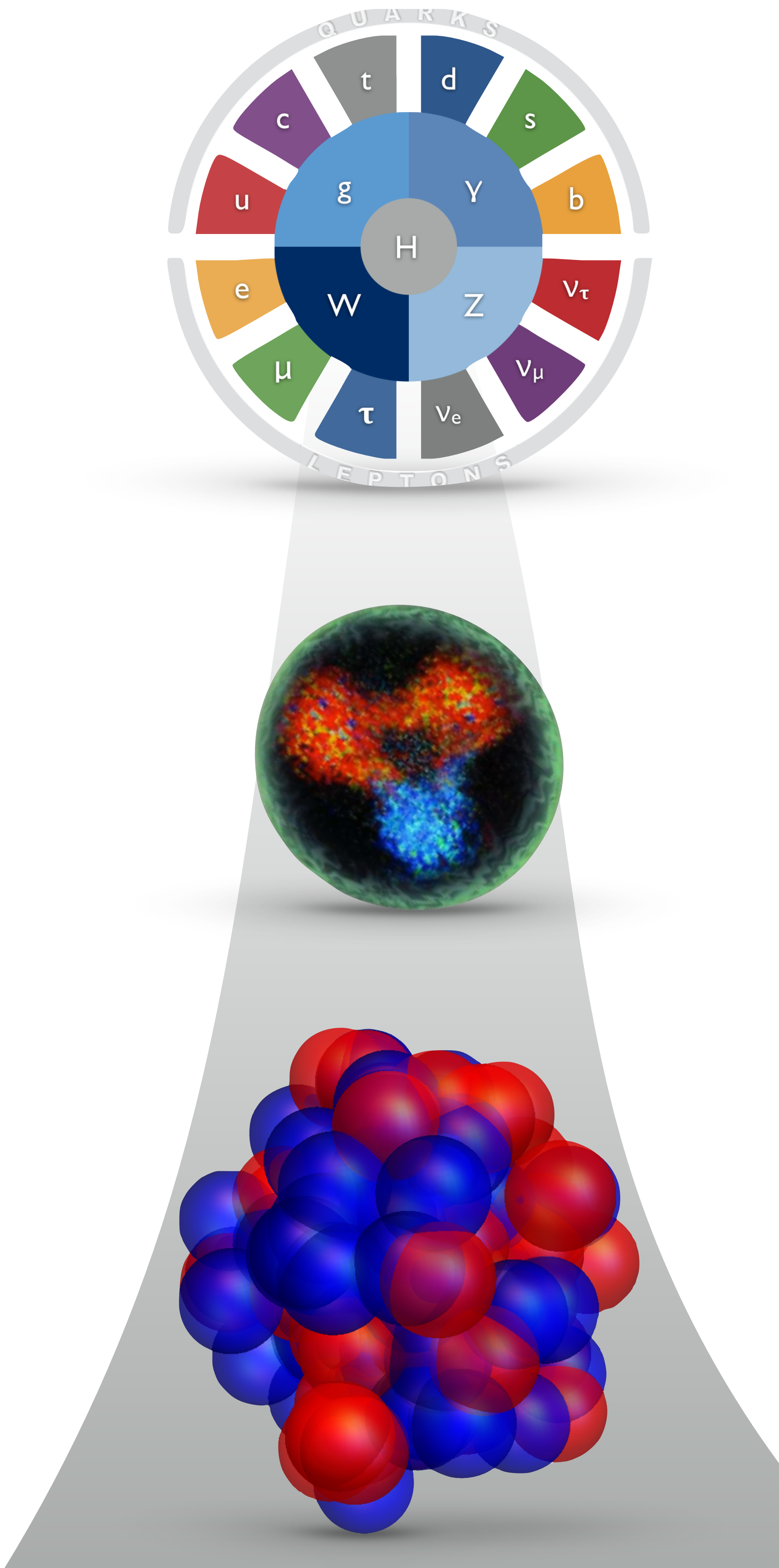


$$T(GT) \sim \sqrt{\sum_f \langle \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \rangle_{i \to f}}$$

$$\langle \boldsymbol{\sigma\tau} \rangle = \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}^k_{\pm} || i \rangle}{\sqrt{2J_i + 1}}$$

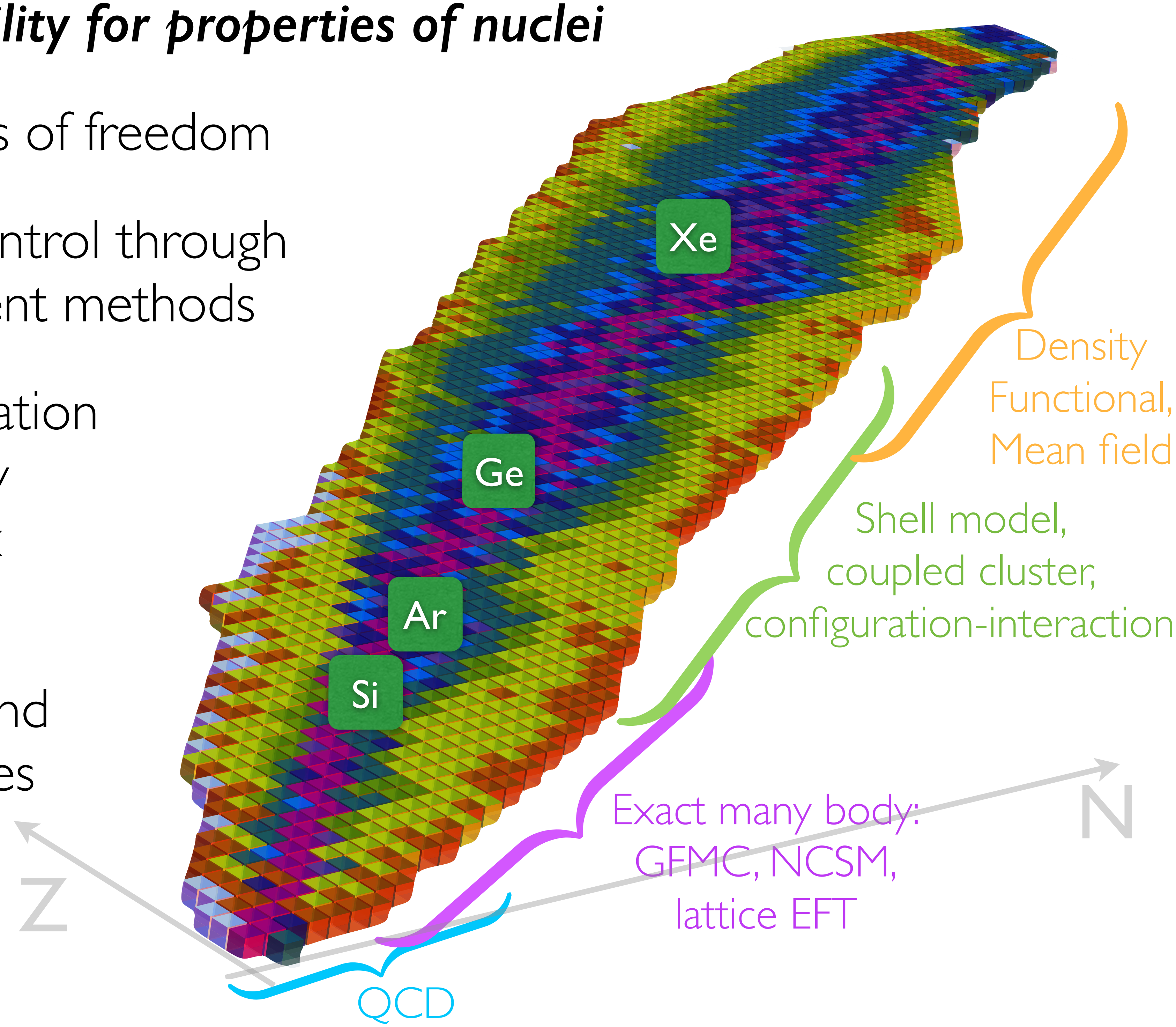
Nuclear theory at the intensity frontier

- Coming need for precision determinations of nuclear matrix elements
  - Must be based on the Standard Model (no hand-waving)
  - Must have fully quantified uncertainties
  - Timeframe and precision goals set by experiment
- Current state is far from this
- Need to develop appropriate tools



Precision Nuclear Physics

- ***Goal: Predictive capability for properties of nuclei***
- Exploit effective degrees of freedom
- Establish quantitative control through linkages between different methods
  - QCD forms a foundation determines few body interactions & matrix elements
  - Match existing EFT and many body techniques onto QCD

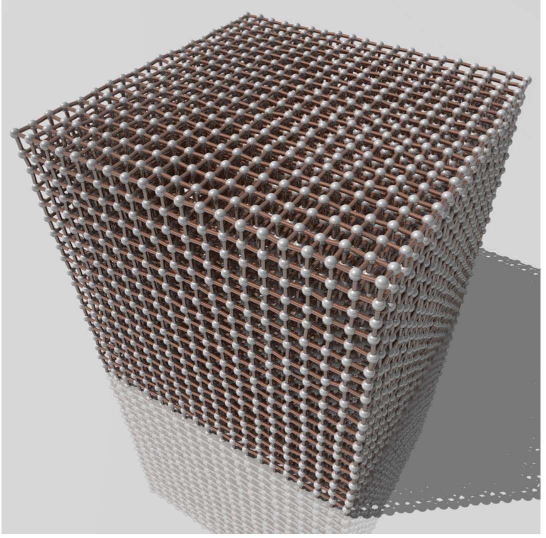


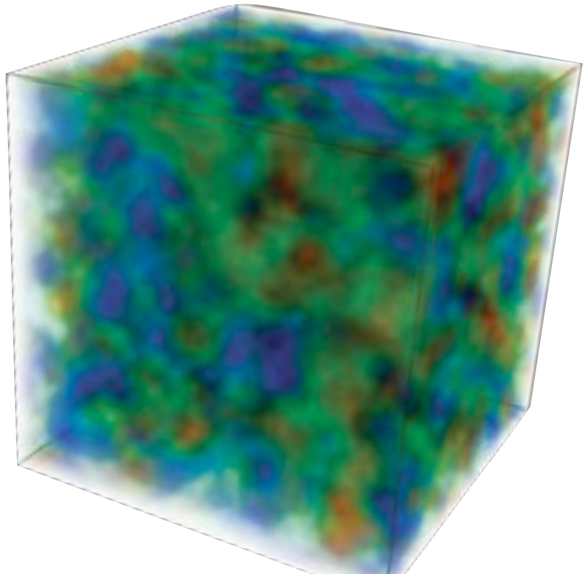
Quantum Chromodynamics

- Lattice QCD: tool to deal with quarks and gluons
  - Formulate problem as functional integral over quark and gluon d.o.f. on R_4

$$\langle \mathcal{O} \rangle = \int dA_\mu dq d\bar{q}\, \mathcal{O}[q, \bar{q}, A] e^{-S_{QCD}[q, \bar{q}, A]}$$

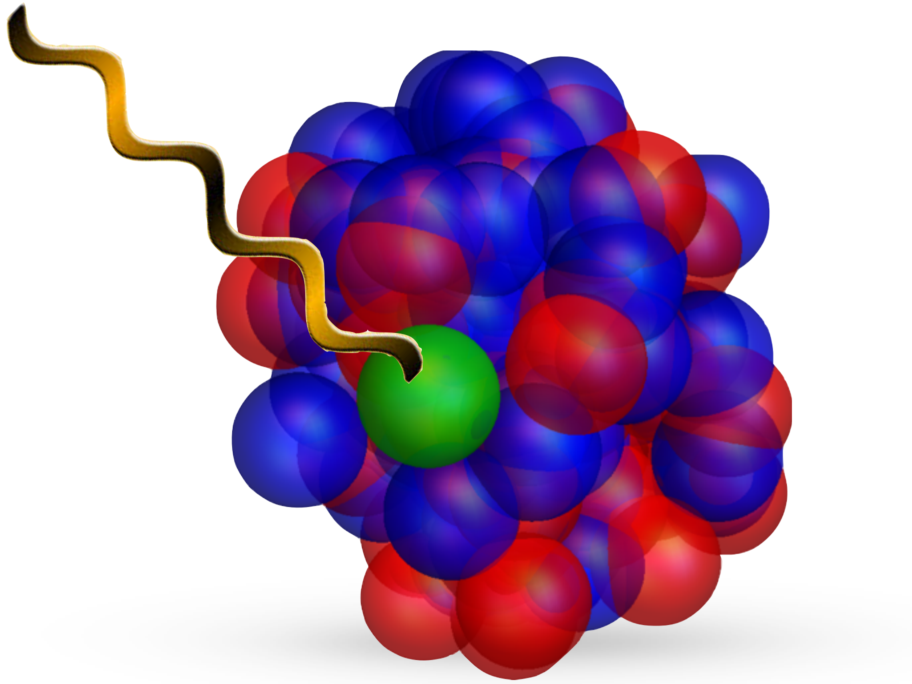
  - Discretise and compactify system
    - Finite but large number of d.o.f $(\sim 10^{10})$
  - Integrate via importance sampling (average over important configurations)
  - Undo the harm done in previous steps

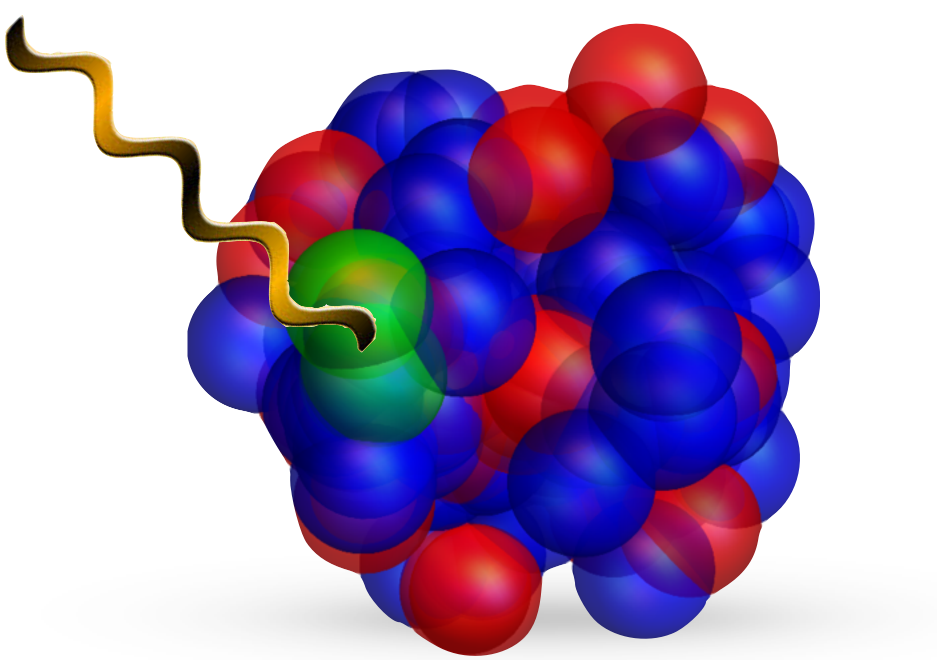




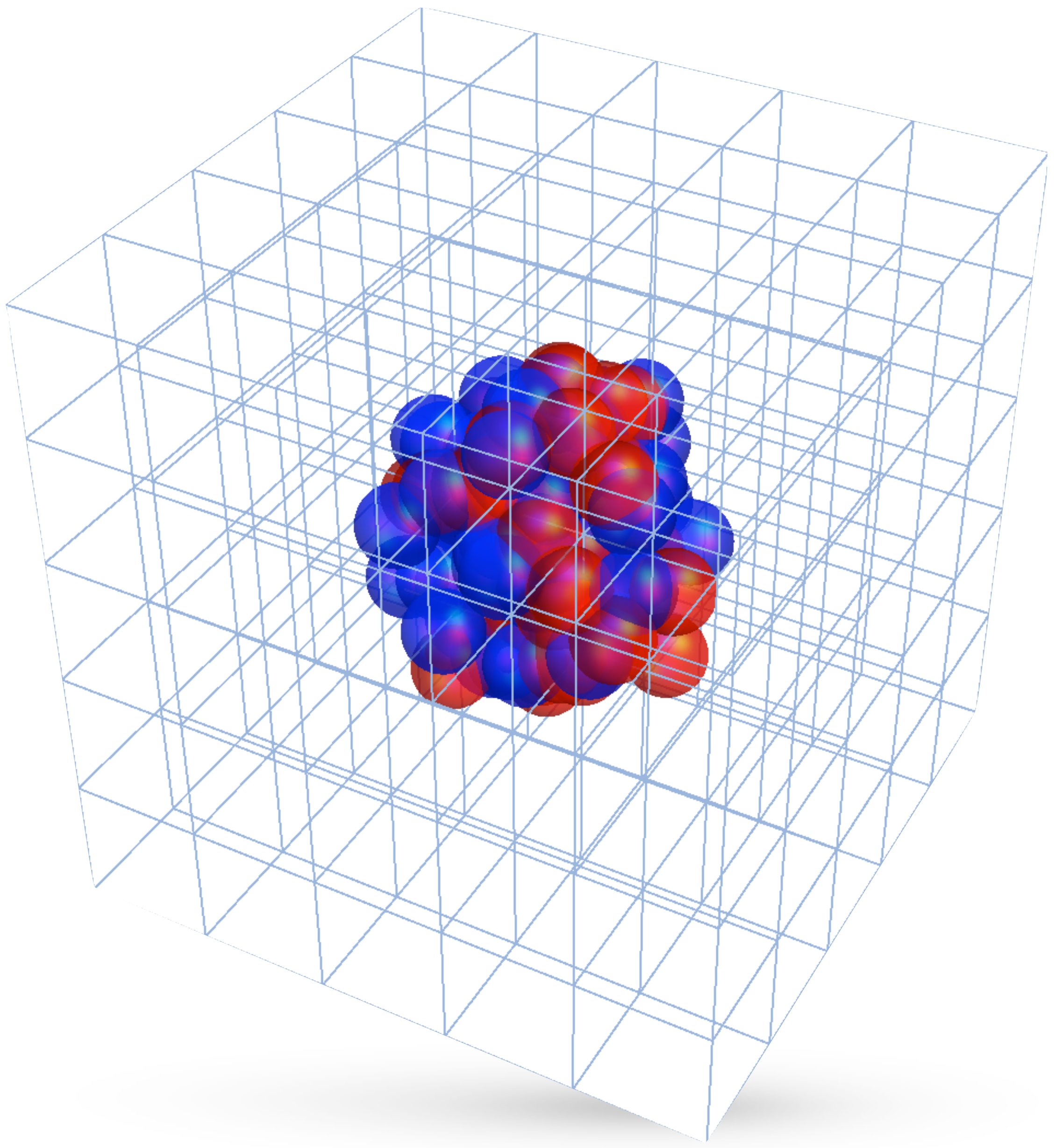
External currents and nuclei

- Xe in LQCD not likely any time soon
- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading *but non-negligible*
- LQCD: determine one body current from single nucleon
- LQCD: determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to make predictions for larger nuclei

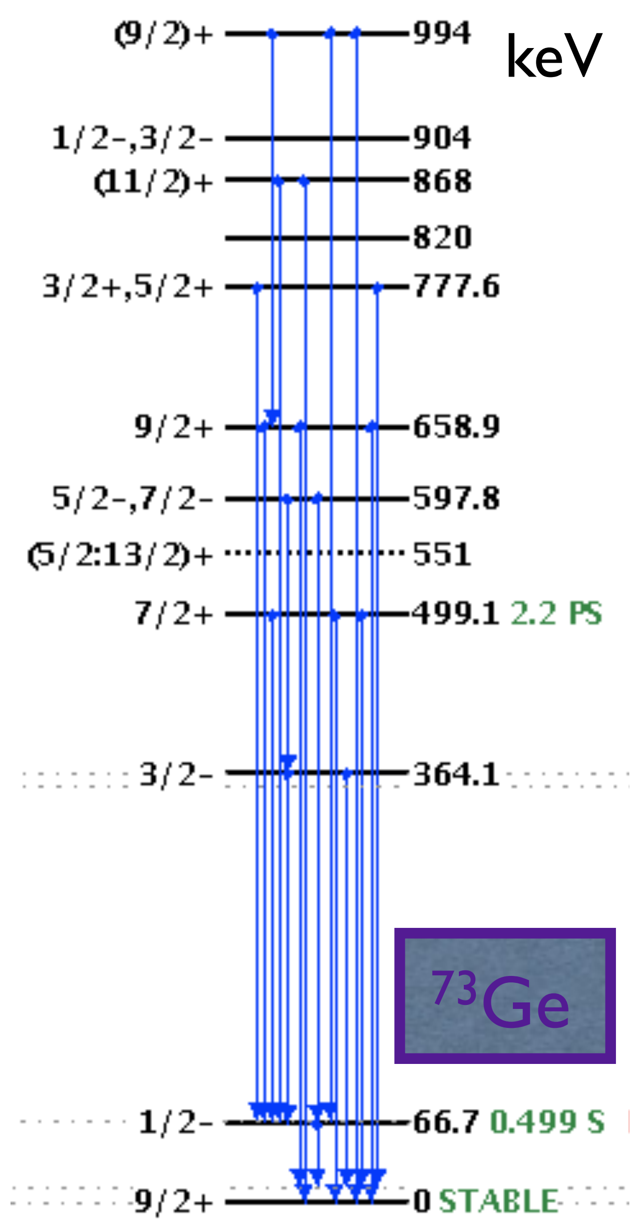




- Nuclei in LQCD are a hard

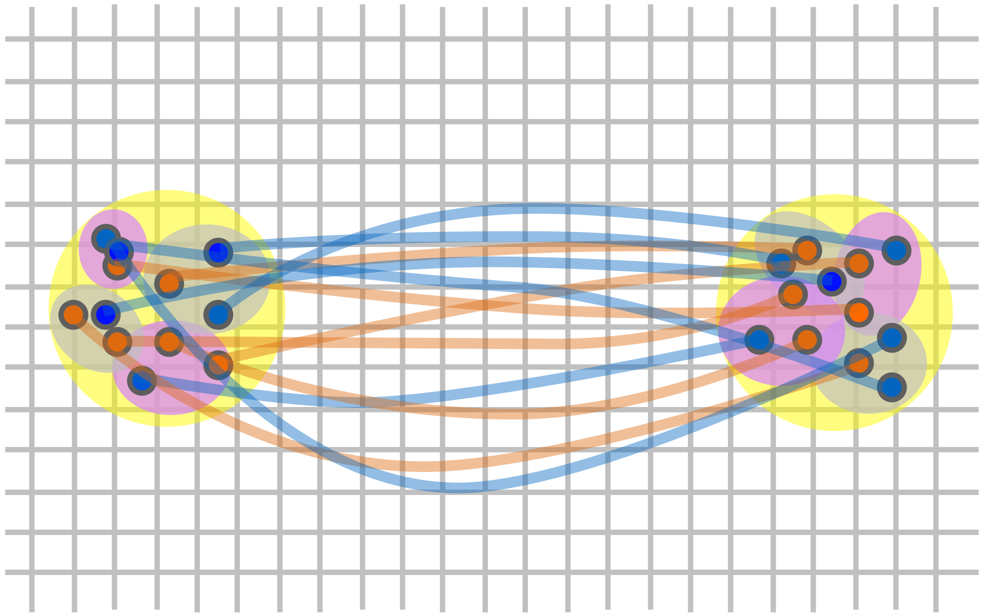


- Nuclei in LQCD are a hard
- Physics at multiple scales



QCD for Nuclear Physics

- Nuclei in LQCD are a hard
- Physics at multiple scales
- Two exponentially difficult challenges for LQCD
  - Contraction complexity grows factorially
  - Probabilistic method statistical uncertainty grows exponentially with A (naively)



Unphysical nuclei

- NPLQCD collaboration
- Case study QCD with

$$m_u = m_d = m_s^{\mathrm{phys}}$$

$$\begin{aligned} m_\pi &\sim 800 \text{ MeV} \\ m_p &\sim 1,600 \text{ MeV} \end{aligned}$$

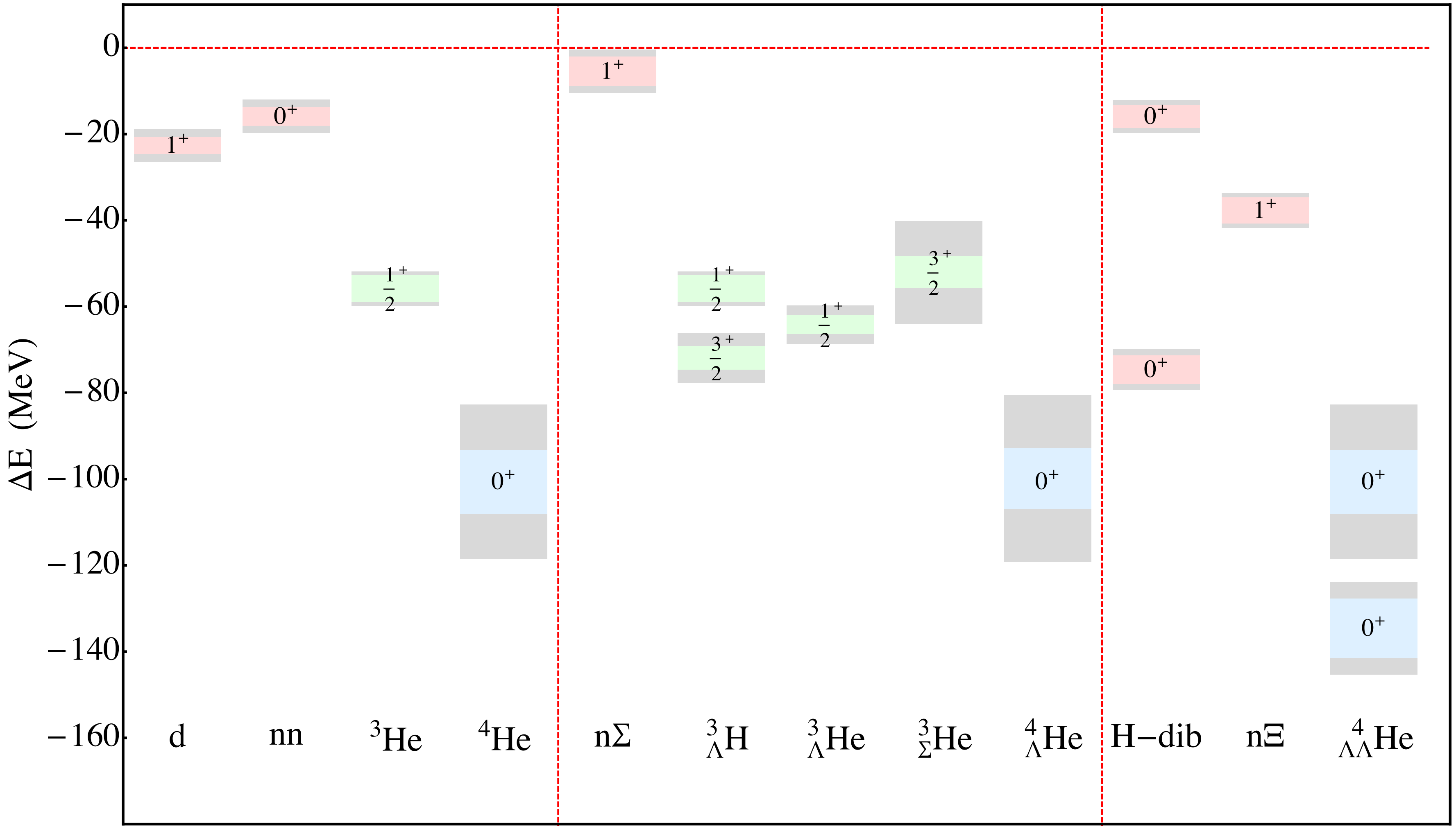
1. Spectrum of light nuclei (A<5); NN interactions
   [PRD **87** (2013), 034506, PRC **88** (2013), 024003]]

2. Nuclear structure: magnetic moments, polarisabilities (A<5)
   [PRL **113**, 252001 (2014)]

3. Nuclear reactions: np→dγ
   [PRL **115**, 132001 (2015)]

Light nuclei

- Light hypernuclear binding energies @ m_π=800 MeV



- More states bound; deeper bindings;

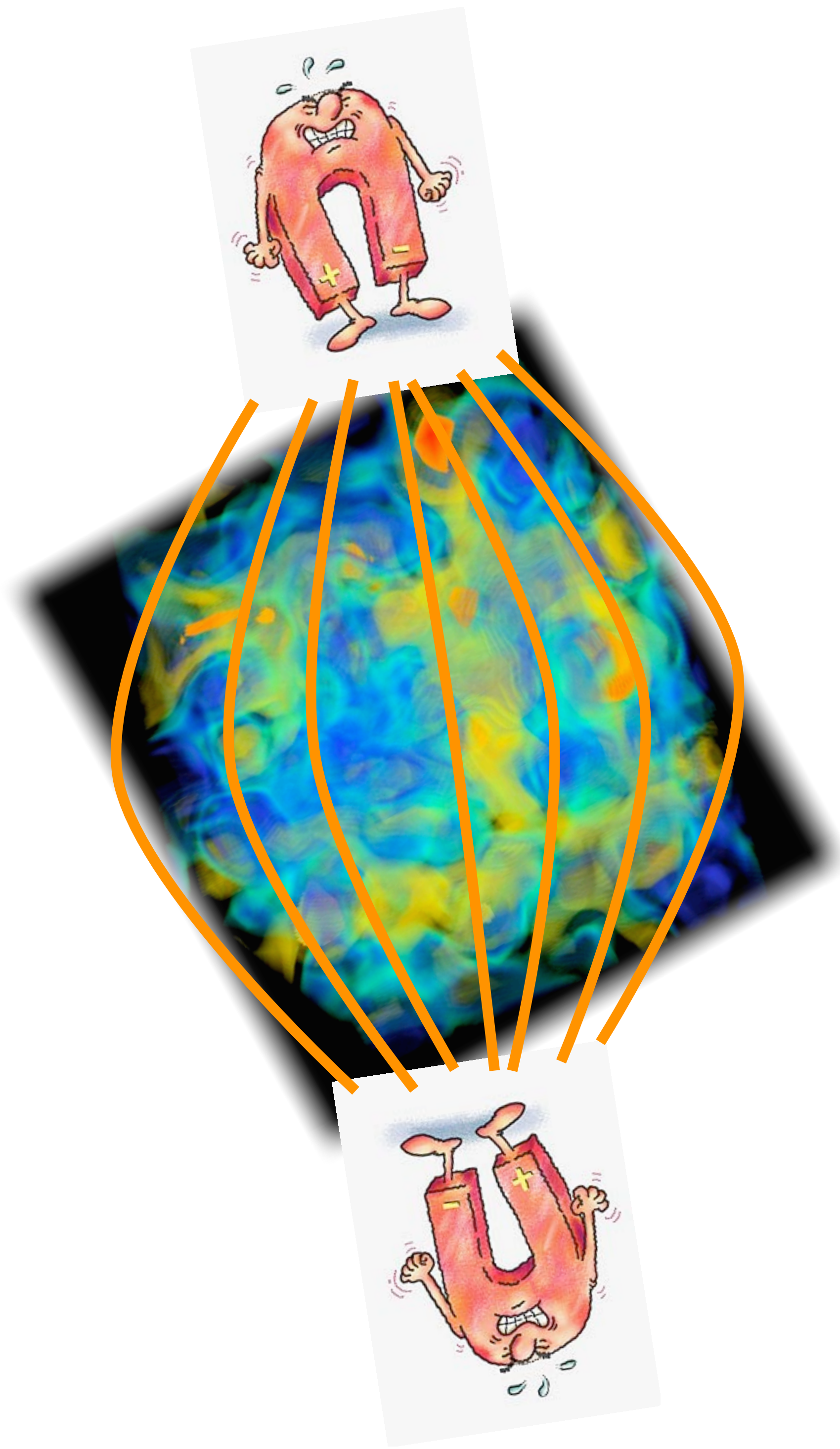
[NPLQCD PRD **87** (2013), 034506]

Magnetic moments of nuclei

- Hadron/nuclear energies are modified by presence of fixed external fields
- Eg: fixed B field

$$E_{h;j_z}(\mathbf{B}) \;=\; \sqrt{M_h^2 + (2n+1)|Q_h e B|} - \boldsymbol{\mu}_h \cdot \mathbf{B} \\ \quad - 2\pi \beta_h^{(M0)} |\mathbf{B}|^2 - 2\pi \beta_h^{(M2)} \langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- QCD calculations with multiple fields enable extraction of coefficients of response
  - Eg: magnetic moments, polarisabilities, …
  - Not restricted to simple EM fields (axial, twist-2,…)



[NPLQCD PRL **113**, 252001 (2014)]

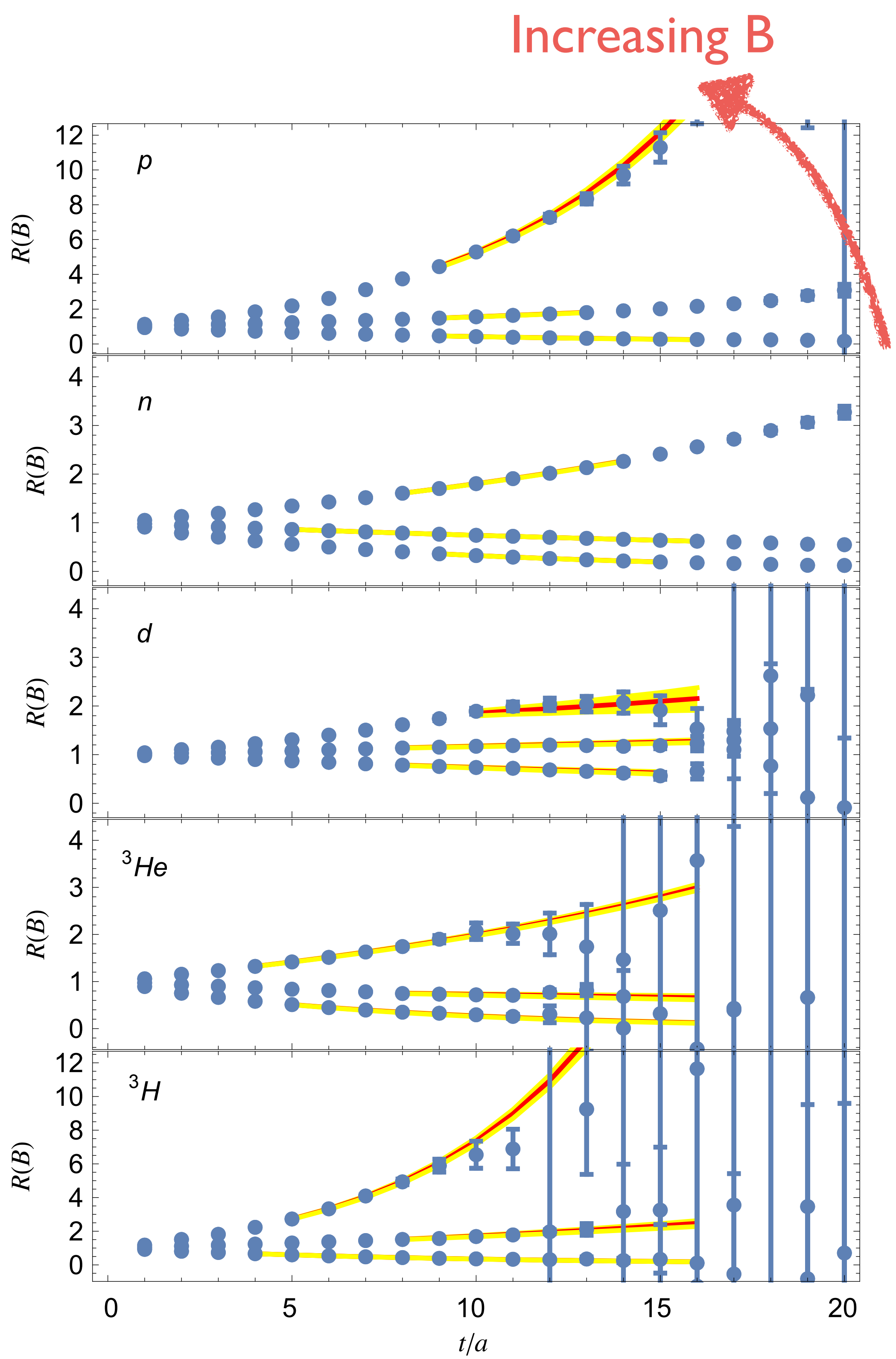
- Magnetic field in *z*-direction (strength quantised by lattice periodicity)
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E^{(B)}_{+j} - E^{(B)}_{-j} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of correlation functions

$$R(B) = \frac{C^{(B)}_j(t)\; C^{(0)}_{-j}(t)}{C^{(B)}_{-j}(t)\; C^{(0)}_j(t)} \overset{t\to\infty}{\longrightarrow} Z e^{-\delta E^{(B)} t}$$

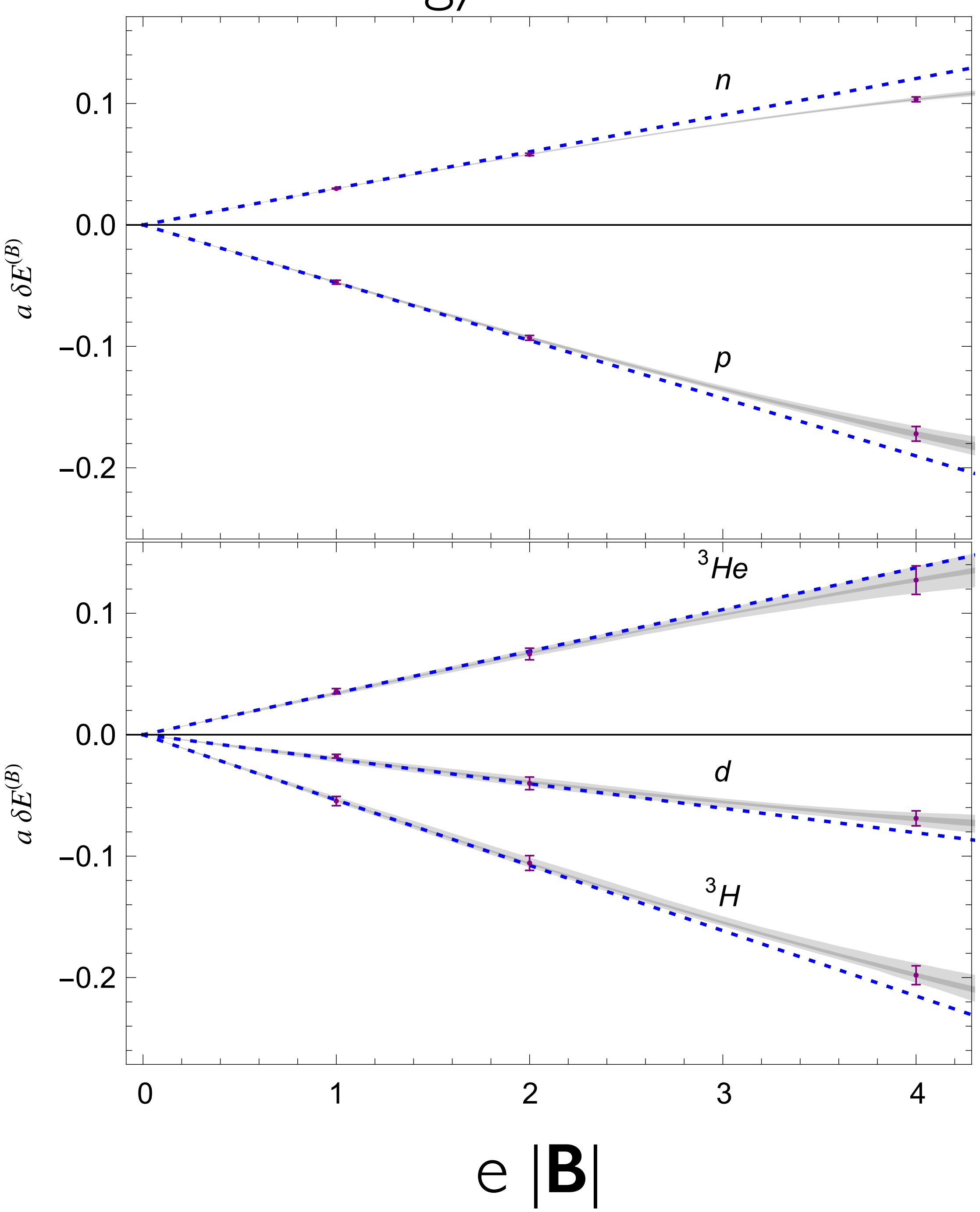
- Careful to be in single exponential region of each correlator

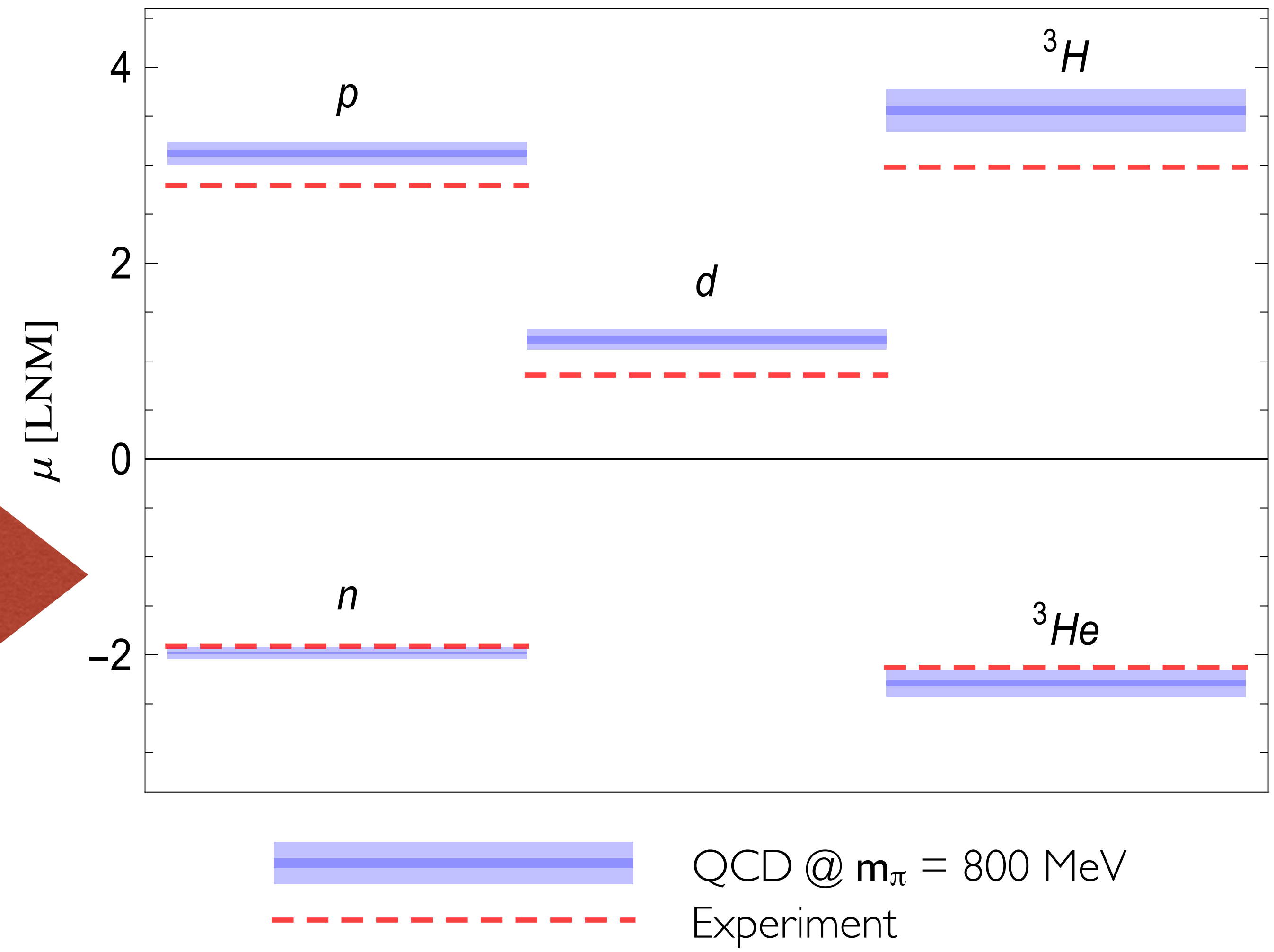


[NPLQCD PRL **113**, 252001 (2014)]

Magnetic moments of nuclei

Energy shift vs B





	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

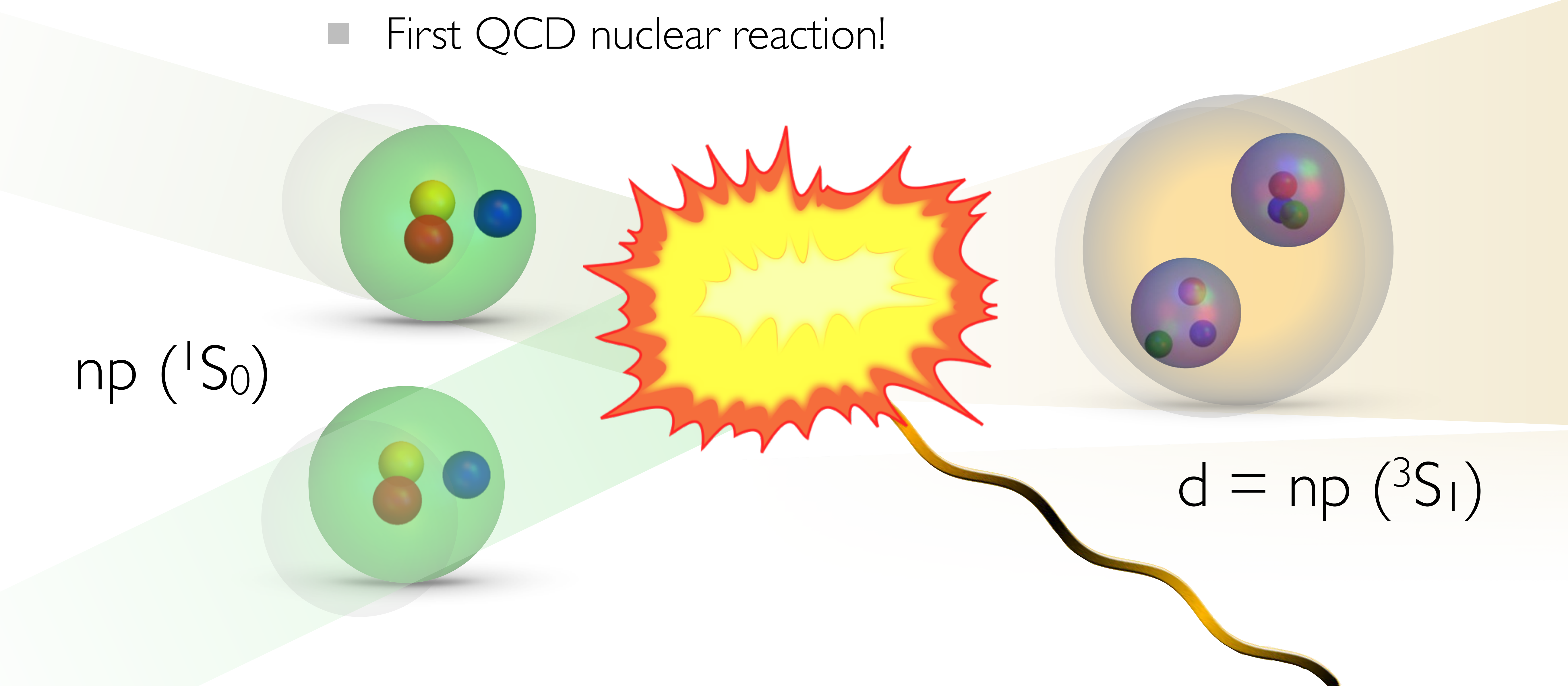
In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL **113**, 252001 (2014)]

Thermal Neutron Capture Cross-Section

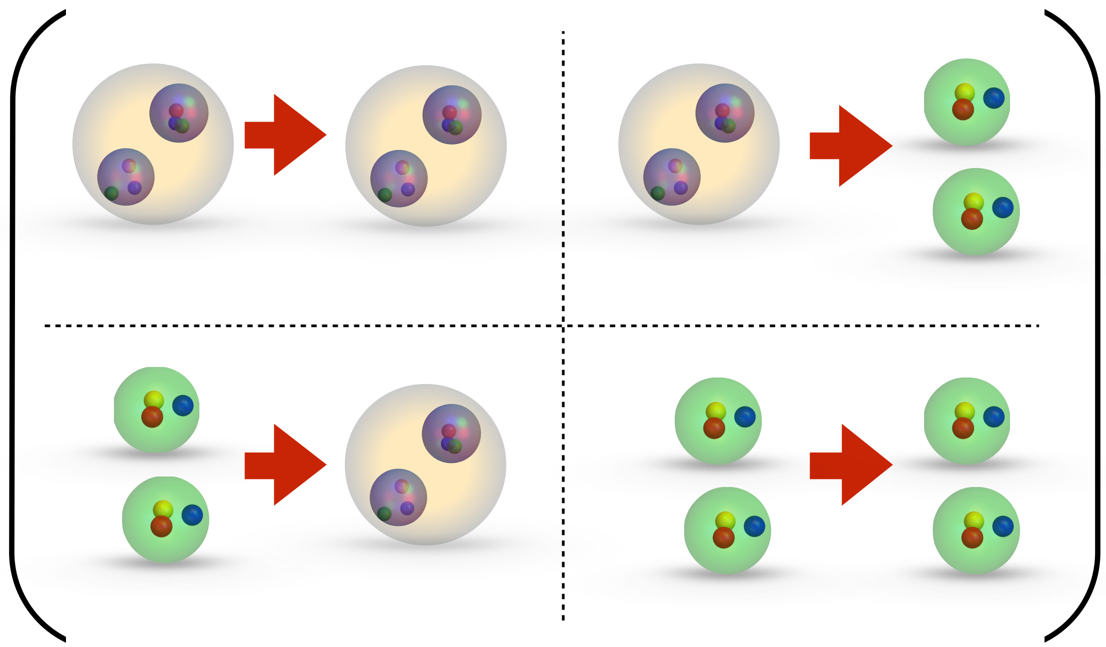
[NPLQCD PRL 115, 132001 (2015)]

- Thermal neutron capture cross-section: np→dγ
  - Critical process in Big Bang Nucleosynthesis
  - Historically important: nucleus is not just nucleons
  - First QCD nuclear reaction!



np→dγ

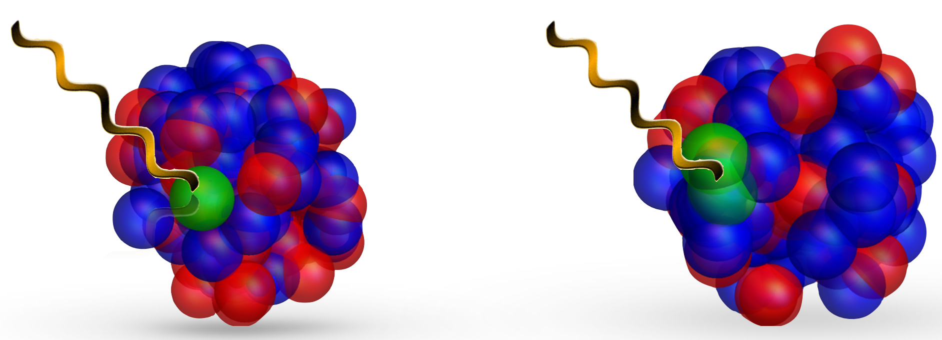
- Presence of magnetic field mixes $I_z=J_z=0$ $^3S_1$ and $^1S_0$ *np* systems

H = 

- Calculate energies in presence of B fields

- Shift of eigenvalues determined by transition amplitude
  [WD, & M Savage 2004]

$$\Delta E_{^3S_1,{}^1S_0} = \mp\left(\kappa_1 + \overline{L}_1\right)\frac{eB}{M} + \ldots$$



[NPLQCD PRL **115**, 132001 (2015)]

np→dγ

Lattice correlator with $^3S_1$ source and $^1S_0$ sink

- Iz=Jz=0 correlation matrix

$$\mathbf{C}(t;\mathbf{B}) = \begin{pmatrix} C_{^3S_1,^3S_1}(t;\mathbf{B}) & C_{^3S_1,^1S_0}(t;\mathbf{B}) \\ C_{^1S_0,^3S_1}(t;\mathbf{B}) & C_{^1S_0,^1S_0}(t;\mathbf{B}) \end{pmatrix}$$

- Generalised eigenvalue problem

$$[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}\mathbf{C}(t;\mathbf{B})[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}v = \lambda(t;\mathbf{B})v$$

- Ratio of correlator ratios to extract 2-body

$$R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{\lambda_+(t;\mathbf{B})}{\lambda_-(t;\mathbf{B})} \xrightarrow{t\to\infty} \hat{Z}\exp\left[2\ \Delta E_{^3S_1,^1S_0} t\right]$$

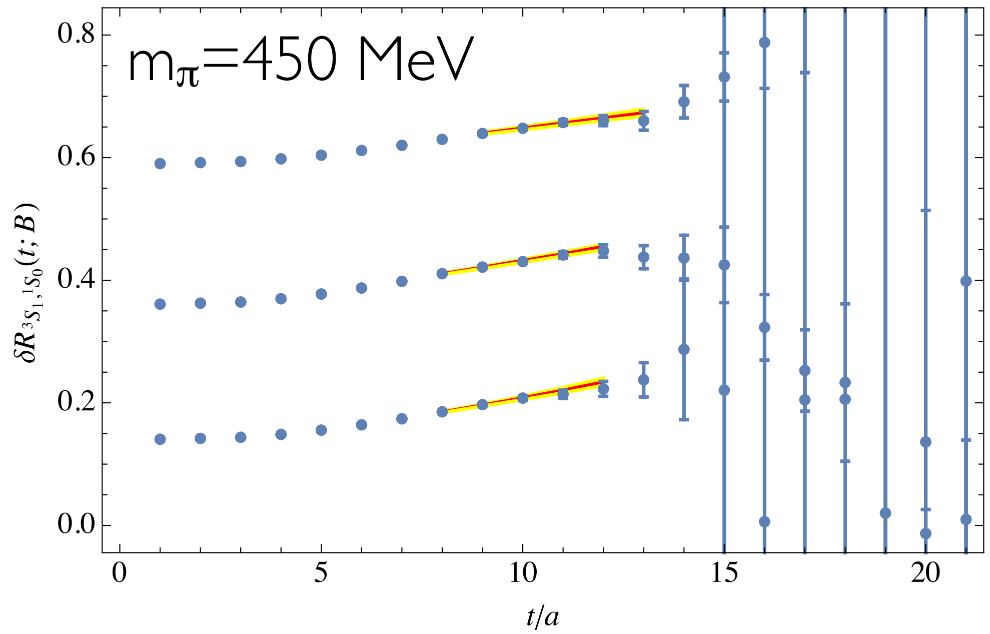
$$\delta R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{R_{^3S_1,^1S_0}(t;\mathbf{B})}{\Delta R_p(t;\mathbf{B})/\Delta R_n(t;\mathbf{B})} \rightarrow A\ e^{-\delta E_{^3S_1,^1S_0}(\mathbf{B})t}$$

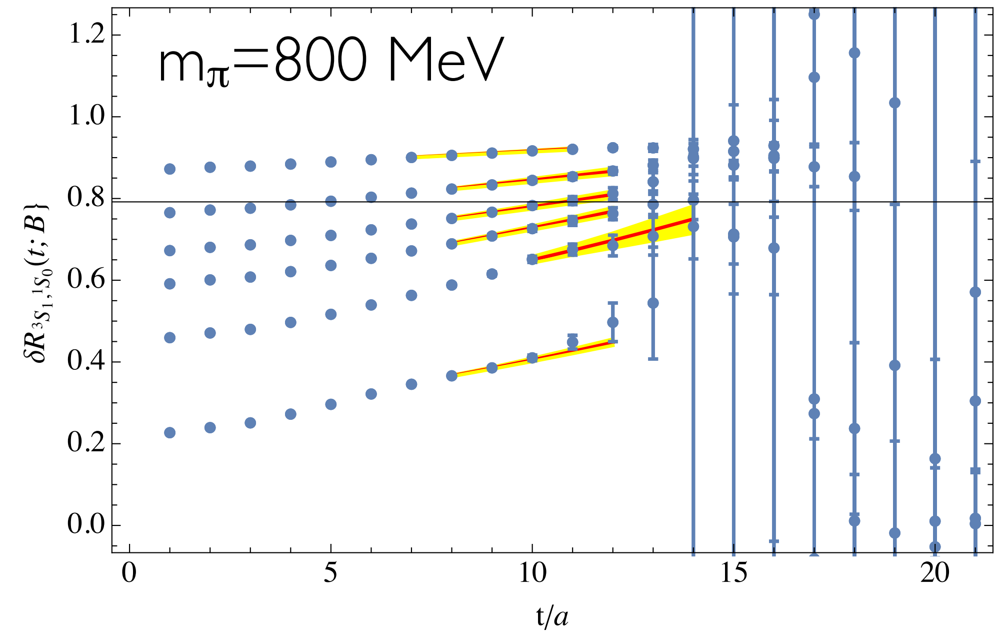
$$\begin{aligned} \delta E_{^3S_1,^1S_0} &\equiv \Delta E_{^3S_1,^1S_0} - [E_{p,\uparrow} - E_{p,\downarrow}] + [E_{n,\uparrow} - E_{n,\downarrow}] \\ &\rightarrow 2\overline{L}_1|e\mathbf{B}|/M + \mathcal{O}(\mathbf{B}^2) \end{aligned}$$

[NPLQCD PRL **115**, 132001 (2015)]

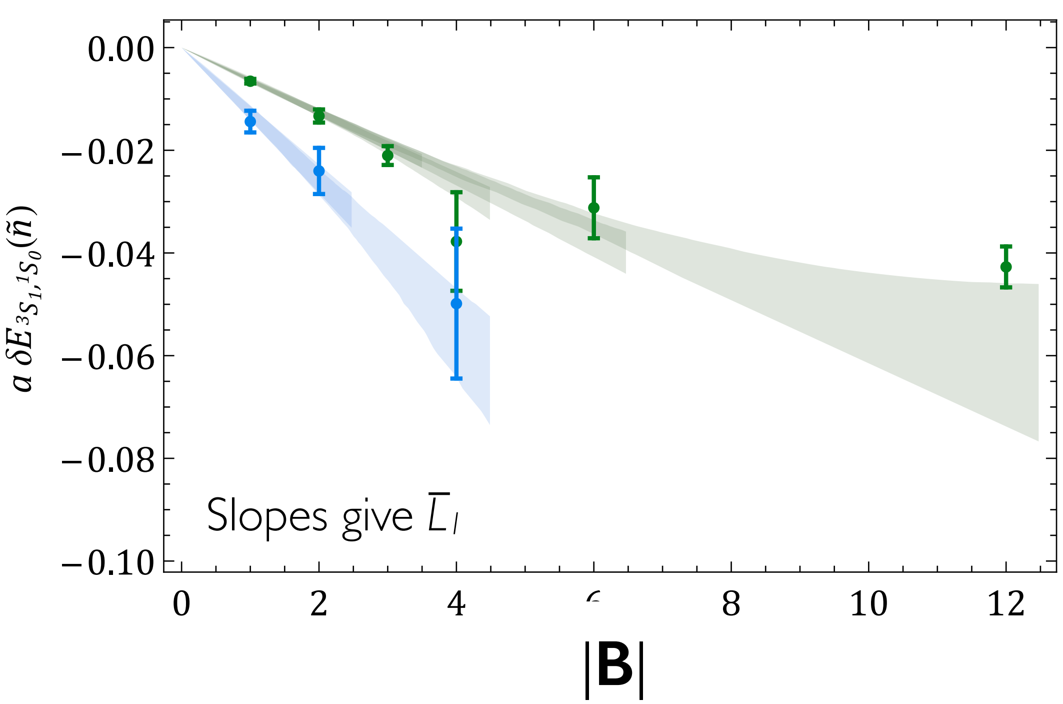
np→dγ

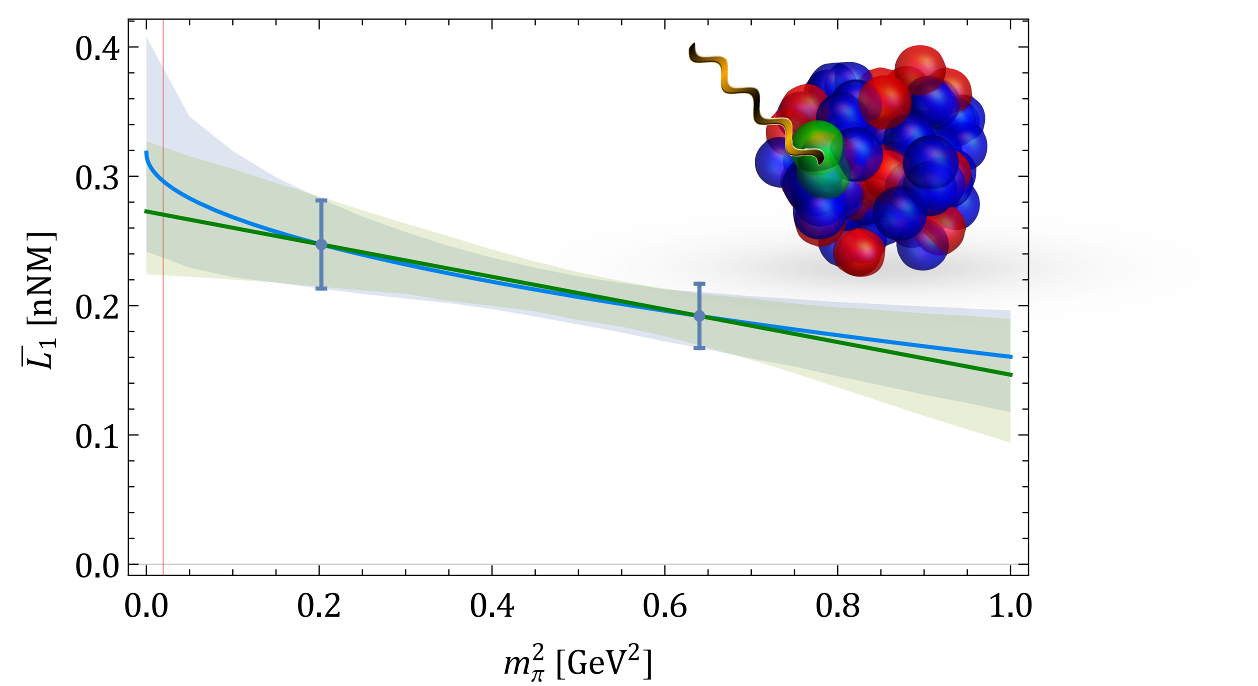
- Correlator ratios





- Field strength & mass dependence





[NPLQCD PRL **115**, 132001 (2015)]

np→dγ

- Key point: extract short-distance contribution at physical mass

$$\overline{L}_1^{\rm lqcd} = 0.285(\,^{+63}_{-60}\,) \quad \rm NM$$

- Use EFT to combine with phenomenological nucleon magnetic moment, scattering parameters

$$\sigma^{\rm lqcd}(np \to d\gamma) = 307.8(1 + 0.273\ \overline{L}_1^{\rm lqcd})\ \rm mb$$

$$\sigma^{\rm lqcd}(np \to d\gamma) = 332.4(\,^{+5.4}_{-4.7}\,)\ \rm mb$$

c.f. phenomenological value

$$\sigma^{\rm expt}(np \to d\gamma) = 334.2(0.5)\ \rm mb$$

- NB: at m_π=800 MeV, use LQCD for all inputs (ab initio)

$$\sigma^{800\ \rm MeV}(np \to d\gamma) \sim 10\ \rm mb$$

[NPLQCD PRL **115**, 132001 (2015)]

QCD for nuclei

- Nuclei are under serious study directly from QCD
  - Spectroscopy of light nuclei and exotic nuclei
  - Structure: magnetic moments and polarisabilities
  - Electromagnetic interactions: thermal capture cross-section
  - Weak interactions: *M Savage, Neutrinos 09:52 am Sat*
- Prospect of a quantitative connection to QCD makes this a very exciting time for nuclear physics
  - Same techniques work for IF relevant matrix elements
  - Critical role in current and upcoming intensity frontier experimental program

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