Light Flavor Physics from Domain Wall Lattice QCD

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Domain Wall Lattice QCD

- Non-perturbative QCD calculations by Monte Carlo integration

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\overline{\psi}\, \mathcal{O}\, e^{-S_E[U,\overline{\psi},\psi]} \rightarrow \frac{1}{N} \sum_{i=1}^{N} \mathcal{O} e^{-S_E[U_i]}$$

- Simulate $2+1$ quark flavors with Iwasaki gauge action
- Domain wall fermions retain excellent chiral symmetry on lattice
- Large range of simulated masses, volumes, and cutoffs allow precise extraction of physical limits with controlled systematics

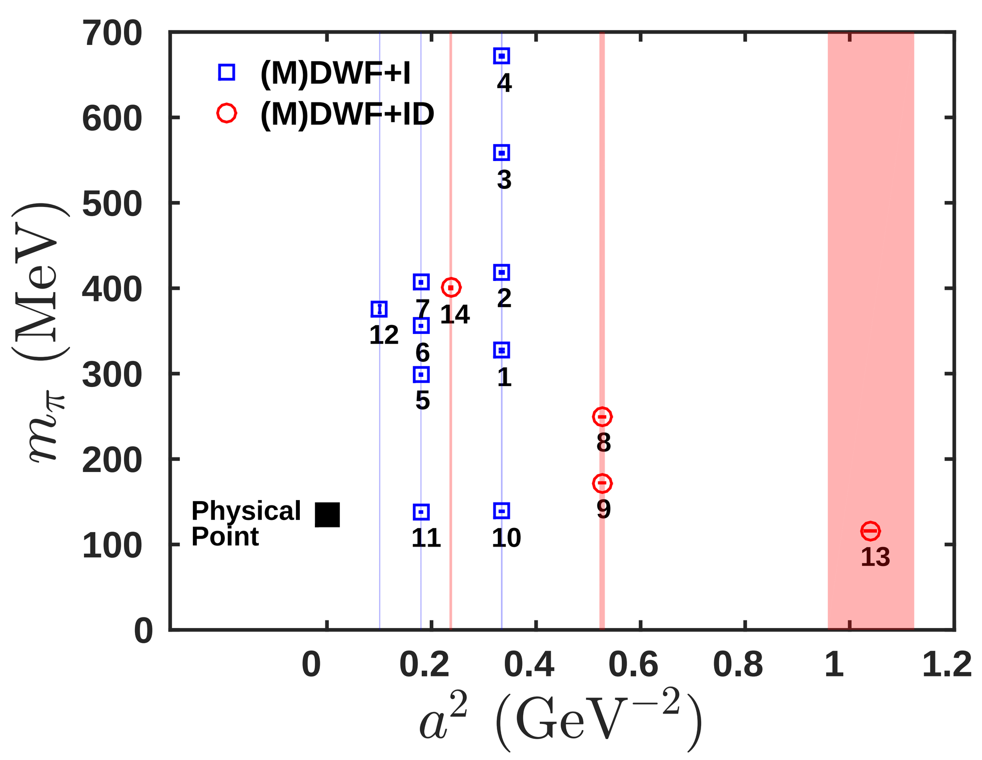

Figure: A ensembles: 1, 2, 3, 4, 10. C ensembles: 5, 6, 7, 11.

Low Energy QCD Spectrum [1]

- On each ensemble we compute: m_π, m_K, m_Ω, f_π, f_K, B_K, $\ldots$
- Predictions from chiral/continuum fit to NLO $SU(2)$ ChPT
  - m_π, m_K, and m_Ω used to determine m_{ud}, m_s, and lattice scales
  - Other quantities are predictions
  - Physical mass ensembles (10,11) weighted to dominate chiral interpolation

Quantity	RBC/UKQCD Prediction	PDG [2]
f_π	130.2(9) MeV	130.4(2) MeV $(*)$
f_K	155.5(8) MeV	156.2(7) MeV $(*)$
f_K/f_π	1.194(4)	1.198(6)
$m_{ud}^{\overline{\rm MS}}(3\,{\rm GeV})$	3.00(4)(3) MeV	—
$m_s^{\overline{\rm MS}}(3\,{\rm GeV})$	81.6(8)(9) MeV	—
$B_K^{\overline{\rm MS}}(3\,{\rm GeV})$	0.529(2)(11)	—

Table: $(*)$ Experimental values are for $f_{\pi^\pm}$ and $f_{K^\pm}$.

References

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Semileptonic Kaon Decay ($K_{\ell 3}$) [3]

- Standard Model: $\Gamma = \frac{G_F^2 m_K^5}{384\pi^3} I S_{\rm EW} \left(1 + \delta_{SU(2)} + \delta_{\rm EM}\right) |V_{us}|^2 \left|f_+^{K\pi}(0)\right|^2$
- Lattice input: non-perturbative QCD matrix elements (related by Ward-Takahashi identity)

$$\langle \pi(p_\pi)|V_\mu|K(p_K)\rangle = f_+^{K\pi}(q^2)\,(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)\,(p_K-p_\pi)_\mu$$

$$\langle \pi(p_\pi)|S|K(p_K)\rangle|_{q^2=0} = \frac{m_K^2 - m_\pi^2}{m_s - m_u} f_+^{K\pi}(0)$$

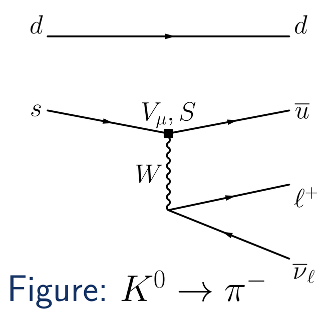


Figure: $K^0 \rightarrow \pi^-$ semileptonic decay.

- Tune p_π using twisted boundary conditions to perform calculation directly at $q^2 = 0$
- We compute and fit to vector and scalar matrix elements simultaneously
- Vector current renormalized by imposing conservation of electric charge ($f_+^{\pi\pi}(0) = f_+^{KK}(0) = 1$)

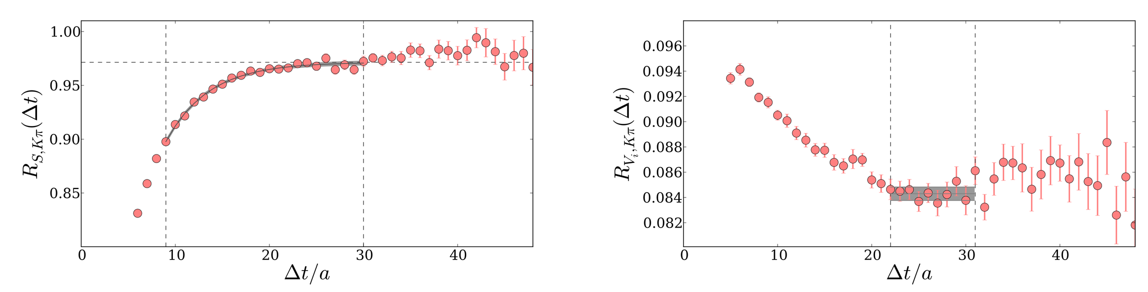
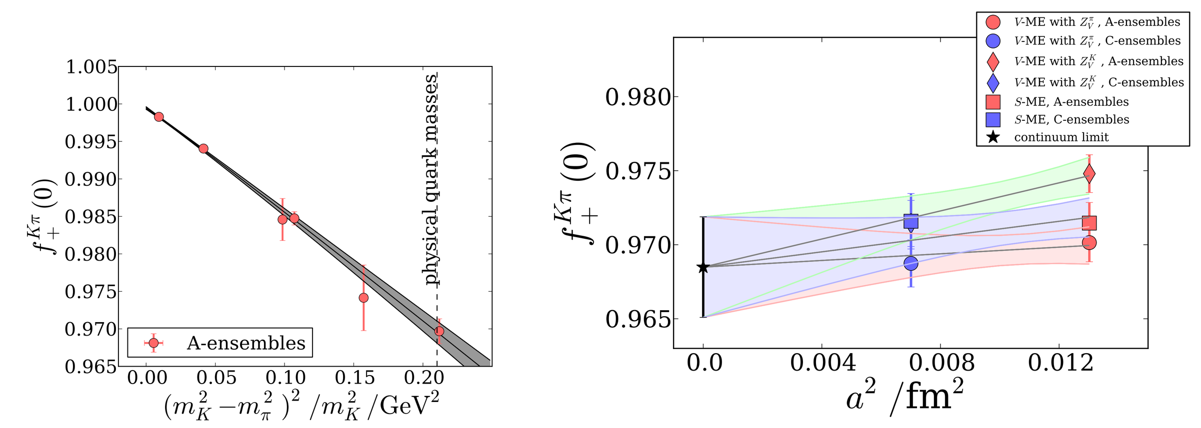


Figure: Fits to S (left) and V_i (right) three-point function on ensemble 11.

- Two step chiral/continuum extrapolation:
  1. Independent interpolations to physical m_π, m_K on A and C ensembles using polynomial in $(m_K^2 - m_\pi^2)$
  2. Joint $a^2 \rightarrow 0$ extrapolation to common continuum limit



(a) Example chiral interpolation (A ens.) (b) Continuum extrapolation

- Ongoing calculation at third lattice spacing ($a^{-1} = 2.77\,{\rm GeV}$)

Quantity	RBC/UKQCD Prediction	FLAG [4]
$f_+^{K\pi}(0)$	$0.9685(34)_{\rm stat}(14)_{\rm sys}$	0.9677(27)

CKM Matrix Elements

- $|V_{ud}|$ and $|V_{us}|$ can be extracted from lattice f_π, f_K, and $f_+^{K\pi}(0)$ together with experimental $K_{\ell 2}$ and $K_{\ell 3}$ decay rates
- First-row CKM unitarity test: $\delta_u \equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2$
- $|V_{ub}| \sim \mathcal{O}(10^{-3})$ not needed for δ_u given current error bounds

Quantity	RBC/UKQCD Prediction	PDG [2]
$\lvert V_{ud}\rvert$	$0.965(1)_{\rm expt}(5)_{\rm latt}$	0.9742(2) $(*)$
$\lvert V_{us}\rvert$	$0.2235(4)_{\rm expt}(8)_{\rm latt}$	0.2253(8)
δ_u $(**)$	$0.0010(4)_{\rm expt}(4)_{\rm latt}$	0.0002(5)

Table: $(*)$ Experimental $|V_{ud}|$ from average of 20 superallowed nuclear β decays. $(**)$ δ_u computed with more precise experimental $|V_{ud}|$.

$SU(2)$ and $SU(3)$ Chiral Perturbation Theory [5, 6]

- Systematic study of next-to-next-to leading order (NNLO) $SU(2)$ and $SU(3)$ partially quenched chiral perturbation theory (PQChPT) fits to RBC/UKQCD lattice data
- Fits include pseudoscalar masses, decay constants, and m_Ω
- Chiral ansatz:

$$X(m_q, L, a^2) \simeq X_0 \Big(1 + \underbrace{X^{\rm NLO}(m_q) + X^{\rm NNLO}(m_q)}_{\text{NNLO Continuum PQChPT}} + \underbrace{\Delta_X^{\rm NLO}(m_q, L)}_{\text{NLO FV corrections}} + \underbrace{c_X a^2}_{\text{Lattice spacing}} \Big)$$

- Determines 2+9+8 ($SU(2)$) [5] and 2+9+10 ($SU(3)$) [6] linear combinations of PQ low energy constants (LECs) at LO+NLO+NNLO
- Self-consistent probe of hierarchy of terms and region of applicability of chiral expansion
- Update: lattice calculation of $m_\pi a_0^2$ further overconstrains LECs

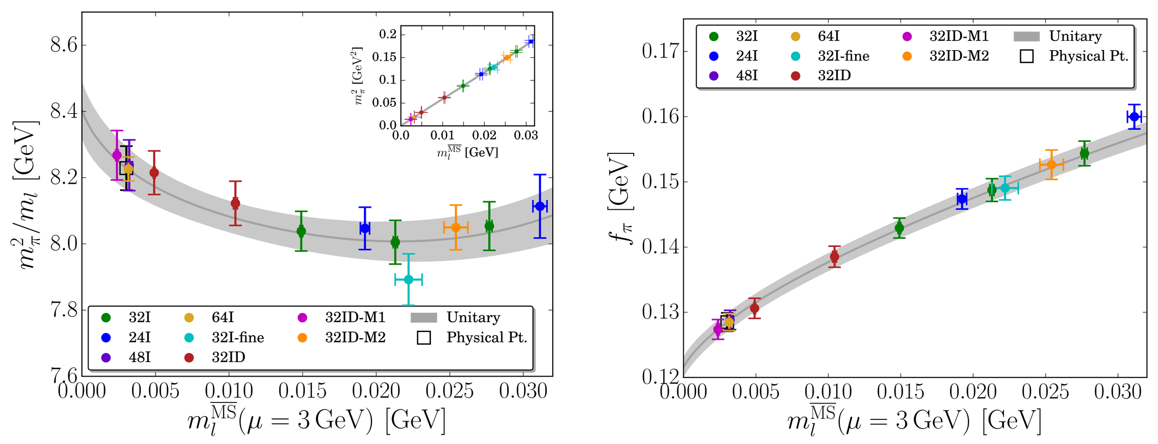


Figure: Example NNLO $SU(2)$ chiral fits (unitary data shown).

- Size of chiral corrections at physical quark masses:

Theory	Quantity	NLO/LO	NNLO/LO
$SU(2)$	m_π^2	-0.0245(41)	0.0034(10)
	f_π	0.0586(35)	-0.0011(7)
$SU(3)$	m_π^2	-0.029(34)	0.061(34)
	m_K^2	-0.130(43)	0.090(41)
	f_π	0.110(19)	0.021(20)
	f_K	0.315(33)	0.035(30)

- Observe breakdown of $SU(2)$ ($SU(3)$) expansion at scale $m_\pi \sim$ 450 MeV (500 MeV): NNLO corrections to f_π are 50% of NLO

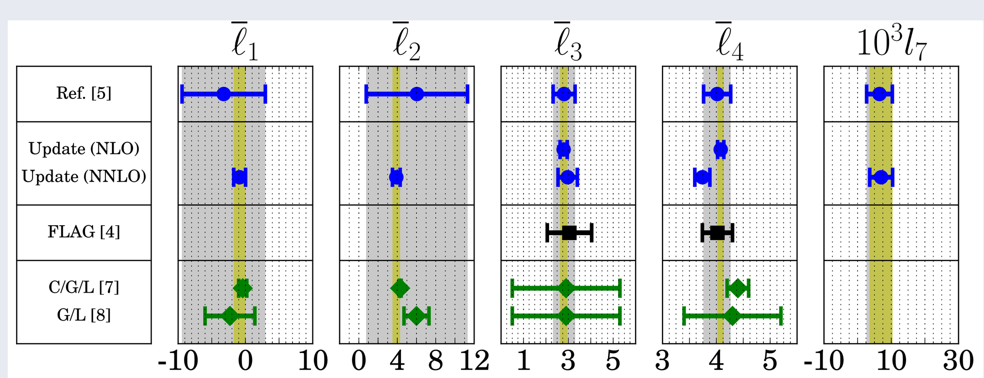


Figure: Results for scale-independent NLO $SU(2)$ LECs.

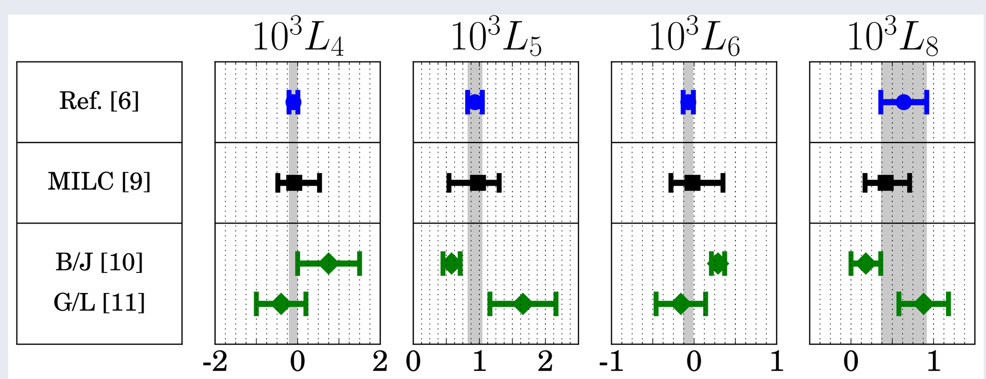


Figure: Results for NLO $SU(3)$ LECs at $\mu = 770$ MeV [6].

$I = 2$ $\pi\pi$ Scattering

- Lüscher formalism relates infinite volume scattering matrix to spectrum of two-particle states in finite box [12]
- Lattice calculation of m_π and energy of two-particle $|\pi\pi^I\rangle$ state ($E_{\pi\pi}^I$) for isospin $I = 2$
- S-wave scattering lengths (a_0^I) extracted from $1/L$ expansion:

$$E_{\pi\pi}^I - 2m_\pi = -\frac{4\pi a_0^I}{mL^3}\left(1 + \sum_{n=1}^{\infty} c_n \left(\frac{a_0^I}{L}\right)^n\right)$$

- Physical prediction from chiral/continuum fit

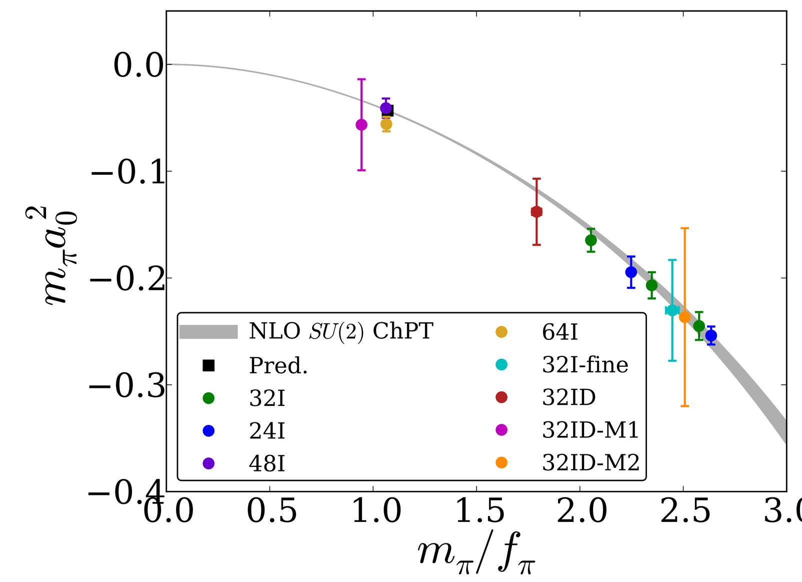


Figure: Example chiral fit.

Quantity	RBC/UKQCD Prediction	Expt. [13]
$m_\pi a_0^2$	-0.0433(2)	-0.043(5)

Other Predictions from ChPT Fits

- Use results for LECs to make other NLO ChPT predictions:

Theory	Quantity	RBC/UKQCD Pred.	Expt.
$SU(2)$	$m_\pi a_0^0$	0.195(1)	0.26(5) [14]
	$m_\pi b_0^0$	0.233(5)	0.25(3) [14]
	$m_\pi a_1^1$	0.0356(7)	0.038(2) [14]
	$m_\pi b_0^2$	-0.083(1)	-0.082(8) [14]
	$m_{\pi^\pm}^2 - m_{\pi^0}^2$ $(*)$	1195(112) MeV^2	1261.2(2) MeV^2 [2]
$SU(3)$	$m_\pi a_0^{1/2}$	0.138(9)	—
	$m_\pi a_0^{3/2}$	-0.067(8)	—
	$m_\pi a_0^-$	0.068(4)	0.11(+9)(-4) [15]

Table: $SU(2)$ ($SU(3)$) predictions are for $\pi\pi$ (πK) scattering lengths (a_ℓ^I) and slopes (b_ℓ^I). $a_0^- \equiv |a_0^{1/2} - a_0^{3/2}|/3$ is isospin-odd combination. $(*)$ LO E&M + NLO QCD isospin breaking.

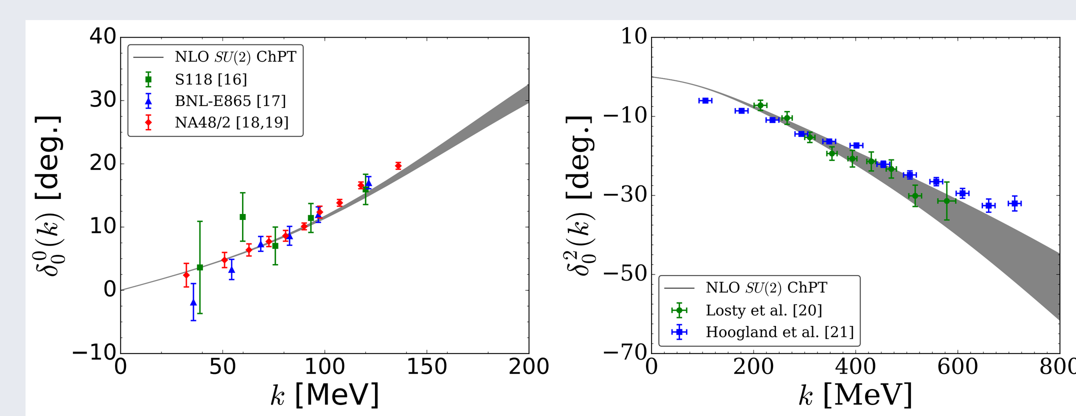


Figure: Predictions for s-wave $\pi\pi$ phase shifts.