

# New Exclusive Cross Section Measurements for $e^+e^- \rightarrow \text{hadrons}$

at low energies:  $e^+e^- \rightarrow \pi^+\pi^-\eta$ ,  $\pi^+\pi^-\pi^0\pi^0$ , and more  
at 10.58 GeV via Initial State Radiation (ISR)

input for precision calculations of g-2

*Roland Waldi, Rostock University  
for the BABAR Collaboration*

*ICHEP, Chicago, Aug 2016*



# Motivation

QED = precision physics

- $a=(g-2)/2$  is the testbed for QED
- experiment: electron: ✓
- muon:  
currently still  $>3\sigma$  discrepancy  
theory/experiment

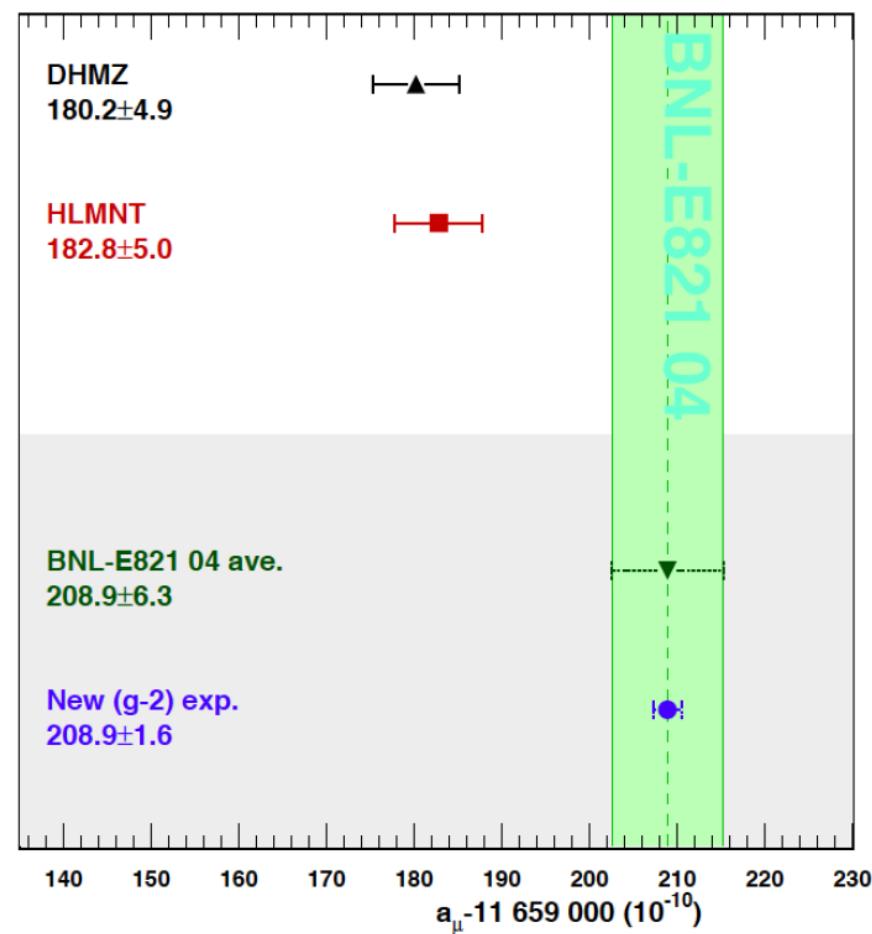
$$a_{\mu}^{\text{theo}} \cdot 10^{10} = 11659180.2 \pm 4.9$$

$$a_{\mu}^{\text{exp}} \cdot 10^{10} = 11659208.9 \pm 6.3$$

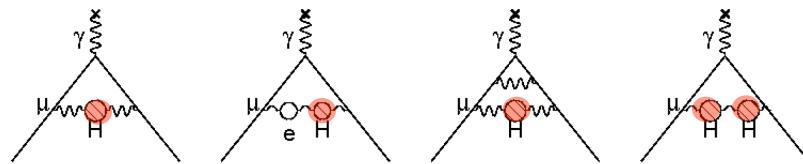
$$\Delta a_{\mu} \cdot 10^{10} = 28.7 \pm 8.0$$

- New Physics?  
**test SM**

new expt.s @ Fermilab, J-PARC



# Motivation



muon sensitive to  
hadronic vacuum polarisation:

$$a_\mu^{\text{had,LO}} \cdot 10^{10} = 692.3 \pm 4.2$$

[Davier et al., EPJ C71, 1515]

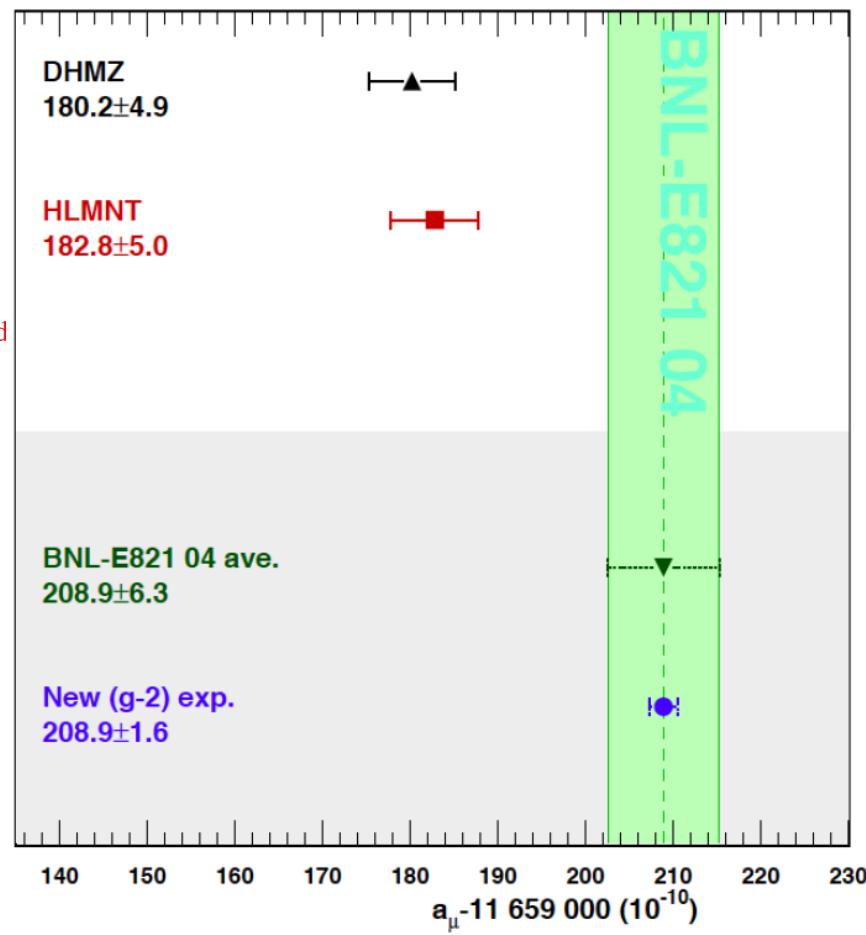
$$a_\mu^{\text{had,LO}} \cdot 10^{10} = 694.9 \pm 4.3$$

$$a_\mu^{\text{had,NLO}} \cdot 10^{10} = -9.84 \pm 0.06_{\text{exp}} \pm 0.04_{\text{rad}}$$

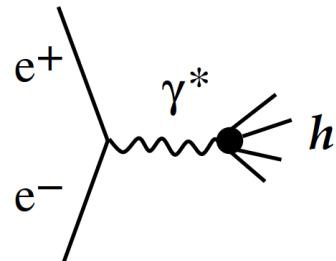
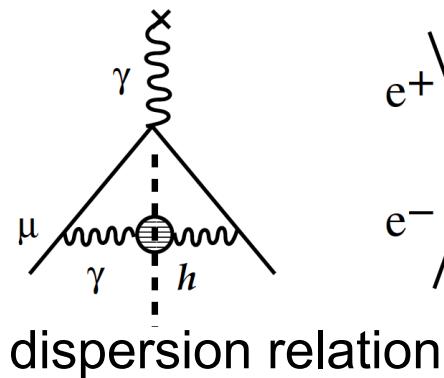
[Hagiwara et al., JP G38, 085003]

QED = precision physics  
QCD = the weak factor

- ab initio calculations difficult
- experimental input required:  
 $\sigma(e^+e^- \rightarrow \text{hadrons})$



# Motivation



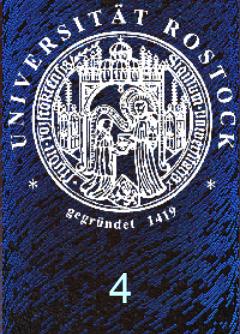
analytical function

$$a_\mu^{\text{had,LO}} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_\pi^2}^\infty ds \frac{K(s)}{s} R(s)$$

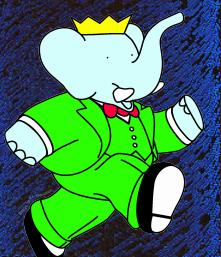
experiment input

$$R \equiv \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

dominated by low-energy cross sections  
improve precision by measuring **exclusive** final states

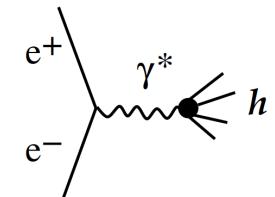


# Motivation



(a) at low-energy storage rings (scan)

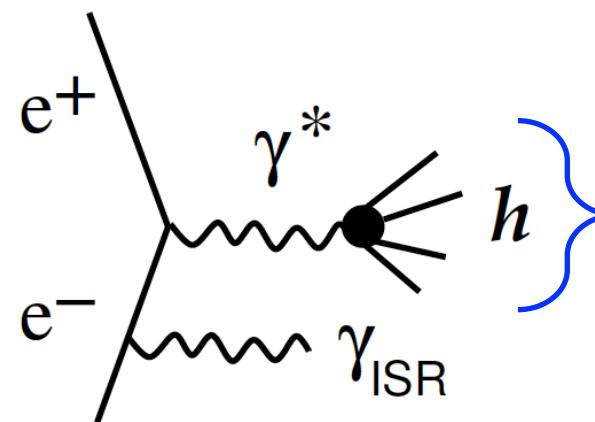
$$E(e^+e^-) = E_{CM}$$



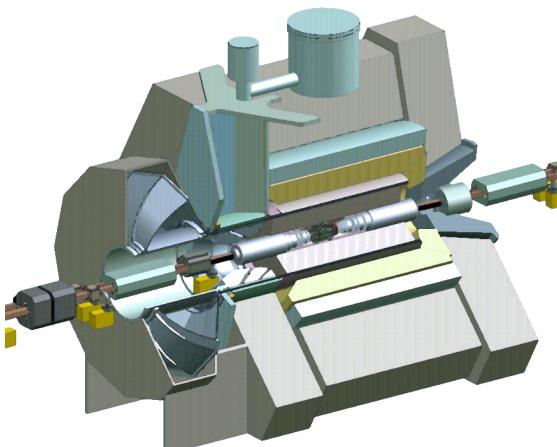
(b) at fixed high energy, using radiative events  
 $e^+e^- \rightarrow \gamma + \text{hadrons}$

BABAR:

$$E(e^+e^-) = 10.58 \text{ GeV}$$



$E_{CM} = M_{\text{had}}$   
continuous  
recoiling against  
ISR photon



$$L_{int} = 460 \text{ events/fb}$$



# $\pi^+\pi^-\eta$ : 4C fit

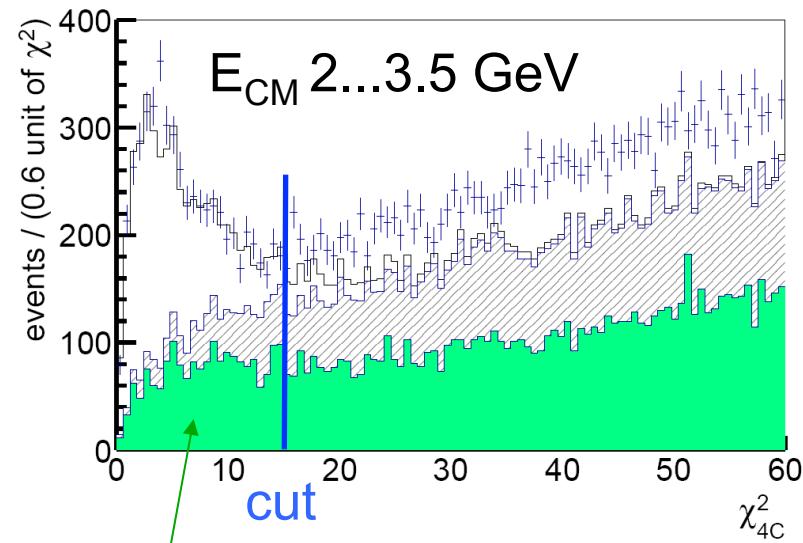
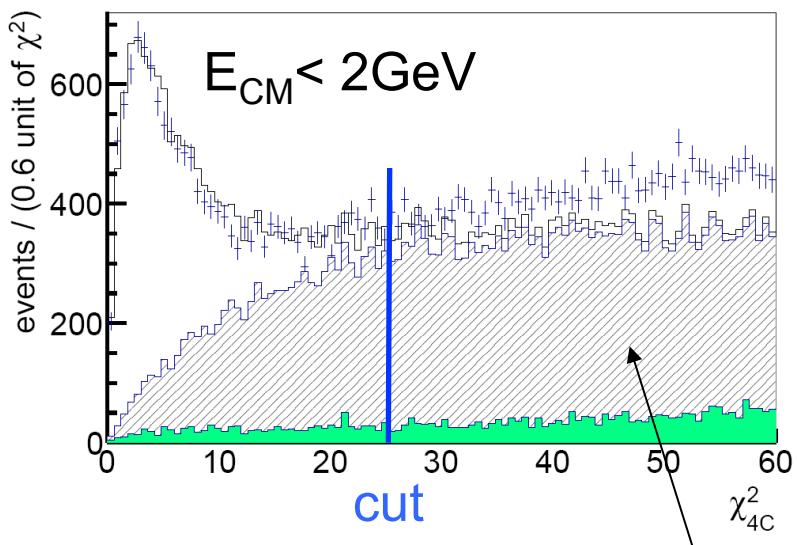


energy/momentum conservation  $e^+e^- \rightarrow \pi^+\pi^-\eta + \gamma$

4C-fit

$\eta \rightarrow \gamma\gamma$

→ (min) chisquare distribution:



ISR- / non-ISR-background  
subtracted using MC calibrated with data

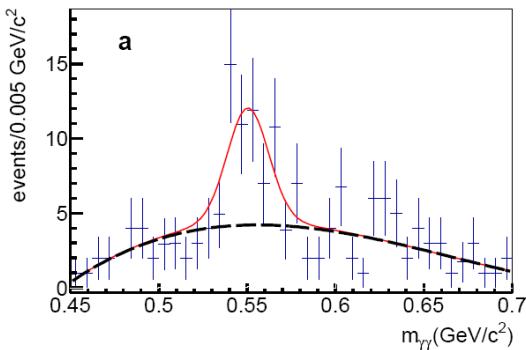
R. Waldi



# $\pi^+\pi^-\eta$



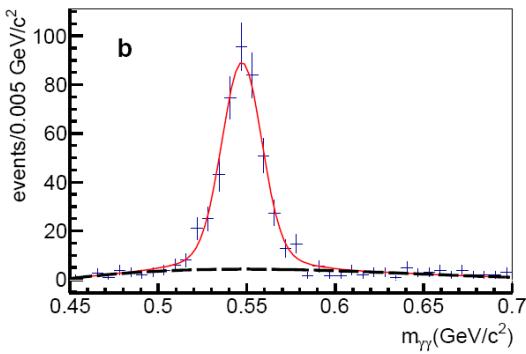
BABAR



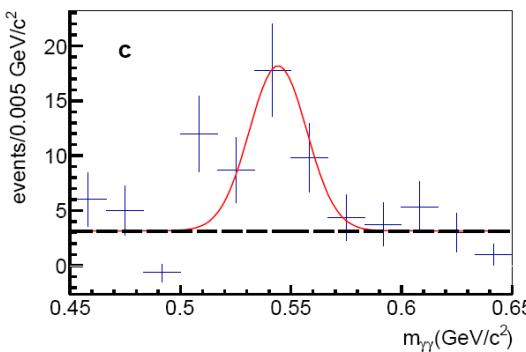
$e^+e^- \rightarrow \pi^+\pi^-\eta + \gamma$

$\eta \rightarrow \gamma\gamma$

fit of  $m_{\gamma\gamma}$  in bins of  $E_{\text{CM}} = M(\eta\pi\pi)$   
1.300-1.325 GeV



1.500-1.525 GeV



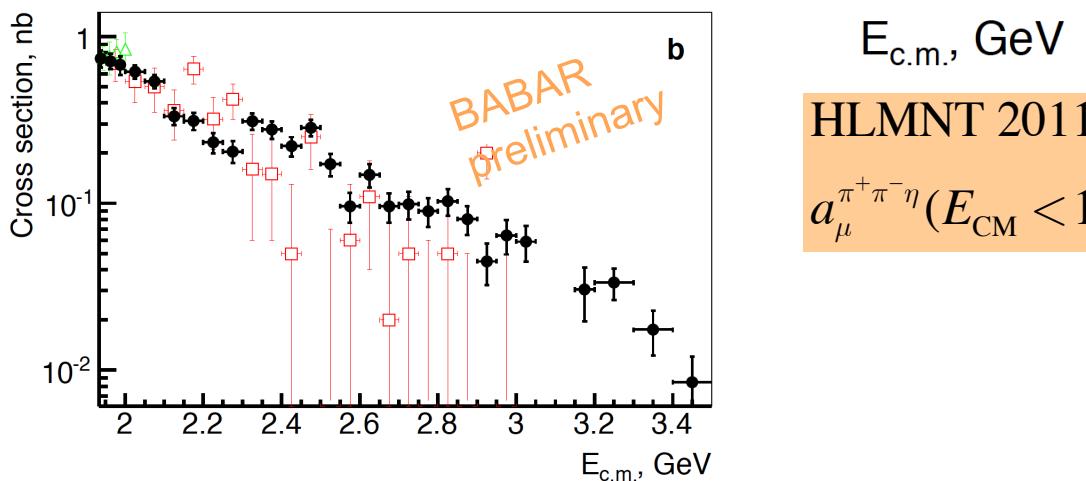
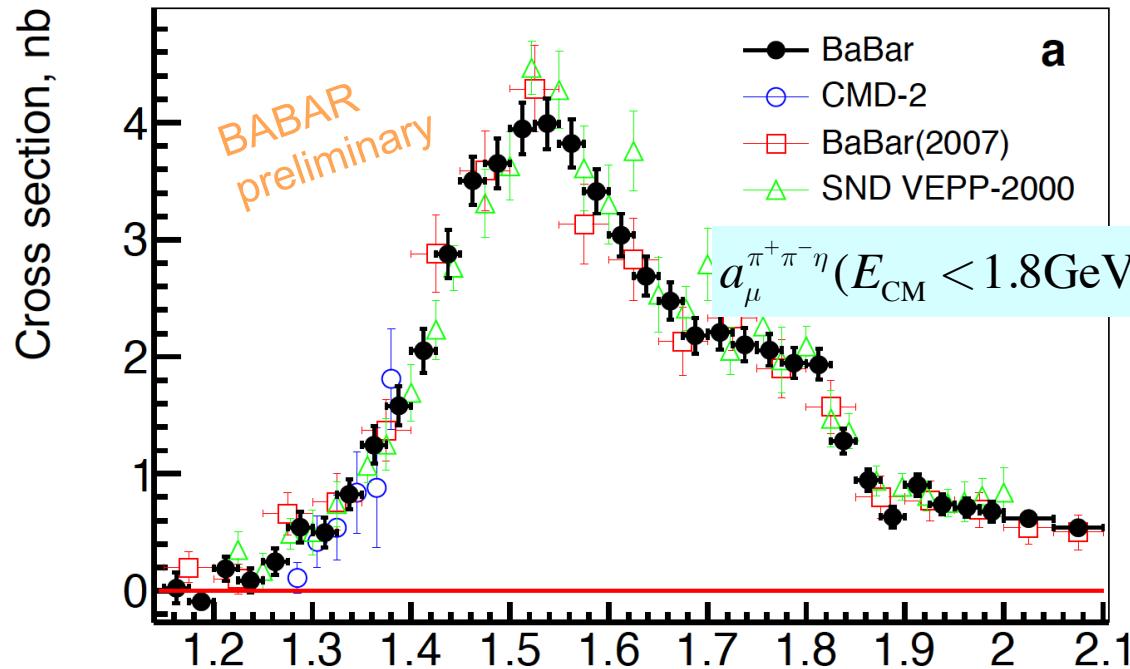
3.4-3.5 GeV



# $\pi^+ \pi^- \eta$



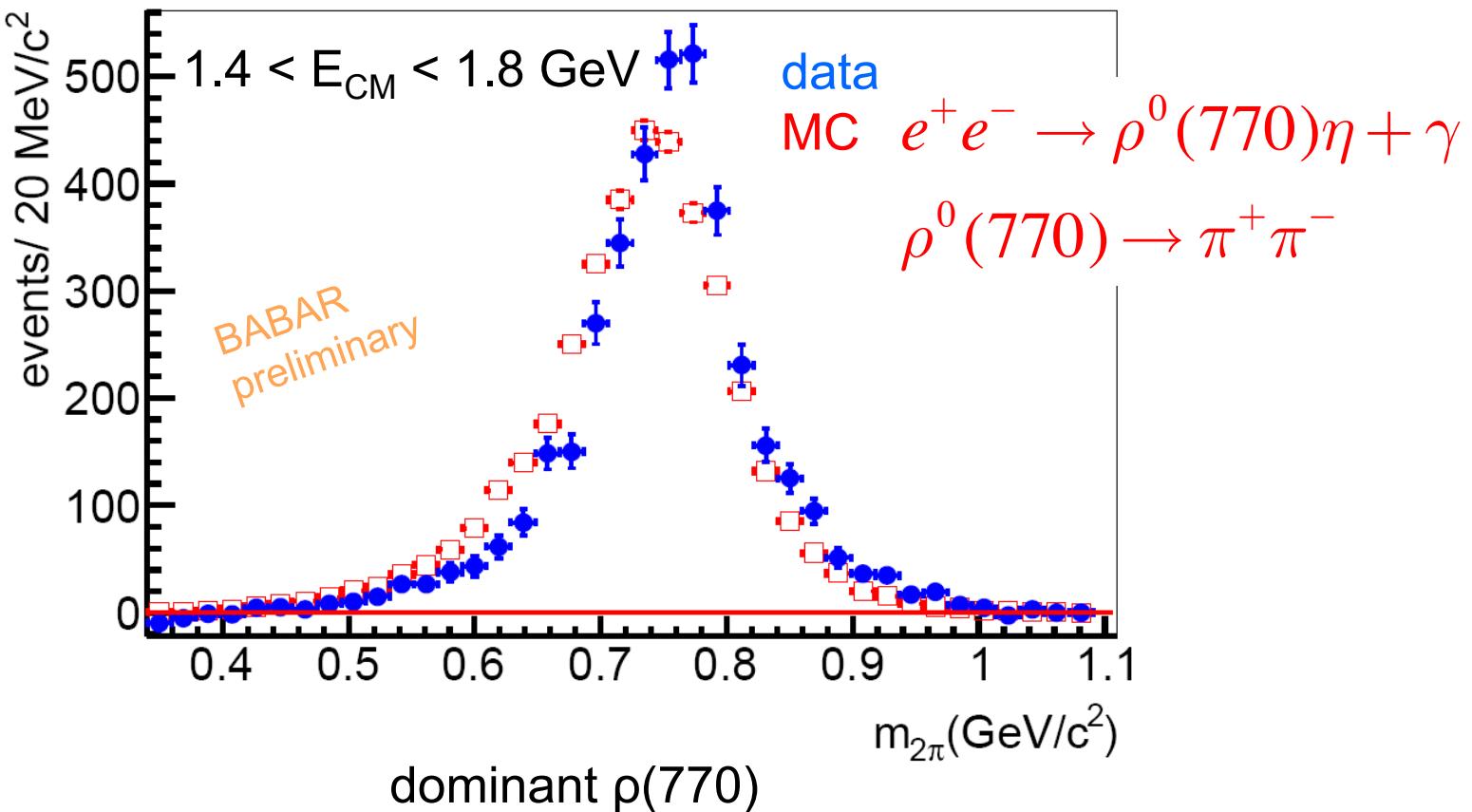
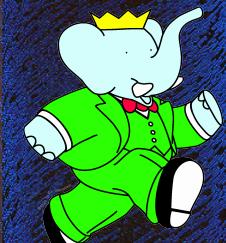
BABAR



HLMNT 2011:

$$a_\mu^{\pi^+ \pi^- \eta} (E_{\text{CM}} < 1.8 \text{ GeV}) \cdot 10^{10} = 0.88 \pm 0.10$$

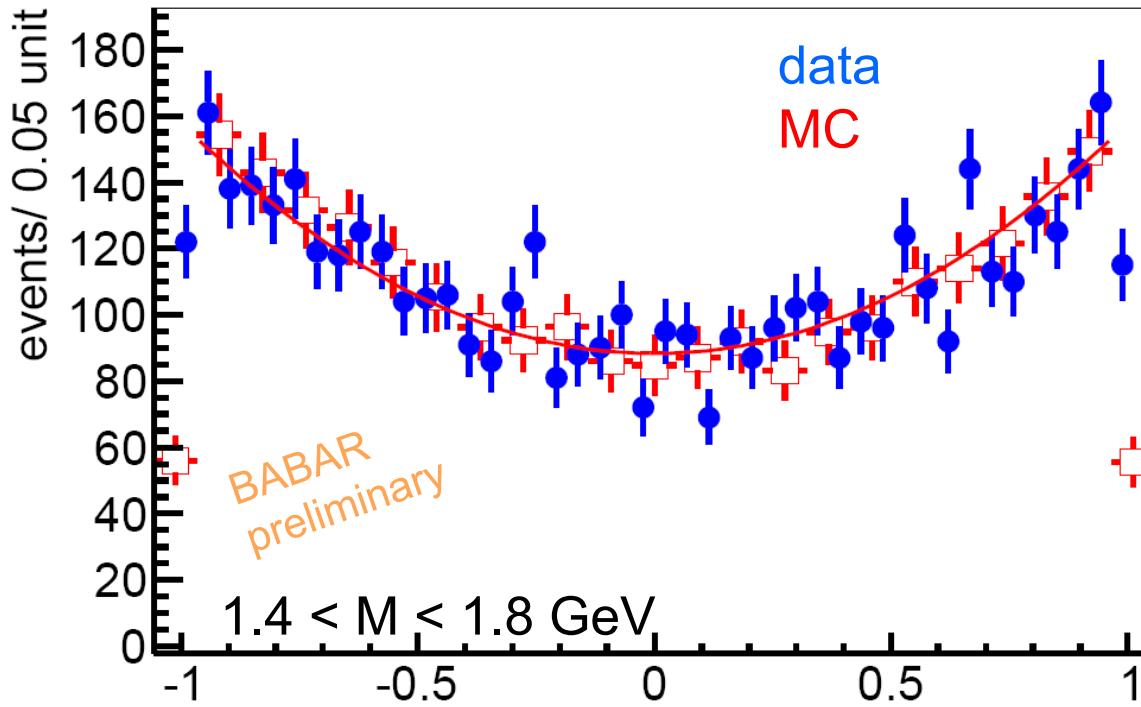
$\pi^+ \pi^- \eta$



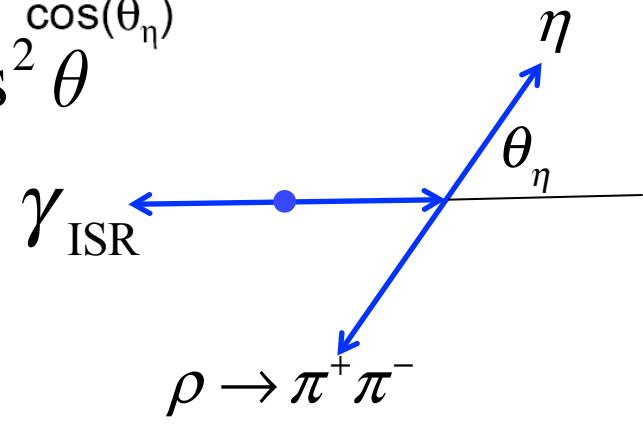
possible  $\rho(770)$ - $\rho(1450)$  interference



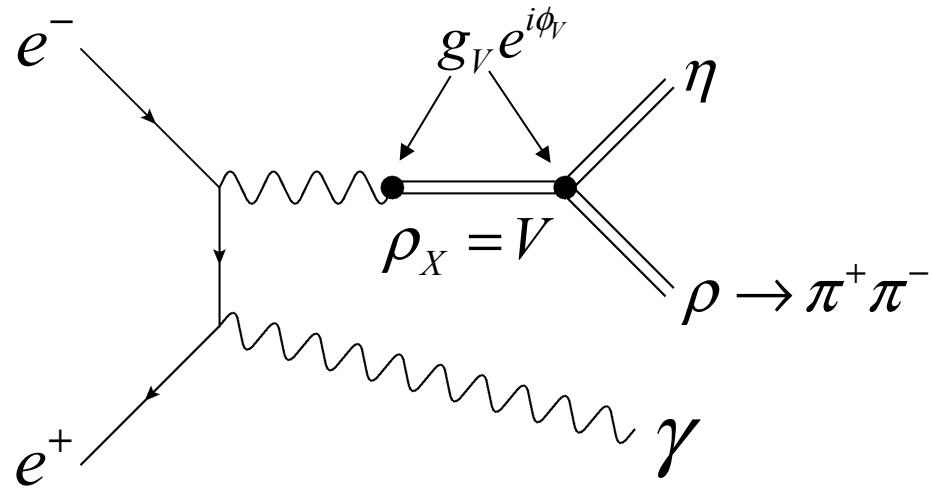
# $\pi^+\pi^-\eta$ Angular Distribution



$$f(\cos \theta) \sim 1 + (0.92 \pm 0.09) \cos^2 \theta$$



# Vector Meson Dominance



$$\sigma(s) = \frac{4\alpha^2}{3} \frac{1}{s\sqrt{s}} |F(s)|^2 G(s), \quad G(s) = \int_{4m_\pi^2}^{(\sqrt{s}-m_\eta)^2} dq^2 \frac{\sqrt{q^2} \Gamma_\rho(q^2) p_\eta^3(s, q^2)}{(q^2 - m_\rho^2)^2 + (\sqrt{q^2} \Gamma_\rho(q^2))^2}$$

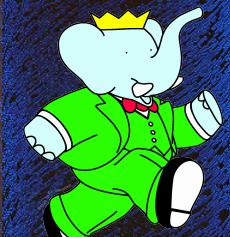
$$\Gamma_\rho(q^2) = \Gamma_\rho(m_\rho^2) \frac{m_\rho^2}{q^2} \left( \frac{p_\pi^2(q^2)}{p_\pi^2(m_\rho^2)} \right)^{\frac{3}{2}}$$

$$F(s) = \sum_V \frac{m_V^2 g_V e^{i\phi_V}}{s - m_V^2 + i\sqrt{s} \Gamma_V(s)}$$

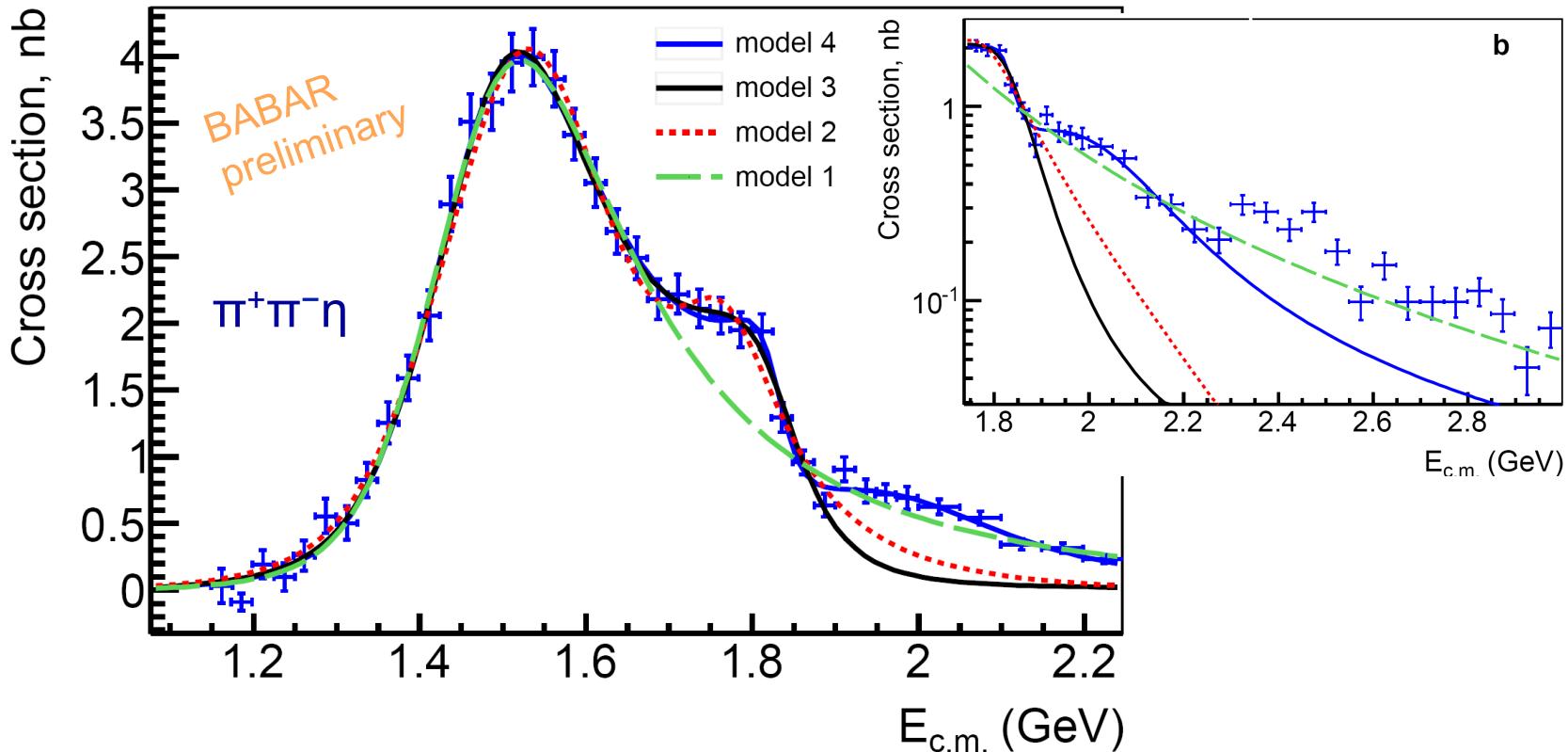
R. Waldi



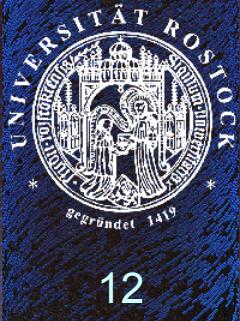
# Vector Meson Dominance



BABAR



R. Waldi



model 1:  $\rho(770) - \rho(1450)$  fits  $E_{CM} < 1.7$  GeV

model 2:  $\rho(770) - \rho(1450) - \rho(1700)$  fits  $< 1.9$  GeV

model 3:  $\rho(770) - \rho(1450) + \rho(1700)$  fits  $< 1.9$  GeV

model 4:  $\rho(770) - \rho(1450) + \rho(1700) + \rho(2150)$  fits  $< 2.2$  GeV

relative phases 0 (+) and 180° (-)

# $\pi^+\pi^-\eta$ and CVC



compare  $e^+e^- \rightarrow \pi^+\pi^-\eta$  with  $\tau \rightarrow \nu\pi^0\pi^-\eta$  (vector current)

CVC-prediction **from cross section**  $e^+e^- \rightarrow \pi^+\pi^-\eta$ :

$$\mathcal{B}(\tau \rightarrow \nu\pi^0\pi^-\eta) = (0.160 \pm 0.009)\%$$

combined with  $\eta \rightarrow 3\pi$  result [BABAR 2007]:

$$\mathcal{B}(\tau \rightarrow \nu\pi^0\pi^-\eta) = (0.162 \pm 0.008)\%$$

**direct measurement** [PDG14]:

$$\mathcal{B}(\tau \rightarrow \nu\pi^0\pi^-\eta) = (0.139 \pm 0.010)\%$$

isospin-breaking?

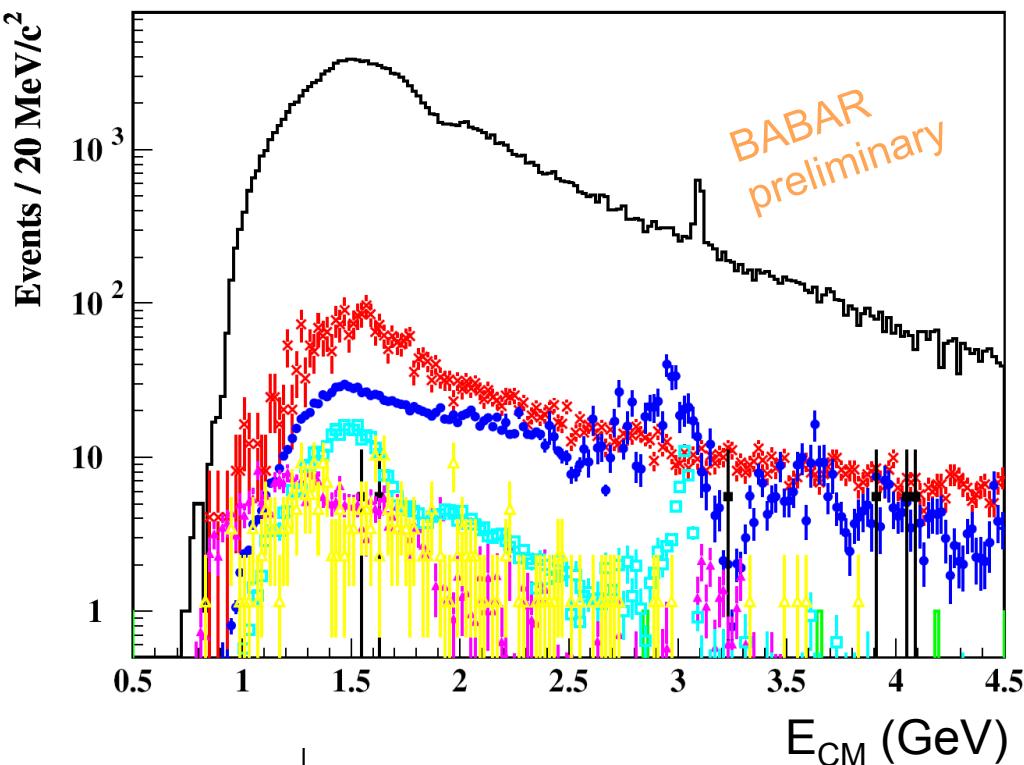
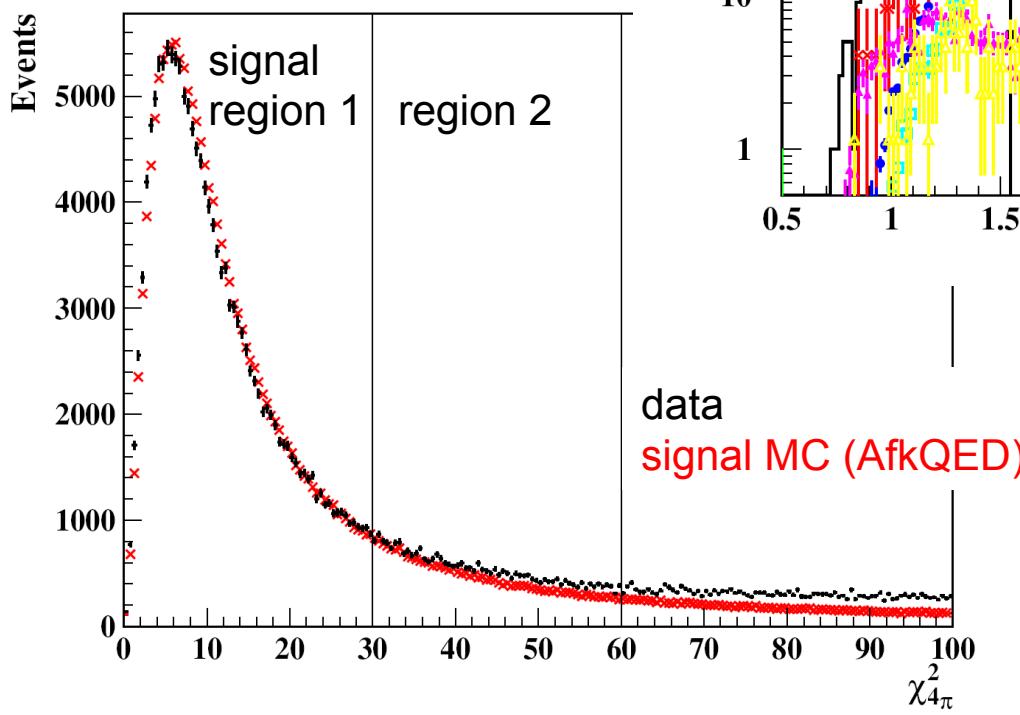
R. Waldi



# $\pi^+\pi^-\pi^0\pi^0$

MC simulated background  
subtracted,

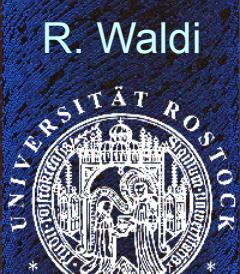
cross-checked with  
6C-fit ( $E, \vec{p}, 2 \times m_{\pi^0}$ )  
ratio  $\chi^2$  region 1/2



$q\bar{q}$ ,  $\tau\tau$ ,  $\gamma\pi^+\pi^-3\pi^0$ ,  
 $\gamma K_s K\pi$ ,  $\gamma K^+ K^- 2\pi^0$ ,  
 $\gamma 3\pi$ ,  $\gamma 4\pi^\pm 2\pi^0$ ,



BABAR



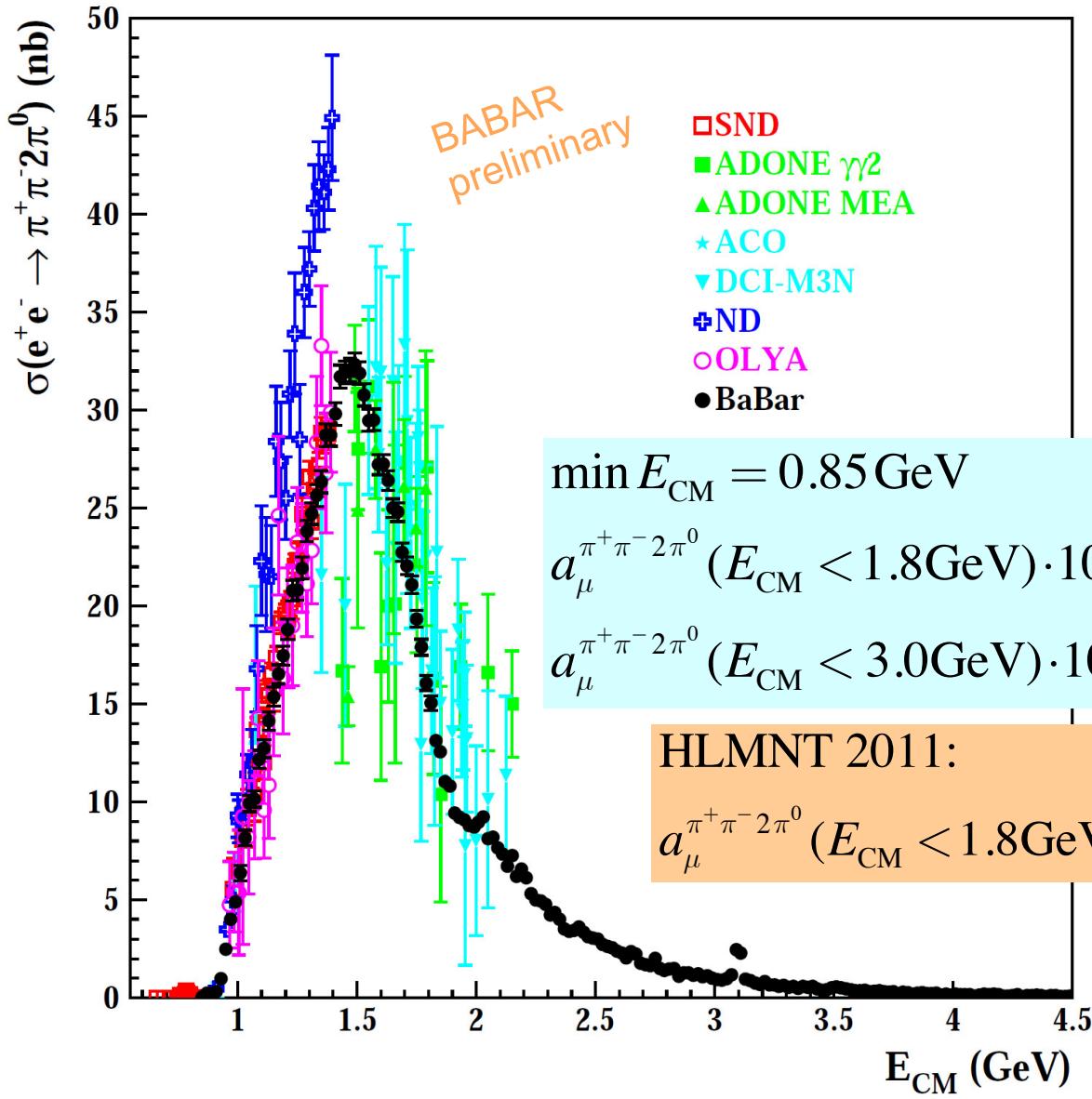
R. Waldi

# $\pi^+\pi^-\pi^0\pi^0$



BABAR

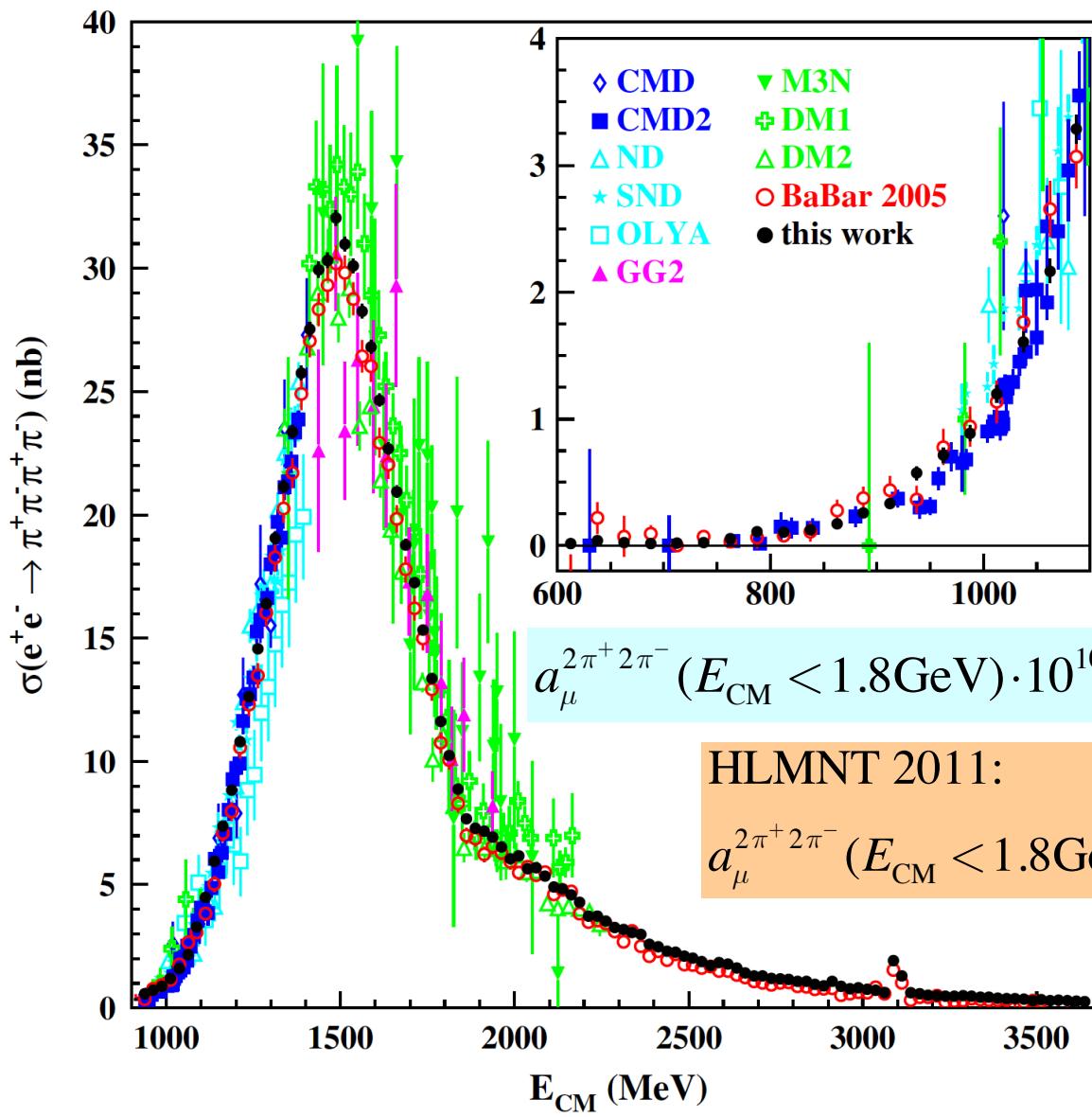
R. Waldi



# $\pi^+\pi^-\pi^+\pi^-$

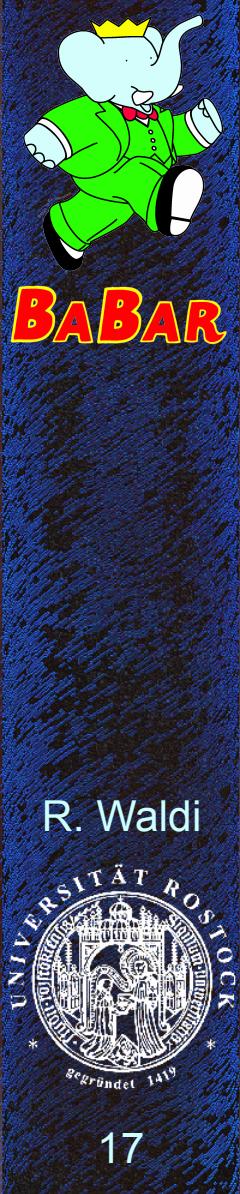


BABAR

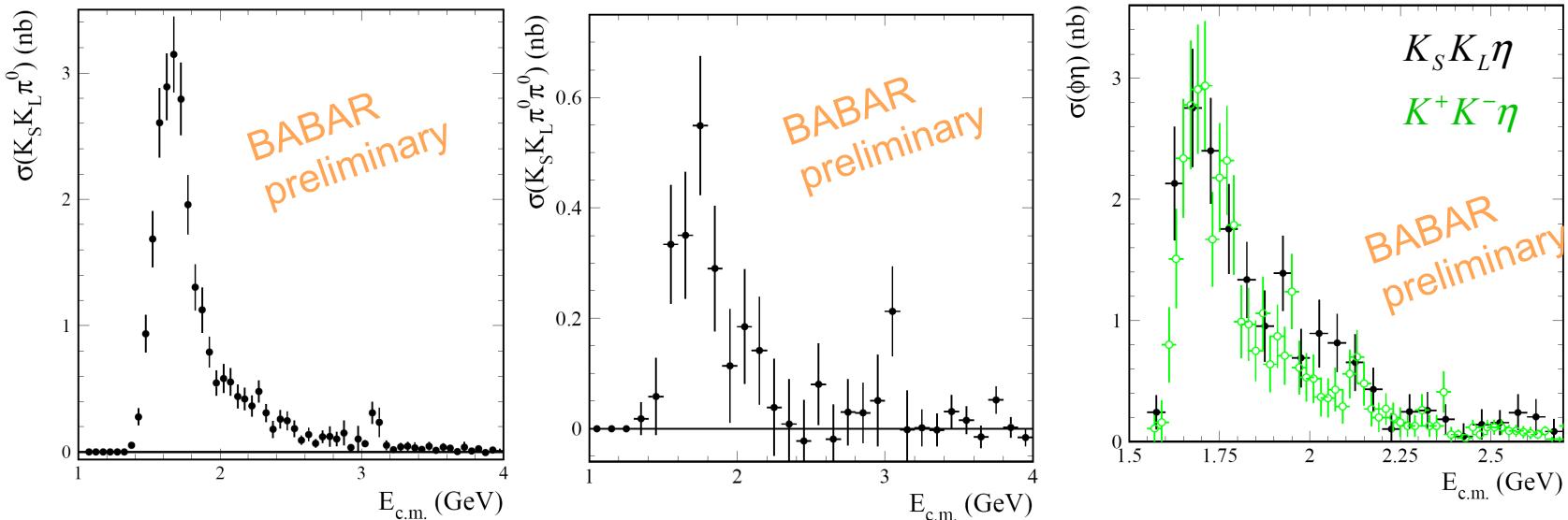


BABAR 2012  
[PR D85, 112009]

# $K_S K_L \pi^0$ , $K_S K_L \pi^0 \pi^0$ , $\phi \eta$



see poster by Alessandro Pilloni



$$a_\mu^{\text{all } KK\pi\pi}(m < 1.8 \text{ GeV}) \cdot 10^{10} = 0.85 \pm 0.05$$

$$a_\mu^{\text{all } KK\pi\pi}(m < 2.0 \text{ GeV}) \cdot 10^{10} = 2.41 \pm 0.11$$

[Evgeni Solodov]

HLMNT 2011:

$$a_\mu^{KK\pi\pi}(E_{\text{CM}} < 2.0 \text{ GeV}) \cdot 10^{10} = 3.31 \pm 0.58$$

# J/ $\psi$ Branching Fractions



$$\sigma(e^+e^- \rightarrow X) \propto \Gamma(J/\psi \rightarrow e^+e^-) \cdot \mathcal{B}(J/\psi \rightarrow X)$$

ISR cross section

$$\mathcal{B}(J/\psi \rightarrow \pi^+\pi^-\eta) = (0.042 \pm 0.008)\%$$

PDG 14

$$(0.040 \pm 0.017)\%$$

$$\mathcal{B}(J/\psi \rightarrow K_S K_L \pi^0) = (0.206 \pm 0.026)\%$$

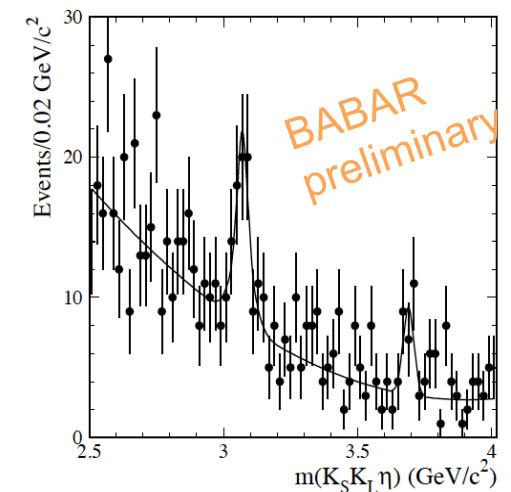
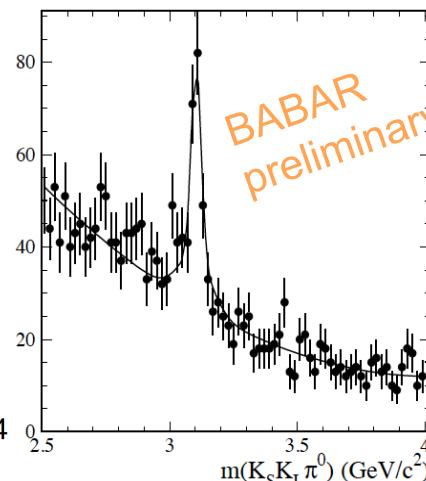
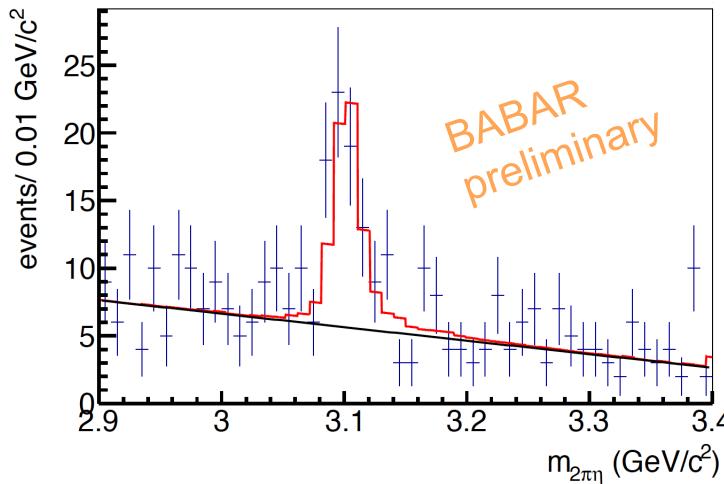
$$\mathcal{B}(J/\psi \rightarrow K_S K_L \eta) = (0.145 \pm 0.033)\%$$

$$K^+K^-\eta: (0.085 \pm 0.014)\%$$

$$\mathcal{B}(J/\psi \rightarrow K_S K_L \pi^0 \pi^0) = (0.186 \pm 0.044)\%$$

$$K^+K^-\pi^0\pi^0: (0.235 \pm 0.041)\%$$

...



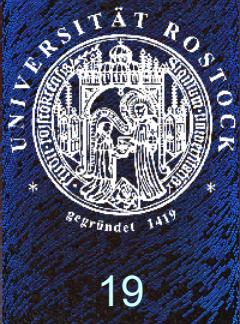
# Summary & Conclusions

- measurements of  $\sigma(e^+e^- \rightarrow \text{hadrons})$  add precision and increased energy range to the hadronic vacuum polarisation
- new channels  $\pi^+\pi^-\eta$ ,  $\pi^+\pi^-\pi^0\pi^0$ ,  $K_S K_L \pi^0$ ,  $K_S K_L \pi^0\pi^0$ ,  $K_S K_L \eta$
- specific channels:  $a_\mu^{\Sigma X} \cdot 10^{10} = 36.3 \pm 1.4$  (HLMNT)  $\rightarrow 35.1 \pm 0.7$  (new)
- small changes do not solve theory/expt. discrepancy
- also improves precision on running  $\alpha_{\text{QED}}(m_Z)$
- side results: test of CVC  $\rightarrow$  isospin breaking
- BFs of rare  $J/\psi$  decays

Thank you!



R. Waldi



# Extra Info

---



**BABAR**

R. Waldi



# Cross Section Measurement

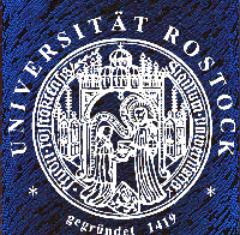


measured

$$\frac{d\sigma_{\pi^+\pi^-2\pi^0\gamma}(M)}{dM} = \frac{2M}{s} \cdot W(s, x, C) \cdot \sigma_{\pi^+\pi^-2\pi^0}(M)$$

↑  
radiator function,  
 $x$  = energy fraction of ISR photon  
 $|\cos \theta_\gamma| < C$

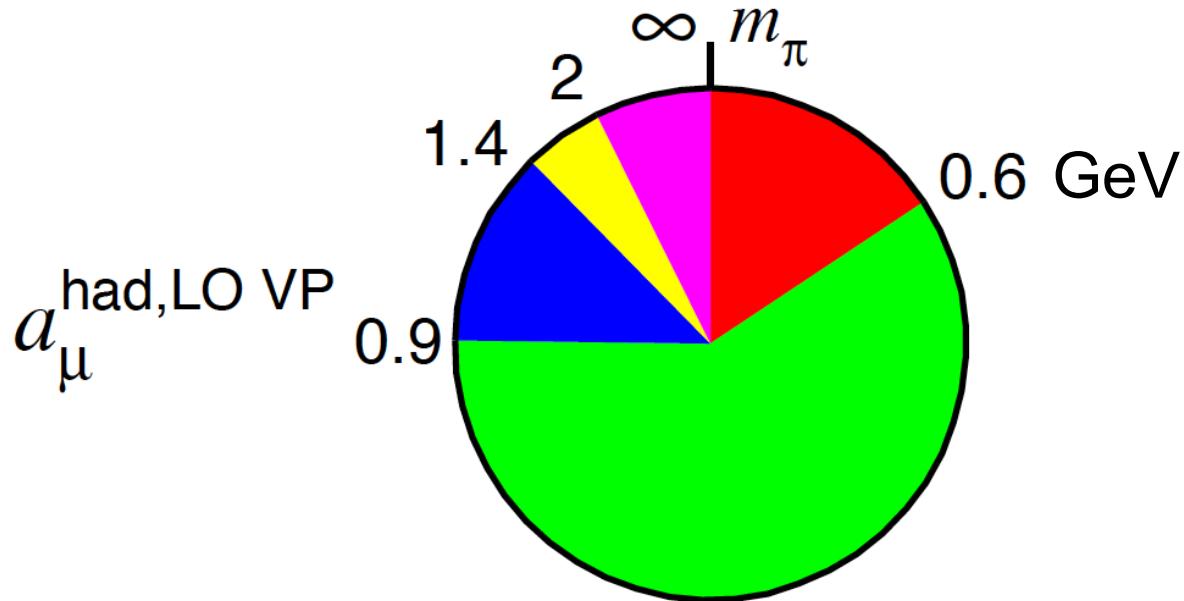
R. Waldi



# Hadronic Contributions to g-2

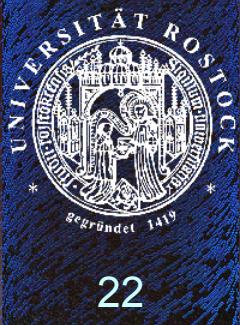


BABAR



dominantly low energy

R. Waldi



# Exclusive Contributions to $a_\mu$



$\eta\pi^+\pi^-$	$0.88 \pm 0.10$	$K\bar{K}\pi$	$2.77 \pm 0.15$
$K^+K^-$	$22.09 \pm 0.46$	$K\bar{K}2\pi$	$3.31 \pm 0.58$
$K_S^0 K_L^0$	$13.32 \pm 0.16$	$K\bar{K}3\pi$	$0.08 \pm 0.04$
$\omega\pi^0$	$0.76 \pm 0.03$	$\omega(\rightarrow\pi^0\gamma)K\bar{K}$	$0.01 \pm 0.00$
$\pi^+\pi^-$	$505.65 \pm 3.09$	$2\pi^+2\pi^-\pi^0$ (no $\eta$ )	$1.20 \pm 0.10$
$2\pi^+2\pi^-$	$13.50 \pm 0.44$	$\pi^+\pi^-3\pi^0$ (no $\eta$ )	$0.60 \pm 0.05$
$3\pi^+3\pi^-$	$0.11 \pm 0.01$	$\omega(\rightarrow\pi^0\gamma)2\pi$	$0.11 \pm 0.02$
$\pi^+\pi^-\pi^0$	$47.38 \pm 0.99$	$2\pi^+2\pi^-2\pi^0$ (no $\eta$ )	$1.80 \pm 0.24$
$\pi^+\pi^-2\pi^0$	$18.62 \pm 1.15$	$\pi^+\pi^-4\pi^0$ (no $\eta$ )	$0.28 \pm 0.28$
$\pi^0\gamma$	$4.54 \pm 0.14$	$\omega(\rightarrow\pi^0\gamma)3\pi$	$0.22 \pm 0.04$
$\eta\gamma$	$0.69 \pm 0.02$	$\eta\pi^+\pi^-$ (data)	$0.98 \pm 0.24$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.00$	$\eta\omega$ (data)	$0.42 \pm 0.07$
$\eta\omega$	$0.38 \pm 0.06$	$\eta\phi$ (data)	$0.46 \pm 0.03$
$\eta\phi$	$0.33 \pm 0.03$	$\eta 2\pi^+2\pi^-$ (data)	$0.11 \pm 0.02$
$\phi(\rightarrow \text{unaccounted})$	$0.04 \pm 0.04$	$\eta\pi^+\pi^-2\pi^0$	$0.11 \pm 0.06$

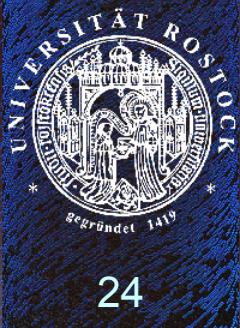
# $\pi^+\pi^-\eta$ Vector Dominance



## BABAR model fit results:

Parameter	Model 1	Model 2	Model 3	Model 4
$g_{\rho(770)}$ , $\text{GeV}^{-1}$	$1.1 \pm 0.3$	$2.4 \pm 0.3$	$1.9 \pm 0.3$	$1.7 \pm 0.3$
$g_{\rho(1450)}$ , $\text{GeV}^{-1}$	$0.48 \pm 0.02$	$0.36 \pm 0.05$	$0.44 \pm 0.02$	$0.46 \pm 0.03$
$g_{\rho(1700)}$ , $\text{GeV}^{-1}$	—	$0.049 \pm 0.020$	$0.086 \pm 0.012$	$0.015 \pm 0.008$
$g_{\rho''}$ , $\text{GeV}^{-1}$	—	—	0.	$0.10 \pm 0.02$
$m_{\rho(1450)}$ , $\text{GeV}/c^2$	$1.487 \pm 0.016$	$1.54 \pm 0.01$	$1.50 \pm 0.01$	$1.49 \pm 0.01$
$m_{\rho(1700)}$ , $\text{GeV}/c^2$	—	$1.76 \pm 0.01$	$1.83 \pm 0.01$	$1.83 \pm 0.01$
$m_{\rho''}$ , $\text{GeV}/c^2$	—	—	—	$2.00 \pm 0.04$
$\Gamma_{\rho(1450)}$ , $\text{GeV}$	$0.33 \pm 0.02$	$0.31 \pm 0.03$	$0.28 \pm 0.02$	$0.29 \pm 0.02$
$\Gamma_{\rho(1700)}$ , $\text{GeV}$	—	$0.18 \pm 0.04$	$0.17 \pm 0.02$	$0.08 \pm 0.03$
$\Gamma_{\rho''}$ , $\text{GeV}$	—	—	—	$0.42 \pm 0.10$
$\phi_V$	$0, \pi$	$0, \pi, \pi$	$0, \pi, 0$	$0, \pi, 0, 0$
$\chi^2/ndf$	13/18	28/23	17/23	23/28

R. Waldi



# Systematic Errors

TABLE I: Summary of the efficiency corrections and systematic uncertainties.

Source	Correction, %	Systematic uncertain
Selection criteria		2.5
Background subtraction		
$m_{\pi^+\pi^-\eta} < 1.35$		9
$1.35 < m_{\pi^+\pi^-\eta} < 1.80$		2
$1.80 < m_{\pi^+\pi^-\eta} < 2.50$		5
$2.50 < m_{\pi^+\pi^-\eta} < 3.10$		10.5
$3.10 < m_{\pi^+\pi^-\eta} < 3.50$		11
$\eta$ reconstruction	-2	1.0
Track reconstruction	-1.1	1.0
ISR photon efficiency	-1.1	1.0
Trigger and filters	-1.5	1.6
Radiative correction		1.0
Luminosity		1.0
Total		
$m_{\pi^+\pi^-\eta} < 1.35$	-5.7	10
$1.35 < m_{\pi^+\pi^-\eta} < 1.80$	-5.7	4.5
$1.80 < m_{\pi^+\pi^-\eta} < 2.50$	-5.7	6.5
$2.50 < m_{\pi^+\pi^-\eta} < 3.10$	-5.7	11
$3.10 < m_{\pi^+\pi^-\eta} < 3.50$	-5.7	12

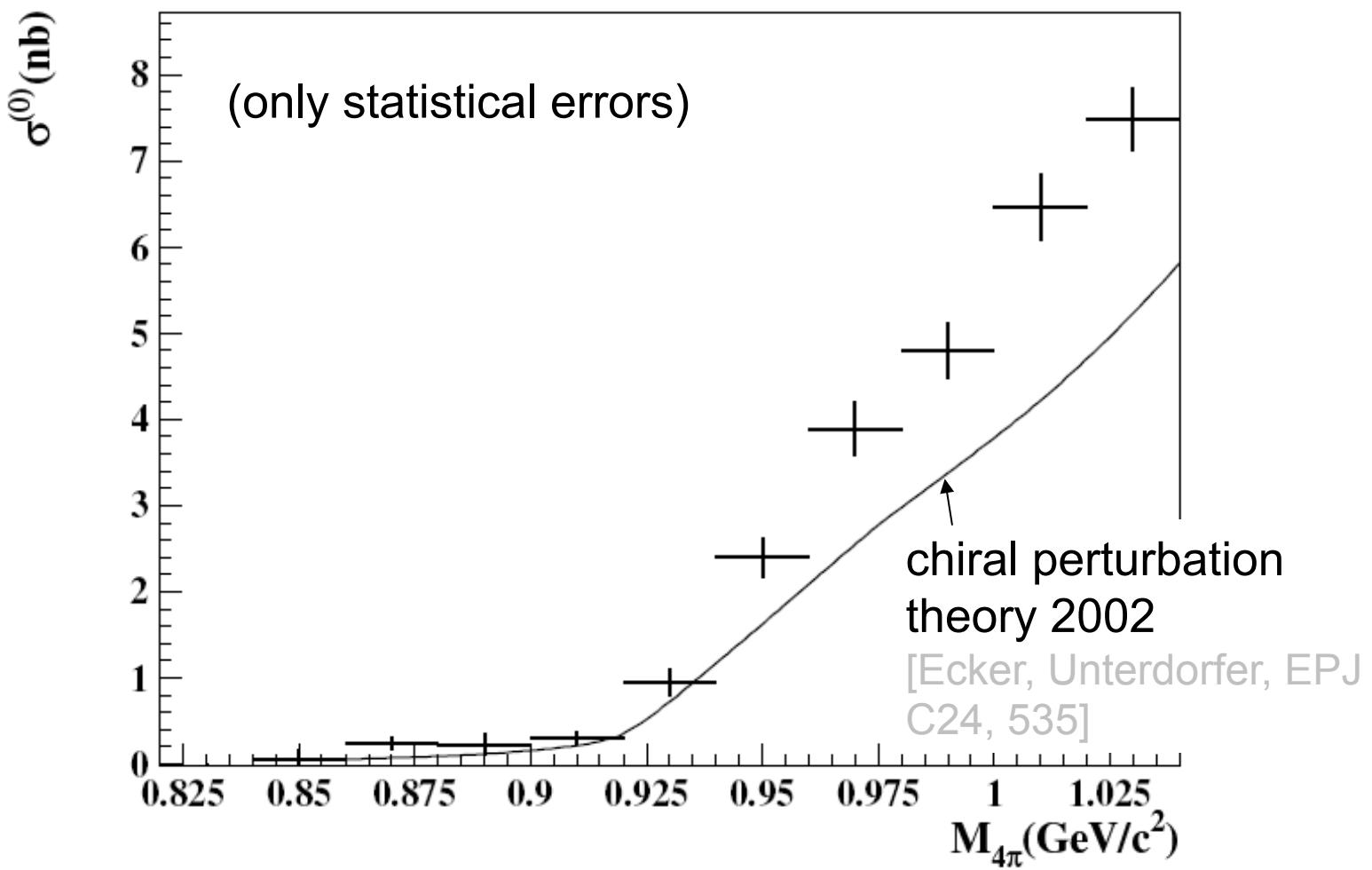


BABAR

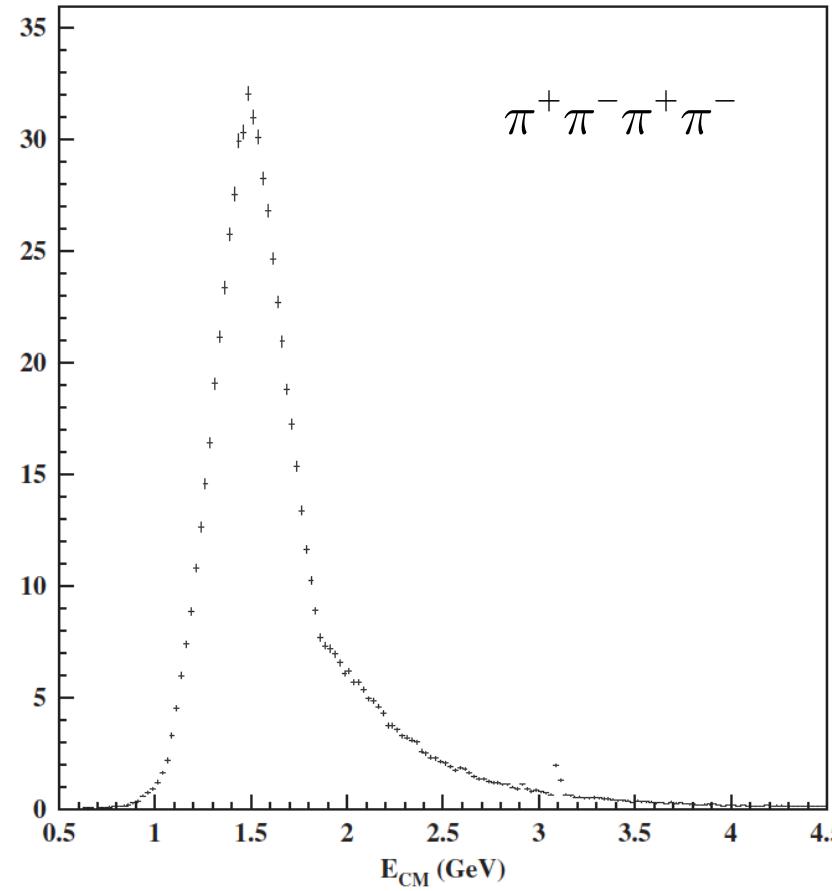
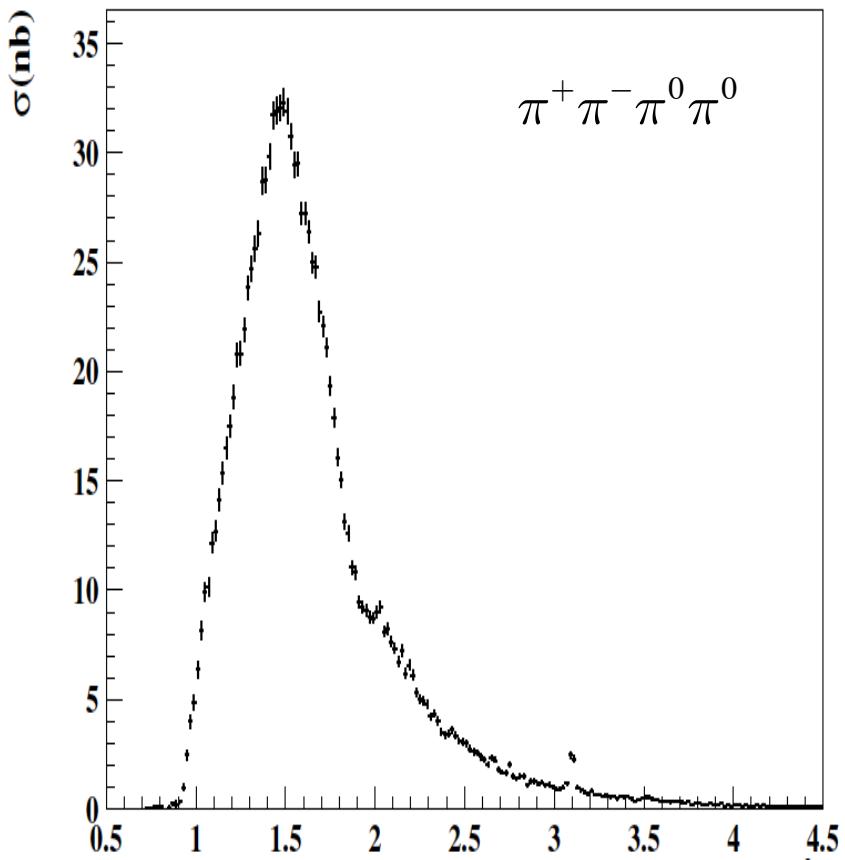
R. Waldi



# $\pi^+\pi^-\pi^0\pi^0$ at Low Energies



# $4\pi$



BABAR

R. Waldi



# Systematic Errors

TABLE I. Systematic uncertainties for different mass ranges.  
For the exact total uncertainties in the low-mass region  
 $M(\pi^+\pi^-2\pi^0) < 1.2 \text{ GeV}/c^2$  see Tab.II.

$M(\pi^+\pi^-2\pi^0)(\text{GeV}/c^2)$	< 1.2	1.2 – 2.7	2.7 – 3.2	> 3.2
Tracking eff.	0.8%	0.8%	0.8%	0.8%
$\gamma$ eff.	0.4%	0.4%	0.4%	0.4%
$2\pi^0$ eff.	2.0%	2.0%	2.0%	2.0%
$\chi^2_{4\pi\gamma}$ eff.	0.4%	0.4%	0.4%	0.4%
Resonances in AfkQED	0.4%	0.4%	0.4%	0.4%
Mass res.	0.3%	0.3%	0.3%	0.3%
FSR	1.0%	1.0%	1.0%	1.0%
NLO ISR	0.5%	0.5%	0.5%	0.5%
ISR luminosity	1.0%	1.0%	1.0%	1.0%
continuum Bkg	1.0%	1.0%	1.0%	2.0%
ISR Background	1 – 100%	1.0%	6.0%	6.0%
Proton PID	0.2%	0.2%	0.2%	0.2%
Kaon PID	0.5%	0.5%	0.5%	0.5%
Muon PID	0%	0%	0%	2.0%
total	3 – 100%	3.1%	6.7%	7.2%



BABAR

R. Waldi

