How to Resolve the Proton Radius Puzzle?

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Introduction: The proton radius puzzle

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)|\sum_q e_q\,\bar{q}\gamma^\mu q|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2(q^2)q^\nu\right]u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\boxed{\langle r^2\rangle_E^p = 6\frac{dG_E^p}{dq^2}\bigg|_{q^2=0}}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2\rangle_E^p}$

- The proton *magnetic* radius

$$\langle r^2\rangle_M^p = \frac{6}{G_M^p(0)}\frac{dG_M^p(q^2)}{dq^2}\bigg|_{q^2=0}$$

Charge radius from atomic physics

$$\langle p(p_f)| \sum_q e_q \, \bar{q}\gamma^\mu q |p(p_i)\rangle = \bar{u}(p_f) \left[\gamma^\mu F_1^p(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2^p(q^2) q^\nu \right] u(p_i)$$

- For a point particle amplitude for $p + \ell \to p + \ell$

$$\mathcal{M} \propto \frac{1}{q^2} \quad \Rightarrow \quad U(r) = -\frac{Z\alpha}{r}$$

- Including q^2 corrections from proton structure

$$\mathcal{M} \propto \frac{1}{q^2} q^2 = 1 \quad \Rightarrow \quad U(r) = \frac{4\pi Z\alpha}{6} \delta^3(r)(r_E^p)^2$$

- Proton structure corrections $\Big(m_r = m_\ell m_p/(m_\ell + m_p) \approx m_\ell\Big)$

$$\boxed{\Delta E_{r_E^p} = \frac{2(Z\alpha)^4}{3n^3} m_r^3 (r_E^p)^2 \delta_{\ell 0}}$$

- **Muonic hydrogen can give the best measurement of $\mathbf{r_E^p}$!**

Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
  more recently $r_E^p = 0.84087(39)$ fm [Antognini et al. Science **339**, 417 (2013)]
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.87680(690)$ fm
  more recently $r_E^p = 0.87750(510)$ fm [Mohr et al. RMP **84**, 1527 (2012)]
  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**

- This is the proton radius puzzle

What could be the reason for the discrepancy?

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2) Hadronic Uncertainty? (Part 2 of this talk)

3) New Physics?

- Declaimer: I will focus on published work I am involved in

- For a review of other approaches see

  [Carlson, Prog. Part. Nucl. Phys. **82**, 59 (2015)]

Part 1: Proton radii from scattering

What does the PDG say?

K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010)

p CHARGE RADIUS

This is the rms charge radius, $\sqrt{\langle r^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.8768±0.0069	MOHR	08	RVUE	2006 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.897 ±0.018	BLUNDEN	05		SICK 03 + 2γ correction
0.8750±0.0068	MOHR	05	RVUE	2002 CODATA value
0.895 ±0.010 ±0.013	SICK	03		$e p \to e p$ reanalysis
0.830 ±0.040 ±0.040	[24] ESCHRICH	01		$e p \to e p$
0.883 ±0.014	MELNIKOV	00		1S Lamb Shift in H
0.880 ±0.015	ROSENFELDR...	00		$e p$ + Coul. corrections
0.847 ±0.008	MERGELL	96		$e p$ + disp. relations
0.877 ±0.024	WONG	94		reanalysis of Mainz $e p$ data
0.865 ±0.020	MCCORD	91		$e p \to e p$
0.862 ±0.012	SIMON	80		$e p \to e p$
0.880 ±0.030	BORKOWSKI	74		$e p \to e p$
0.810 ±0.020	AKIMOV	72		$e p \to e p$
0.800 ±0.025	FREREJACQ...	66		$e p \to e p$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$e p \to e p$

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

[24] ESCHRICH 01 actually gives $\langle r^2 \rangle = (0.69 \pm 0.06 \pm 0.06)$ fm^2.

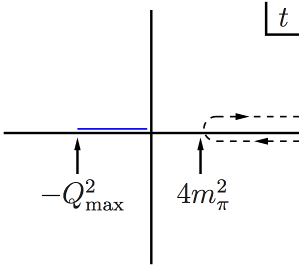
Form Factors: What we don't know

- The form factors are non-perturbative objects.
- **Nobody** knows the *exact* functional form of G_E^p and G_M^p
- They don't have to have a dipole/polynomial/spline
  or any other functional form
- Including such models can bias your extraction of r_E^p and r_M^p

Form Factors: What we do know

- Analytic properties of $G_E^p(t)$ and $G_M^p(t)$ are known
- They are analytic outside a cut $t \in [4m_\pi^2, \infty]$

  [Federbush, Goldberger, Treiman, Phys. Rev. **112**, 642 (1958)]
- $e - p$ scattering data is in $t < 0$ region



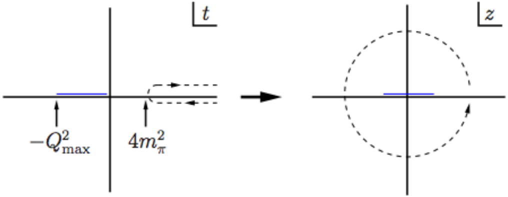
z expansion

- z expansion:

  We can map the domain of analyticity onto the unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

  where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$

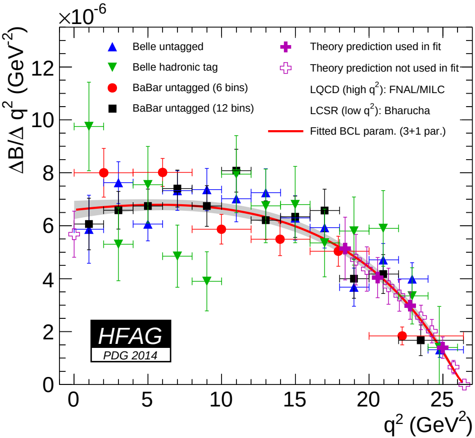


- Expand $G^p_{E,M}$ in a Taylor series in z: $G^p_{E,M}(q^2) = \sum_{k=0}^{\infty} a_k \, z(q^2)^k$

z expansion

- For meson form factors, z expansion **is** the method

- E.g. $|V_{ub}|$ from exclusive $B \to \pi \ell \bar{\nu}$



[Heavy Flavor Averaging Group, arXiv:1412.7515]

Extracting r_E^p using the z expansion

- First use of the z expansion to extract r_E^p
  [Hill, GP PRD **82** 113005 (2010)]

- Proton: $Q^2 < 0.5$ GeV^2

$$r_E^p = 0.870 \pm 0.023 \pm 0.012\,\mathrm{fm}$$

- Proton and neutron data

$$r_E^p = 0.880^{+0.017}_{-0.020} \pm 0.007\,\mathrm{fm}$$

- Proton, neutron and $\pi\,\pi$ data

$$r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$$

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
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Extracting r_M^p using the z expansion

- z expansion study

  [Zachary Epstein, GP, Joydeep Roy PRD **90**, 074027 (2014)]

- Proton data :$r_M^p = 0.91^{+0.03}_{-0.06} \pm 0.02$ fm
- Proton and neutron data: $r_M^p = 0.87^{+0.04}_{-0.05} \pm 0.01$ fm
- Proton, neutron and $\pi\,\pi$ data: $r_M^p = 0.87 \pm 0.02$ fm

- PDG 2014:

- $r_M^p = 0.777 \pm 0.017$ fm [Bernauer et al. PRL **105**, 242001 (2010)]
- $r_M^p = 0.876 \pm 0.019$ fm [Borisyuk NPA **843**, 59 (2010)]
- $r_M^p = 0.854 \pm 0.005$ fm [Belushkin et al. PRC **75**, 035202 (2007)]

- Other non-PDG values:

- $r_M^p = 0.855 \pm 0.035$ fm [Sick Prog.Part.Nucl.Phys. **55**, 440 (2005)]
- $r_M^p = 0.86^{+0.02}_{-0.03}$ fm [Lorenz et al. EPJA **48**, 151 (2012)]
- $r_M^p = 0.78 \pm 0.08$ fm [Karshenboim PRD **90** 053013 (2014) 5]

Latest z expansion fit

- Most recent study using the z expansion
  [Gabriel Lee, Arrington, Hill, PRD **92**, 013013 (2015)]
  Analyze the "Mainz" data set
  [Bernauer et al. PRL **105**, 242001 (2010)]
  and world data (excluding Mainz)

- World data

[Lee, Arrington, Hill '15]	$r_E^p = 0.918 \pm 0.024$ fm
[Hill , GP '10]	$r_E^p = 0.870 \pm 0.023 \pm 0.012$ fm
[Lee, Arrington, Hill '15]	$r_M^p = 0.913 \pm 0.037$ fm
[Epstein, GP, Roy '14]	$r_M^p = 0.910^{+0.030}_{-0.060} \pm 0.020$ fm

- Mainz data
  $r_E^p = 0.895 \pm 0.020$ fm $\quad r_M^p = 0.773 \pm 0.038$ fm

Part 2: Connecting muon-proton scattering and muonic hydrogen

The bottom line

- Scattering:
  - World $e - p$ data [Lee, Arrington, Hill '15]
    $r_E^p = 0.918 \pm 0.024$ fm
  - Mainz $e - p$ data [Lee, Arrington, Hill '15]
    $r_E^p = 0.895 \pm 0.020$ fm
  - Proton, neutron and π data [Hill , GP '10]
    $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$
- Muonic hydrogen
  - [Pohl et al. Nature **466**, 213 (2010)]
    $r_E^p = 0.84184(67)$ fm
  - [Antognini et al. Science **339**, 417 (2013)]
    $r_E^p = 0.84087(39)$ fm
- The bottom line:
  using z expansion scattering disfavors muonic hydrogen
- Is there a problem with muonic hydrogen *theory*?

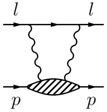
Muonic hydrogen theory

- Is there a problem with muonic hydrogen *theory*?

- Potentially yes!
  [Hill, GP PRL **107** 160402 (2011)]

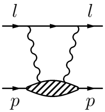
- Muonic hydrogen measures ΔE and translates it to r_E^p
  - [Pohl et al. Nature **466**, 213 (2010)]
    $\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3$ meV
  - [Antognini et al. Science **339**, 417 (2013)]
    $\Delta E = 206.0336(15) - 5.2275(10)(r_E^p)^2 + 0.0332(20)$ meV

- In both cases apart from r_E^p need two-photon exchange



Two photon exchange

- In both cases apart from r_E^p we have two-photon exchange



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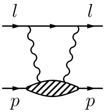

$$\frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \mathbf{k}, s|\, T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|\mathbf{k}, s\rangle$$
$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu-\frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu-\frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

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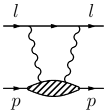

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- Dispersion relations $(\nu = 2k\cdot q,\ Q^2=-q^2)$

$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\, \frac{{\rm Im} W_1(\nu', Q^2)}{\nu'^2(\nu'^2-\nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\, \frac{{\rm Im} W_2(\nu', Q^2)}{\nu'^2-\nu^2}$$

- W_1 requires subtraction...

Two photon exchange

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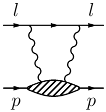

- Imaginary part of TPE related to data:
  form factors, structure functions

Two photon exchange

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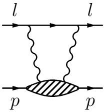

- Imaginary part of TPE related to data:
  form factors, structure functions

- Cannot reproduce it from its imaginary part:
  Dispersion relation requires subtraction

- Need poorly constrained non-perturbative function $W_1(0, Q^2)$
  Can calculate it
- In small Q^2 limit using NRQED [Hill, GP, PRL **107** 160402 (2011)]
- In large Q^2 limit using OPE [J. C. Collins, NPB **149**, 90 (1979)]

Two photon exchange

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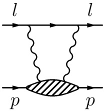

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- Introduces hard to quantify hadronic uncertainty

Variety of approaches

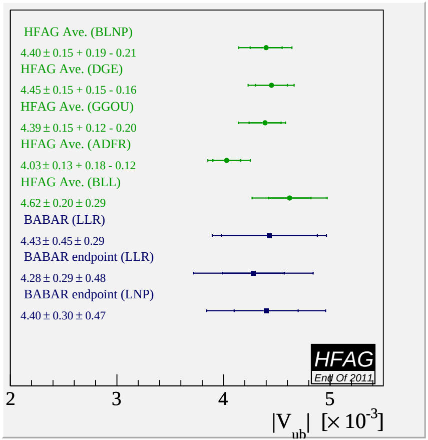
- Considering far-reaching implications of the puzzle we should explore a variety of approaches:

[Nevado, Pineda, PRC **77**, 035202 (2008)]
[Peset, Pineda, EPJA **51**, 32 (2015)]
[Peset, Pineda, NPB **887**, 69 (2014)]
[Carlson, Vanderhaeghen, PRA **84**, 020102 (2011)]
[Birse, McGovern, EPJA **48**, 120 (2012)]
[Miller PLB **718**, 1078 (2013)]
[Gorchtein, Llanes-Estrada, Szczepaniak PRA **87**, 052501 (2013)]
[Alarcon, Lensky, Pascalutsa,EPJC **74**, 2852 (2014)]
[Tomalak, Vanderhaeghen, PRD **90**, 013006 (2014)]
[Tomalak, Vanderhaeghen, EPJC **76**, 125 (2016)]

...

Variety of approaches

- Considering far-reaching implications of the puzzle we should explore a variety of approaches
- What you want to have:



- In the following: $W_i \to$ NRQED

Why NRQED?

- Solving Schrödinger equation for $1/r$ potential

$$E_n = -\frac{1}{2} m_r c^2 \alpha^2 \frac{1}{n^2}$$

  where $m_r = m_\mu m_p/(m_\mu + m_p) \approx m_\mu$

- Muon momentum in muonic hydrogen $p \sim m_\mu c \alpha \sim 1$ MeV

- Muon is non-relativistic

  Can use Non Relativistic QED (NRQED)

  [Caswell, Lepage PLB **167**, 437 (1986); Kinoshita Nio PRD **53**, 4909 (1996); Manohar PRD **56**, 230 (1997)]

- For an introduction to NRQED see [GP MPLA **30**, 1550128 (2015)]

NRQED

$$
\begin{aligned}
\mathcal{L}_p \quad = \quad & \psi_p^\dagger \Big\{ iD_t + \frac{\mathbf{D}^2}{2m_p} + \frac{\mathbf{D}^4}{8m_p^3} + c_F e \frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2m_p} + c_D e \frac{[\boldsymbol{\partial}\cdot\mathbf{E}]}{8m_p^2} \\
& + i c_S e \frac{\boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m_p^2} + c_{W1} e \frac{\{\mathbf{D}^2, \boldsymbol{\sigma}\cdot\mathbf{B}\}}{8m_p^3} \\
& - c_{W2} e \frac{D^i \boldsymbol{\sigma}\cdot\mathbf{B} D^i}{4m_p^3} + c_{p'p} e \frac{\boldsymbol{\sigma}\cdot\mathbf{D}\mathbf{B}\cdot\mathbf{D} + \mathbf{D}\cdot\mathbf{B}\boldsymbol{\sigma}\cdot\mathbf{D}}{8m_p^3} \\
& + i c_M e \frac{\{\mathbf{D}^i, [\boldsymbol{\partial}\times\mathbf{B}]^i\}}{8m_p^3} + c_{A1} e^2 \frac{\mathbf{B}^2 - \mathbf{E}^2}{8m_p^3} - c_{A2} e^2 \frac{\mathbf{E}^2}{16m_p^3} + \ldots \Big\} \psi_p
\end{aligned}
$$

- The $1/m_p^4$ calculated in [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]

- Need also

$$
\mathcal{L}_{\text{contact}} = d_1 \frac{\psi_p^\dagger \boldsymbol{\sigma} \psi_p \cdot \psi_\ell^\dagger \boldsymbol{\sigma} \psi_\ell}{m_\ell m_p} + d_2 \frac{\psi_p^\dagger \psi_p \psi_\ell^\dagger \psi_\ell}{m_\ell m_p}
$$

NRQED

- Matching
- Operators with one photon coupling:
  c_i given by $F_i^{(n)}(0)$

- Operators with only two photon couplings:
  c_{A_i} given by forward and backward Compton scattering

- d_i from two-photon amplitude

- From c_i and d_i determine proton structure correction, e.g.

$$\delta E(n,\ell) = \delta_{\ell 0} \frac{m_r^3 (Z\alpha)^3}{\pi n^3} \left(\frac{Z\alpha\pi}{2m_p^2} c_D^{\mathrm{proton}} - \frac{d_2}{m_\ell m_p} \right)$$

- Bottom line: need $c_D \Leftrightarrow r_E^p$ and d_2
  d_2 suffers from hadronic uncertainty
  What to do?

Hadronic uncertainty d_2

- d_2 suffers from hadronic uncertainty
  What to do?
- Improve modeling [Hill, GP *in progress*]
  How large can it be?
- But even if d_2 *can* be large it does not follow that it *must* be large
- Experimental test: $\mu - p$ scattering
  MUSE (MUon proton Scattering Experiment)

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV

  Both proton and muon non-relativistic

MUSE

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  Muon momentum $\sim m_\mu \sim 100$ MeV

  Muon is relativistic, proton is still non-relativistic

MUSE

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  Both proton and muon non-relativistic

- MUSE:
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- QED-NRQED effective theory:
  - Use QED for muon
  - Use NRQED for proton

  $m_\mu/m_p \sim 0.1$ as expansion parameter

- A *new* effective field theory suggested in
  [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]

QED-NRQED Effective Theory

- Example: TPE at the lowest order in $1/m_p$
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

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- Consider muon-proton scattering $\mu(p) + p(k) \to \mu(p') + p(k')$
- At lowest order in $1/m_p$: $p^0 = p'^0 \Rightarrow \delta(p^0 - p'^0)$
- At the proton rest frame $k = (m_p, \vec{0}) \Rightarrow k^0 = 0$ in NRQED

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- NRQED propagator: $\dfrac{1}{l^0 - \vec{l}^2/2M + i\epsilon}$

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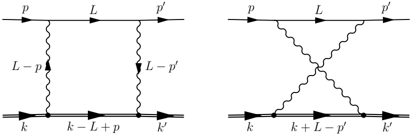
- At the proton rest frame $k = (m_p, \vec{0}) \Rightarrow k^0 = 0$ in NRQED
- NRQED propagator: $\dfrac{1}{l^0 - \vec{l}^2/2M + i\epsilon}$

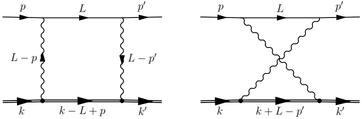


$$\frac{1}{p^0 - L^0 + i\epsilon} + \frac{1}{L^0 - p^0 + i\epsilon} \Rightarrow \delta(L^0 - p^0)$$

- In total

$$\delta(p^0 - p'^0)\,\delta(L^0 - p^0) = \delta(L^0 - p^0)\,\delta(L^0 - p'^0)$$

QED-NRQED Effective Theory



- The amplitude

$$i\mathcal{M}(2\pi)^4\delta^4(k+p-k'-p') = Z^2e^4\int\frac{d^4L}{(2\pi)^4}\frac{1}{(L-p)^2L-p')^2}$$

$$\times \quad \bar{u}(p')\gamma^0\frac{i}{\not{L}-m}\gamma^0u(p)\chi^\dagger\chi(2\pi)\delta(L^0-p^0)\,(2\pi)\delta(L^0-p'^0)$$

$$\times \quad (2\pi)^3\delta^4(\vec{p}-\vec{p}'-\vec{k}')$$

- The cross section

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 4E^2\left(1-v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}\left[1+\frac{Z\alpha\pi v\sin\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)}{1-v^2\sin^2\theta}\right]$$

$Z=1$, $E=$ muon energy, $v=|\vec{p}|/E$, $q=p'-p,\theta$ scattering angle

QED-NRQED Effective Theory

- QED-NRQED result

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 4E^2\left(1 - v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}\left[1 + \frac{Z\alpha\pi v\sin\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)}{1 - v^2\sin^2\theta}\right]$$

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- *Same result* as scattering relativistic lepton off static $1/r$ potential [Dalitz, Proc. Roy. Soc. Lond. **206**, 509 (1951)] reproduced in [Itzykson, Zuber, "Quantum Field Theory"]

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- *Same result* as $m_p \to \infty$ of "point particle proton" QED scattering (For $m_p \to \infty$ only proton charge is relevant)

QED-NRQED Effective Theory beyond $m_p \to \infty$ limit

- QED-NRQED allows to calculate $1/m_p$ corrections
- Example: one photon exchange $\mu + p \to \mu + p$:
  QED-NRQED = $1/m_p$ expansion of form factors
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

Connecting muon-proton scattering to muonic hydrogen

- Matching

QED, QCD $\qquad$ $G_{E,M}$, Structure func., $W_1(0, Q^2)$

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NRQED-NRQED: *muonic H*	r_E^p, $\psi_\mu^\dagger\psi_\mu\psi_p^\dagger\psi_p$

- Need to match QED-NRQED contact interaction, e.g. $\bar{\mu}\gamma^0\mu\psi_p^\dagger\psi_p$ to NRQED-NRQED contact interaction, e.g. $\psi_\mu^\dagger\psi_\mu\psi_p^\dagger\psi_p$ [Dye, Gonderinger, GP *in progress*]

Connecting muon-proton scattering to muonic hydrogen

- To do list:

1) Relate QED-NRQED contact interactions to NRQED contact interactions and $W_1(0, Q^2)$

2) Calculate $d\sigma(\mu + p \to \mu + p)$ and asymmetry in terms of r_E^p and d_2

3) *Direct* relation between μ–p scattering and muonic H

Conclusions

- Proton radius puzzle: $> 5\sigma$ discrepancy between
- r_E^p from muonic hydrogen
- r_E^p from hydrogen and $e-p$ scattering

- After 6 years the origin is still not clear

1) Is it a problem with the electronic extraction?
2) Is it a hadronic uncertainty?
3) is it new physics?

- Motivates a reevaluation of our understanding of the proton

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Conclusions

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1) Extraction of proton radii from scattering:
   Use an established tool of the z expansion
   Studies disfavor the muonic hydrogen value

2) Direct connection between muon-proton scattering and muonic hydrogen using a new effective field theory: QED-NRQED

- Much more work to do!

- Thank you