Grand unification and CP violation

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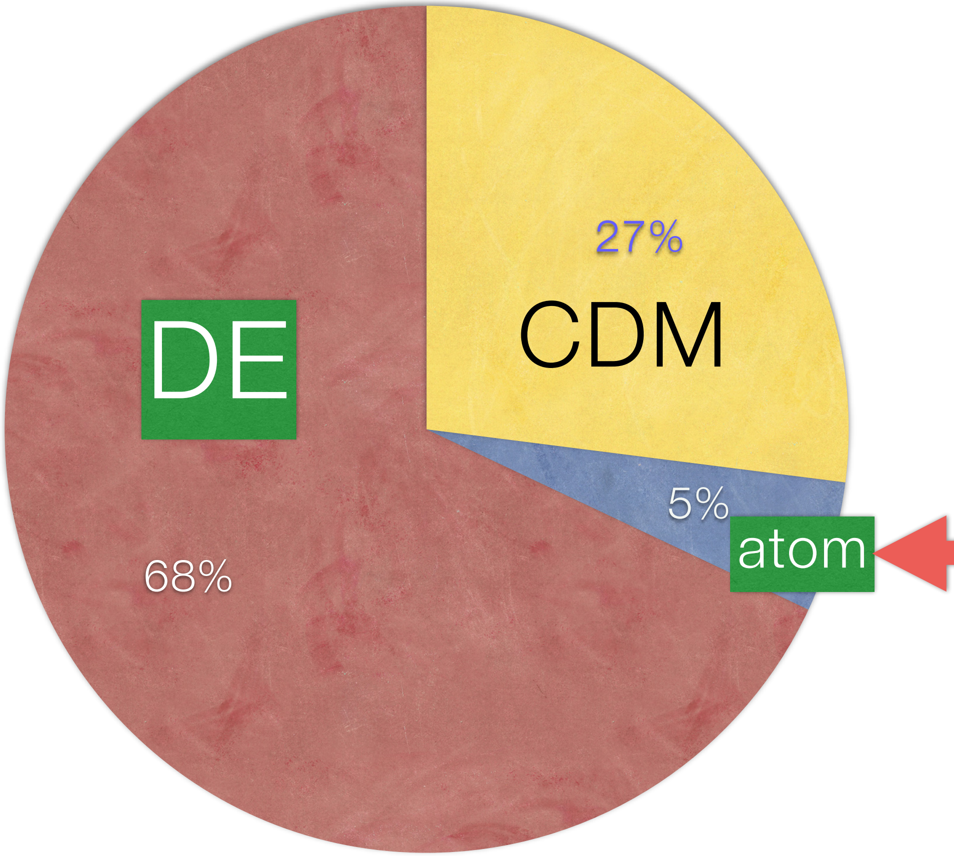
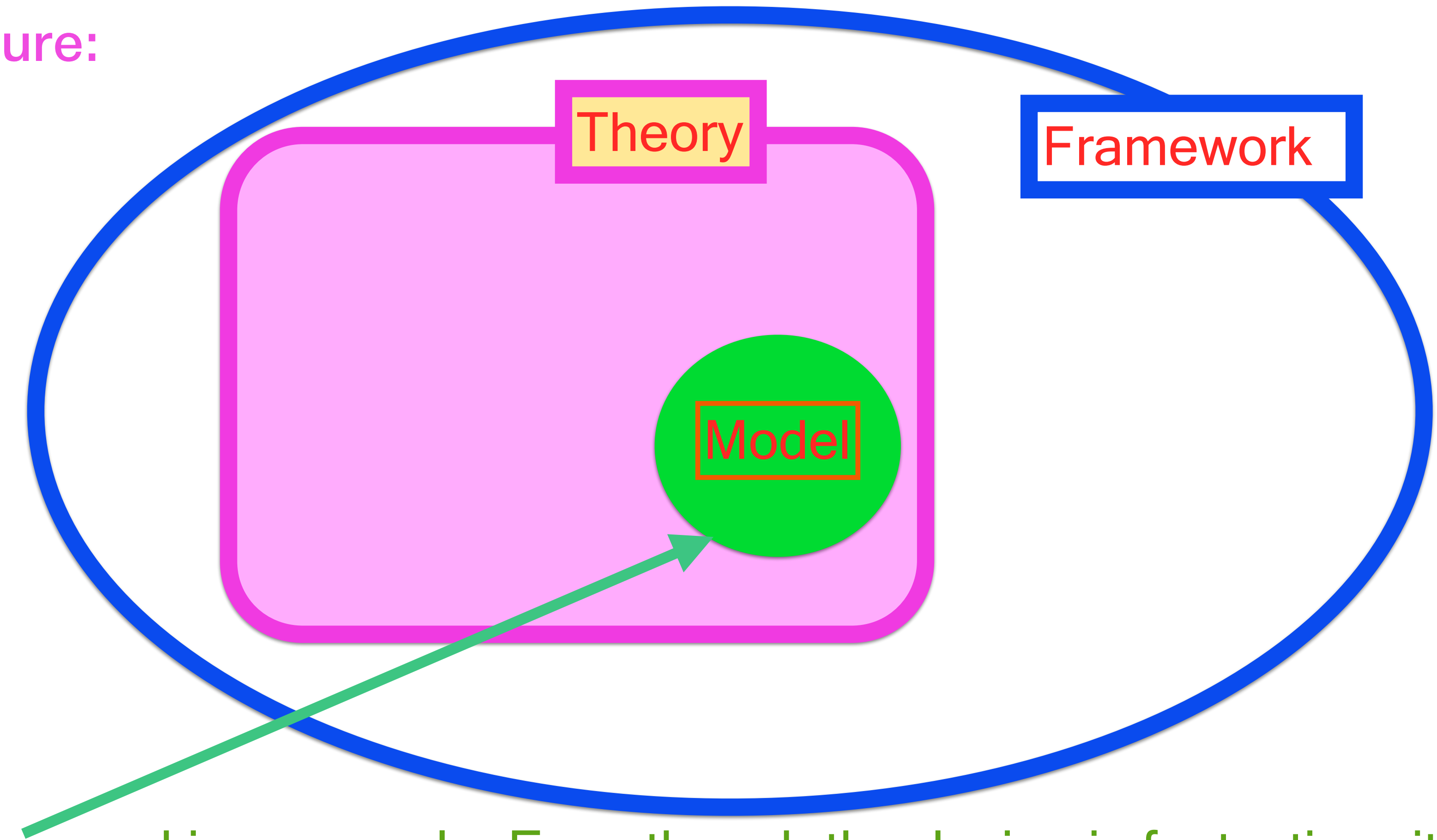


Chiral fields at GUT scale
SU(5), SU(7) GUTs

UGUTF:
Kim, PRL 45, 1916 (1980);
arXiv:1503.03104.

Gross's picture:



"Model" is a working example. Even though the design is fantastic, without a model example some will say that it is a religion.
Efforts to find a working model is our job toward THEORY/FRAMEWORK.

CP violation is the needed ingredient for our existence.

1. Jarlskog phase in CKM matrix

To have physical effects of CP violation, the J must be non vanishing. Our form for the CKM matrix is, with the 1st row real,

$$\begin{pmatrix} c_1, & s_1 c_3, & s_1 s_3 \\ -c_2 s_1, & e^{-i\delta} s_2 s_3 + c_1 c_2 c_3, & -e^{-i\delta} s_2 c_3 + c_1 c_2 s_3 \\ -e^{i\delta} s_1 s_2, & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta}, & c_2 c_3 + c_1 s_2 s_3 e^{i\delta} \end{pmatrix}$$

The individual element of determinant is

$$\begin{aligned}
V_{11}V_{22}V_{33} &= c_1^2 c_2^2 c_3^2 + c_1^2 s_2^2 s_3^2 + 2c_1 c_2 c_3 s_2 s_3 \cos\delta \\
&\quad - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
-V_{11}V_{23}V_{32} &= c_1^2 c_2^2 s_3^2 + c_1^2 s_2^2 c_3^2 - 2c_1 c_2 c_3 s_2 s_3 \cos\delta \\
&\quad + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
V_{12}V_{23}V_{31} &= s_1^2 s_2^2 c_3^2 - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
-V_{12}V_{21}V_{33} &= s_1^2 c_2^2 c_3^2 + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
V_{13}V_{21}V_{32} &= s_1^2 c_2^2 s_3^2 - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}, \\
-V_{13}V_{22}V_{31} &= s_1^2 s_2^2 s_3^2 + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}.
\end{aligned}$$

Is Im(V_{11} V_{22} V_{33}) the Jarlskog determinant?

The Jarlskog determinant is
J=|Im V_{11} V_{22} V_{12}*V_{21}*|, or |Im $V_{ii}V_{jj}V_{ij}$*V_{ji}*|

With the usual definition on J:
J=|Im V_{11} V_{33} V_{13}*V_{31}*|. Then, on 1=Det V

imaginary part of this is J

$$V_{13}^*V_{22}^*V_{31}^* = |V_{22}|^2 V_{11}V_{33}V_{13}^*V_{31}^* - V_{11}V_{23}V_{32}V_{13}^*V_{31}^*V_{22}^* + |V_{31}|^2 V_{12}V_{23}V_{13}^*V_{22}^* - V_{12}V_{21}V_{33}V_{13}^*V_{31}^*V_{22}^* + |V_{13}|^2 V_{21}V_{32}V_{31}^*V_{22}^* - |V_{13}V_{22}V_{31}|^2.$$

unitarity of V

$$V_{13}^*V_{22}^*V_{31}^* = (1-|V_{21}|^2)V_{11}V_{33}V_{13}^*V_{31}^* + V_{11}V_{23}V_{13}^*V_{21}^*|V_{31}|^2 + (1-|V_{11}|^2)V_{12}V_{23}V_{13}^*V_{22}^* + |V_{13}|^2(V_{12}V_{21}V_{11}^*V_{22}^* + V_{21}V_{32}V_{31}^*V_{22}^*) - |V_{13}V_{22}V_{31}|^2.$$

Similar considerations for other elements give the imaginary part as [(1-$|V_{21}|^2$) -$|V_{31}|^2$ +(1-$|V_{11}|$)^2]J=J

Kim-Seo form of J: $J = \mathrm{Im}\,(V_{31}^* V_{22}^* V_{13}^*)$

JEK, M-S. Seo, PoS DSU2012 (2012) 009 [arXiv:1211.0357[hep-ph]]

JEK, D.Y. Mo, S. Nam, JKPS 66 (2015) 894 [arXiv:1402.2978[hep-ph]]

By looking at the KS form of J, we can see where the physical CP phase appears. In particular, for tree processes

Maximal CP violation in lepton sector?

T2K experiment [S.V. Cao at PASCOS 2016], slightly favors $\delta_{\rm PMNS}$ near -90 degrees.

Determination of $\delta_{\rm PMNS}$ may choose $\delta_{\rm CKM}$ in certain models.

Is $\delta_{\rm PMNS} = \pm\delta_{\rm CKM}$?

JEK + S. Nam, arXiv:1506.08491
JEK + D. Y. Mo + M-S. Seo, arXiv:1506.08984

$$V_{\rm CKM}^{\rm KS} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -c_2 s_1 & e^{-i\delta_{\rm CKM}} s_2 s_3 + c_1 c_2 c_3 & -e^{-i\delta_{\rm CKM}} s_2 c_3 + c_1 c_2 s_3 \\ -e^{i\delta_{\rm CKM}} s_1 s_2 & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\rm CKM}} & c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\rm CKM}} \end{pmatrix}$$

$s_i = \sin\theta_i$ for $i = 1,2,3$ $\quad J = c_1c_2c_3s_1^2s_2s_3 \sin(\mathrm{delta}_{\rm CKM})$

Easy way to calculate J is

$$V_{\rm PMNS}^{\rm KS} = \begin{pmatrix} C_1 & S_1 C_3 & S_1 S_3 \\ -C_2 S_1 & e^{-i\delta_{\rm PMNS}} S_2 S_3 + C_1 C_2 C_3 & -e^{-i\delta_{\rm PMNS}} S_2 C_3 + C_1 C_2 S_3 \\ -e^{i\delta_{\rm PMNS}} S_1 S_2 & -C_2 S_3 + C_1 S_2 C_3 e^{i\delta_{\rm PMNS}} & C_2 C_3 + C_1 S_2 S_3 e^{i\delta_{\rm PMNS}} \end{pmatrix}$$

$S_i = \sin\Theta_i$ for $i = 1,2,3$ $\quad J = C_1C_2C_3S_1^2S_2S_3 \sin(\mathrm{delta}_{\rm PMNS})$

Even though s_i is not equal to S_i, |deltaCKM| and |deltaPMNS| can be equal. We may satisfy the following in this program

$$\delta_{\rm PMNS} \simeq \pm\delta_{\rm CKM}$$

if CP violation is spontaneous a la Froggatt-Nielsen by ONE complex vev of a SM singlet X. [JEK-Nam, 1506.08491]

Comments on Jarlskog determinant [JEK-Mo-Seo]

- There are three possibilities for $\delta_{\rm CKM}$: α, β, γ of PDG book.

- (i) Make Det=1 as in KS. Make the real part of (22) element is very large as in many parametrizations.

  $\delta_{\rm CKM}$ is α (i) If 1st row = real, Kobayashi-Maskawa with a phase multiplied,
  (ii) Kim-Seo parametrization

- (i) If both 1st row = real, and 1st column = real, then

  $\delta_{\rm CKM}$ = γ : Chau-Keung parametrization
  Maiani parametrization with a phase multiplied

- The identity $\delta_{\rm CKM}$ = Imaginary part of V(31)*V(22)*V(13)* was very useful.

12. CKM quark-mixing matrix **15**

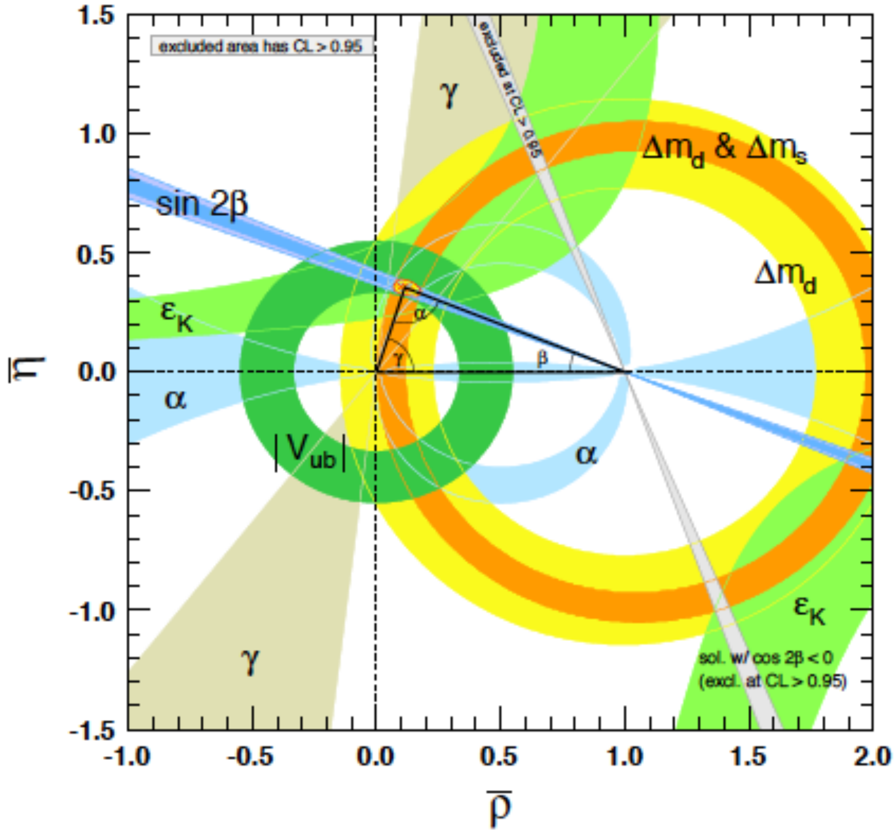


Figure 12.2: Constraints on the $\bar\rho, \bar\eta$ plane. The shaded areas have 95% CL.

and the Jarlskog invariant is $J = (3.06^{+0.21}_{-0.20}) \times 10^{-5}$.

PDG 2014, Ceccucci et al.

$$\alpha = (85.4^{+3.9}_{-3.8})^\circ \quad \gamma = (68.0^{+8.0}_{-8.5})^\circ$$

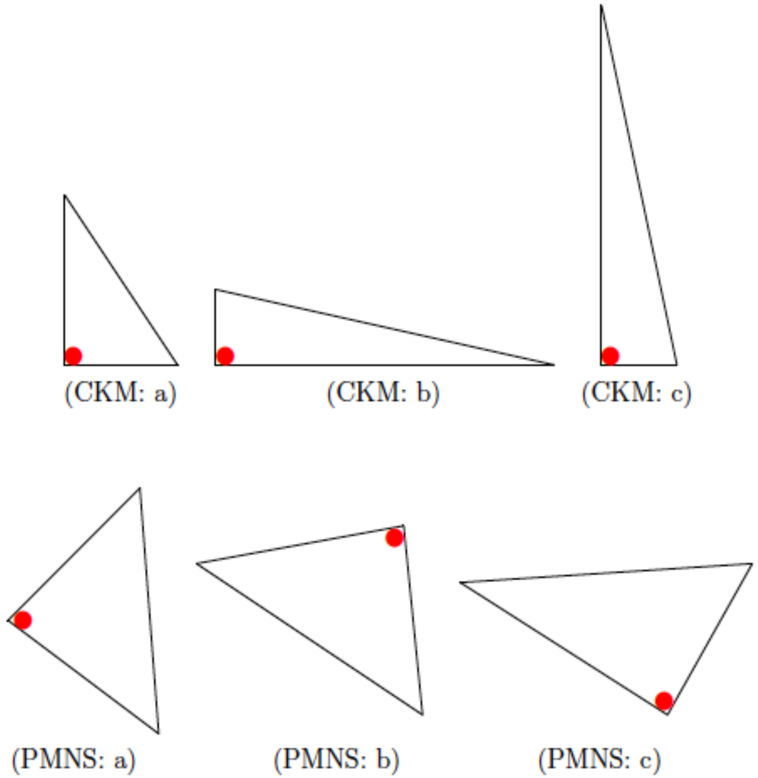
$$\alpha \to \frac{\pi}{2}, \quad \gamma \to \frac{\pi}{2.5}$$

CKM matrix form determines the phase in the matrix.

Quark sector alone cannot fix it.

2. One phase in the theory ?

If there is only one phase, these are the possibility.



Is the angle 90^o ?

There are many phases in concern:
CKM, PMNS, Majorana, and
leptogenesis.

If there is only one phase, all of these
must be expressed in terms of one phase.

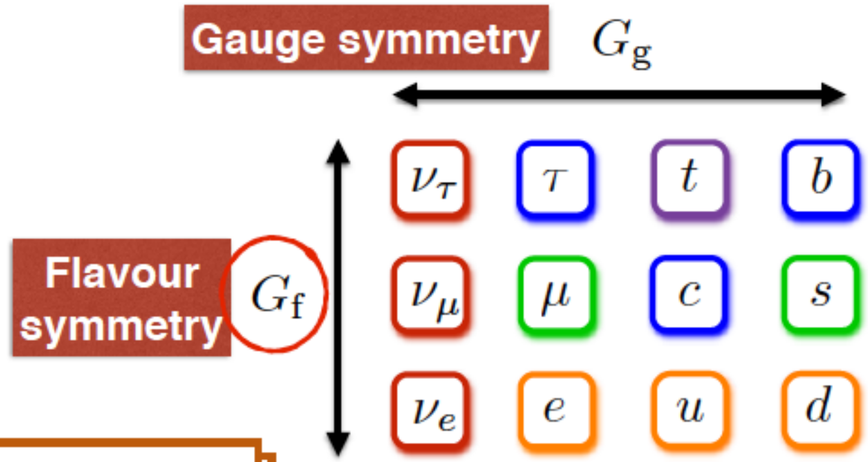
Relation of CKM and PMNS phases: GUTs

But CP phase is a property of families.
Grand unification of families?

FCNC: The family symmetry breaking scale is above 10^5 GeV.

Flavour symmetry

Y-L Zhou, 4 Aug 2016



Useful for CP phase calculation.

Anarchy

	Continuous	Discrete
Abelian	U(1)	Z_n
Non-Abelian	SU(3), SO(3),...	A_4, S_4, A_5, $\Delta(48)$, ...

I show just a flavor in flavor session

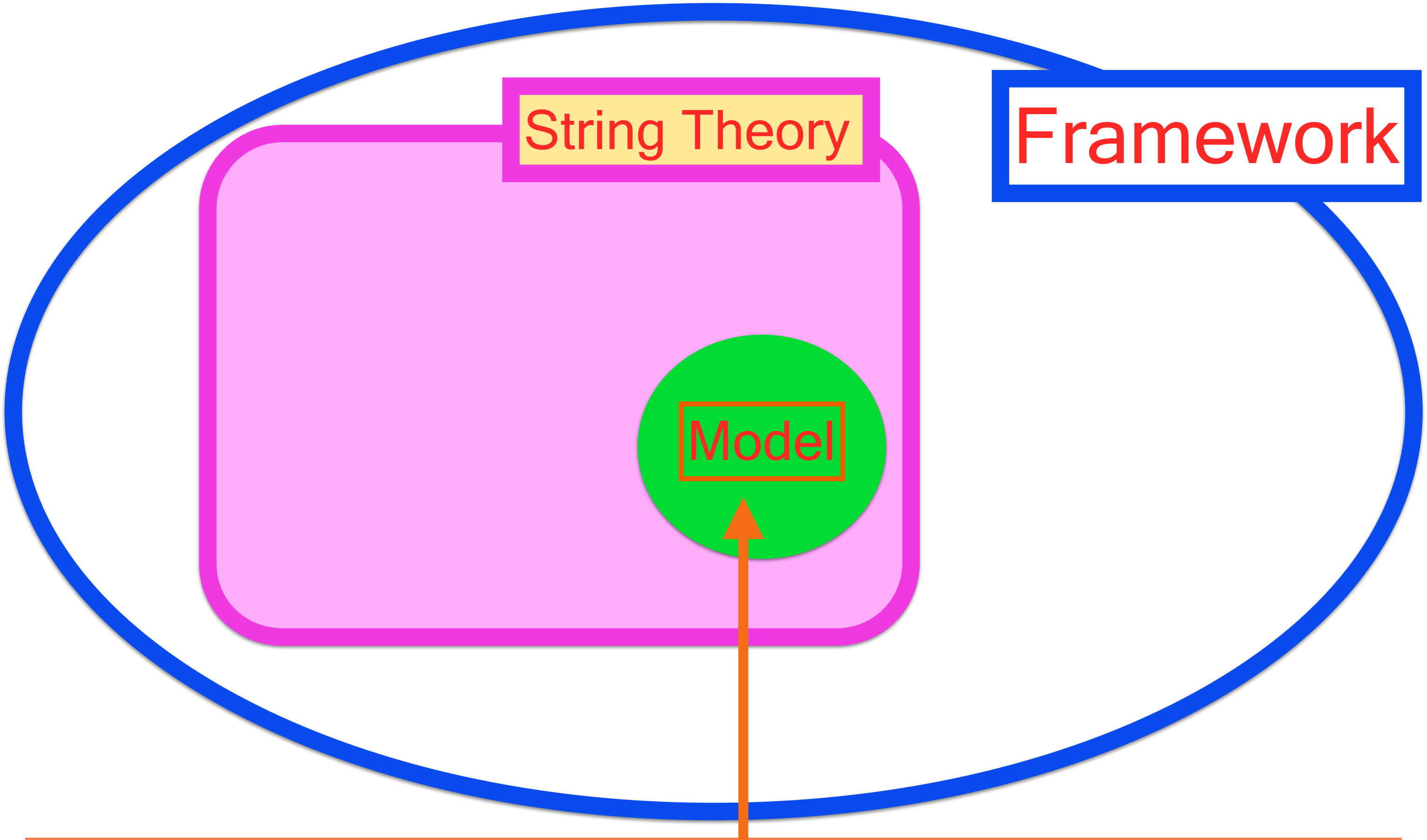
- (i) SM x (Family symmetry)
- (ii) GUT x (Family symmetry)
- (iii) Unification of GUT families in a simple gauge group!!!

UGUTFs: SU(7) in string compactification [JEK, 1503.03104].
One CP phase is given in [JEK+Nam, 1506.08491].
Froggatt-Nielsen form,

$$\tilde{M}^{(u)} = \left(\begin{array}{c|ccc} & u_R(+5) & c_R(+4) & t_R(+2) \\ \hline \bar{q}_1(+1) & cX^{u6}_{-1} & -cX^{u5}_{-1} & \kappa_t X^{u3}_{-1} \\ \bar{q}_2(0) & -cX^{u5}_{-1} & cX^{u4}_{-1} & -\kappa_t X^{u2}_{-1} \\ \bar{q}_3(-2) & \kappa_t X^{u3}_{-1} & -\kappa_t X^{u2}_{-1} & 1 \end{array}\right) v_u, \qquad \tilde{M}^{(d)} = \left(\begin{array}{c|ccc} & d_R(-5) & s_R(0) & b_R(+2) \\ \hline \bar{q}_1(+1) & dX^{d4}_{+1} & 0 & 0 \\ \bar{q}_2(0) & 0 & sX^{d}_{+1}X^{d}_{-1} & \kappa_b X^{d2}_{-1} \\ \bar{q}_3(-2) & 0 & \kappa_b X^{d2}_{+1} & 1 \end{array}\right) v_d$$

Entries must have different phases

These are real, for example



But there must be working examples, consistent with the grand design, framework.

CKM matrix: maximal CP phase +-90°
JEK: PLB704, 360 (2011)

Z_4 symmetry

CKM phase=- PMNS phase
JEK+Nam: EPJC75, 619 (2015)

$$\delta_{\rm PMNS} = \pm\delta_{\rm CKM}$$

$Z_{12\text{-}I}$ orbifold compactification

The FN field may have terms in the potential as

$$V \ni \Phi^4, \Phi^2, \Phi^3, \text{ etc.}$$

Let the phase of FN field be $\frac{2\pi}{N}$.
The phase of VEV should not appear in the above terms. For the quartic term for example, it must be N=2,4. If 4, Z_4 symmetry is implied, with FN phase of +- 90^o.

$\boxed{\text{Model}}$

$$\mathbf{Z}_{6-I}:\begin{cases}2\bar{c}_k: & \frac{3}{2}(k=1),\ \frac{4}{3}(k=2),\ \frac{3}{2}(k=3),\\ 2c_k: & \frac{1}{2}(k=1),\ \frac{1}{3}(k=2),\ \frac{1}{2}(k=3),\end{cases}$$

$$\mathbf{Z}_{6-II}:\begin{cases}2\bar{c}_k: & \frac{25}{18}(k=1),\ \frac{14}{9}(k=2),\ \frac{3}{2}(k=3),\\ 2c_k: & \frac{7}{18}(k=1),\ \frac{5}{9}(k=2),\ \frac{1}{2}(k=3),\end{cases}$$

$$\mathbf{Z}_{8-I}:\begin{cases}2\bar{c}_k: & \frac{47}{32}(k=1),\ \frac{11}{8}(k=2),\ \frac{47}{32}(k=3),\ \frac{3}{2}(k=4),\\ 2c_k: & \frac{15}{32}(k=1),\ \frac{3}{8}(k=2),\ \frac{15}{32}(k=1),\ \frac{1}{2}(k=3),\end{cases}$$

$$\mathbf{Z}_{8-II}:\begin{cases}2\bar{c}_k: & \frac{45}{32}(k=1),\ \frac{13}{8}(k=2),\ \frac{45}{32}(k=1),\ \frac{3}{2}(k=4),\\ 2c_k: & \frac{13}{32}(k=1),\ \frac{5}{8}(k=2),\ \frac{13}{32}(k=1),\ \frac{1}{2}(k=4).\end{cases}$$

$$\mathbf{Z}_{12-I}:\begin{cases}2\bar{c}_k: & \frac{210}{144}(k=1),\ \frac{216}{144}(k=2),\ \frac{234}{144}(k=3),\ \frac{192}{144}(k=4),\ \frac{210}{144}(k=5),\ \frac{216}{144}(k=6),\\ 2c_k: & \frac{11}{24}(k=1),\ \frac{1}{2}(k=2),\ \frac{5}{8}(k=3),\ \frac{1}{3}(k=4),\ \frac{11}{24}(k=5),\ \frac{1}{2}(k=6).\end{cases}$$

$$\mathbf{Z}_{12-II}:\begin{cases}2\bar{c}_k: & \frac{103}{72}(k=1),\ \frac{31}{18}(k=2),\ \frac{11}{8}(k=3),\ \frac{14}{9}(k=4),\ \frac{103}{72}(k=5),\ \frac{3}{2}(k=6),\\ 2c_k: & \frac{31}{72}(k=1),\ \frac{13}{18}(k=2),\ \frac{3}{8}(k=3),\ \frac{5}{9}(k=4),\ \frac{31}{72}(k=5),\ \frac{1}{2}(k=6).\end{cases}$$

$$V^a=\begin{cases}V_0=(\frac{-5}{12},\frac{-5}{12},\frac{-5}{12},\frac{-5}{12},\frac{-5}{12},\frac{-5}{12},\frac{-5}{12};\frac{+5}{12})\,(\frac{4}{12},\frac{4}{12},\frac{4}{12},\frac{4}{12},0,\frac{4}{12},\frac{7}{12},\frac{3}{12})',\ V_0^2=\frac{338}{144},\\ a_3=a_4=(\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{3};\frac{1}{3})(\frac{-1}{3},\frac{-1}{3},\frac{-1}{3},\frac{-1}{3},0,0,\frac{5}{3},-1)'.\end{cases}$$

	$\mathcal{P}\times$(rep.)	Sector	Weight	V_a^k	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
(a)	$\Psi_R^{[ABC]}$	U_1	$(\underline{-----+++};+)\,(0^8)'$	0	$\frac{-6}{12}$	$\frac{6}{12}$	0	0	0	0	0
(b)	$2\,\Psi_R^{[AB]}$	T_3	$\left(\underline{\tfrac{3}{4}\,\tfrac{3}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}\,\tfrac{-1}{4}};\tfrac{1}{4}\right)(0^6\,\tfrac{-1}{4}\,\tfrac{-1}{4})'$	V_0^3	$\frac{3}{12}$	$\frac{3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(c)	$8\,\Psi_{[A]R}$	T_3	$\left(\underline{\tfrac{-3}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}};\tfrac{-1}{4}\right)(0^6\,\tfrac{-1}{4}\,\tfrac{-1}{4})'$	V_0^3	$\frac{9}{12}$	$\frac{-3}{12}$	0	0	0	$\frac{-3}{12}$	$\frac{-3}{12}$
(d)	$\Psi_{[A]R}$	T_5^+	$(\tfrac{11}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12};\tfrac{-3}{12})\,(00000\tfrac{4}{12}\,\tfrac{-3}{12}\,\tfrac{-3}{12})'$	V_+^5	$\frac{5}{12}$	$\frac{-3}{12}$	0	0	$\frac{4}{12}$	$\frac{-3}{12}$	$\frac{-3}{12}$
(e)	$\Psi_R^{[A]}$	T_6	$(\underline{-1,0^6};0)\,(0^6,\tfrac{1}{2},\tfrac{1}{2})'$	V_0^6	$\frac{-12}{12}$	0	0	0	0	$\frac{6}{12}$	$\frac{6}{12}$
(f)	$40\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_3	$\left(\underline{\tfrac{-3}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}\,\tfrac{1}{4}};\tfrac{-1}{4}\right)(0^6\,\tfrac{-1}{4}\,\tfrac{-1}{4})'\oplus$ H.c.	V_0^3	0	0	0	0	0	0	0
(g)	$5\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_6	$(\underline{-10^6}\,0)\,(0^6\,\tfrac{1}{2}\,\tfrac{1}{2})'\oplus$ H.c.	V_0^6	0	0	0	0	0	0	0
(h)	$10\left(\Phi_{[A]R}+\Phi_R^{[A]}\right)$	T_5^+	$(\tfrac{11}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12}\,\tfrac{-1}{12};\tfrac{-3}{12})\,(00000\tfrac{4}{12}\,\tfrac{-3}{12}\,\tfrac{-3}{12})'\oplus$ H.c.	V_+^5	0	0	0	0	0	0	0
				$\sum_i$	$\frac{35}{12}$	$\frac{63}{12}$	0	0	$\frac{4}{12}$	$\frac{-51}{12}$	$\frac{-51}{12}$
(a')	$\Psi_{[\alpha']R}$	U_3	$(0^8)\,(\underline{-+++};-+++)'$	0	0	0	$\frac{1}{12}$	$\frac{-1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$
(b')	$\Psi_R^{[\alpha']}$	T_1^0	$((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{1}{12};\tfrac{-3}{12}\right)'$	V_0^1	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{1}{12}$	$\frac{-3}{12}$
(c')	$\Psi_{[\alpha']R}$	T_4^0	$((\tfrac{-1}{6})^7;\tfrac{1}{6})\left(\underline{\tfrac{-1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}};0\,\tfrac{1}{3}\,\tfrac{1}{3}\,0\right)'$	V_0^4	$\frac{-14}{12}$	$\frac{2}{12}$	$\frac{8}{12}$	0	$\frac{4}{12}$	$\frac{4}{12}$	0
(d')	$\Psi_R^{[\alpha']}$	T_5^0	$((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{-5}{12}\,\tfrac{3}{12}\right)'$	V_0^5	$\frac{7}{12}$	$\frac{-1}{12}$	$\frac{4}{12}$	$\frac{-6}{12}$	$\frac{-2}{12}$	$\frac{-5}{12}$	$\frac{3}{12}$
(e')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_1^0	H.c.$\oplus((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{1}{12};\tfrac{-3}{12}\right)'$	V_0^1	0	0	0	0	0	0	0
(f')	$5\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_4^0	$((\tfrac{-1}{6})^7;\tfrac{1}{6})\left(\underline{\tfrac{-1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3}};0\,\tfrac{1}{3}\,\tfrac{1}{3}\,0\right)'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
(g')	$10\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_5^0	H.c.$\oplus((\tfrac{1}{12})^7;\tfrac{-1}{12})\left(\underline{\tfrac{10}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}\,\tfrac{-2}{12}};\tfrac{-6}{12}\,\tfrac{-2}{12}\,\tfrac{-5}{12}\,\tfrac{3}{12}\right)'$	V_0^5	0	0	0	0	0	0	0
(h')	$7\left(\Phi_{[\alpha']R}+\Phi_R^{[\alpha']}\right)$	T_6	$(0^8)\,(\underline{1000};00\,\tfrac{-1}{2}\,\tfrac{-1}{2})'\oplus$ H.c.	V_0^4	0	0	0	0	0	0	0
				$\sum_i$	0	0	$\frac{17}{12}$	$\frac{-25/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$	$\frac{1/2}{12}$

$\mathrm{SU}(5):\quad [2]\to n_f=1$

$\mathrm{SU}(6):\quad [3]\to n_f=0,\quad [2]\to n_f=1$

UGUTF:

$\mathrm{SU}(7):\quad [3]\to n_f=1,\quad [2]\to n_f=1$

$\mathrm{SU}(8):\quad [4]\to n_f=0,\quad [3]\to n_f=2,\quad [2]\to n_f=1$

$\mathrm{SU}(9):\quad [4]\to n_f=5,\quad [3]\to n_f=3,\quad [2]\to n_f=1$

$\mathrm{SU}(11):\quad [5]\to n_f=-5,\quad [4]\to n_f=9,\quad [3]\to n_f=5,\quad [2]\to n_f=1$

Phenomenomogy discussed in JHEP 1506, 114 (2015). FN phase of +- 90$^{\text{o}}$ is shown in EPJC 75, 619 (2015).

6. Conclusion

1. Jarlskog det. is J=Im $V_{31}^* V_{22}^* V_{13}^*$ in the KS form.
2. $\delta_{\rm CKM}$ is maximal.
3. GUT family unification in SU(7)xU(1) from Z(12-I).
4. $\delta_{\rm PMNS} = \pm\delta_{\rm CKM}$ possibility with spontaneous CP violation (cf:T2K on $\text{delta}_{\rm PMNS} = -90^{\rm o}$)
5. Determination of phase: FN phase delta=+-90$^{\rm o}$.