LITTLE CONFORMAL SYMMETRY

arXiv: 1603.00030

Rachel Houtz
ICHEP 2016
Chicago

In Collaboration with John Terning (UC Davis), Kitran Colwell (UC Davis)
125 GeV Higgs

Amazing!

except...

\[ \delta m_h^2 = -\frac{N_C}{16\pi^2} |y_t|^2 \times \left[ 2\Lambda^2 - 6m_t^2 \ln \left( \frac{\Lambda^2 + m_t^2}{m_t^2} \right) + \ldots \right] \]
Cancelling the Divergence

SUSY’s Claim to Fame

\[
\delta m_h^2 = -\frac{N_c}{16\pi^2} |\gamma_t|^2 \times \left[ 2\Lambda^2 - 6m_t^2 \ln\left( \frac{\Lambda^2 + m_t^2}{m_{\tilde{t}}^2} \right) + \ldots \right]
\]

\[
\delta m_h^2 = \frac{\lambda N}{16\pi^2} \left[ 2\Lambda^2 - m_L^2 \ln\left( \frac{\Lambda^2 + m_L^2}{m_L^2} \right) - m_R^2 \ln\left( \frac{\Lambda^2 + m_R^2}{m_R^2} \right) + \ldots \right]
\]

SUSY guarantees such a cancellation

### Where are the superpartners?

**ATLAS SUSY Searches** - 95% CL Lower Limits

Status: July 2016

<table>
<thead>
<tr>
<th>Model</th>
<th>Jets</th>
<th>$E_{T}^{miss}$</th>
<th>Mass limit</th>
<th>Reference</th>
</tr>
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<tbody>
<tr>
<td>MSUGRA/CMSSM</td>
<td>2-10</td>
<td>2-6 jets</td>
<td>$0.85$ TeV</td>
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<tr>
<td>$\tilde{q}_{L} \rightarrow q l^+ l^- $ (compressed)</td>
<td>mono-jet</td>
<td>1-3 jets</td>
<td>$1$ TeV</td>
<td>1605.03814</td>
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<td>2-6 jets</td>
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<td>GGM (bino NLSP)</td>
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<td>$n$</td>
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<td>$1.37$ TeV</td>
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<td>0 mono-jet</td>
<td>0 mono-jet</td>
<td>$900$ GeV</td>
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</table>

### ATLAS Preliminary

$\sqrt{s} = 7, 8, 13$ TeV

*Only a selection of the available mass limits on new states or phenomena is shown.*
Alternative Theory: Introduce a new Gauge Boson

\[ \delta m_h \equiv \frac{2 \Lambda^2}{16\pi^2} \left( -2N_C y_t^2 + 3C_2(S) g_N^2 \right) \]

\[ 0 = -2N_C y_t^2 + 3C_2(S) g_N^2 \]

A cancellation is possible if \( y_t \) and \( g_N \) satisfy this relationship.
Some Problems…

1) How is this not just different fine tuning?

2) \( y_t \) and \( g_N \) run, so a cancellation at one scale will be spoiled

\[
\delta m_h \equiv (-2N_c y_t^2 + 3C_2(S)g_N^2) \frac{\Lambda^2}{16\pi^2} \implies \delta m_h^2 \equiv -2N_c y_t^2 \Lambda_t^2 + 3C_2(S)g_N^2 \Lambda_N^2
\]

3) Why would the cutoffs for both loops be the same?
- Depends on high energy theory

www.nobelprize.org/nobel_prizes/physics/laureates/2004/
Solution:
Little Conformal Symmetry

What if this relation is the result of an underlying symmetry?

\[ 0 = -2N_c y_t^2 + 3C_2(S) g_N^2 \]


- Impose Conformal Symmetry to derive this relationship between couplings
- Allows for a naturally small Higgs mass with superpartners >10 TeV, similar to Little Higgs models

Little Conformal Symmetry: A Simple Toy Model

\[ \mathcal{L} \ni -y_t \bar{t}_R \phi t_L \]

\[ \alpha_N = \frac{g_N^2}{4\pi} \quad \alpha_t = \frac{y_t^2}{4\pi} \]

<table>
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<tr>
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<th>(\text{Su}(N))</th>
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</thead>
<tbody>
<tr>
<td>(t_L)</td>
<td>2x (\text{N}_c)</td>
</tr>
<tr>
<td>(t_R)</td>
<td>(\text{N}_c)</td>
</tr>
<tr>
<td>(\phi)</td>
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</table>

\(10 \text{ TeV}\)

\(\sim 1 \text{ TeV}\)

---

\(\text{Symmetry Breaking Scale}\)

\(\text{SU}(N)\) Gauge Bosons

\(m_t\)

\(m_h\)

---

\(t_L, t_R\)

\(\phi\)
Top Yukawa Fixed Point

\[ \beta(\alpha_t) = \frac{1}{2\pi} \left( \alpha_t^2 - b_N \alpha_N \alpha_t \right) \]

\[ \alpha_t = \frac{b_N}{a} \alpha_N \]

Quadratic Divergence Cancellation

\[ \delta m^2 \equiv -2N_C \alpha_t + 3C_2(\phi)\alpha_N \]

\[ \alpha_t = \frac{3C_2(\phi)}{2N_C} \alpha_N \]

Symmetry Condition

\[ \frac{b_N}{a} = \frac{3C_2(S)}{2N_C} \]

Ensures that both the quadratic divergence is cancelled and the top Yukawa coupling is at a fixed point.
Top Yukawa Fixed point

\[ a = \frac{1}{2} \left( \begin{array}{c} t_R \\ t_L \end{array} \right) - \phi + \begin{array}{c} t_R \\ t_L \end{array} - \phi + \begin{array}{c} t_R \\ t_L \end{array} \]


Symmetry Condition

\[ b_N = 3x \]

\[ \Rightarrow a = \frac{1}{2} N + 4 \quad \& \quad b_N = 3C_2(t_R) \]

- Quadratic Divergence Cancelled
- Top Yukawa at a fixed point
- New Gauge coupling at a fixed point.

\[ \alpha_t = \frac{15}{16} \alpha_N \]
Gauge Coupling Banks-Zaks Fixed Point


\[ \beta(\alpha_N) \propto \left( \sum m^2 + \sum C_{\text{QCD}} \right) + \left( \sum m^2 + \sum C_{\text{QED}} \right) + \ldots \]

\( \beta \)-function coefficients depend on the matter content of the UV theory

<table>
<thead>
<tr>
<th>copies</th>
<th>( SU(N) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_L )</td>
<td>2 NC</td>
</tr>
<tr>
<td>( t_R )</td>
<td>NC</td>
</tr>
<tr>
<td>( \phi )</td>
<td>2</td>
</tr>
<tr>
<td>( f_V )</td>
<td>7</td>
</tr>
<tr>
<td>( S )</td>
<td>4</td>
</tr>
<tr>
<td>( S_A )</td>
<td>10</td>
</tr>
</tbody>
</table>

\[ 10 \text{ TeV} \]

\[ \approx 1 \text{ TeV} \]

\[ \underline{f_V, s, s_A} \]

Symmetry Breaking Scale

SU(\(N\)) Gauge Bosons

\[ m_t \]

\[ m_h \]

\[ \phi \]

\[ t_L, t_R \]
Gauge Coupling Banks-Zaks Fixed Point

• Remember, the values of $\alpha_N$ is already set from our relation.

• In order to make this value of $\alpha_N$ to coincide with its fixed point, we added new matter to the UV theory

Little Conformal Symmetry: More Realistic Toy Model

Problem:

\[
\beta(\alpha_t) = \frac{1}{2\pi} \left( a \alpha_t^2 - b_N \alpha_N \alpha_t - b_3 \alpha_3 \alpha_t - b_2 \alpha_2 \alpha_t - b_1 \alpha_1 \alpha_t \right)
\]

We still want these to cancel

Too big! Drags \(\beta(\alpha_t)\) negative quickly

Idea: Embed \(SU(3) \subset SU(N)\)

More Specifically ...

\(SU(3) \subset SU(3)_L \times SU(3)_R \subset SU(N) \times SU(N')\)

• We have two new gauge couplings instead of one: \(\alpha_N\) and \(\alpha_{N'}\)
• Add another constraint to ensure both gauge groups are at a fixed point
Little Conformal Symmetry: Matter Content

<table>
<thead>
<tr>
<th>SL(N)</th>
<th>SL(N')</th>
</tr>
</thead>
<tbody>
<tr>
<td>qL</td>
<td>1</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
</tr>
<tr>
<td>tR</td>
<td>1</td>
</tr>
</tbody>
</table>

Realistic symmetry breaking scenario:

\[ \frac{1}{\alpha_3} \geq \frac{1}{\alpha_N} + \frac{1}{\alpha_{N'}} \]

Realistic top quark Yukawa coupling:

\[ \alpha_t^{\overline{MS}}(m_t) \text{ within } \pm 5.65 \%

\[
\begin{align*}
\dot{\alpha}_N &= 0 \\
\dot{\alpha}_t &= 0 \\
\dot{\alpha}_{N'} &= 0 \\
\delta m_h^2 &= 0
\end{align*}
\]
Take another look at the cutoffs...

\[ \delta m_{h}^{2} \equiv -2N_{C}\alpha_{t}\Lambda_{t}^{2} + 3C_{2}(S)\alpha_{N}\Lambda_{N}^{2} + 3C_{2}(S)\alpha_{N',}\Lambda_{N'}^{2}, \]

- For a relationship among \( \Lambda_{N}, \Lambda_{N'}, \) and \( \Lambda_{t}, \) a UV theory must be chosen.
- Choice: SUSY with the cutoffs being masses of superpartners
- Let \( \frac{\Lambda_{N}}{\alpha_{N}} = \frac{\Lambda_{N'}}{\alpha_{N'}}, \) as in Gauge-Mediated SUSY Breaking

Conclusions

- Conformal symmetry can produce a cancellation of the Higgs mass quadratic divergence with a new SU(N) gauge boson

- This cancellation prevents the Higgs mass from being sensitive to new physics up to the 10 TeV scale

- More parameter space to explore by requiring fixed points only of ratios of couplings
Back-up Slides
Gauge-Mediated SUSY

\[
M_N = \frac{\alpha_N}{4\pi} S^N_2(m) \frac{\mathcal{F}}{M}
\]

\[
M_M = \frac{\alpha_M}{4\pi} S^M_2(m) \frac{\mathcal{F}}{M}
\]

\[
\langle X \rangle = M + \theta^2 \mathcal{F}
\]

\[
m^2_{\tilde{t}_R} = 2C^N_2 (t_R) \frac{\alpha^2_N}{16\pi^2} S^N_2(m) \left( \frac{\mathcal{F}}{M} \right)^2
\]

\[
m^2_{\tilde{t}_L} = 2C^M_2 (t_L) \frac{\alpha^2_M}{16\pi^2} S^M_2(m) \left( \frac{\mathcal{F}}{M} \right)^2
\]

For \( N_m \) messengers in the fundamental representation:

\[
m^2_N = \frac{S^N_2(m)(\alpha_N)^2}{S^M_2(m)(\alpha_M)^2}
\]

\[
m^2_M = \frac{S^M_2(m)}{S^M_2(m)} \left( \frac{1}{2} \right)
\]
