Fluctuations and correlations in finite temperature QCD

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for the Wuppertal-Budapest collaboration.

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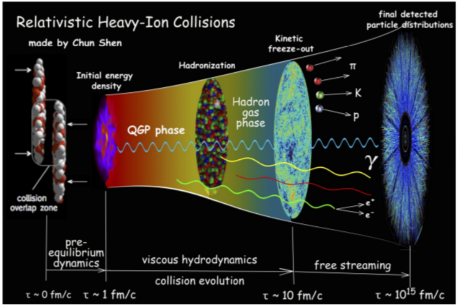
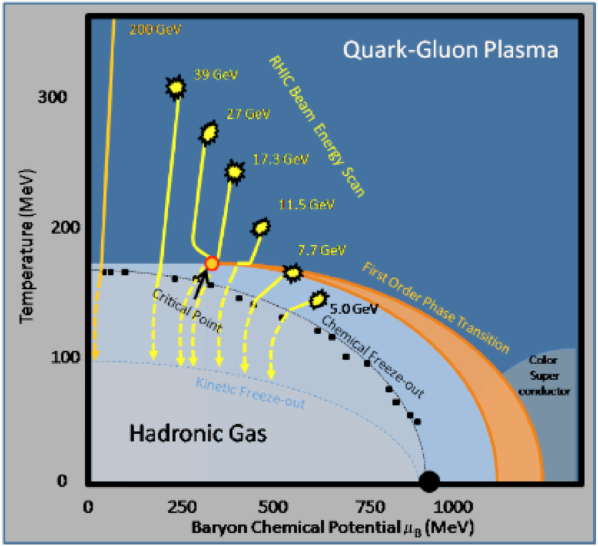
The Standard Model of Heavy Ion Collisions



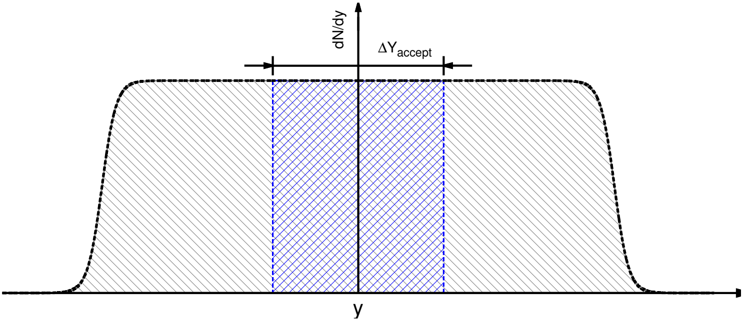
Stage	Examples of QCD input
Pre-equilibrium evolution	How perturbative is the medium at $T \sim 3T_c$?
Hydrodynamic evolution	Equation of State
Hadronization, chemical freeze-out	T_c, fluctuations below T_c

The RHIC beam energy scan



QCD in the grand canonical ensemble

Do conserved charges fluctuate in HIC?



Acceptance cut in rapidity and transverse momentum $\rightarrow$ we have a sub-volume, so the grand canonical ensemble applies

QCD in the grand canonical ensemble

The expectation value of a conserved charge:

$$\langle N_q \rangle = T \frac{\partial \log \mathcal{Z}}{\partial \mu_q}$$

The response to μ_q is given by the fluctuations of the conserved charge:

$$\frac{\partial \langle N_i \rangle}{\partial \mu_j} = T \frac{\partial^2 \log \mathcal{Z}}{\partial \mu_i \partial \mu_j} = \frac{1}{T} \left(\langle N_i N_j \rangle - \langle N_i \rangle \langle N_j \rangle \right)$$

The higher order susceptibilities:

$$\chi^{u,d,s,c}_{i,j,k,l} = \frac{\partial^{i+j+k+l} \left(p/T^4\right)}{(\partial \hat{\mu}_u)^i (\partial \hat{\mu}_d)^j (\partial \hat{\mu}_s)^k (\partial \hat{\mu}_c)^l} \quad \chi^{B,S,Q}_{i,j,k} = \frac{\partial^{i+j+k} \left(p/T^4\right)}{(\partial \hat{\mu}_B)^i (\partial \hat{\mu}_S)^j (\partial \hat{\mu}_Q)^k}$$

where $\hat{\mu} = \mu/T$. The relationship between the chemical potentials:

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \qquad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \qquad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

Lattice details

The 4stout staggered action

- 2+1+1 dynamical flavors
- 4 levels of stout smearing in the fermion action, with the smearing parameter $\rho = 0.125$
- The masses are set by bracketing both the pion and the kaon masses within a few percent, keeping $m_c/m_s = 11.85$
- The scale is set 2 ways: f_π and w_0 (with Wilson flow). The scale setting procedure is one of the sources of the systematic error in all of the plots.

Ensembles

- For $\mu = 0$ we have $N_t = 8, 10, 12, 16, 20, 24$. With aspect ratios $LT = 3, 4$ at lower temperatures, and a fixed volume $LT_c \approx 2$ at higher temperatures $(T > 300\mathrm{MeV})$.
- For imaginary μ we have $N_t = 8, 10, 12, 16$, aspect ratios $LT = 3, 4$, and no high temperature configurations.

Model estimates at low and high temperatures

Low temperatures: Hadron Resonance Gas

The interaction of the hadrons are introduced by adding all their resonances to the heat bath, as free particles.

$$\frac{p^{\mathrm{HRG}}}{T^4} = \frac{1}{VT^3}\left(\sum_{i\in\mathrm{meson}} \log \mathcal{Z}^M\left(T,V,m_i,\{\mu\}\right) + \sum_{i\in\mathrm{baryon}} \log \mathcal{Z}^B\left(T,V,m_i,\{\mu\}\right)\right)$$

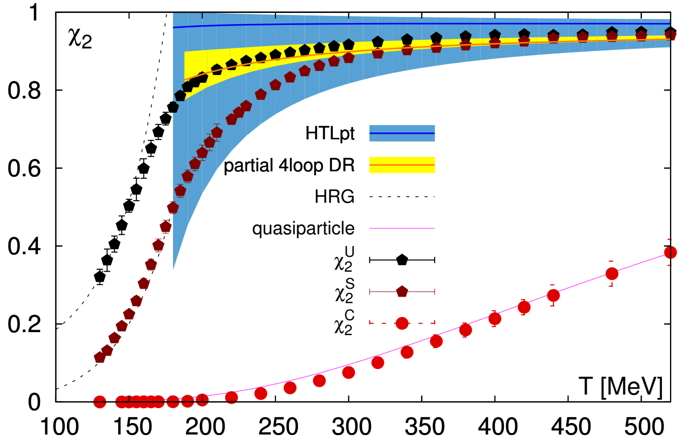
High temperature: weakly interacting quarks

For an ideal gas we have:

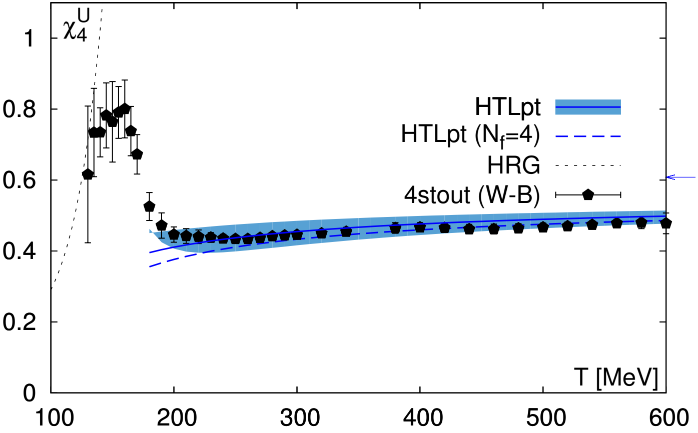
$$\frac{p}{T^4} = \frac{8\pi^2}{45} + \frac{7\pi^2}{60}N_f + \frac{1}{2}\sum_f\left(\frac{\mu_f^2}{T^2} + \frac{\mu_f^4}{2\pi^2T^4}\right)$$

This means e.g. that $\chi_4^u = 0.608$ or $\chi_{11}^{ud} = 0$ etc. This estimate can be improved with resummed PT: Hard Thermal Loop, Dimensional Reduction

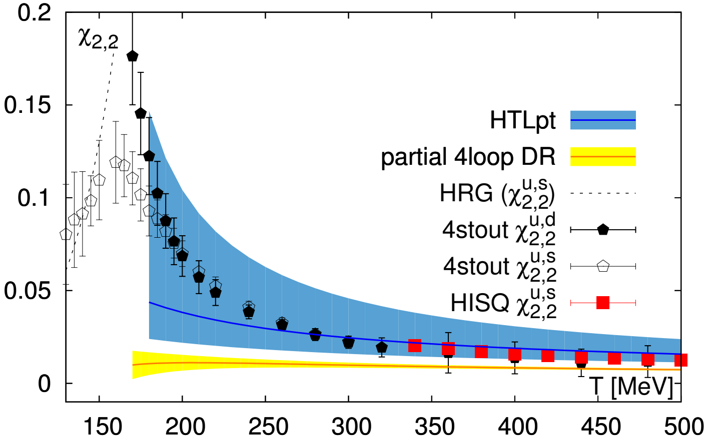
Second order diagonal quark susceptibilities



4th order susceptibilities at high temperature



4th order susceptibilities at high temperature



Two methods to calculate fluctuations

Direct method/Taylor expansion

Calculate the μ derivatives directly at $\mu = 0$.

- Pro: No additional systematic error coming from fitting.
- Con: Higher derivatives are very noisy. (Sign problem.)

Analytical continuation

Simulate at imaginary μ. Do a fit for the μ dependence of different observables, and deduce the derivatives that way.

- Pro: Higher accuracy possible with the same amount of computer time.
- Con: Has systematic errors coming from fitting, as at imaginary μ you have an exact result, containing all orders of the Taylor expansion.

Equation of state from the fluctuations

EoS at finite but small density

At general values of μ_B, μ_Q, μ_S we have:

$$\frac{p}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BSQ}(T) \hat{\mu}_B^i \hat{\mu}_S^j \hat{\mu}_Q^k$$

If we restrict outselves to conditions present in HIC:
$\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$:

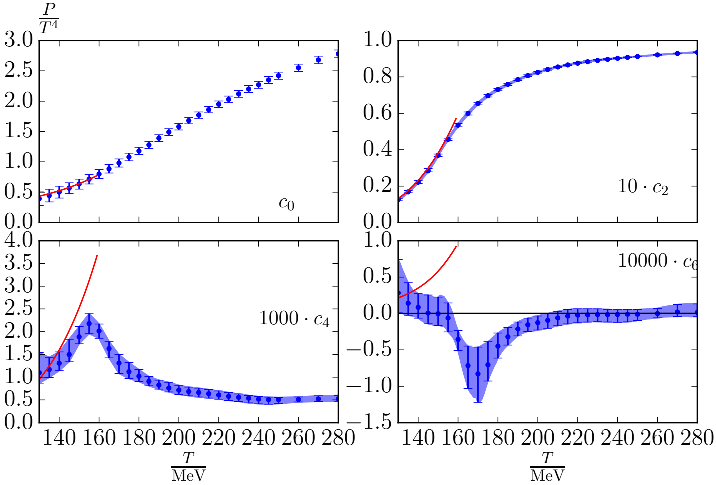
$$\frac{p}{T^4} = c_0(T) + c_2(T) \cdot \hat{\mu}_B^2 + c_4(T) \cdot \hat{\mu}_B^4 + c_6(T) \cdot \hat{\mu}_B^6 + \dots$$

The state-of-the-art at the moment is $\mathcal{O}\left(\mu_B^6\right)$. The expansion is under control for $\mu_B/T \leq 2$, or in terms of the RHIC beam energy scan, for:

$$\sqrt{s} = 200,\ 62.4,\ 39,\ 27,\ 19.6,\ 14.5 \mathrm{GeV}$$

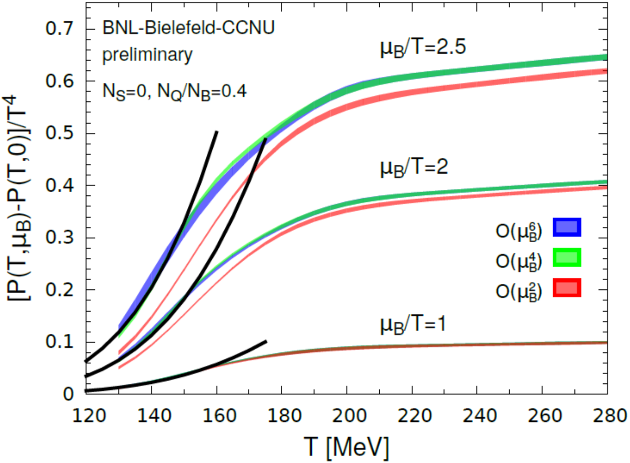
Taylor coefficients of the pressure

Results from analytical continuation.



Consistent with direct evaluation, but with smaller errorbars.

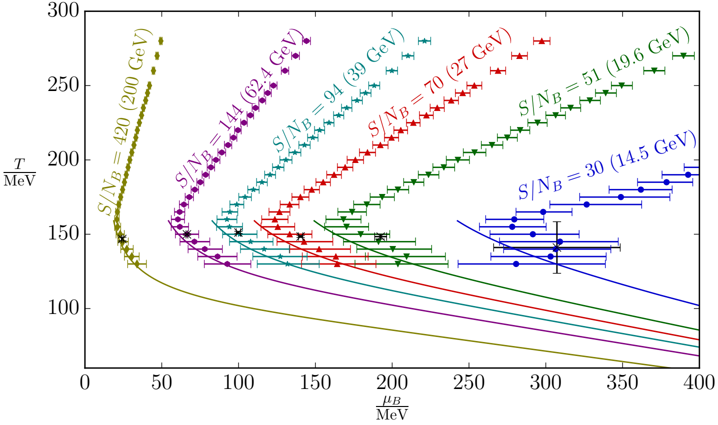
Different orders in the EoS



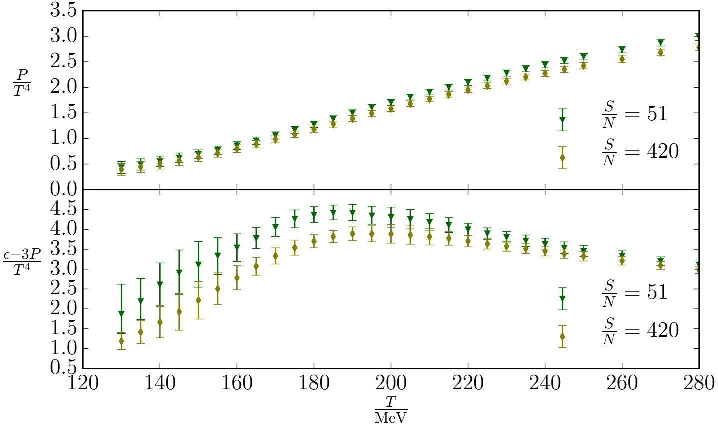
From direct method, plot from BNL-Bielefeld-CCNU collaboration.

Isentropic trajectories

In ideal hydrodynamics, we have (entropy)/(baryon number)=fixed. These trajectories can be readily calculated from the EoS.



EoS along the trajectories



Summary

- Second and fourth order fluctuations can be continuum extrapolated with the direct method. Results in both the low- and high temperature regime: hep-lat/1507.04627 (WB), Results in the high temperature regime with a different discretization: hep-lat/1507.06637 (HotQCD)
- HRG agrees with lattice data up to $T \approx 150..155\mathrm{MeV}$. ($\rightarrow$ good news for models of chemical freezout)
- HTL agrees with lattice from $T \approx 250\mathrm{MeV}$. ($\rightarrow$ good news for HTL based/kinetic theory approximations)
- For sixth order fluctuations, analytical continuation works better.
- Equation of state up to $\mathcal{O}(\mu_B^6)$ in the continuum from analytical continuation: hep-lat/1607.02493
- This allows us to have the phenomoenlogically relevant equation of state for beam energies down to $\sqrt{s} = 14.5\mathrm{GeV}$.
- Results with different staggered discretization are compatible within errors.