Fluctuations and correlations in finite temperature QCD

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hep-lat/1507.04627
hep-lat/1607.02493

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The Standard Model of Heavy Ion Collisions

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The RHIC beam energy scan
QCD in the grand canonical ensemble

Do conserved charges fluctuate in HIC?

Acceptance cut in rapidity and transverse momentum \( \rightarrow \) we have a sub-volume, so the grand canonical ensemble applies
QCD in the grand canonical ensemble

The expectation value of a conserved charge:

\[ \langle N_q \rangle = T \frac{\partial \log Z}{\partial \mu_q} \]

The response to \( \mu_q \) is given by the fluctuations of the conserved charge:

\[ \frac{\partial \langle N_i \rangle}{\partial \mu_j} = T \frac{\partial^2 \log Z}{\partial \mu_i \partial \mu_j} = \frac{1}{T} (\langle N_i N_j \rangle - \langle N_i \rangle \langle N_j \rangle) \]

The higher order susceptibilities:

\[ \chi_{u,d,s,c}^{i,j,k,l}(p/T^4) = \frac{\partial^{i+j+k+l}(\partial \hat{\mu}_u)^i(\partial \hat{\mu}_d)^j(\partial \hat{\mu}_s)^k(\partial \hat{\mu}_c)^l}{(\partial \hat{\mu}_B)^i(\partial \hat{\mu}_S)^j(\partial \hat{\mu}_Q)^k} \]

where \( \hat{\mu} = \mu/T \). The relationship between the chemical potentials:

\[ \mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q \quad \mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q \quad \mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S \]
Lattice details

The 4stout staggered action

- 2+1+1 dynamical flavors
- 4 levels of stout smearing in the fermion action, with the smearing parameter $\rho = 0.125$
- The masses are set by bracketing both the pion and the kaon masses within a few percent, keeping $m_c/m_s = 11.85$
- The scale is set 2 ways: $f_\pi$ and $w_0$ (with Wilson flow). The scale setting procedure is one of the sources of the systematic error in all of the plots.

Ensembles

- For $\mu = 0$ we have $N_t = 8, 10, 12, 16, 20, 24$. With aspect ratios $LT = 3, 4$ at lower temperatures, and a fixed volume $LT_c \approx 2$ at higher temperatures ($T > 300\text{MeV}$).
- For imaginary $\mu$ we have $N_t = 8, 10, 12, 16$, aspect ratios $LT = 3, 4$, and no high temperature configurations.
Model estimates at low and high temperatures

Low temperatures: Hadron Resonance Gas

The interaction of the hadrons are introduced by adding all their resonances to the heat bath, as free particles.

\[
\frac{p_{HRG}^{T^4}}{T^4} = \frac{1}{VT^3} \left( \sum_{i \in \text{meson}} \log Z^M (T, V, m_i, \{\mu\}) + \sum_{i \in \text{baryon}} \log Z^B (T, V, m_i, \{\mu\}) \right)
\]

High temperature: weakly interacting quarks

For an ideal gas we have:

\[
\frac{p}{T^4} = \frac{8\pi^2}{45} + \frac{7\pi^2}{60} N_f + \frac{1}{2} \sum_f \left( \frac{\mu_f^2}{T^2} + \frac{\mu_f^4}{2\pi^2 T^4} \right)
\]

This means e.g. that \( \chi_u^4 = 0.608 \) or \( \chi_{11}^{ud} = 0 \) etc. This estimate can be improved with resummed PT: Hard Thermal Loop, Dimensional Reduction
Second order diagonal quark susceptibilities
4th order susceptibilities at high temperature
4th order susceptibilities at high temperature

![Graph showing 4th order susceptibilities](image)

- **HTLpt**
- **partial 4loop DR**
- **HRG** ($\chi_{2,2}^{u,s}$)
- **4stout** $\chi_{2,2}^{u,d}$
- **4stout** $\chi_{2,2}^{u,s}$
- **HISQ** $\chi_{2,2}^{u,s}$
Two methods to calculate fluctuations

Direct method/Taylor expansion

Calculate the $\mu$ derivatives directly at $\mu = 0$.

- Pro: No additional systematic error coming from fitting.
- Con: Higher derivatives are very noisy. (Sign problem.)

Analytical continuation

Simulate at imaginary $\mu$. Do a fit for the $\mu$ dependence of different observables, and deduce the derivatives that way.

- Pro: Higher accuracy possible with the same amount of computer time.
- Con: Has systematic errors coming from fitting, as at imaginary $\mu$ you have an exact result, containing all orders of the Taylor expansion.
Equation of state from the fluctuations

EoS at finite but small density

At general values of $\mu_B, \mu_Q, \mu_S$ we have:

$$\frac{p}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BSQ}(T) \hat{\mu}_B^i \hat{\mu}_S^j \hat{\mu}_Q^k$$

If we restrict ourselves to conditions present in HIC: $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$:

$$\frac{p}{T^4} = c_0(T) + c_2(T) \cdot \hat{\mu}_B^2 + c_4(T) \cdot \hat{\mu}_B^4 + c_6(T) \cdot \hat{\mu}_B^6 + \ldots$$

The state-of-the-art at the moment is $\mathcal{O}(\mu_B^6)$. The expansion is under control for $\mu_B/T \leq 2$, or in terms of the RHIC beam energy scan, for:

$$\sqrt{s} = 200, \ 62.4, \ 39, \ 27, \ 19.6, \ 14.5\text{GeV}$$
Taylor coefficients of the pressure

Results from analytical continuation.

Consistent with direct evaluation, but with smaller errorbars.
Different orders in the EoS

From direct method, plot from BNL-Bielefeld-CCNU collaboration.
Isentropic trajectories

In ideal hydrodynamics, we have \( \frac{\text{entropy}}{\text{baryon number}} = \text{fixed} \). These trajectories can be readily calculated from the EoS.
EoS along the trajectories
Second and fourth order fluctuations can be continuum extrapolated with the direct method. Results in both the low- and high temperature regime: hep-lat/1507.04627 (WB), Results in the high temperature regime with a different discretization: hep-lat/1507.06637 (HotQCD)

HRG agrees with lattice data up to $T \approx 150..155\text{MeV}$. (→ good news for models of chemical freezout)

HTL agrees with lattice from $T \approx 250\text{MeV}$. (→ good news for HTL based/kinetic theory approximations)

For sixth order fluctuations, analytical continuation works better.

Equation of state up to $O(\mu_B^6)$ in the continuum from analytical continuation: hep-lat/1607.02493

This allows us to have the phenomenologically relevant equation of state for beam energies down to $\sqrt{s} = 14.5\text{GeV}$.

Results with different staggered discretization are compatible within errors.