The Fate of the Higgs Vacuum

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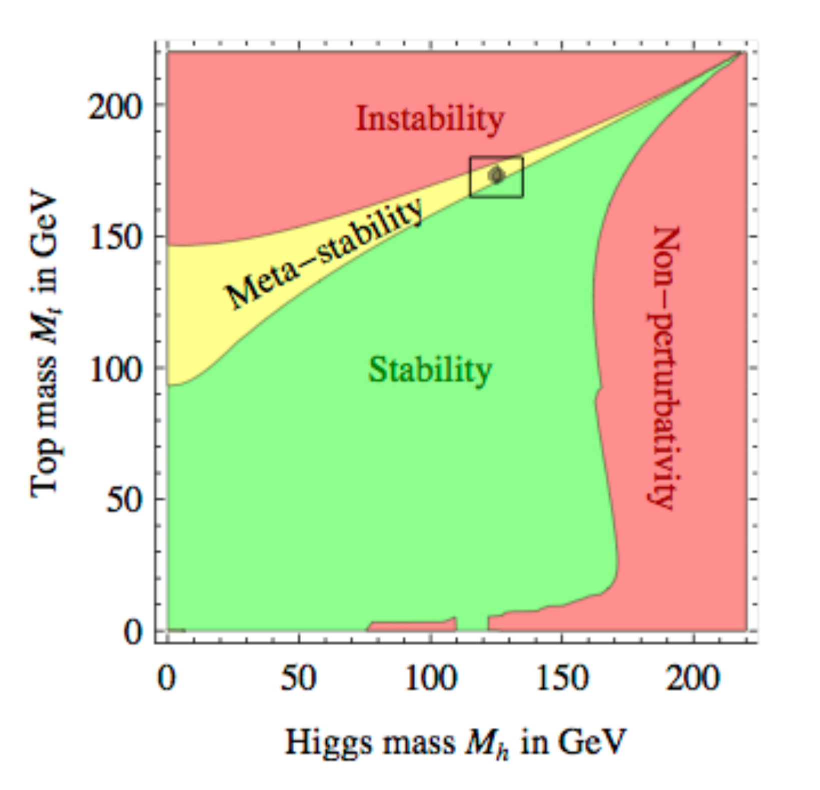
Ian Moss and Ben Withers,1401.0017
Philipp Burda, Ian Moss 1501.04937, 1503.07331, 1601.02152

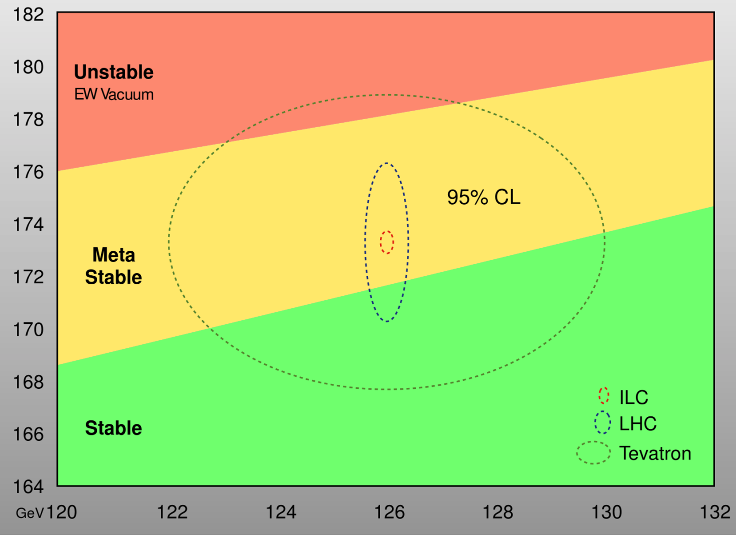
Outline

- Rationale
- Euclidean method and tunneling
- Gravity and tunneling
- Breaking the symmetry with black holes
- Outlook

How Stable Is Our Vacuum?

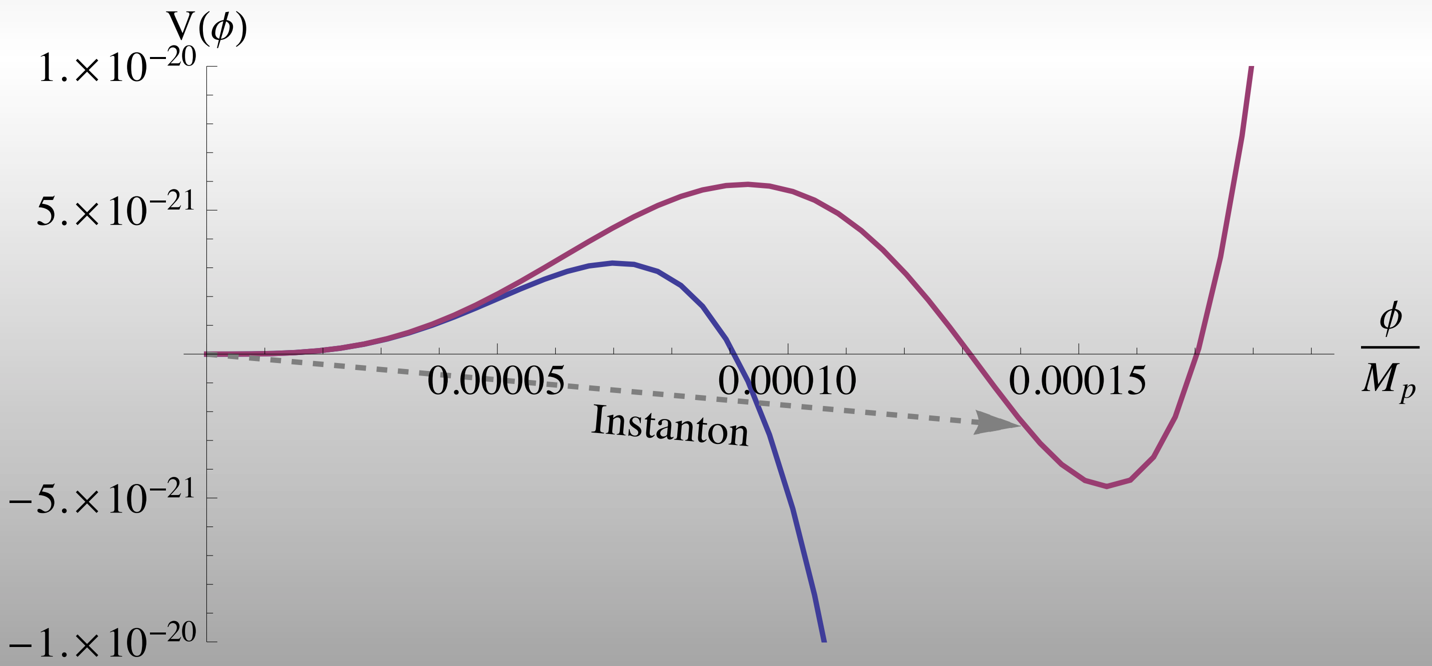
Calculating the running of the Higgs coupling tells us that we seem to be in a sweet spot between stability and instability – metastability.





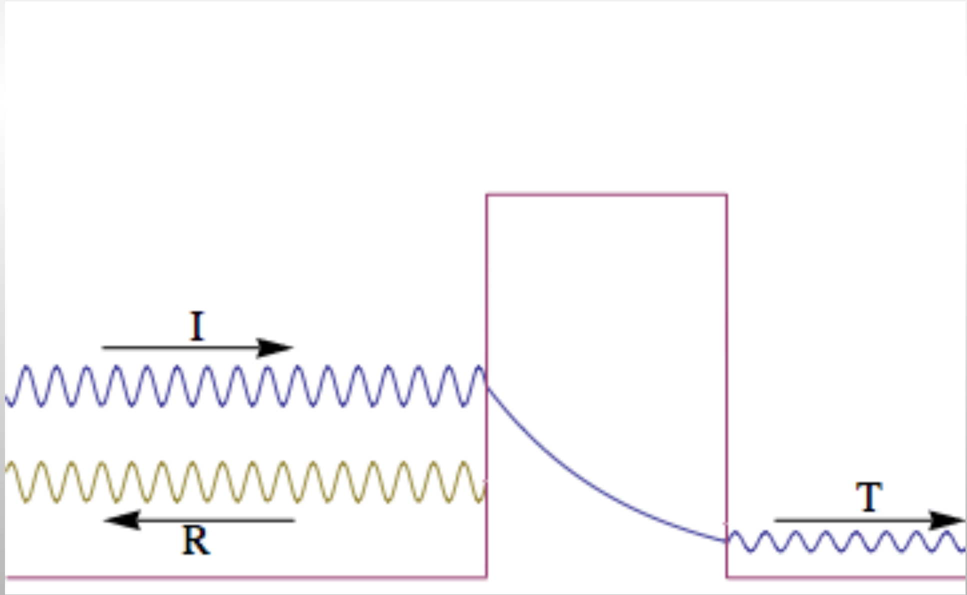
Higgs Potential

At high energies, the Higgs self-coupling becomes negative, opening the possibility of vacuum tunnelling.



Quantum Tunneling

Standard 1+1 Schrodinger tunneling exactly soluble. Recall tunnelling probabilities exponentially suppressed.



$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}} \approx e^{-2\Omega d}$$

$$\Omega^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$\Omega d = \frac{1}{\hbar}\int_0^d \sqrt{2m(V_0 - E)}dx$$

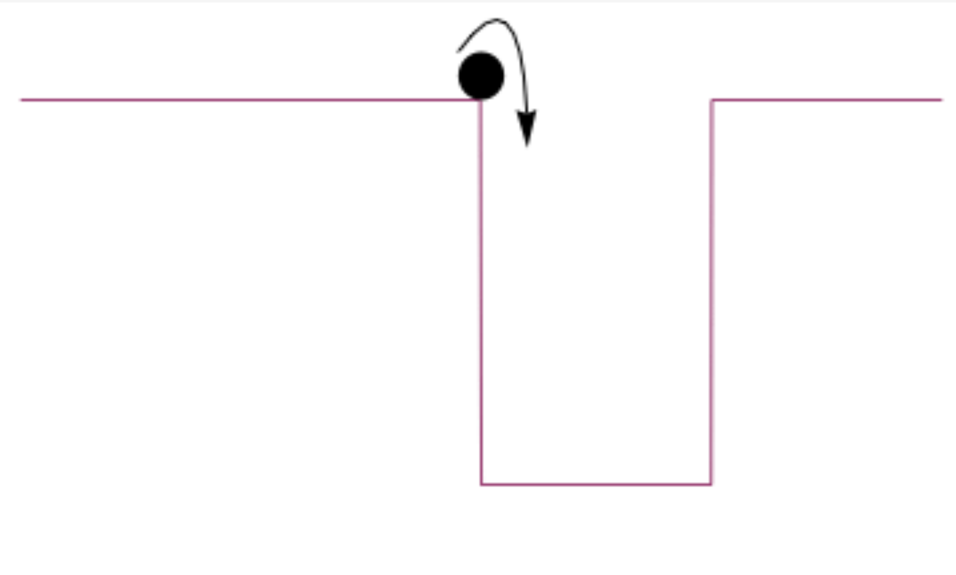
Euclidean Perspective

Rotate problem to imaginary time:

$$t \to i\tau$$

A classical particle moving in imaginary time has kinetic energy equal to the potential drop, so the amplitude $|T|^2$ now looks like the action integral for this classical motion.

$$\frac{1}{2}\dot{x}^2 = \Delta V$$



$$\int \sqrt{2\Delta V}\, dx = \int 2\Delta V\, d\tau$$
$$= \int \left(\Delta V + \frac{1}{2}\dot{x}^2\right) d\tau$$

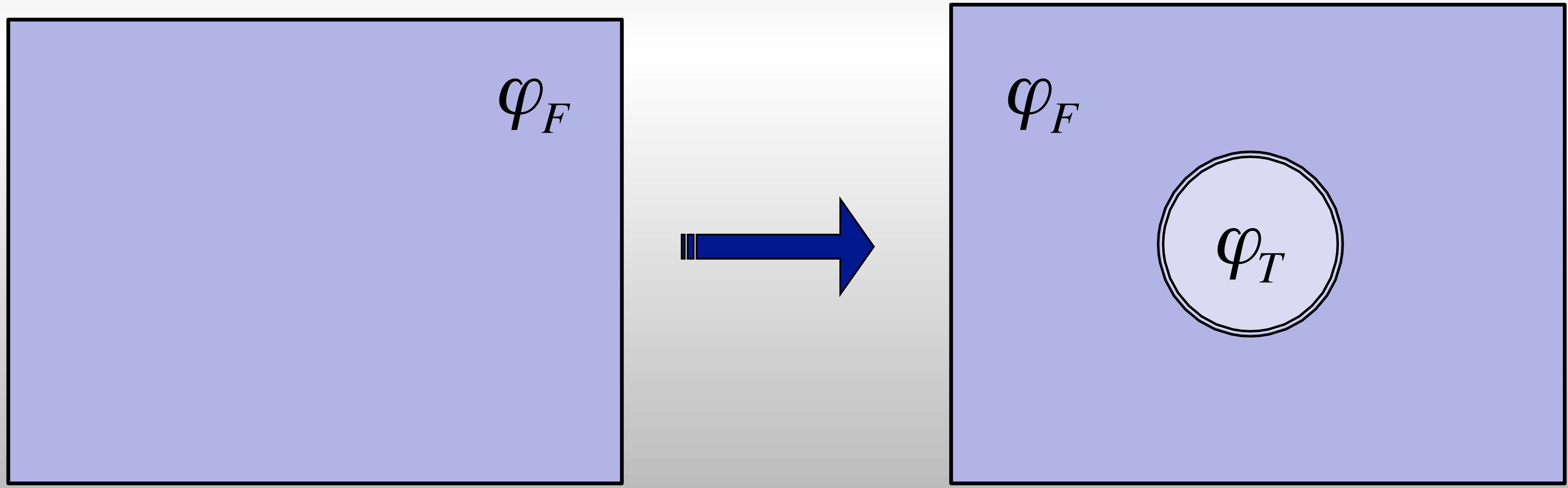
Euclidean Trick

Generally, to compute leading behaviour of a tunneling amplitude take action of a classical particle moving in an inverted potential. The particle rolls from the (now) unstable point to the “exit” and back again – a “bounce”.

The action of this bounce gives the exponent in the amplitude of the wavefunction – a nice way of computing tunneling probability.

Coleman Bounce

Coleman described this in field theory by the Euclidean solution of a bubble of true vacuum inside false vacuum separated by a “thin wall” (cf the Euclidean tunneling)



We gain energy from moving to true vacuum, but the bubble wall costs energy

Euclidean Action

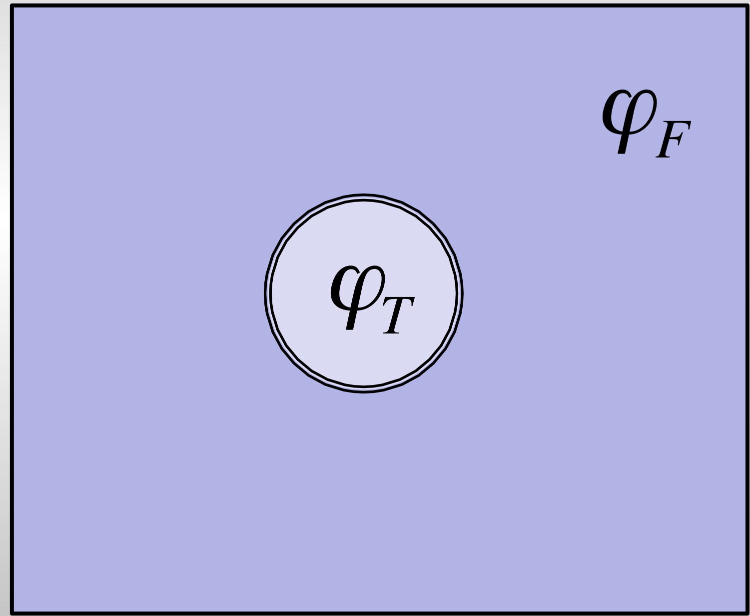
Amplitude determined by action of Euclidean tunneling solution: “The Bounce”

$$\mathcal{B} = \varepsilon \int d^4x\sqrt{g} - \sigma \int d^3x\sqrt{h}$$

$$\sim \frac{\pi^2}{2}\varepsilon R^4 - 2\pi^2\sigma R^3$$

GAIN FROM VACUUM

COST OF WALL



Coleman

Since the bounce is a solution to eqns of motion, it should be stationary under variation of R:

$$R = \frac{3\sigma}{\varepsilon} \quad , \quad \mathcal{B} = \frac{27\pi^2\sigma^4}{2\varepsilon^3}$$

Tunneling amplitude:

$$\mathcal{P} \sim e^{-\mathcal{B}/\hbar}$$

(Notice, R is big, so justifies use of the “thin wall” approximation.)

Coleman de Luccia (CDL)

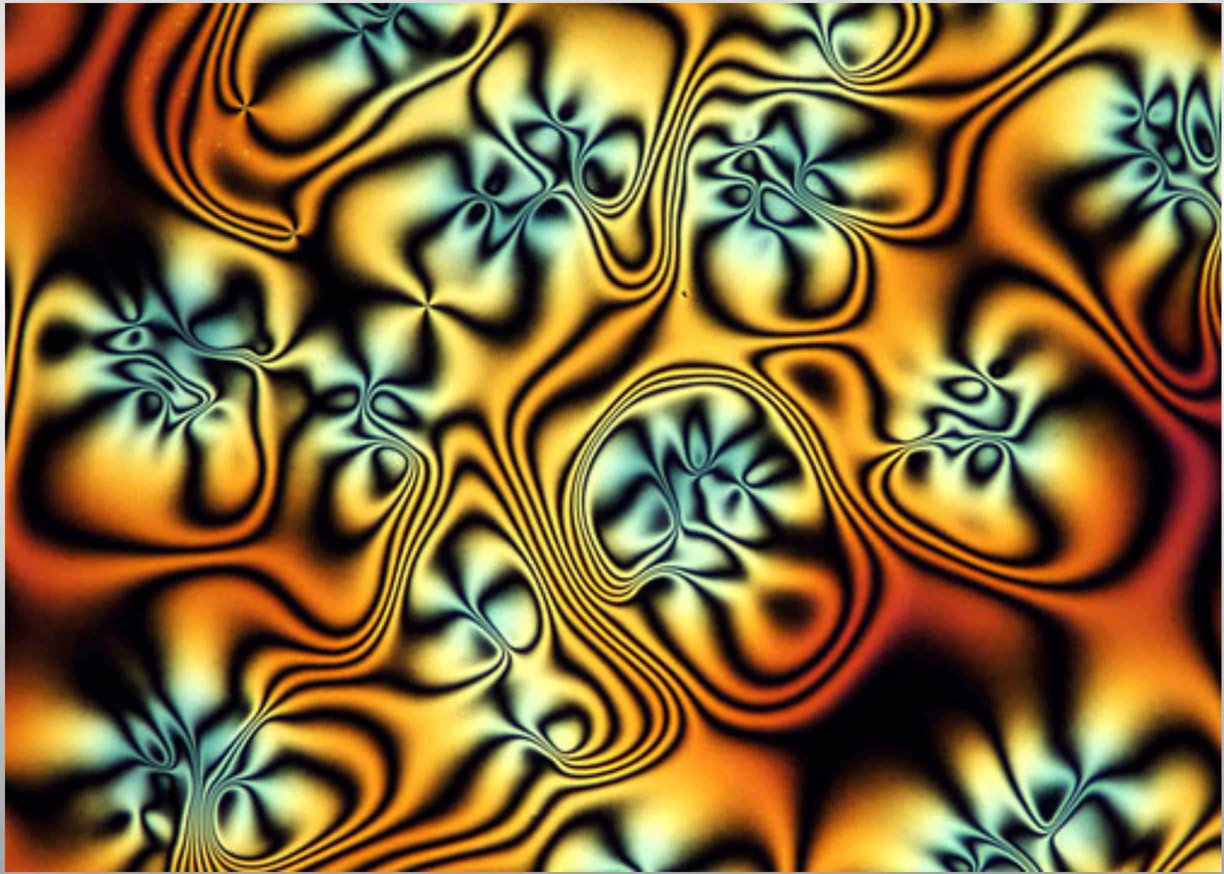
Vacuum energy gravitates – e.g. our current universe is accelerating – so we must add gravity to our picture.

Coleman and de Luccia showed how to do this with a bubble wall.

o The instanton is a solution of the Euclidean Einstein equations with a bubble of flat space separated from dS space by a thin wall.
o The wall radius is ***determined*** by the Israel junction conditions
o The action of the bounce is the difference of the action of this wall configuration and a pure de Sitter geometry.

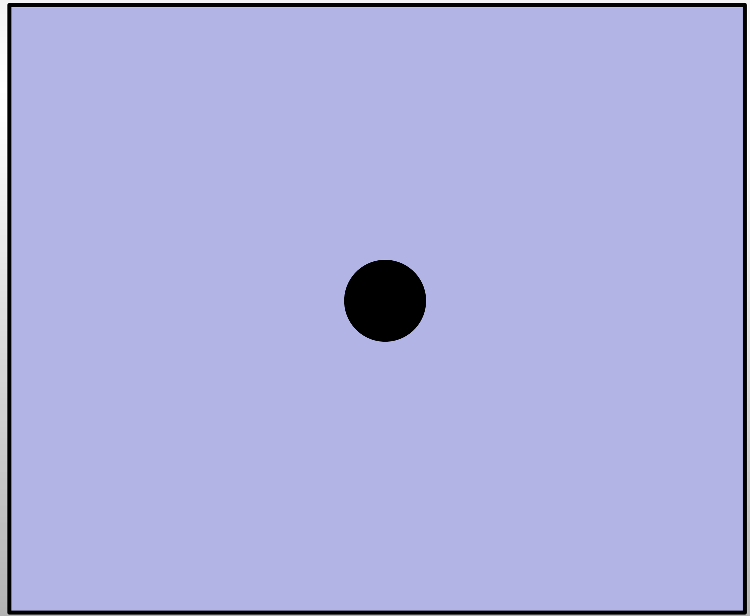
Coleman and de Luccia, PRD21 3305 (1980)

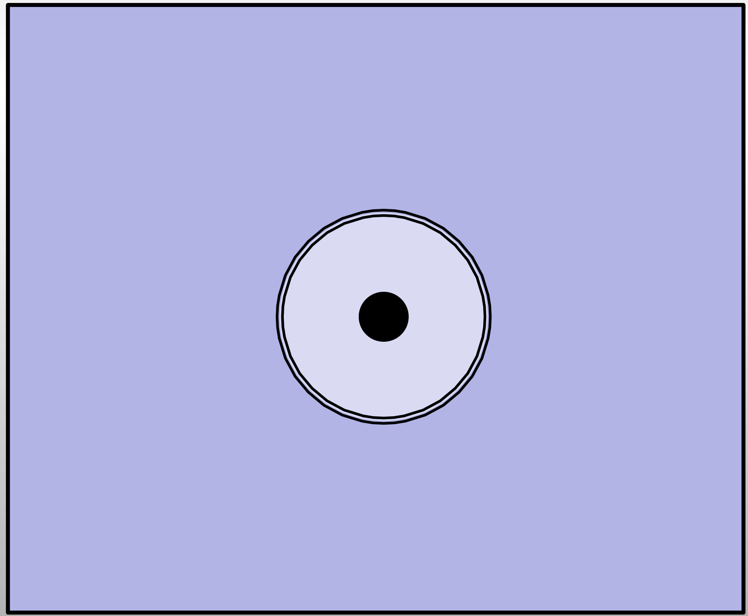
The universe is complex – so how dependent are our results on the assumptions of homogeneity and isotropy? Phase transitions in nature are more “dirty” – how does that affect modelling?



Tweaking CDL

The bubble of true vacuum has a spherical symmetry, so we can add a black hole at "minimal expense"!





RG, Moss & Withers, 1401.0017

A more general Thin Wall bubble

Straightforward to find solutions. Israel junction conditions determine the equation of motion of bubble wall with the black hole.

In each case we have to calculate the difference between the background black hole action and the effect of the bubble.

Need to deal with conical singularities (sometimes).

The general action with a black hole on each side is (details vary with Lambda):

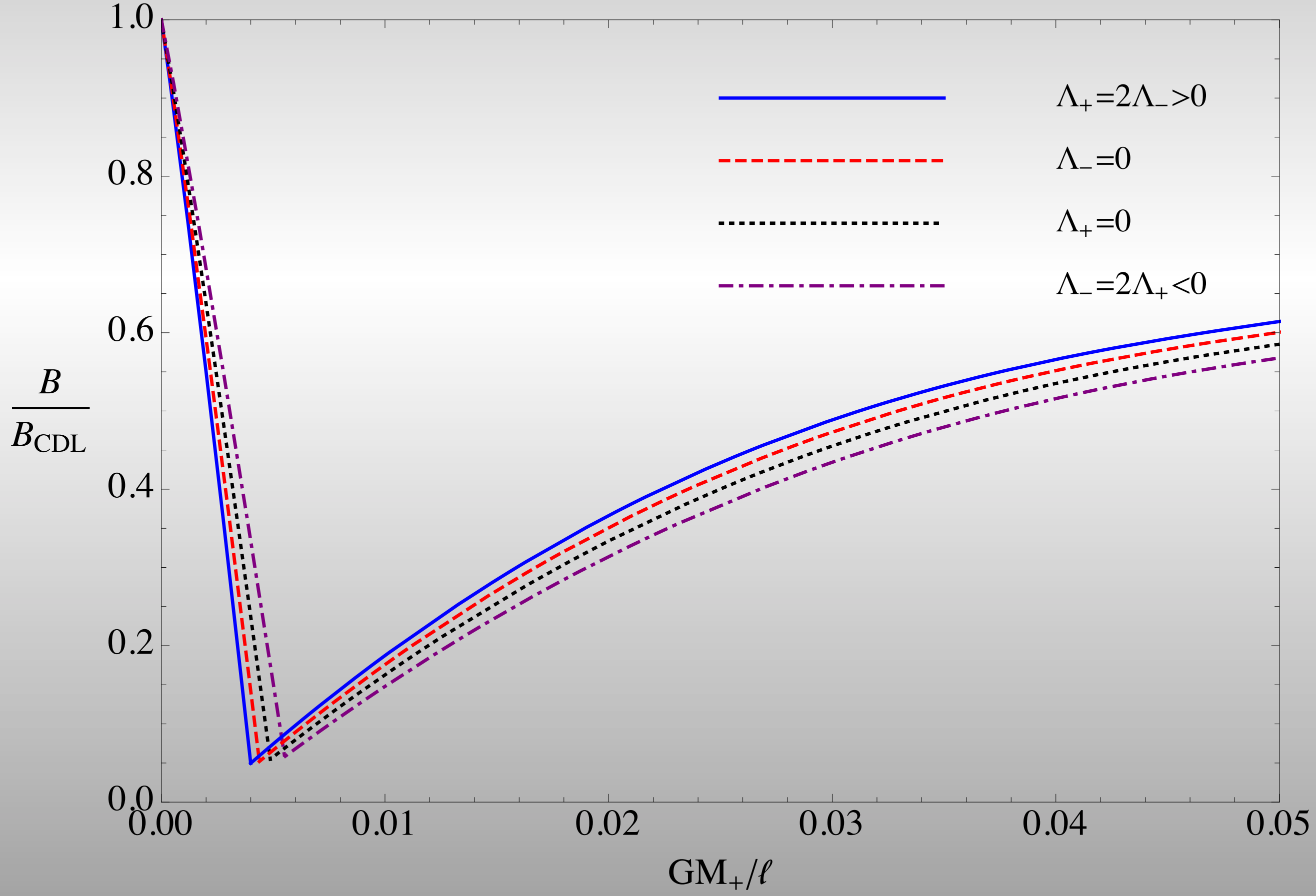
$$\mathcal{B} = \underbrace{\frac{\pi(r_+^2 - r_-^2)}{G}}_{\text{Geometry}} \underbrace{- \frac{\bar{\sigma}}{G} \int d\lambda R^2 - \frac{1}{4G} \int d\lambda R^2 \left(f'_+ \dot{\tau}_+ - f'_- \dot{\tau}_- \right)}_{\text{Bubble}}$$

General Bounce

- The general solution has a black hole inside the bubble (remnant) and a mass term outside (seed).
- The solution in general depends on time, but for each seed mass there is a unique bubble with lowest action.
- For small seed masses this is time dependent – a perturbed CDL – with no remnant black hole.
- For larger seed masses this is static and has a remnant black hole.
- For a special Mcrit, there is a static bubble with no remnant.
- Large range of solutions with $B<B_{CDL}$

Generic thin wall Tunneling

Main change is the value of lambda on each side, this changes the action ratio surprisingly little.



Tunneling v Evaporation:

Black holes can also evaporate – so we must check which process wins. Compare the evaporation rate:

$$\Gamma_H \approx 3.6 \times 10^{-4} (G^2 M_*^3)^{-1}$$

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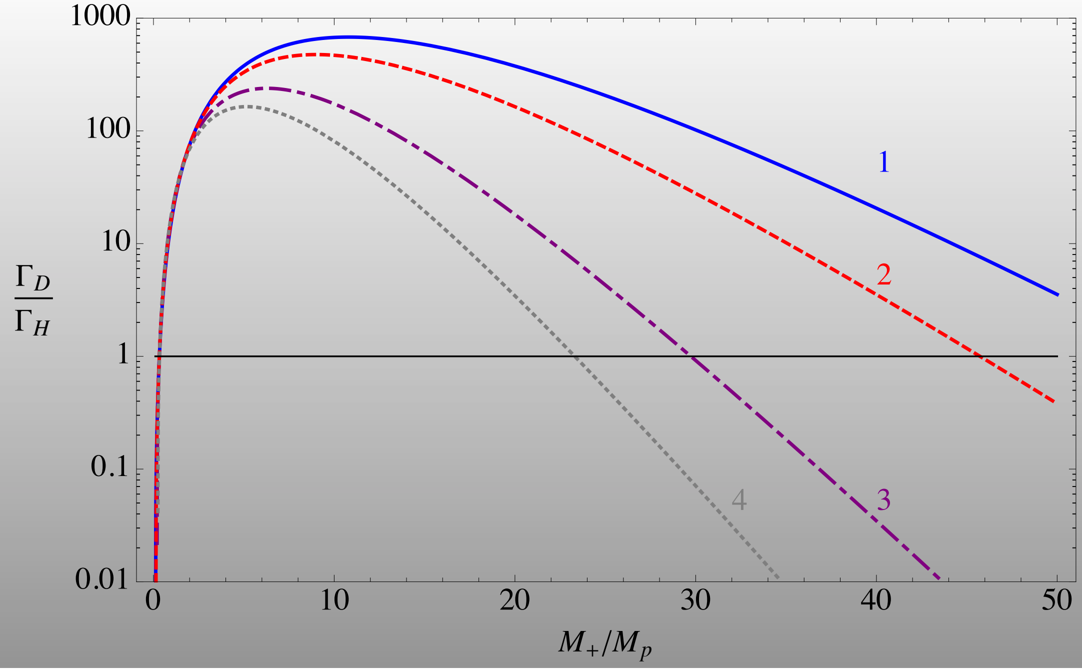
to our calculated tunneling rate



Primordial Black Holes

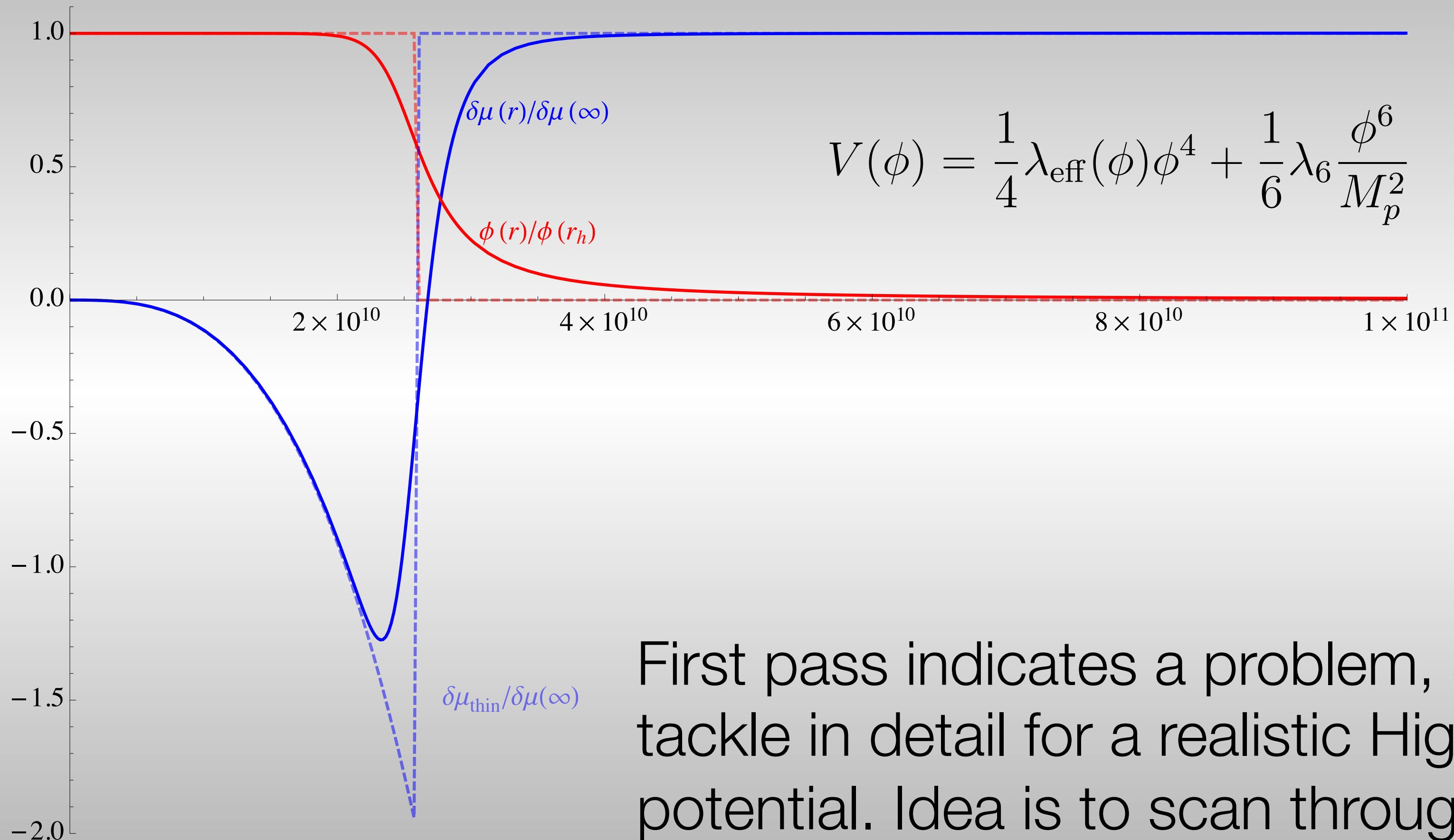
Plot shows that evaporation (perturbative) is much stronger than decay (nonperturbative) until the black holes are very small.

Decay NOT an issue for astrophysical black holes.

Primordial black holes have a temperature above the CMB, so these do evaporate over time. Eventually, they become light enough that they hit the "danger range" for vacuum decay and WILL catalyse it.

For thin wall, parameter values push credulity – however – provide proof of principle.

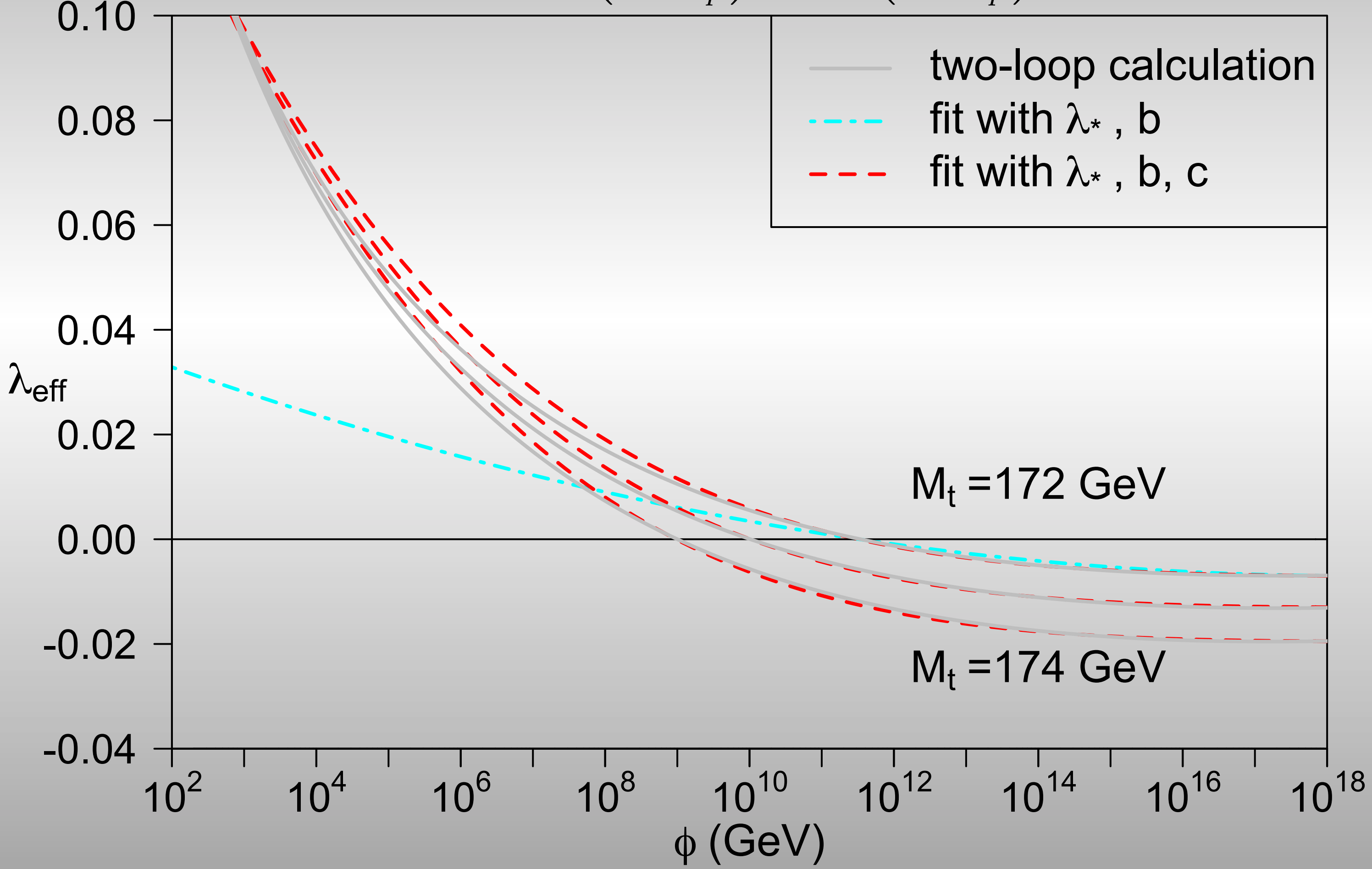
Thin to Thick wall



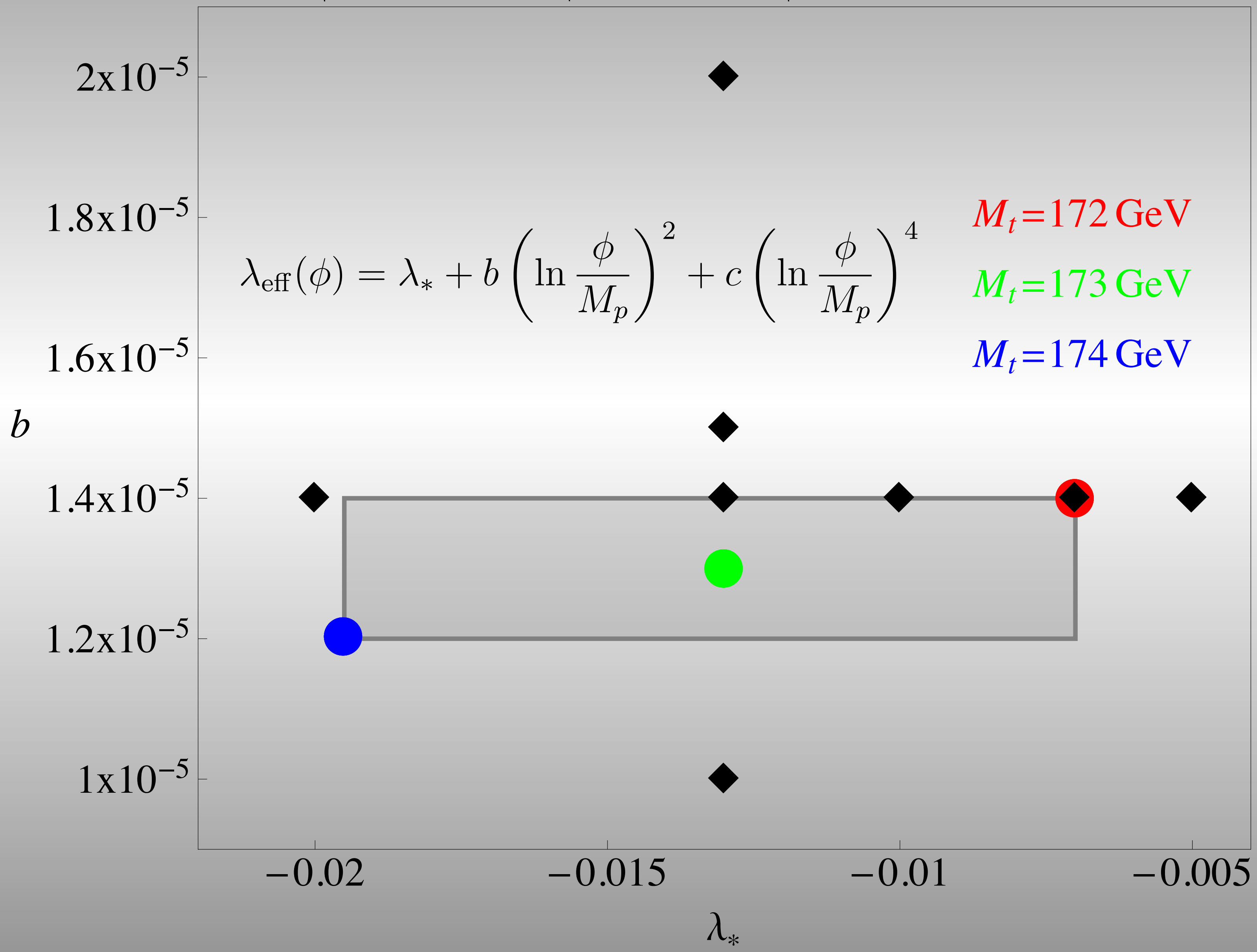
First pass indicates a problem, so tackle in detail for a realistic Higgs potential. Idea is to scan through parameter space (beyond standard model) to see how robust result it.

Fitting the Potential

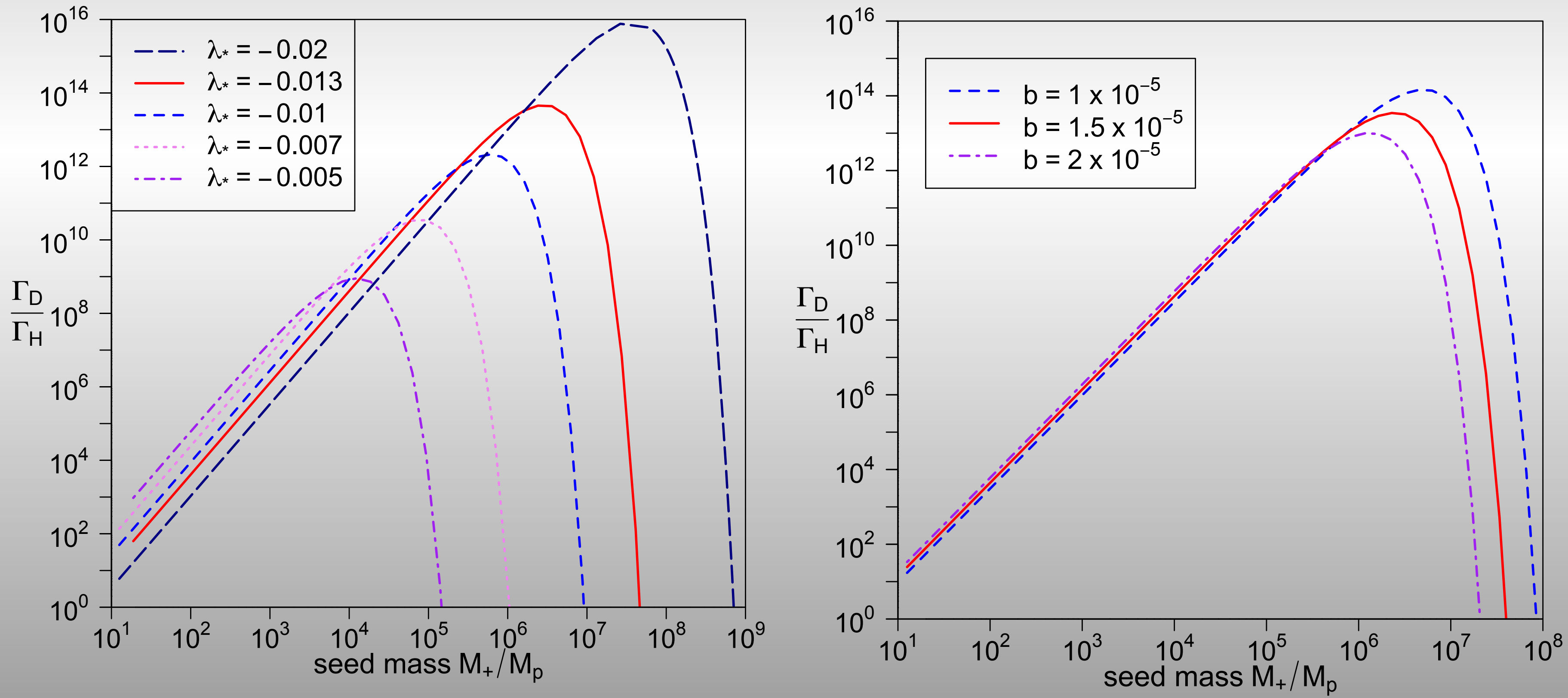
$$\lambda_{\rm eff}(\phi) = \lambda_* + b\left(\ln\frac{\phi}{M_p}\right)^2 + c\left(\ln\frac{\phi}{M_p}\right)^4$$

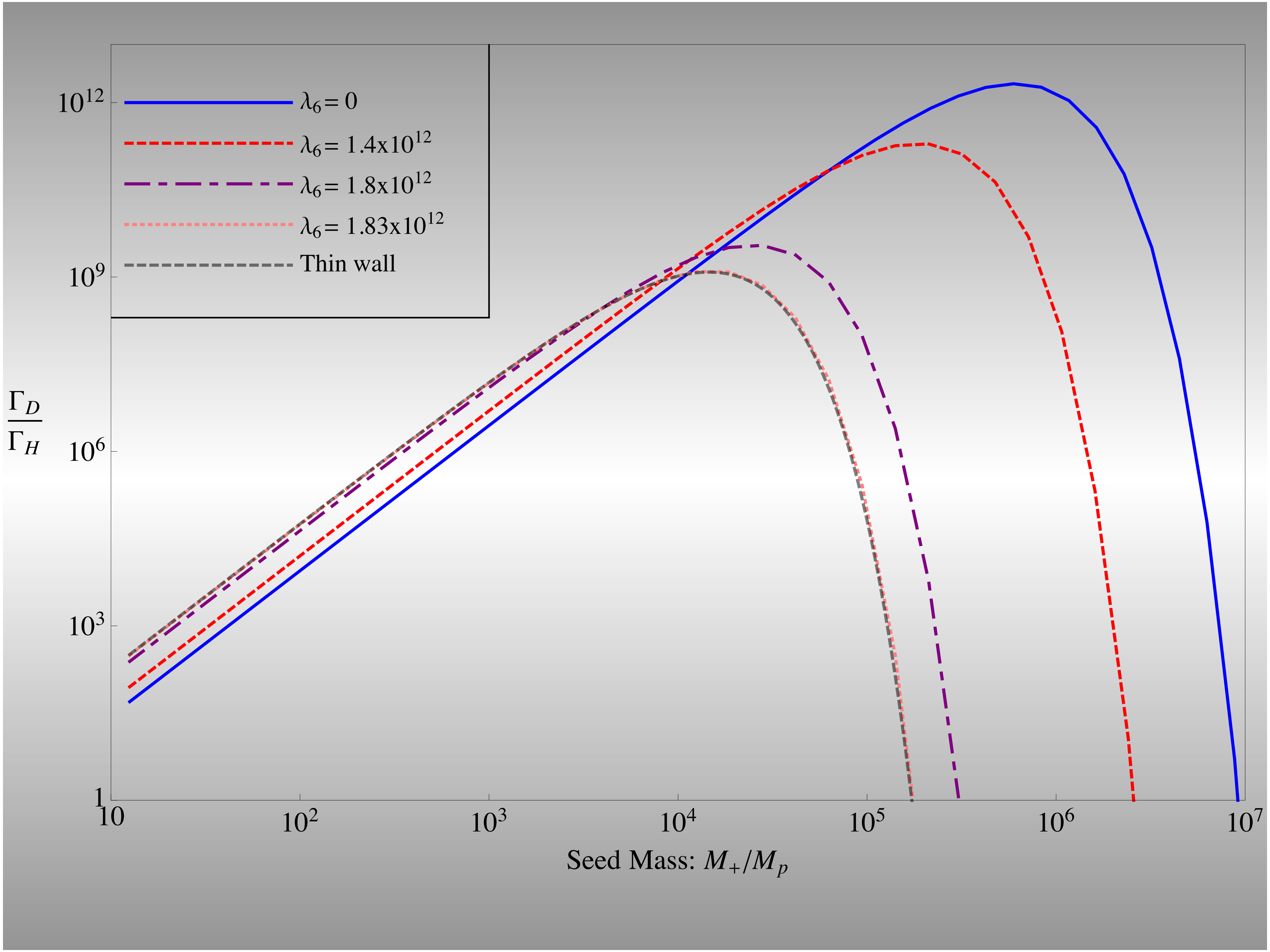


Numerical Integration



Thickening the wall increases the effectiveness of the instanton – the primordial black hole will hit the danger zone much sooner, and the decay will proceed rapidly.





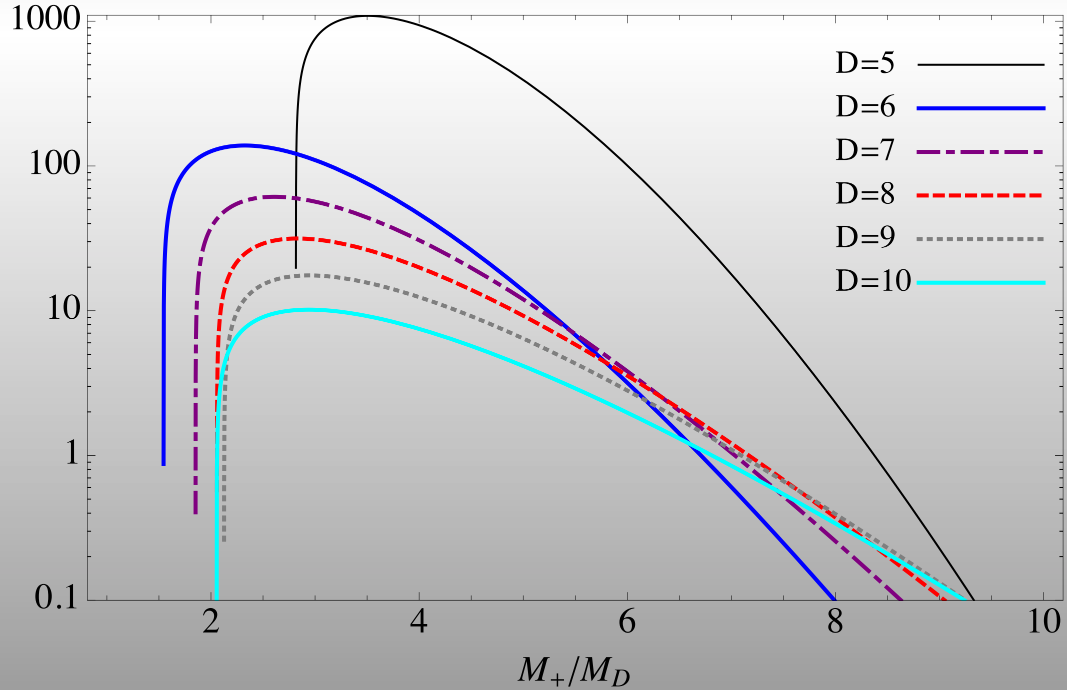
λ6 = 0
λ6 = 1.4x10^12
λ6 = 1.8x10^12
λ6 = 1.83x10^12
Thin wall
10^12
10^9
10^6
10^3
1
ΓD/ΓH
10
10^2
10^3
10^4
10^5
10^6
10^7
Seed Mass: M+/Mp

Primordial black holes start out with small enough mass to evaporate and will eventually hit these curves.

Can view as a constraint on PBH's or (weak) on corrections to the Higgs potential.

Small black holes also possible in theories with Large Extra Dimensions.

(but the branching ratio seems to drop with D – shown here $\frac{\Gamma_D}{\Gamma_H}$ for thin wall)



Summary

- Depending on higher energy physics, the Higgs vacuum may be unstable.
- We can construct an instanton to describe the decay process – even including gravity.
- Tunneling amplitude significantly enhanced in the presence of a black hole – bubble forms around black hole and can remove it altogether.
- Very efficient for small black holes, so either they don't exist – or the vacuum is stable.