



Highlight new
results since
ICHEP 2014!

Lattice QCD for precision particle physics

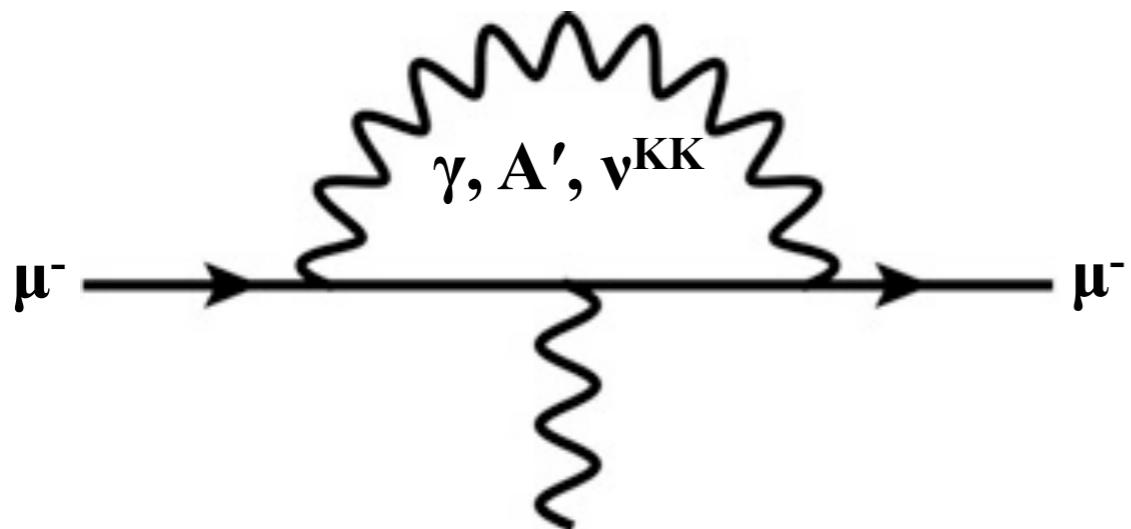
Ruth Van de Water
Fermilab

38th International Conference on High-Energy Physics
August 9, 2016

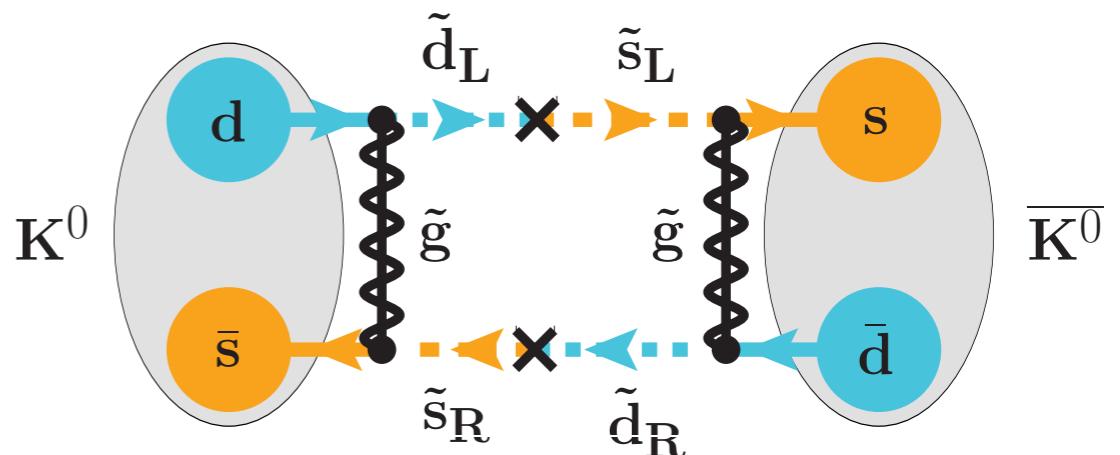
Where is the new physics?

- ❖ Experimental high-energy physics community searching broadly for direct and indirect signs of **new particles and forces** over wide range of energy scales and areas within particle physics
 - ❖ High-precision measurements sensitive to **quantum-mechanical effects of new particles that would give rise to tiny deviations from Standard-Model expectations**
- ★ Revealing new Physics requires reliable and precise theory!

muon anomalous magnetic moment
sensitive to SUSY, X-dimensions,
dark photons, ...



$s \rightarrow d$ & $b \rightarrow d, s$ flavor-changing neutral currents sensitive to SUSY, flavor-changing Z', leptoquarks, 4th generation, composite Higgs, ...



The lattice-QCD HEP program

Lattice high-energy-physics effort aims to **reduce QCD uncertainties to at-or-below measurement errors** to maximize discovery potential of high-precision experiments

- ♦ U.S. DOE HEPAP Particle Physics Prioritization Panel (P5) identified five "compelling lines of inquiry [that] show great promise for discovery over the next 10 to 20 years":
 - (1) Explore the unknown: **new particles, interactions, & physical principles**
 - (2) Use the **Higgs boson** as a new tool for discovery
 - (3) Pursue the physics associated with **neutrino mass**
 - (4) Identify the new physics of **dark matter**
 - (5) Understand **cosmic acceleration**: dark energy & inflation
- ♦ **Lattice-QCD calculations supporting all of these areas underway!**

(See appendix slides on the role of lattice QCD for cosmic-frontier experiments)



Precision lattice QCD

"[An] area of striking progress has been lattice gauge theory. ... It is now possible to compute the spectrum of hadrons with high accuracy, and lattice computations have been crucial in the measurement of the properties of heavy quarks. Continuing improvements in calculational methods are anticipated in coming years."

– **Snowmass 2013 Executive Summary (1401.6075)**

Quantum ChromoDynamics

$$\mathcal{L}_{\text{QCD}} = \frac{1}{2g^2} \text{tr} [F_{\mu\nu} F^{\mu\nu}] - \sum_{f=1}^{n_f} \bar{\psi}_f (\not{D} + m_f) \psi_f + \underbrace{\frac{i\bar{\theta}}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{tr} [F_{\mu\nu} F_{\rho\sigma}]}_{\text{violates } CP}$$

- ♦ QCD Lagrangian contains $1 + n_f + 1$ parameters that can be fixed from equal number of experimental inputs

FUNDAMENTAL PARAMETER

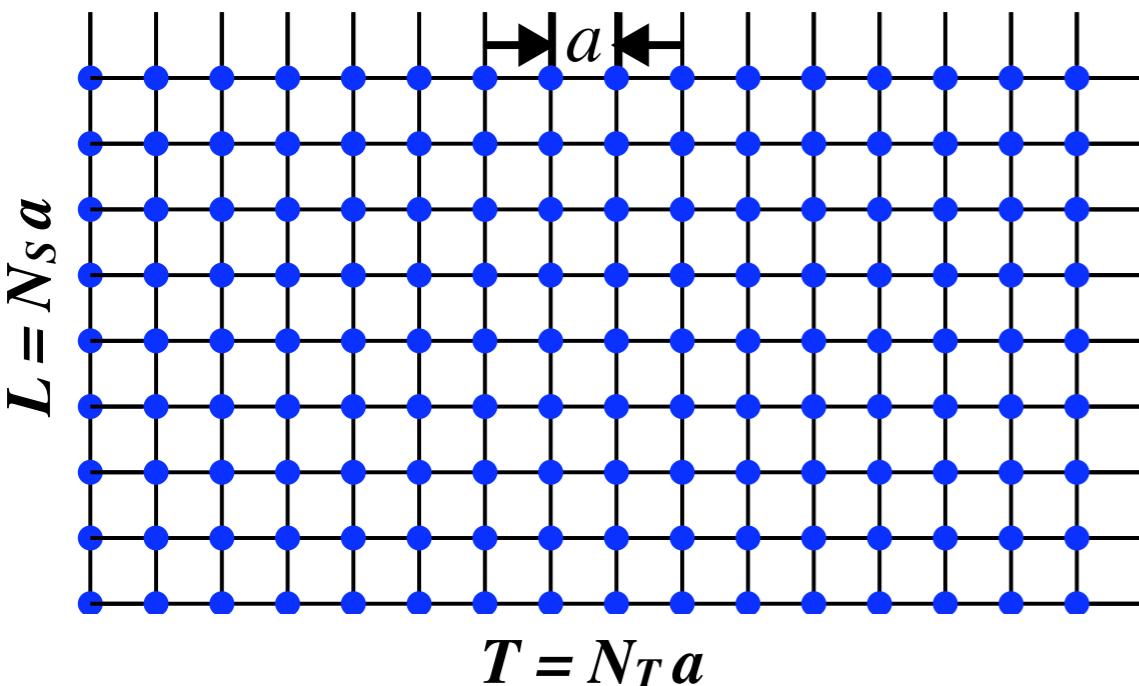
- ❖ Gauge coupling g^2
- ❖ n_f quark masses m_f
- ❖ $\theta = 0$
- ♦ Once the parameters are fixed, everything else is a prediction of the theory
- ♦ Calculations of hadronic parameters challenging in practice because low-energy QCD is nonperturbative

EXPERIMENTAL INPUT

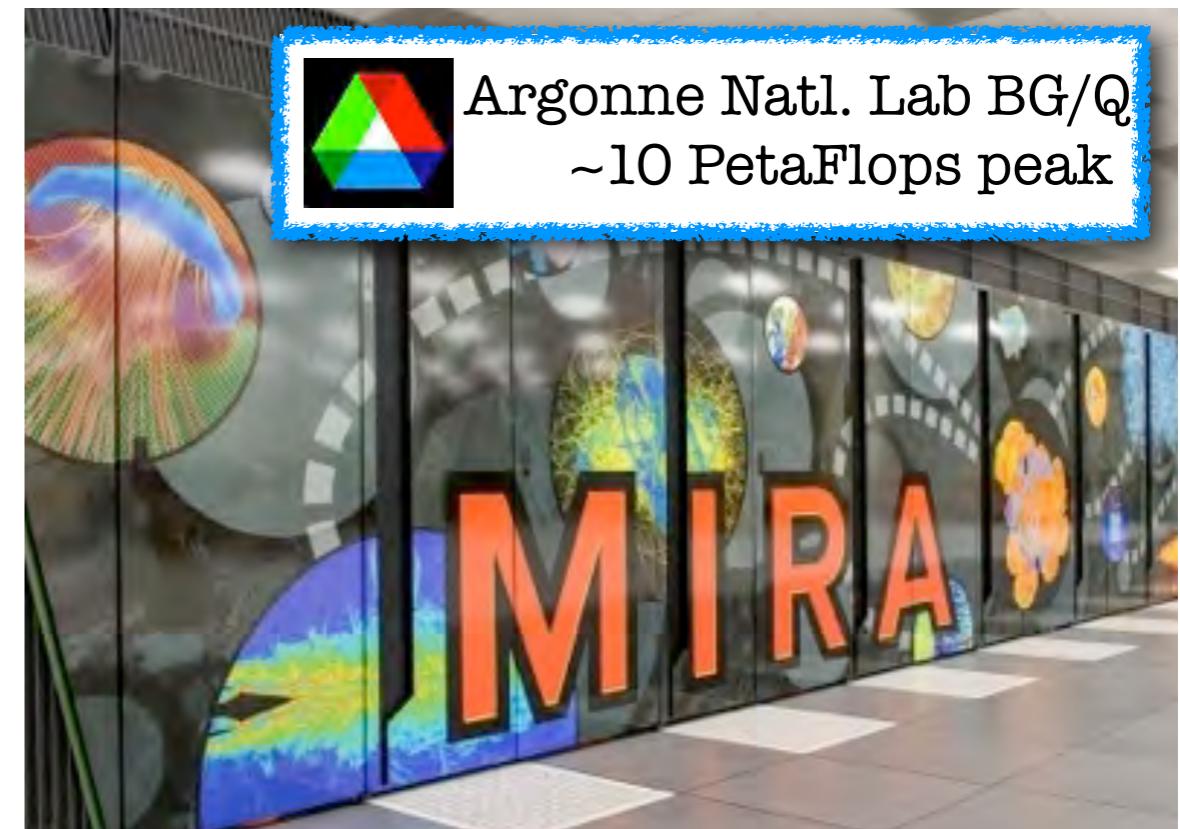
- $r_1, m_\Omega, Y(2S-1S),$ or f_π
- $m_\pi, m_K, m_{J/\psi}, m_Y, \dots$
- neutron EDM ($|\theta| < 10^{-11}$)

Numerical lattice QCD

- ◆ Systematic method for calculating hadronic parameters from QCD first principles
- ◆ Define QCD on (Euclidean) spacetime lattice and solve path integral numerically
 - ❖ Recover QCD when lattice spacing $a \rightarrow 0$ and box size $L \rightarrow \infty$

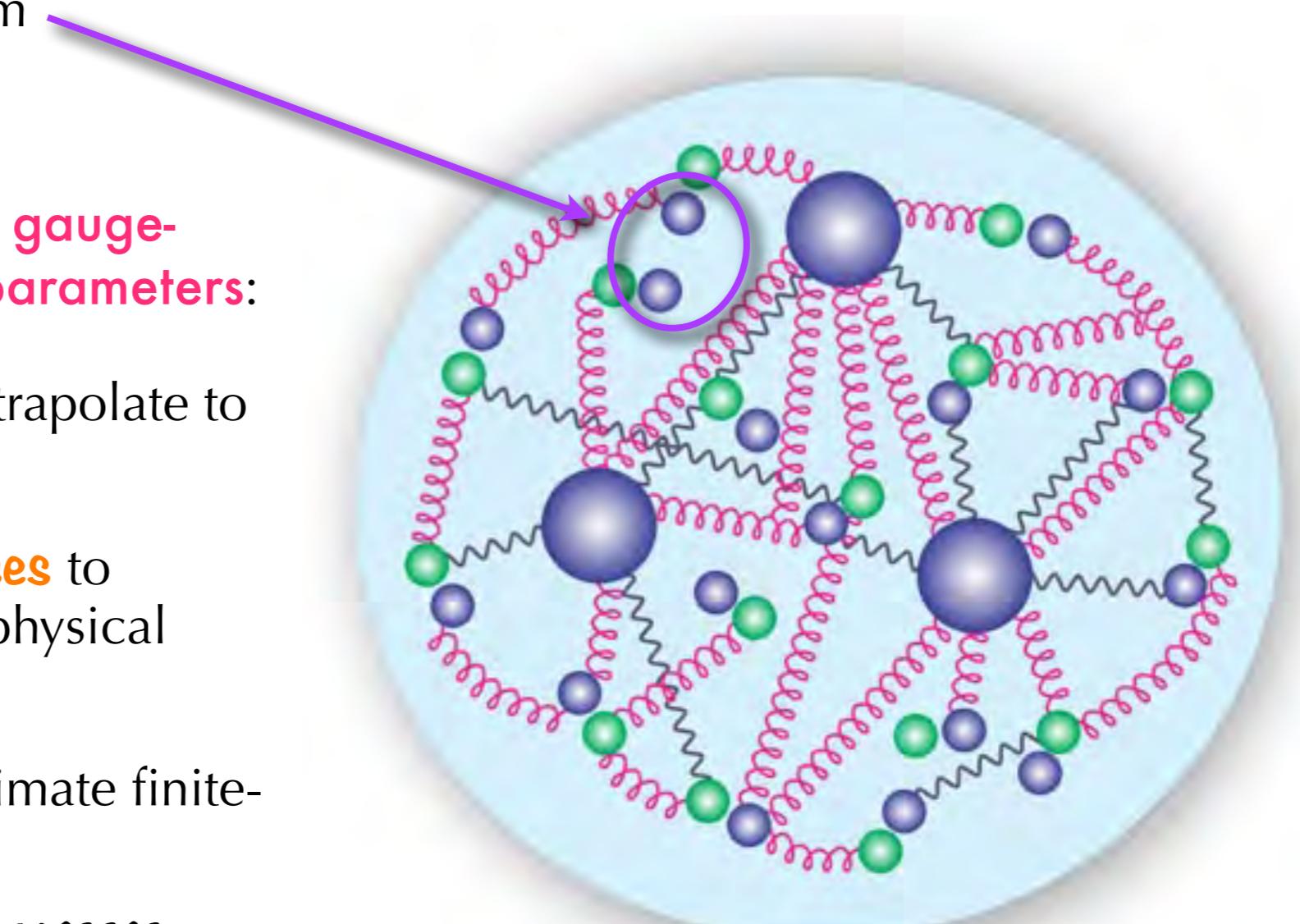


- ◆ Simulate using Monte-Carlo methods and importance sampling
 - ❖ Sample from all possible field configurations using a distribution given by $\exp(-S_{\text{QCD}})$
- ◆ Run codes upon supercomputers and dedicated clusters



Modern lattice-QCD simulations

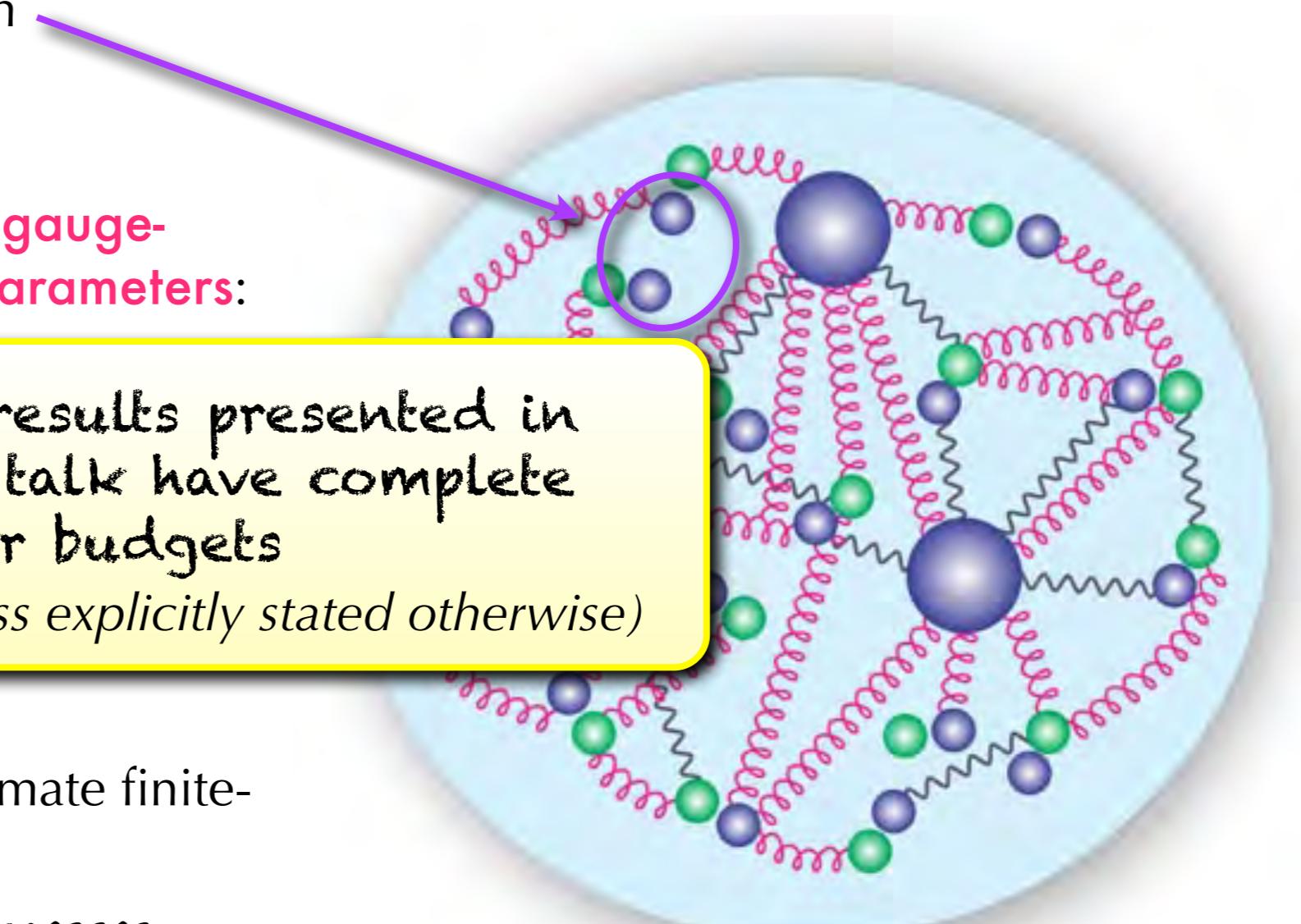
- ◆ Standard simulations include dynamical u, d, s (& c) quarks in the vacuum
 - ❖ (Typically sea $m_u=m_d$)
- ◆ Control systematic errors using gauge-field ensembles with different parameters:
 - ❖ **Multiple lattice spacings** to extrapolate to continuum limit ($a \rightarrow 0$)
 - ❖ **Multiple up/down-quark masses** to interpolate or extrapolate to physical $M_\pi = 135$ MeV
 - ❖ **Multiple spatial volumes** to estimate finite-size effects
- ◆ Most precise results for simple processes with single (stable) initial hadron & at most 1 final-state hadron



Modern lattice-QCD simulations

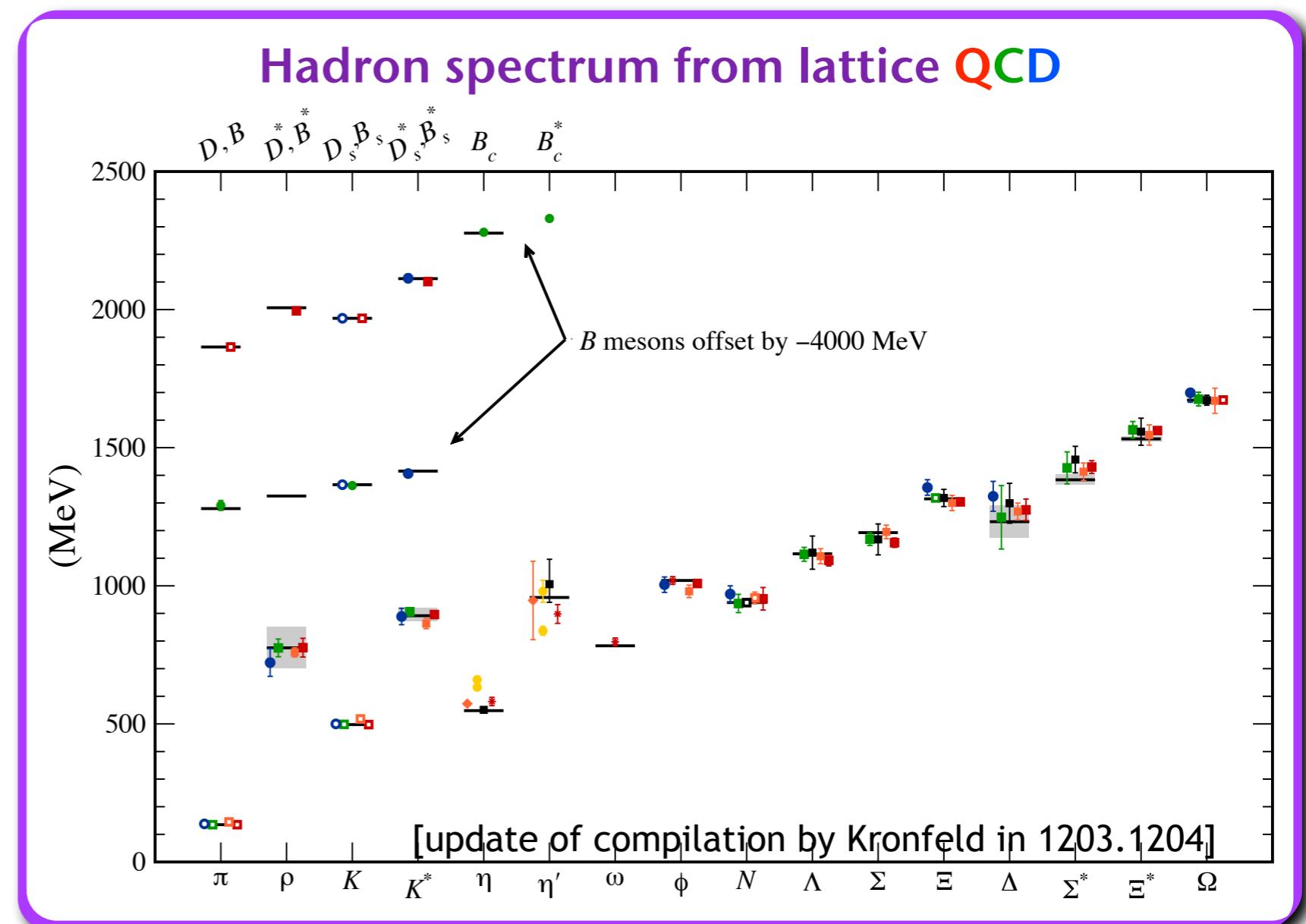
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- ◆ Control systematic errors using gauge-field ensembles with different parameters:
 - ❖ Multiple lattice spans continuum limit (a)
 - ❖ Multiple up/down-interpolate or extrapolate $M_\pi = 135 \text{ MeV}$
 - ❖ Multiple spatial volumes to estimate finite-size effects
- ◆ Most precise results for simple processes with single (stable) initial hadron & at most 1 final-state hadron

★ All results presented in this talk have complete error budgets
(unless explicitly stated otherwise)



Lattice-QCD validation

- ♦ Lattice-QCD results agree with experimental measurements for wide variety of hadron properties including light- and heavy-hadron spectrum & proton-neutron mass difference
- ♦ Lattice-QCD predictions of B_c mass, $D_{(s)}$ decay constants, & $D \rightarrow K\ell\nu$ form factor appeared *before* measurements with comparable precision
- ♦ Independent lattice-QCD calculations using different actions & methods provide confirmation for matrix elements inaccessible by experiment
- ♦ Demonstrate that calculations are reliable with controlled errors!



Lattice **QCD** for particle physics: *recent results & future prospects*

"In the last five years lattice QCD has matured into a precision tool. ...

The ultimate aim of lattice-QCD calculations is to reduce errors in hadronic quantities to the level at which they become subdominant either to experimental errors or other sources of error."

– Snowmass 2013 Quark-flavor WG report (1311.1076)

Recent lattice-QCD results for quark-flavor physics

Explore the unknown: **new particles,
interactions**, and physical principles

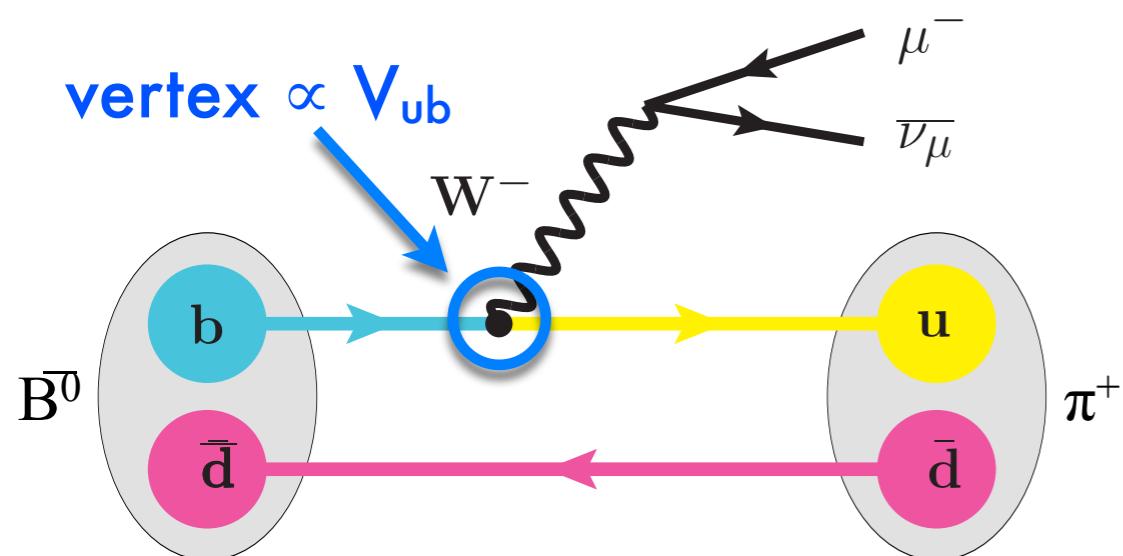


Quark-flavor physics

- ◆ Most Standard-Model extensions have additional sources of flavor & CP violation in the quark sector
- ◆ Lattice-QCD quark-flavor effort has two main thrusts:

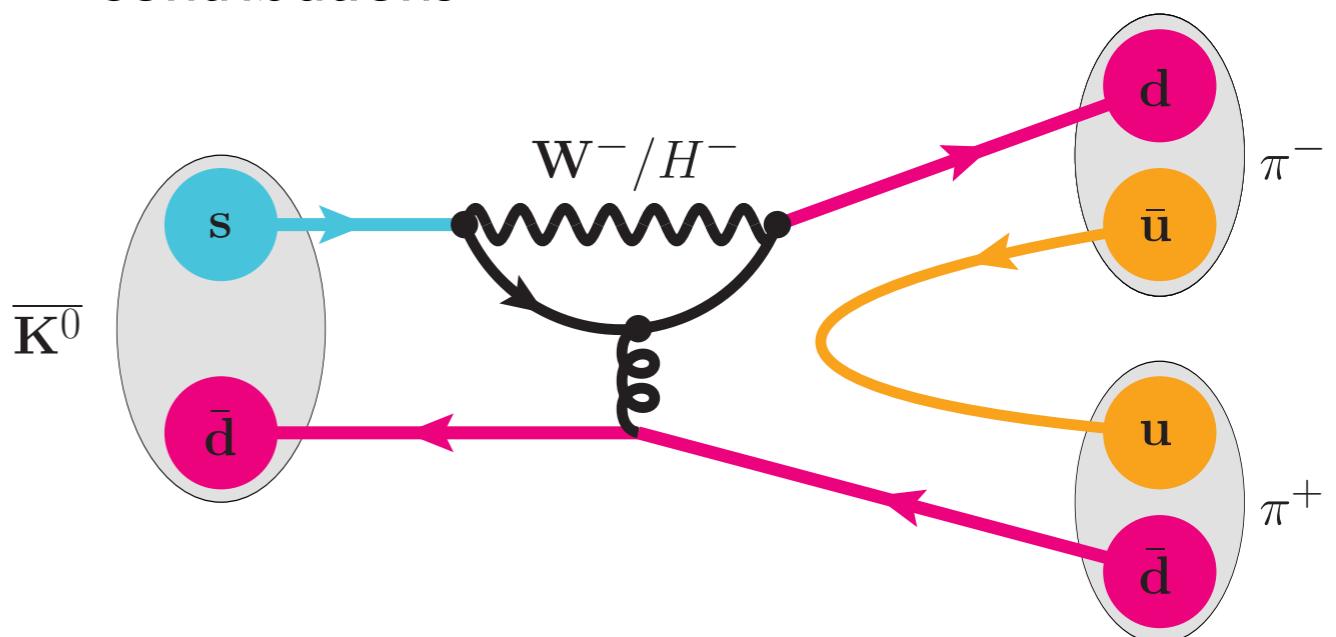
(1) Determination of CKM quark-mixing matrix elements

- ◆ Use tree-level decays unlikely to receive substantial new-physics contributions



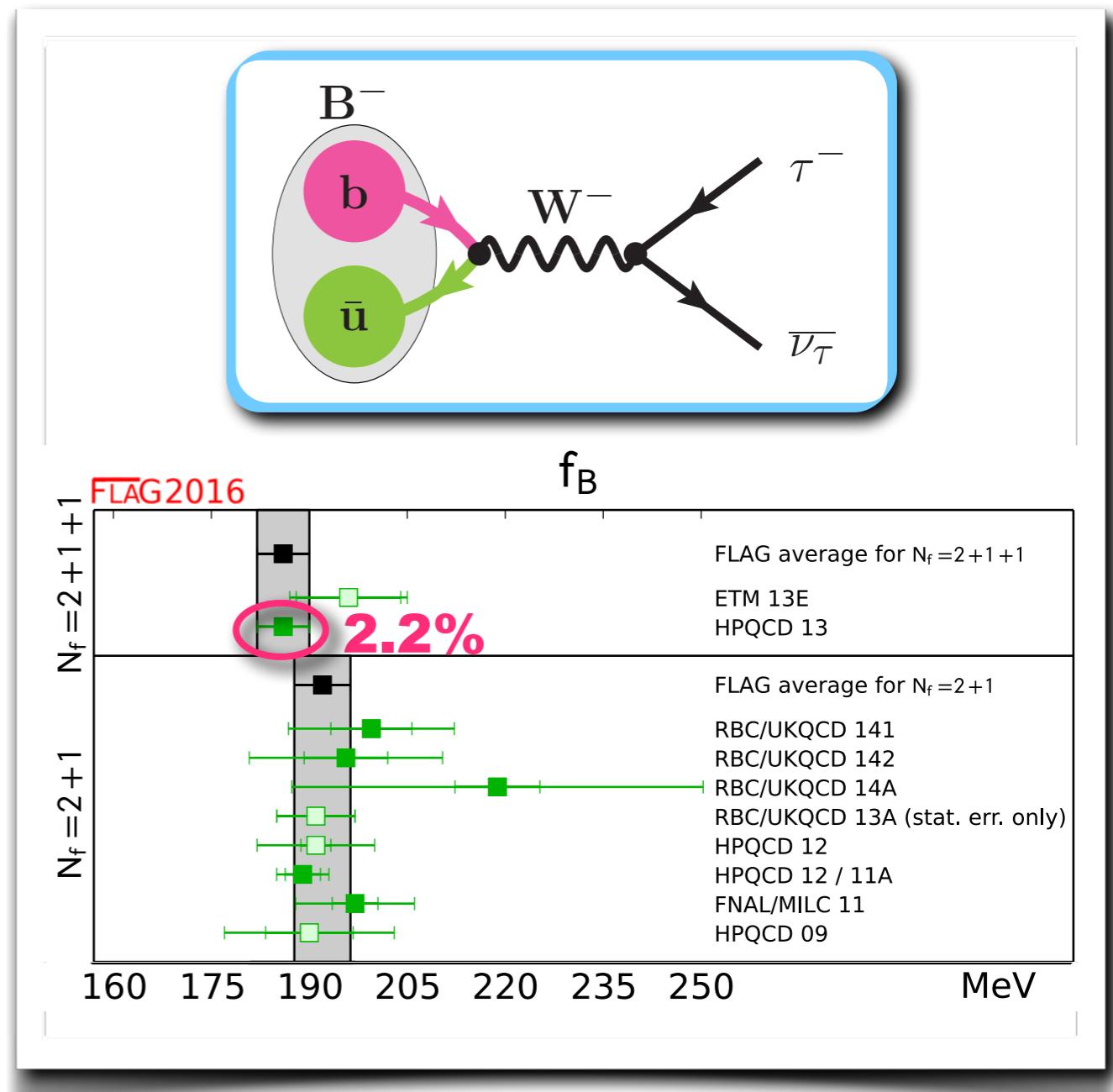
(2) New-physics searches in rare decays & mixing

- ◆ Study (primarily) loop-level processes sensitive to beyond-the-Standard-Model contributions



b-quark physics in lattice QCD

- ♦ In this talk focus on bottom quarks because many new results since ICHEP 2014
 - ♦ Appendices present details & additional results for [light \(u,d,s\)](#) & [heavy flavors \(c,b\)](#)
- ♦ LQCD B-physics calculations rapidly improving precision & expanding scope
- ♦ For $B_{(s)}$ -meson leptonic decay constants, confirmation from several calculations [see Rosner, Stone, RV, 1509.02220 (PDG 2016)]
 - ♦ **First f_B at physical pion mass** [HPQCD, PRL110, 222003 (2013)]
 - ♦ For form factors & mixing matrix elements, still only 1 or 2 calculations with ≥ 3 dynamical quarks...
 - ♦ In few years, expect $<1\%$ $f_{B(s)}$ errors and percent-level form factors & mixing parameters from finer lattice spacings + improved actions at physical m_b



Lattice-QCD CKM *highlights* 2015/16

- ♦ Lattice-QCD community has mature & successful program to calculate weak matrix elements needed to obtain Cabibbo-Kobayashi-Maskawa quark-mixing matrix elements (see [recent lattice reviews](#))

$$\left(\begin{array}{ccc} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \rightarrow \ell\nu & K \rightarrow \ell\nu & B \rightarrow \ell\nu \\ & K \rightarrow \pi\ell\nu & B \rightarrow \pi\ell\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \rightarrow \ell\nu & D_s \rightarrow \ell\nu & B \rightarrow D\ell\nu \\ D \rightarrow \pi\ell\nu & D \rightarrow K\ell\nu & B \rightarrow D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d | \bar{B}_d \rangle & \langle B_s | \bar{B}_s \rangle & \\ B \rightarrow \pi\ell\ell & B \rightarrow K\ell\ell & \end{array} \right)$$

Simple processes in LQCD
enable determinations of all CKM
elements & phase except $|V_{tb}|$

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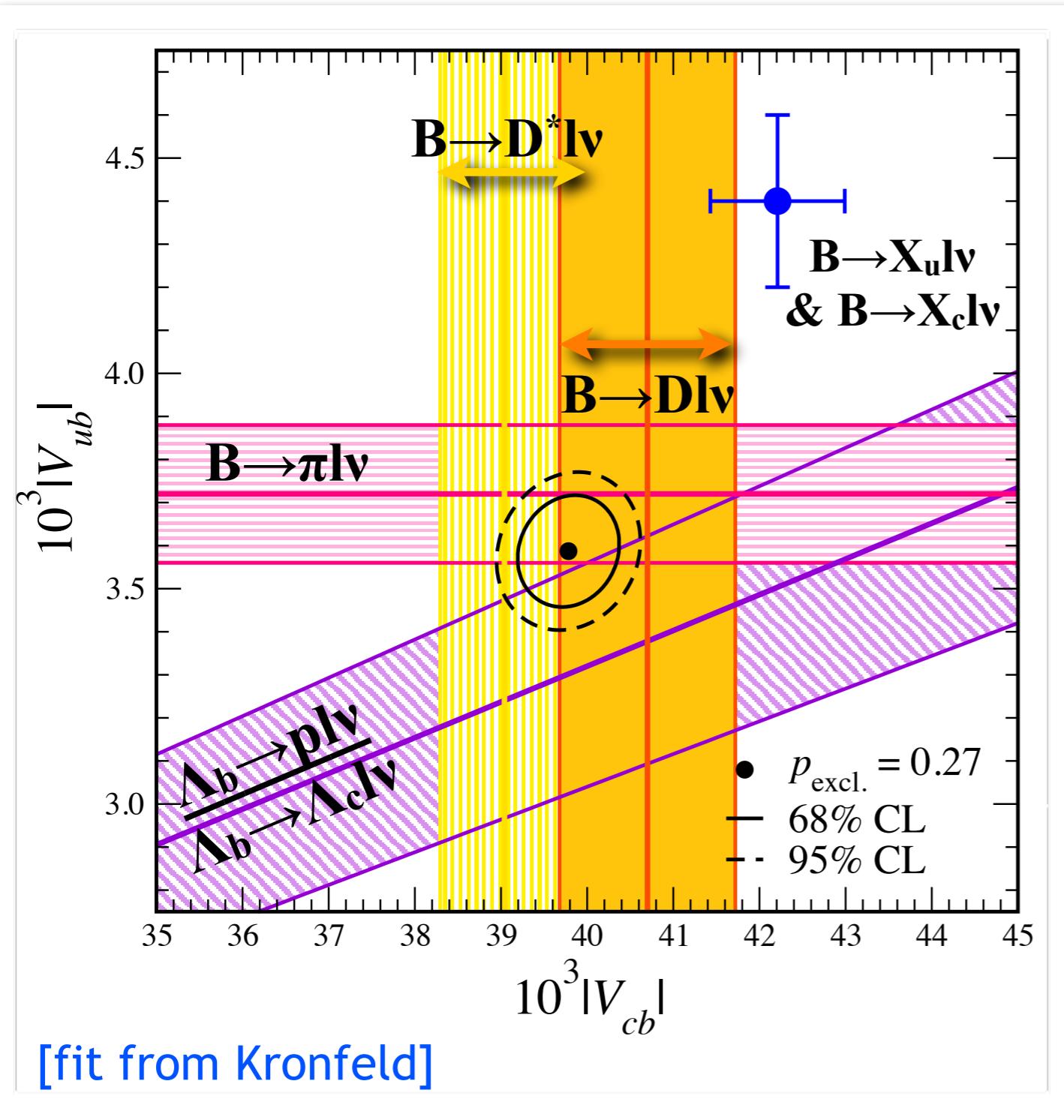
V_{ud}	V_{us}	V_{ub}
$\pi \rightarrow \ell\nu$	$K \rightarrow \ell\nu$	$B \rightarrow \ell\nu$
	$K \rightarrow \pi\ell\nu$	$B \rightarrow \pi\ell\nu$
V_{cd}	V_{cs}	V_{cb}
$D \rightarrow \ell\nu$	$D_s \rightarrow \ell\nu$	$B \rightarrow D\ell\nu$
$D \rightarrow \pi\ell\nu$	$D \rightarrow K\ell\nu$	$B \rightarrow D^*\ell\nu$
V_{td}	V_{ts}	V_{tb}
$\langle B_d \bar{B}_d \rangle$	$\langle B_s \bar{B}_s \rangle$	
$B \rightarrow \pi\ell\ell$	$B \rightarrow K\ell\ell$	

Simple processes in LQCD enable determinations of all CKM elements & phase except $|V_{tb}|$

Several new B -meson & and b -baryon semileptonic form-factor results since ICHEP 2014 including:

- (1) Two independent calculations of $B \rightarrow \pi\ell\nu$ form factors [RBC/UKQCD, PRD91 (2015) 7, 074510; Fermilab/MILC [PRD92 (2015) 1, 014024]
- (2) First three-flavor $B \rightarrow D\ell\nu$ form factors over full kinematic range (nonzero recoil) [Fermilab/MILC, PRD92, 034506 (2015); HPQCD, PRD92, 054510 (2015)]
- (3) First three-flavor $\Lambda_b \rightarrow p\ell\nu$ and $\Lambda_b \rightarrow \Lambda_c\ell\nu$ form factors with physical-mass b quarks [Detmold, Lehner, & Meinel, PRD92, 034503 (2015)]

Implications for $|V_{ub}|$ & $|V_{cb}|$



$|V_{ub}|$:

- ◆ 2× smaller error on $|V_{ub}|$ from $B \rightarrow \pi l \nu$
- ◆ Determinations from inclusive & exclusive semileptonic B decays continue to differ by $>2\sigma$...

$|V_{cb}|$:

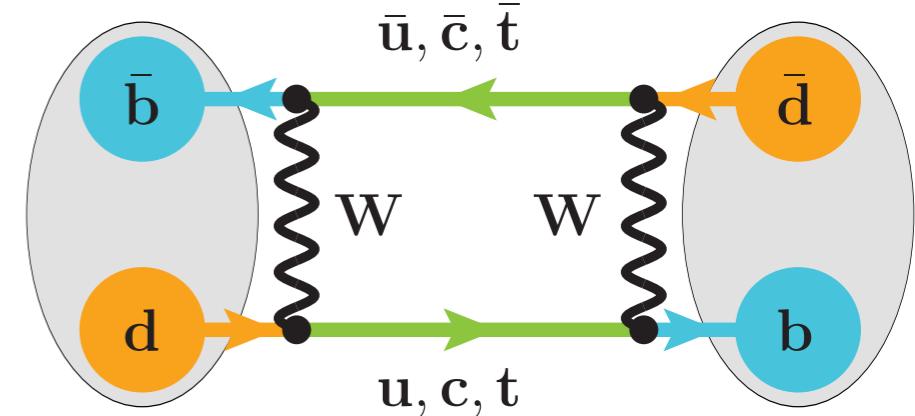
- ◆ Exclusive $|V_{cb}|$ from $B \rightarrow D l \nu$ over full kinematic range *agrees* with inclusive value
- ◆ Lattice-QCD calculation of $B \rightarrow D^* l \nu$ at nonzero recoil in progress ... perhaps “puzzle” will resolve itself?

Lattice-QCD FCNC *highlights* 2015/16

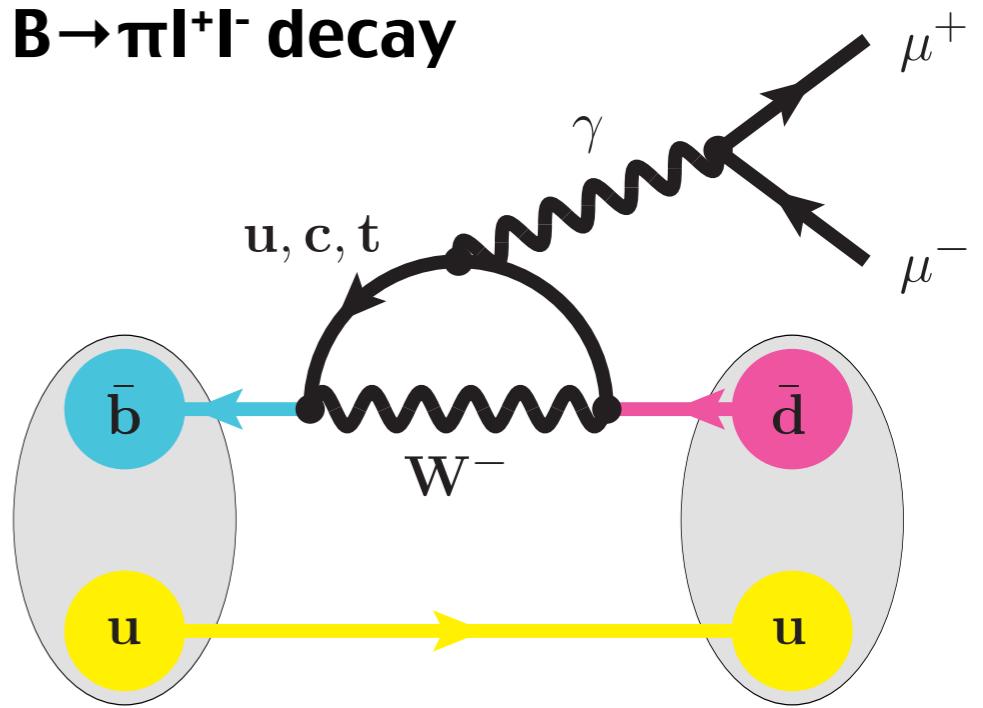
Several new results for $b \rightarrow d$ & $b \rightarrow s$ flavor-changing-neutral current processes sensitive to new physics:

- (1) First complete 3-flavor calculation of **neutral $B_{d,s}$ -mixing matrix elements** [Fermilab/MILC, PRD93, 113016]
 - ◆ Combination with experimental oscillation frequencies $\Delta M_{d,s}$ yields CKM combinations $|V_{td} V_{tb}^*|$ & $|V_{ts} V_{tb}^*|$ and constrains **apex $(\bar{\rho}, \bar{\eta})$ of CKM unitarity triangle**
- (2) First LQCD result for **$B \rightarrow \pi$ tensor form factor** [Fermilab/MILC PRL115, 152002 (2015)]
- (3) First lattice-QCD calculation of **$\Lambda_b \rightarrow \Lambda$ form factors with relativistic physical-mass b quarks** [Detmold & Meinel, PRD93, 074501(2016)]

B^0_d -meson mixing

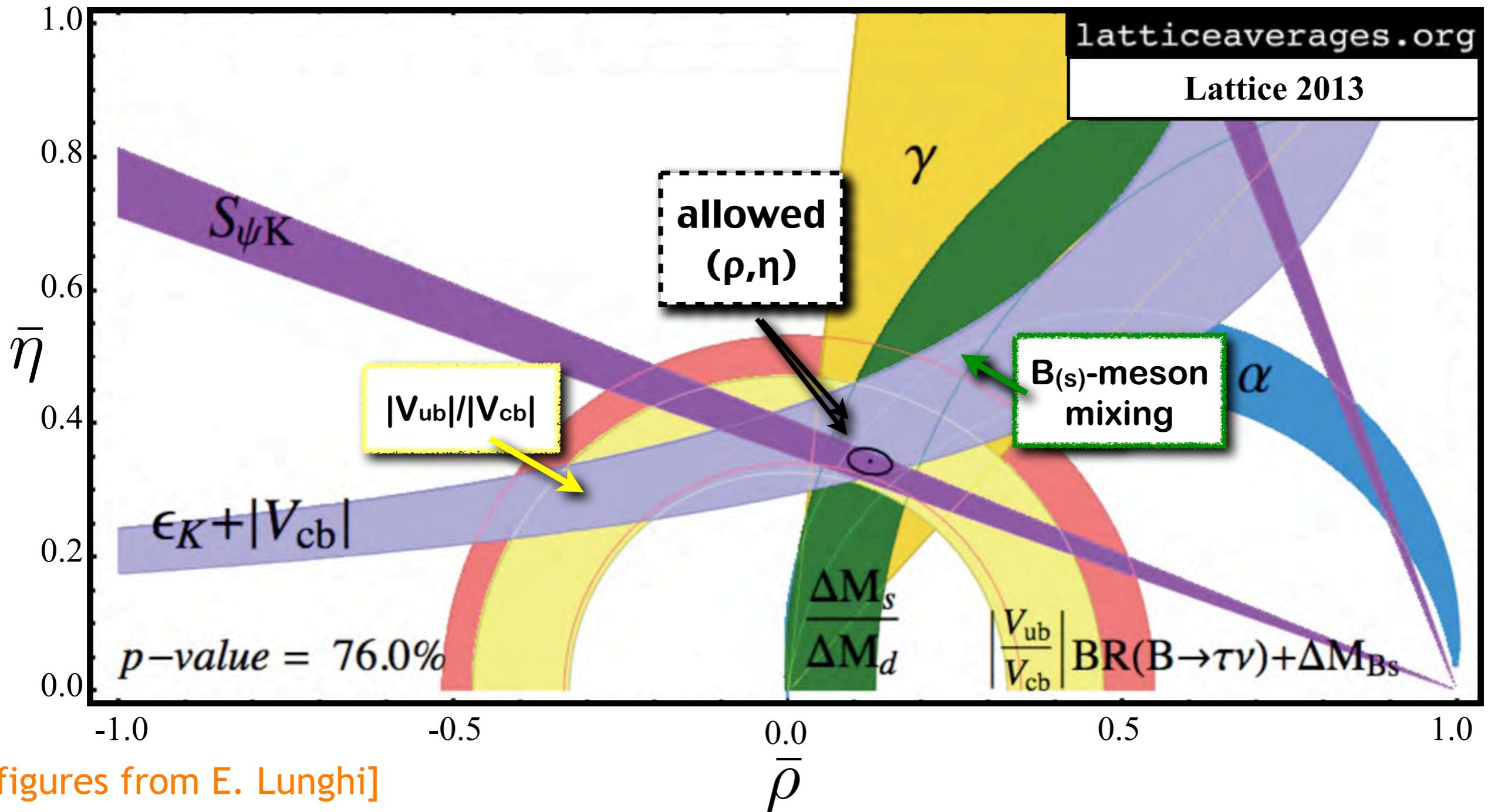


$B \rightarrow \pi l^+ l^-$ decay



Impact on CKM unitarity-triangle fit

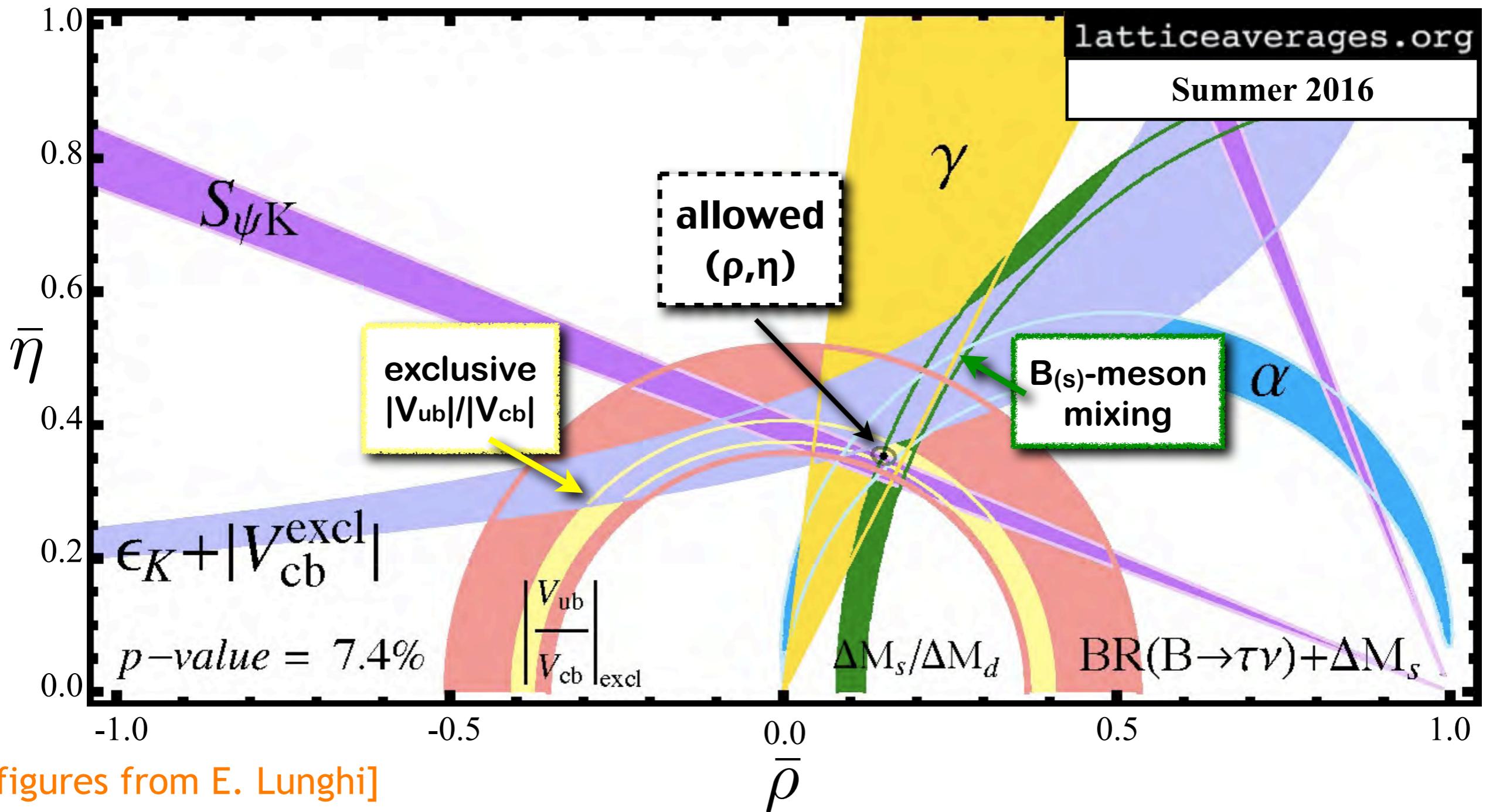
ICHEP 2014



[figures from E. Lunghi]

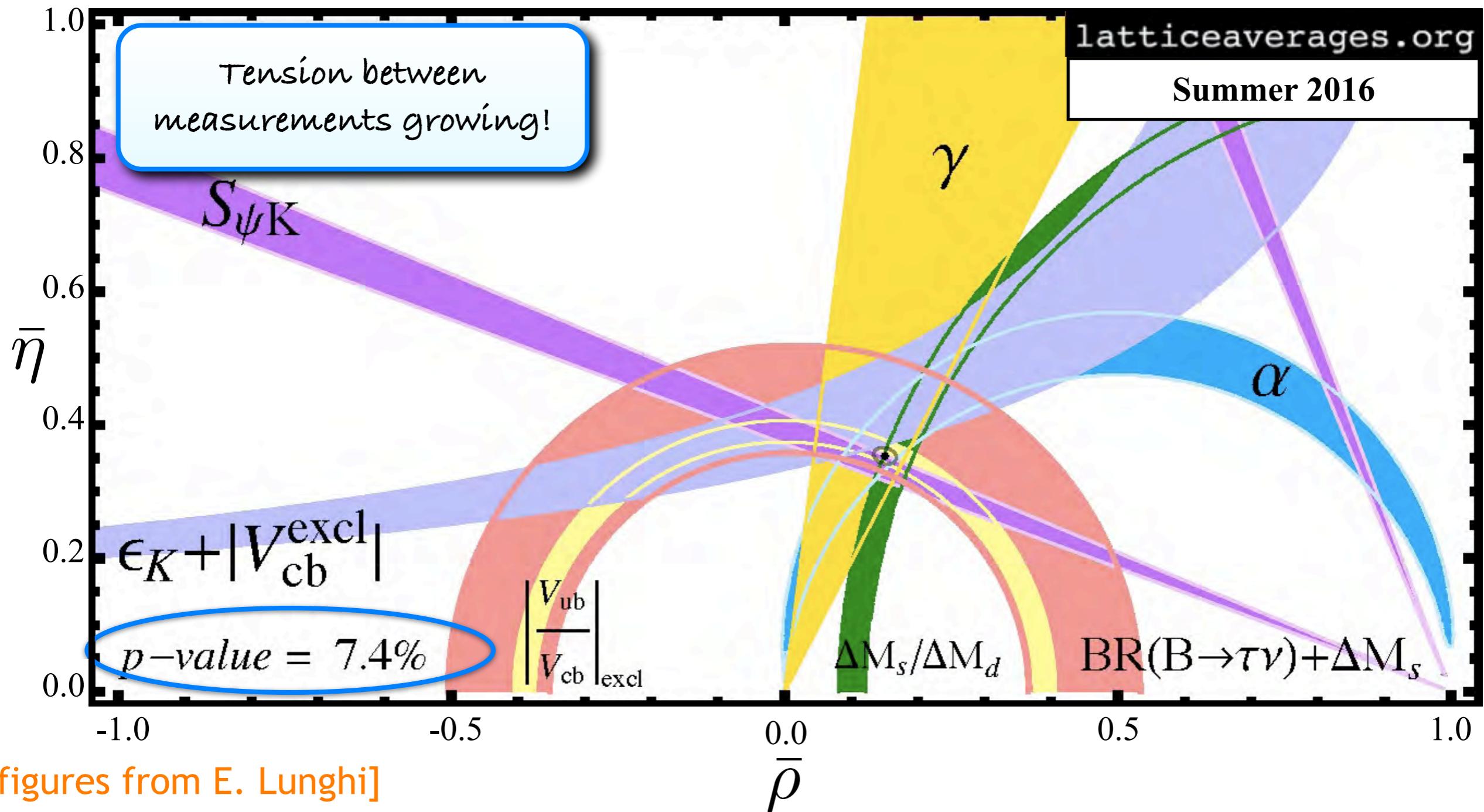
Impact on CKM unitarity-triangle fit

ICHEP 2016

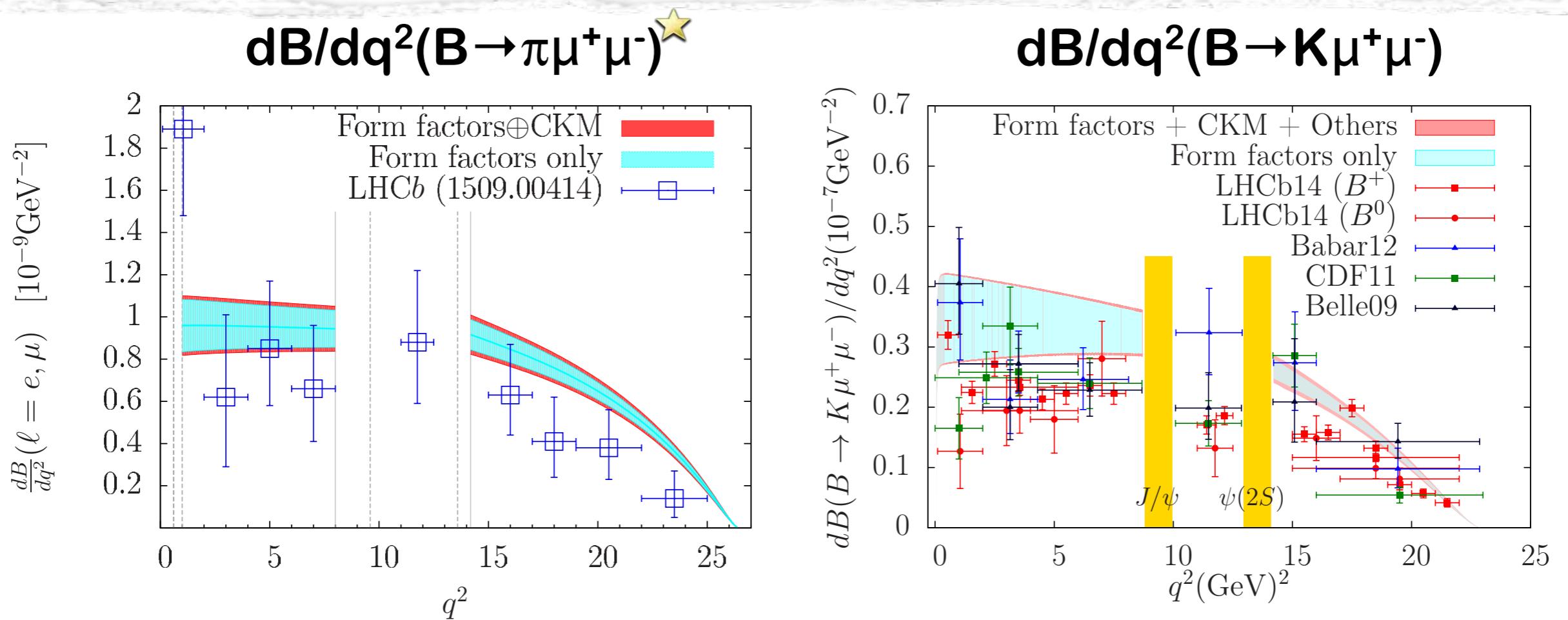


Impact on CKM unitarity-triangle fit

ICHEP 2016

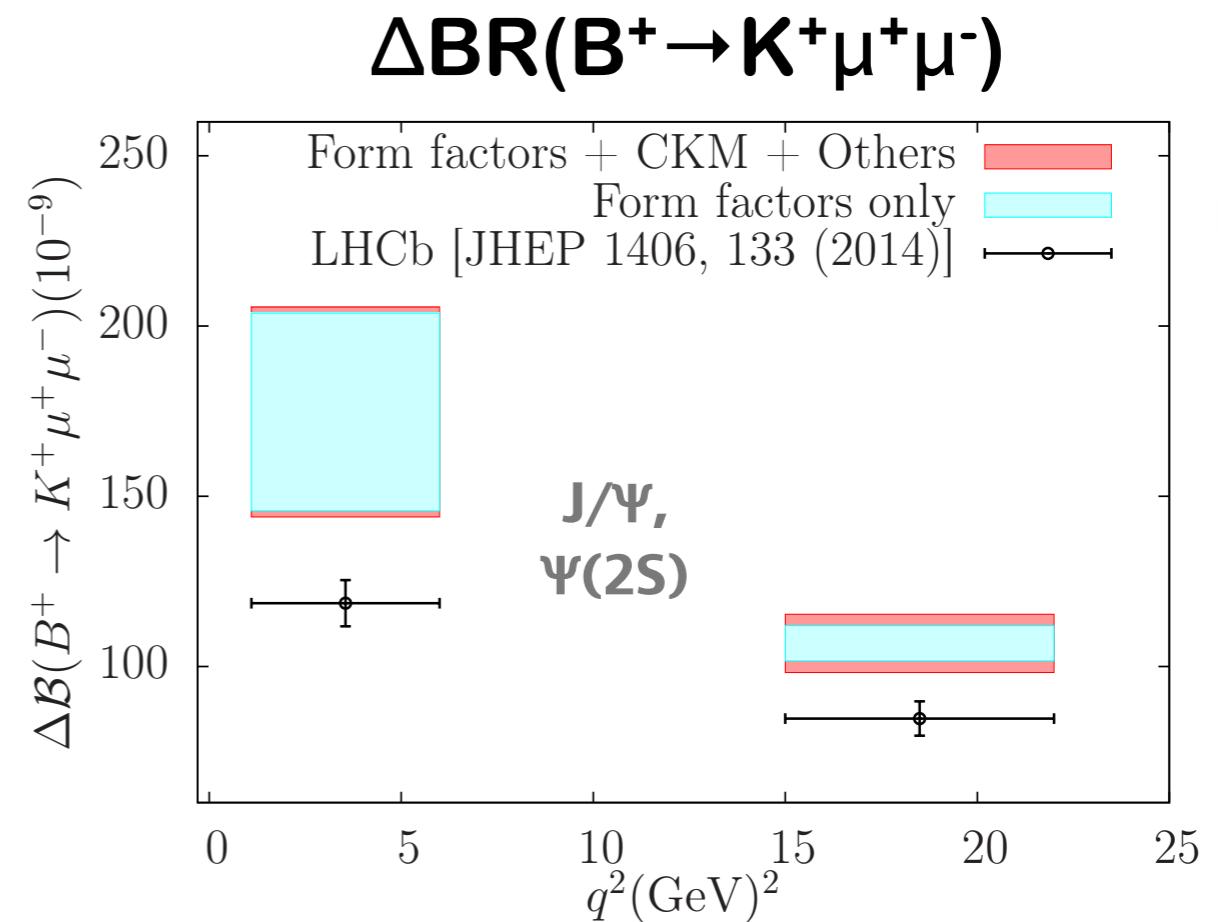
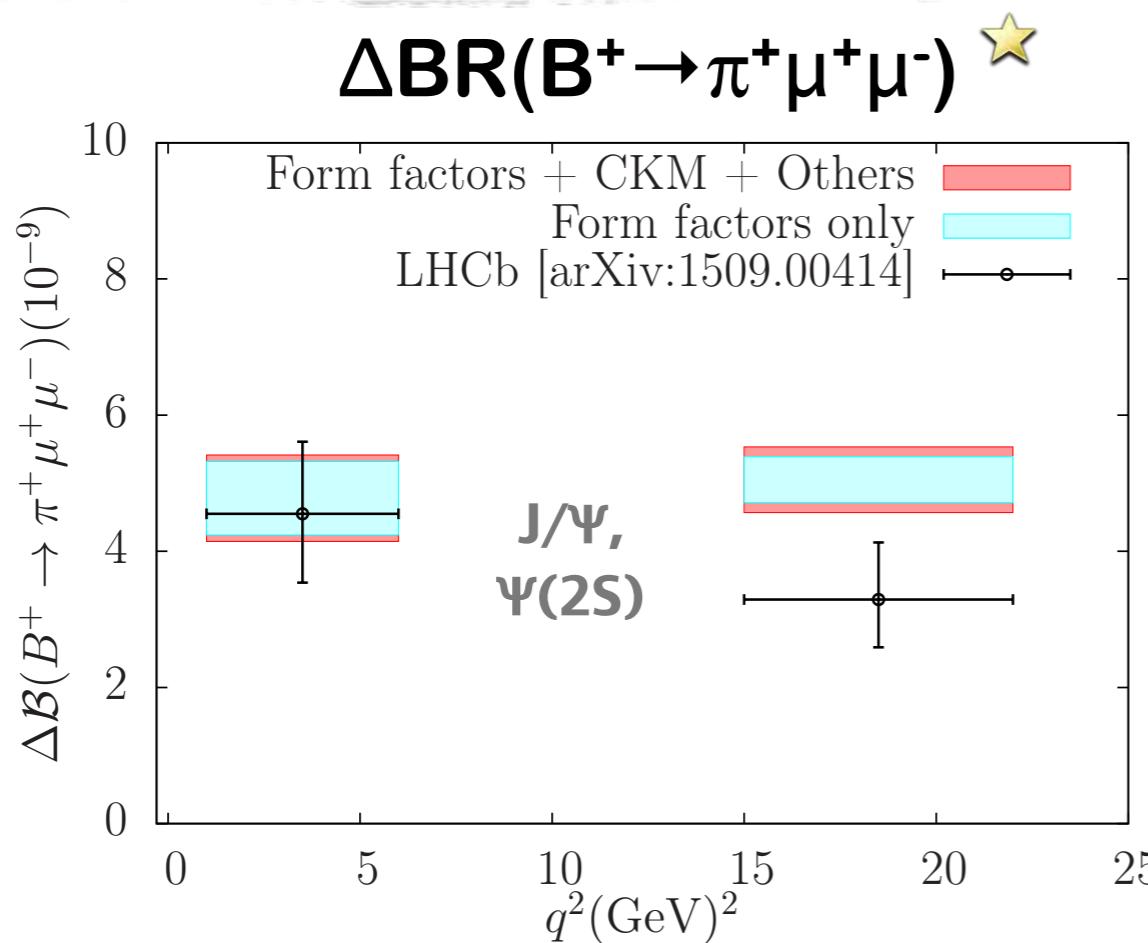


SM expectations for $b \rightarrow (d,s)l^+l^-$ decays



- ◆ Theoretical & experimental q^2 shapes for $B \rightarrow \pi(K)\mu^+\mu^-$ differential branching fractions consistent, but measurements lie slightly below Standard-Model expectations
- ★ Lattice-QCD prediction for $d\mathcal{B}(B \rightarrow \pi \mu^+ \mu^-)/dq^2$ appeared before LHCb measurement!

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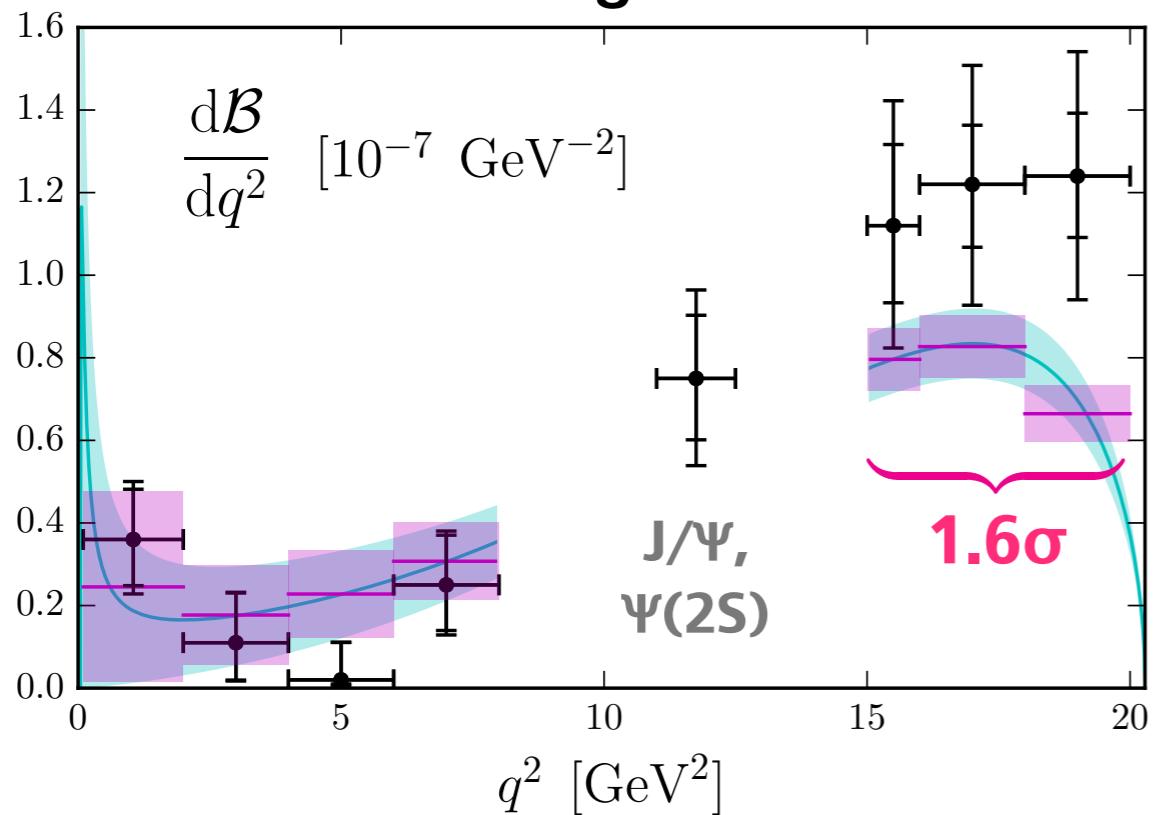


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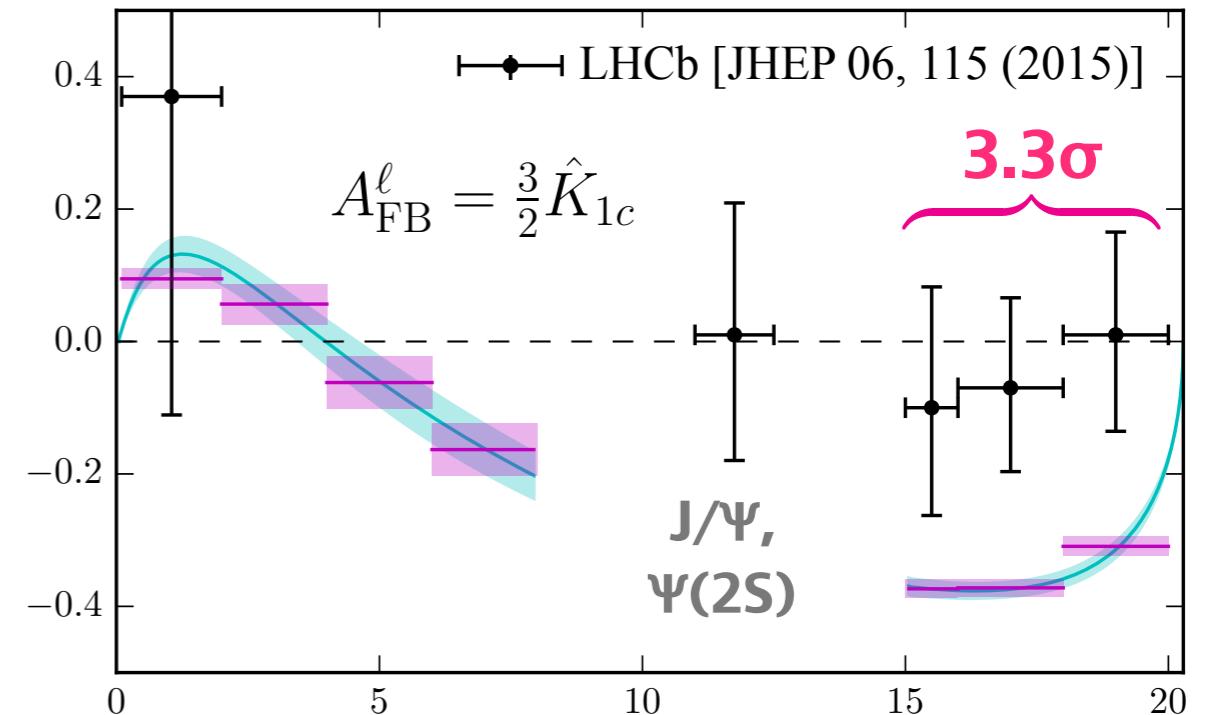
Measurements in four wide q^2 bins in **1.7 σ combined tension** with Standard Model

SM expectations for $b \rightarrow (d,s)l^+l^-$ decays

$\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ differential branching fraction



$\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ lepton forward-backward asymmetry

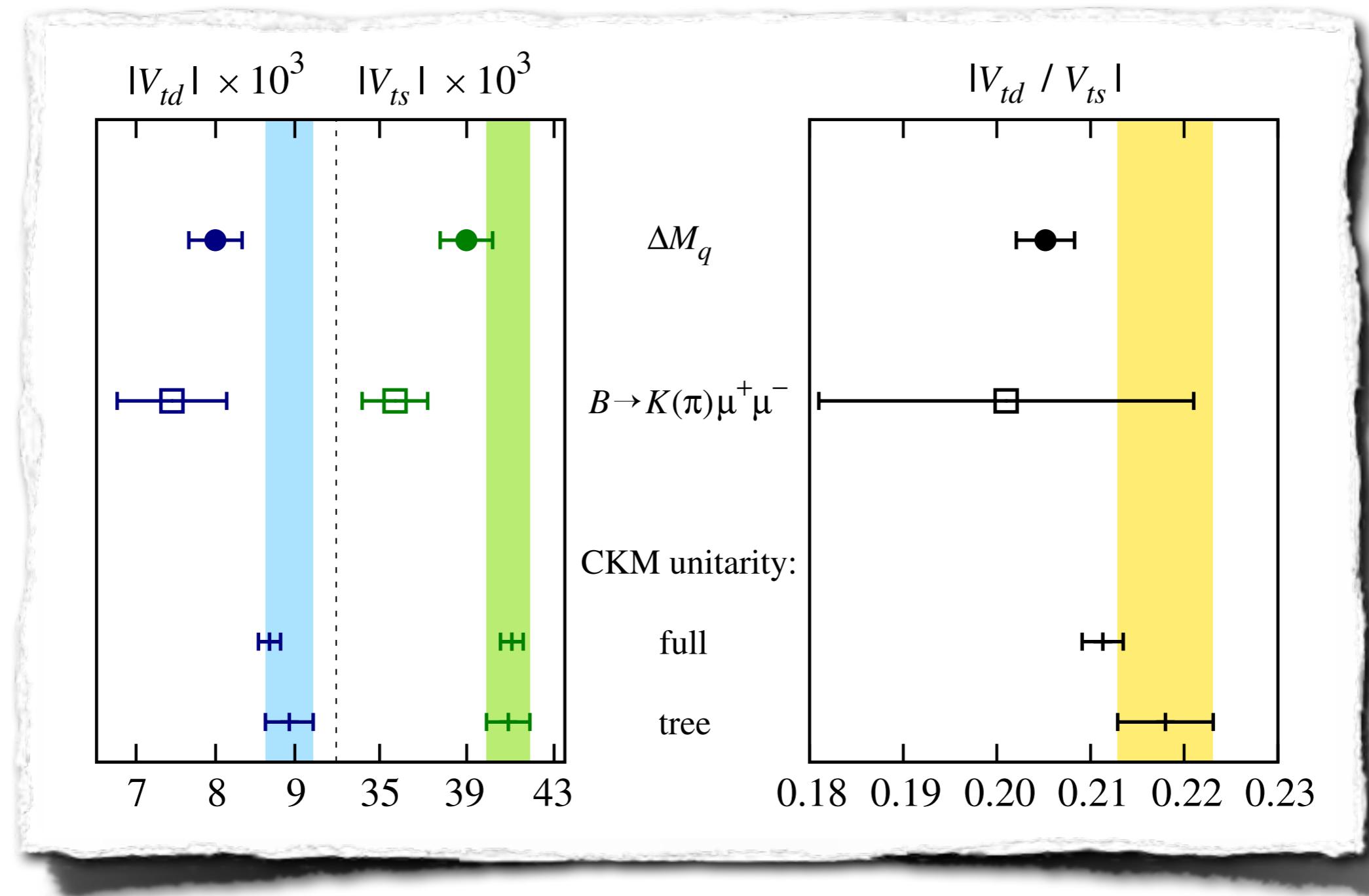


- ◆ Nonzero baryon spin leads to nontrivial angular distributions for $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay
- ★ Pattern of measurements can disentangle contributions from individual operators in the $b \rightarrow sll$ effective Hamiltonian

Measurements in $[15 \text{ GeV}^2, 20 \text{ GeV}^2]$ bin above Standard Model

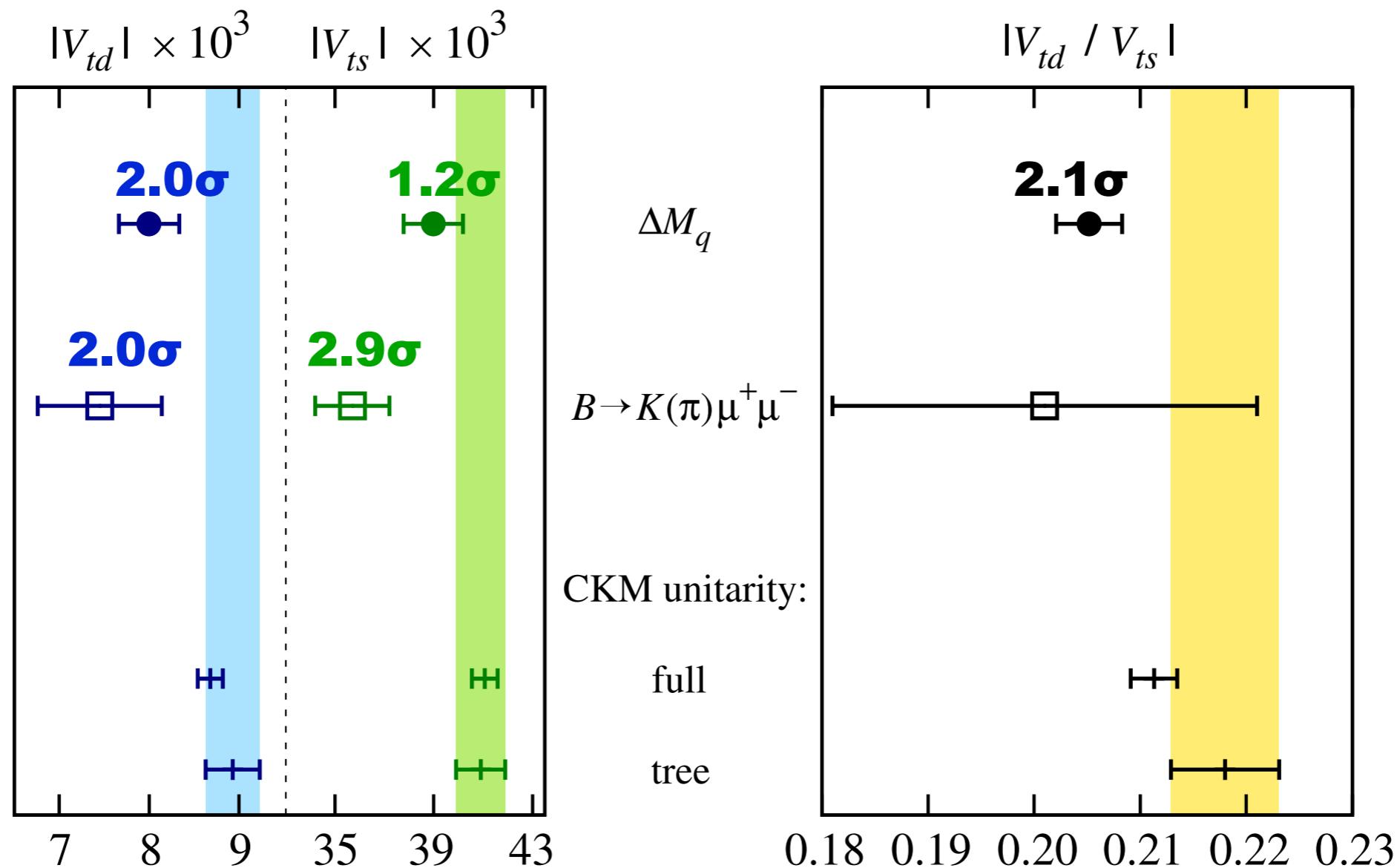
Implications for $|V_{td}|$ & $|V_{ts}|$

- ◆ $|V_{tq}|$ from $B_{d,s}$ -mixing ~2-3x more precise, but still limited by hadronic matrix elements
- ◆ $|V_{ts}|$ from $B \rightarrow K\mu\mu$ >2x more precise, with commensurate theory & experimental errors



Implications for $|V_{td}|$ & $|V_{ts}|$

Determinations from flavor-changing-neutral current processes differ by $\sim 2\sigma$ from values implied by tree-level processes + CKM unitarity



[plot from
Bouchard]



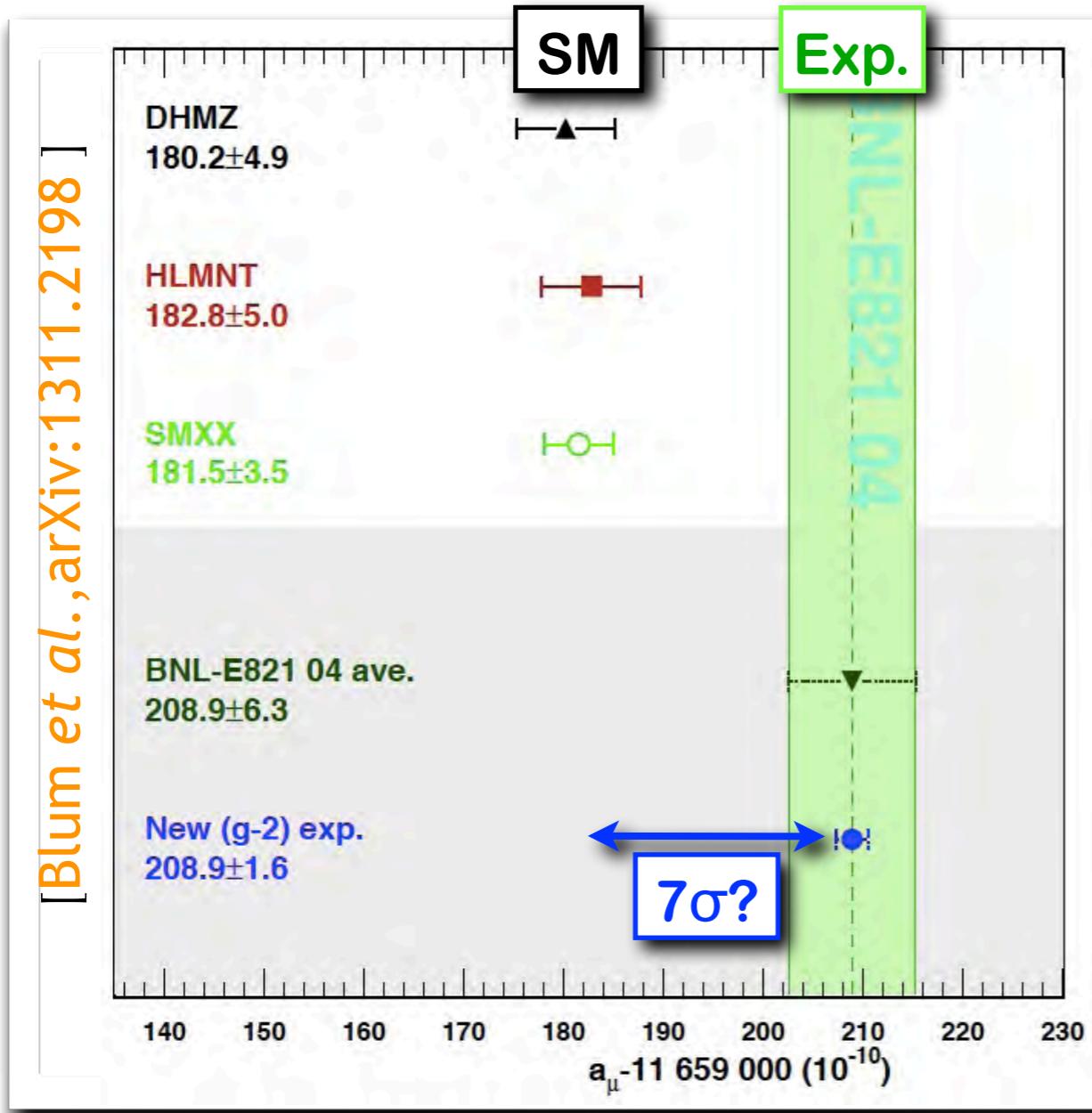
Lattice-QCD progress on muon g-2

Explore the unknown: **new particles,
interactions**, and physical principles

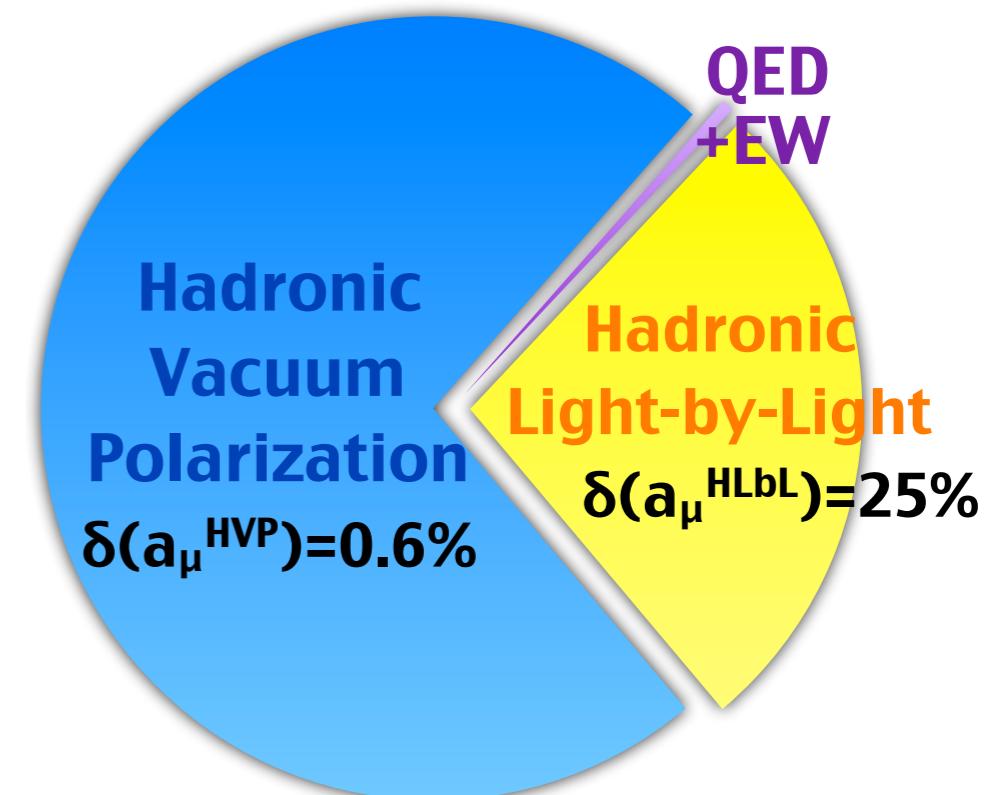


Muon anomalous magnetic moment ($g-2$)

- ♦ Fermilab Muon $g-2$ Experiment will reduce experimental error by 4 to explore $\geq 3\sigma$ tension between BNL measurement & Standard Model



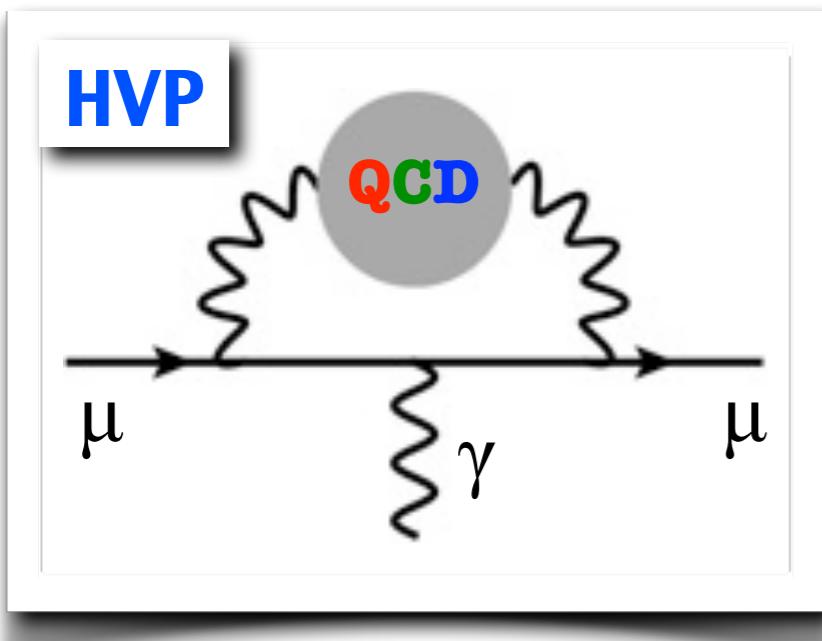
- ♦ To fully leverage new measurement, must bring Standard-Model theory error to commensurate precision



- ♦ Hadronic contributions are calculable (in principle) with lattice QCD

Target hadronic uncertainties are
 $\delta(a_\mu^{\text{HVP}}) \lesssim 0.2\%$ & $\delta(a_\mu^{\text{HLbL}}) \lesssim 10\%$

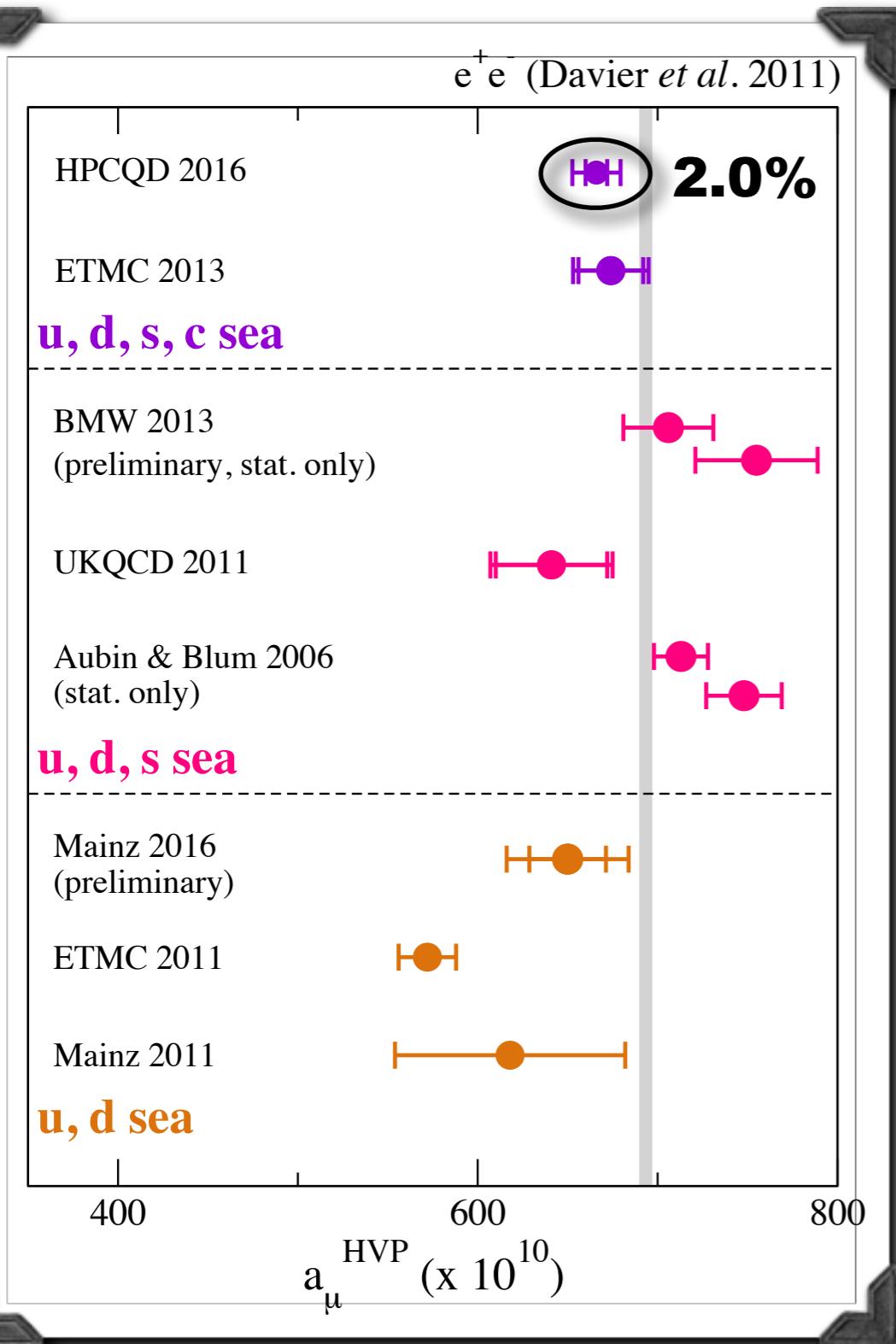
Lattice-QCD progress on a_μ^{HVP}



- ◆ Can calculate nonperturbative vacuum polarization function $\Pi(Q^2)$ directly in lattice QCD from simple 2-point correlation function of EM quark current [Blum, PRL 91 (2003) 052001]

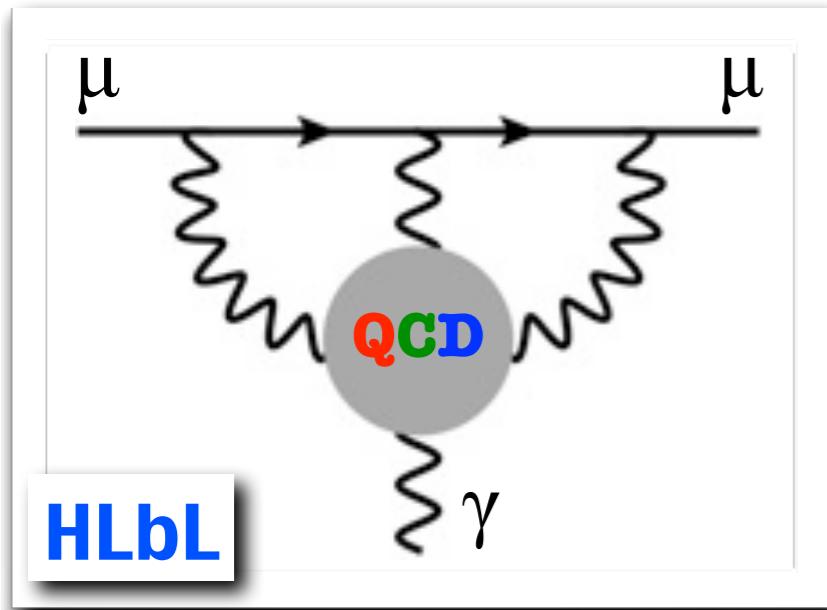
- ◆ Lowest-order contribution of $\mathcal{O}(\alpha_{\text{EM}})$
- ◆ Current determination of $\Pi(Q^2)$ and a_μ^{HVP} from $e^+e^- \rightarrow \text{hadrons}$ limited by experimental uncertainties

Lattice-QCD progress on a_μ^{HVP}



- ◆ Can calculate nonperturbative vacuum polarization function $\Pi(Q^2)$ directly in lattice QCD from simple 2-point correlation function of EM quark current [Blum, PRL 91 (2003) 052001]
- ◆ Several ongoing lattice efforts yielding new results since ICHEP 2014 including:
 - (1) First calculation of quark-disconnected contribution [RBC/UKQCD, PRL 116, 232002 (2016)]
 - (2) Second complete calculation of leading-order a_μ^{HVP} [HPQCD, arXiv:1601.03071]
 - ❖ First to reach precision needed to observe significant deviation from experiment
- ◆ ~1% total uncertainty by 2018 possible
- ◆ Sub-percent precision will require inclusion of isospin breaking & QED, and hence take longer

Lattice-QCD progress on a_μ^{HLbL}



- ◆ Lowest-order contribution of $O(\alpha_{\text{EM}}^2)$
- ◆ Current estimate from QCD models subjective and somewhat controversial
[Glasgow consensus, Prades, de Rafael, Vainshtein, 0901.0306]

- ◆ New method combines dynamical QCD gauge-field configurations with exact analytic formulae for photon propagators [Blum *et al.*, PRD93, 014503 (2016)]
 - ❖ Exploits stochastic methods for position-space sums to control computational cost
 - ❖ Obtain $\leq 10\%$ statistical errors at the physical pion mass in ballpark of Glasgow consensus value $a_\mu^{\text{HLbL,GC}} \times 10^{10} = 10.5(2.6)$

$$a_\mu^{\text{HLbL}} \times 10^{10} = \begin{cases} 11.60(0.96)_{\text{stat.}} & \text{connected} \\ -6.25(0.80)_{\text{stat.}} & \text{disconnected} \end{cases}$$

[L. Jin, Lattice 2016; preliminary update of Blum *et al.*]

- ◆ Full study of systematic errors including lattice-spacing and finite-volume effects still needed ... but *initial results encouraging!*

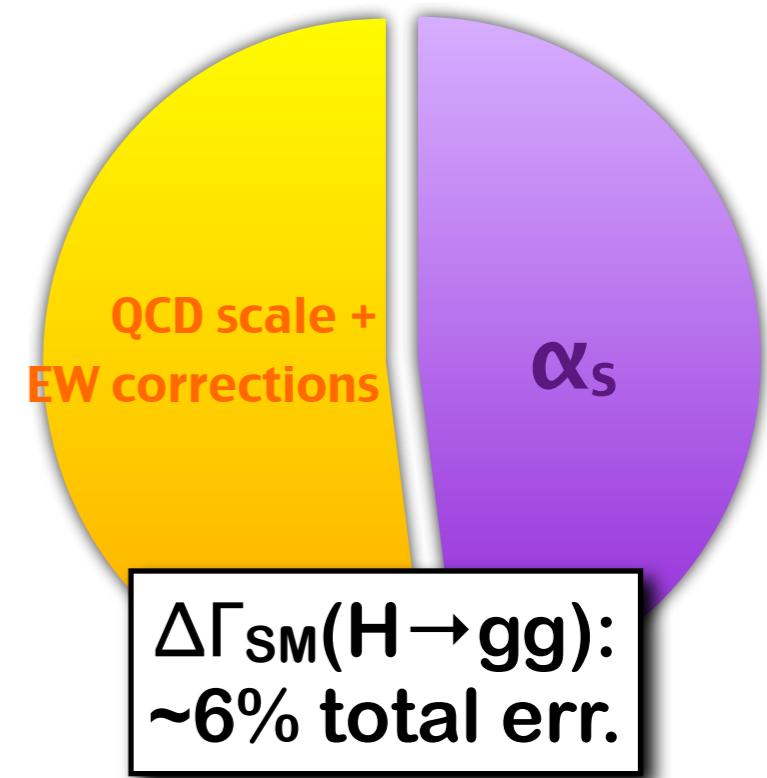
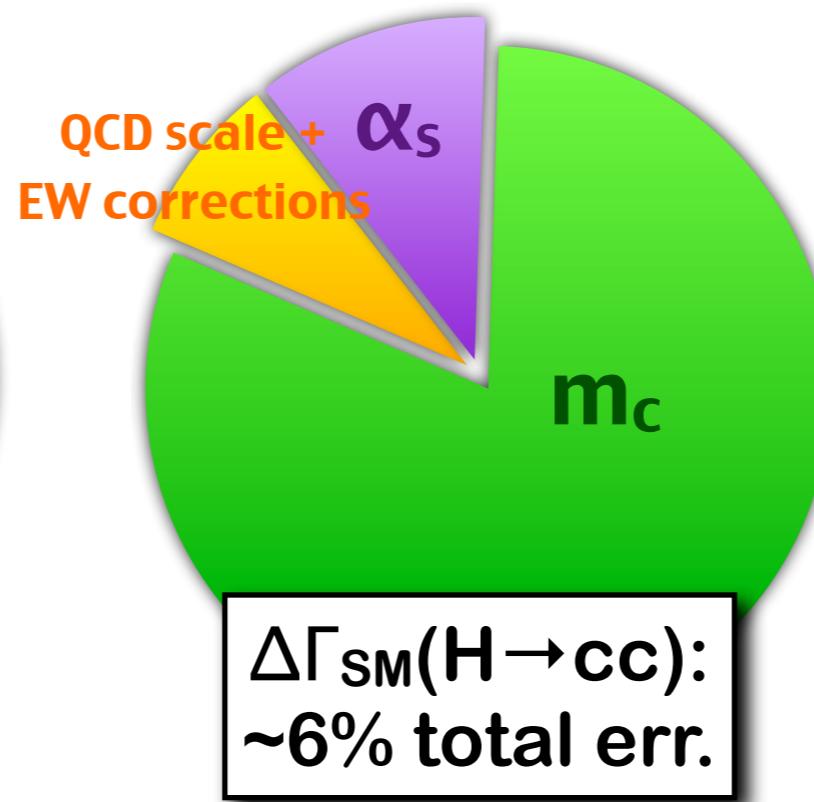
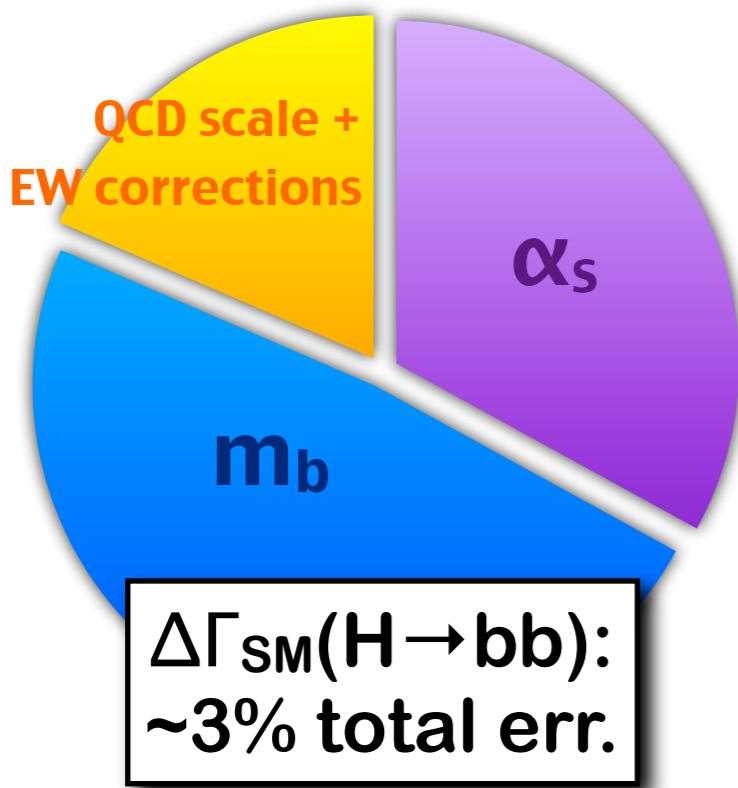
Lattice-QCD results & prospects for Higgs physics

Use the **Higgs boson** as a
new tool for discovery



Higgs physics

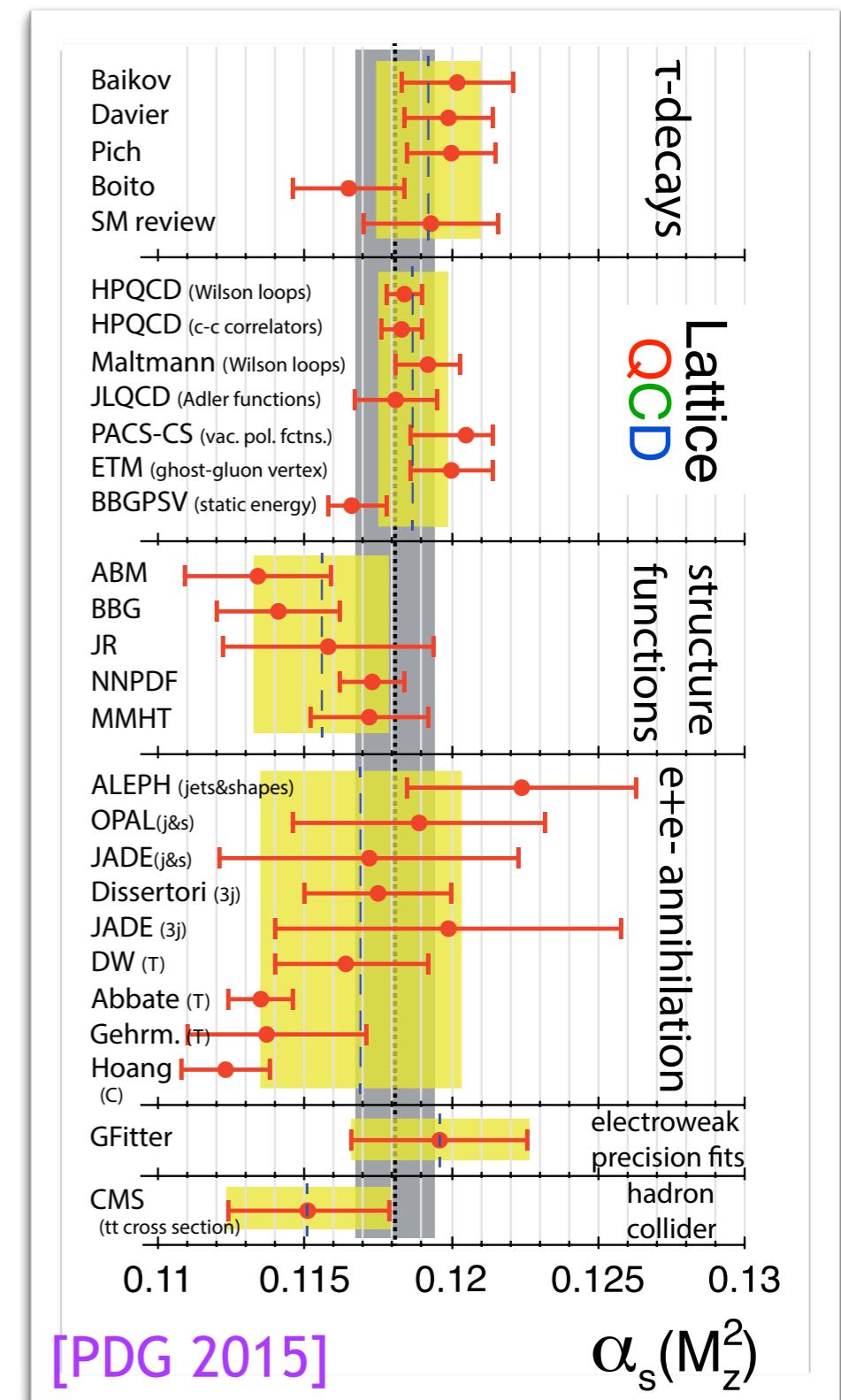
- ◆ Next-generation high-luminosity colliders will measure Higgs partial widths to sub-percent precision to look for deviations from Standard-Model expectations
 - ◆ Full exploitation of measurements needs theory predictions with same precision
- ◆ Parametric errors from m_c , m_b , & α_s are largest sources of uncertainty in SM Higgs partial widths for many decay modes [LHCHXSWG-DRAFT-INT-2016-008]
- ◆ b,c-quark masses & strong coupling can be calculated to needed precision with lattice QCD



Quark masses & strong coupling from LQCD

- ◆ b, c -quark masses from lattice QCD agree with non-lattice results with competitive errors
 - ❖ **m_b error dominated by discretization effects** → anticipate significant improvement with simulations using finer lattice spacings
- ◆ There are several independent lattice-QCD methods available to obtain α_s
 - ❖ Results consistent, and each is more precise than from non-lattice methods
- ◆ In next few years, increased corroboration from independent calculations will reduce α_s , m_b , & m_c uncertainties & make lattice-QCD determinations very robust

Obtaining m_c , m_b , & α_s to precisions needed by high-luminosity ILC **straightforward with existing lattice methods** + anticipated computing resources
 [see Lepage, Mackenzie, & Peskin, 1404.0319]



Lattice-QCD needs for neutrino physics

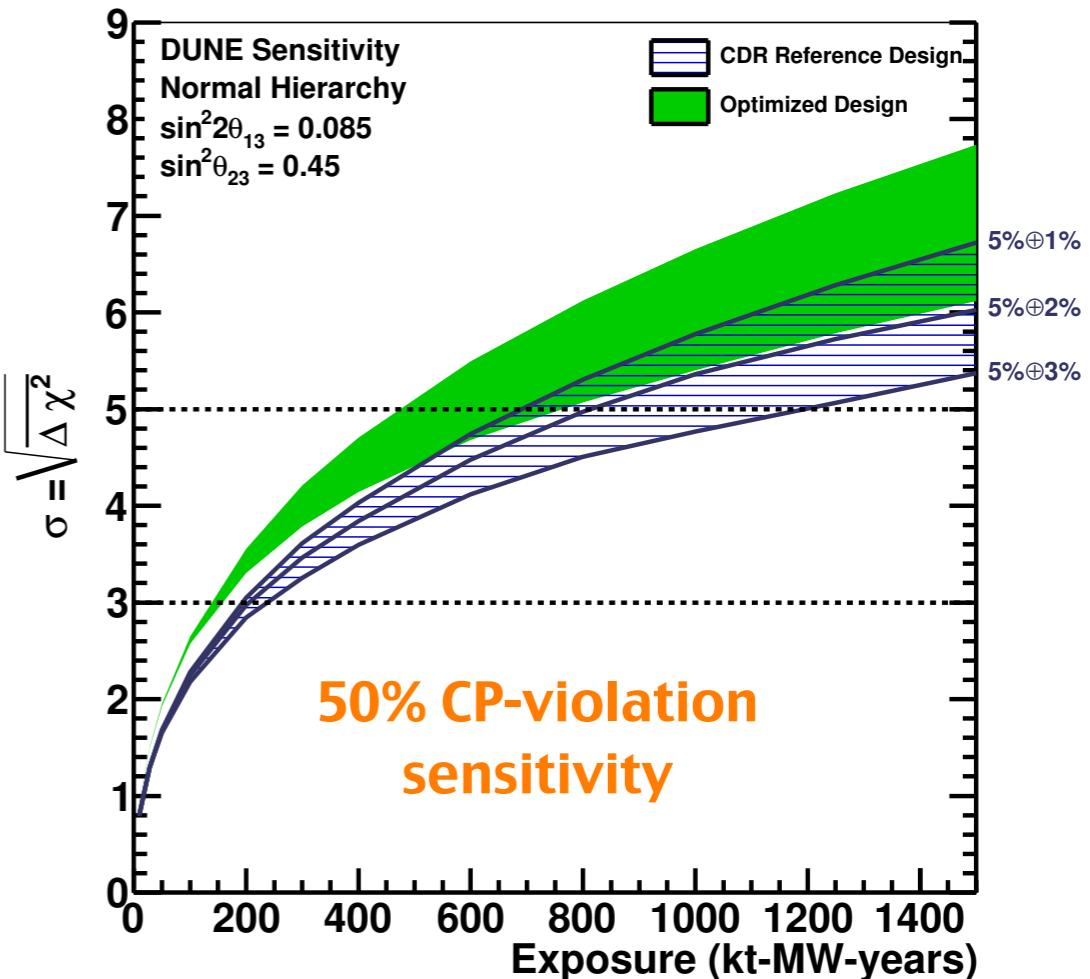
Pursue the physics associated
with **neutrino mass**



Long-baseline neutrino experiments

- ♦ Aim to improve precision on mass-squared splittings & mixing parameters, determine the mass hierarchy, and observe CP violation
- ♦ **Detect neutrinos via scattering off atomic nuclei** such as carbon (NOvA), oxygen (T2K), or argon (DUNE)
 - ➡ Interpretation of measurements requires knowledge of ν -nucleus interactions
- ♦ Underlying processes are ν -proton & ν -neutron scattering, as modified by the presence of nucleon inside atomic nucleus
 - ❖ Nucleon-level matrix elements can be calculated with controlled uncertainties in lattice **QCD**

LBNF/DUNE Conceptual Design Report
[arXiv:1512.06148]

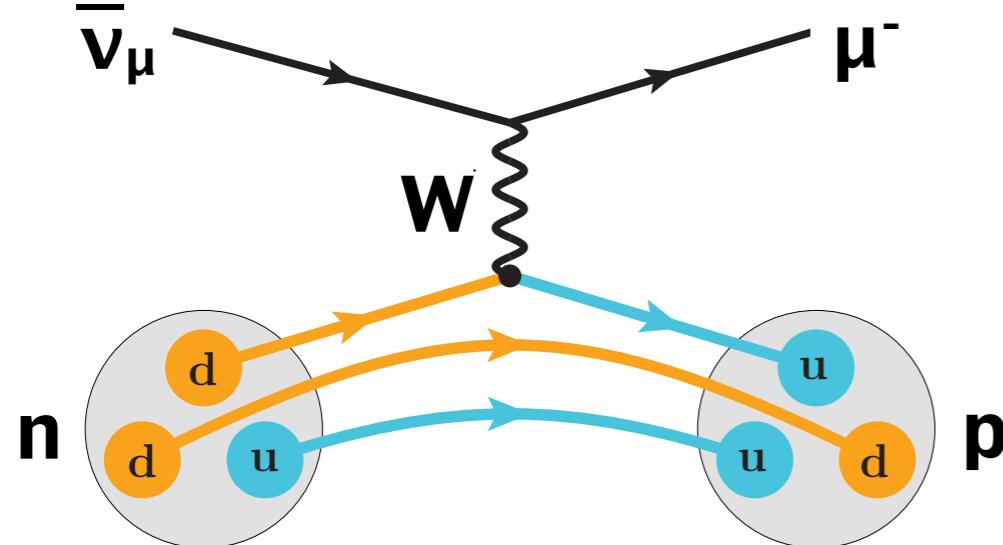


Reaching DUNE sensitivity goals for mass hierarchy & δ_{CP} requires reduced $\nu(\bar{\nu})$ -Ar X-section uncertainties

Useful lattice-QCD calculations for DUNE

Nucleon axial-vector form factor

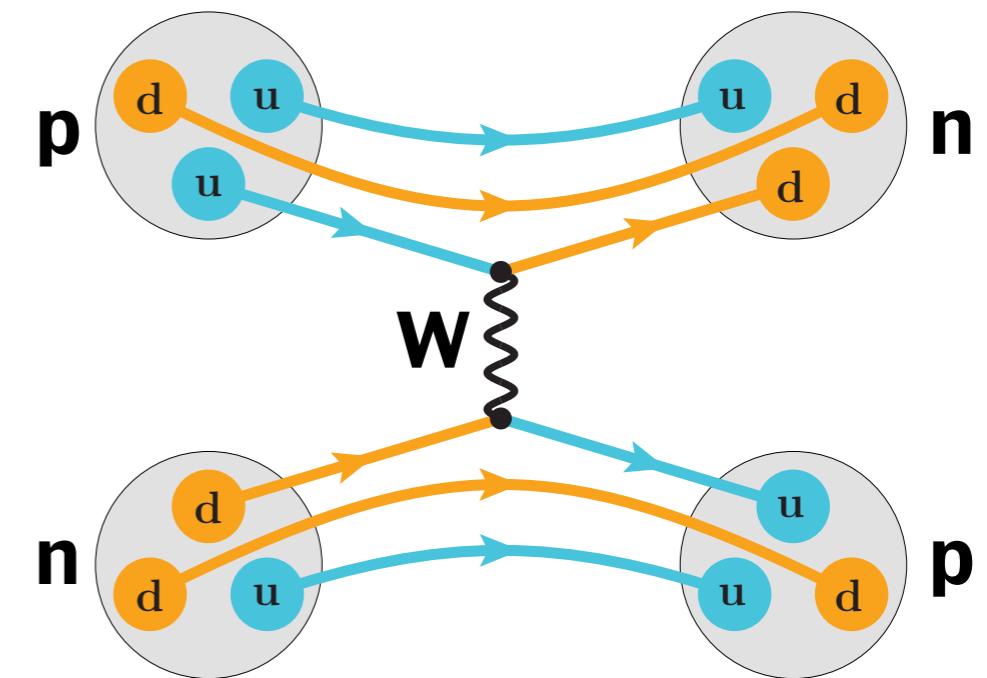
- ◆ Gives dominant contribution to charged-current quasielastic ν -nucleus scattering



- ◆ Obtain $F_A(q^2)$ in lattice QCD from matrix element $\langle p(p_f) | \bar{u} \gamma_\mu \gamma_5 d | n(p_i) \rangle$
- ◆ Nuclear model calculation still needed to relate nucleon form factor to ν -Ar cross-section

N-body nucleon currents

- ◆ Enter effective theories of ν -nucleus scattering



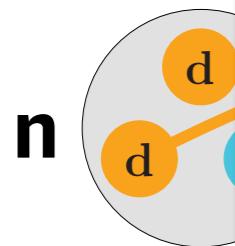
- ◆ $(N \rightarrow N)$ -nucleon scattering matrix elements from lattice QCD can constrain nuclear EFT parameters
- ◆ First LQCD calculations begun this year
[Savage @ ICHEP 2016]

Useful lattice-QCD calculations for DUNE

Nucleon axial-vector form factor

- ◆ Gives dominant contribution to charged-current quasielastic ν -nucleus scattering

$\bar{\nu}_\mu \rightarrow$



Lattice-QCD calculations of hadronic matrix elements will:

- Cleanly separate nucleon & nuclear effects and uncertainties
- Provide reliable input to nuclear-theory calculations and test nuclear models
- Improve precision & reliability of needed X-sections & matrix elements

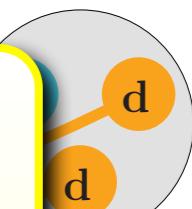
- ◆ Obtain F_A

element $\langle p(p_f) | \bar{u} \gamma_\mu \gamma_5 d | n(p_i) \rangle$

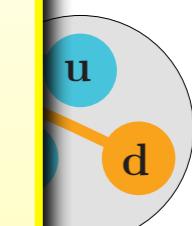
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N-body nucleon currents

- ◆ Enter effective theories of ν -nucleus scattering



n



p

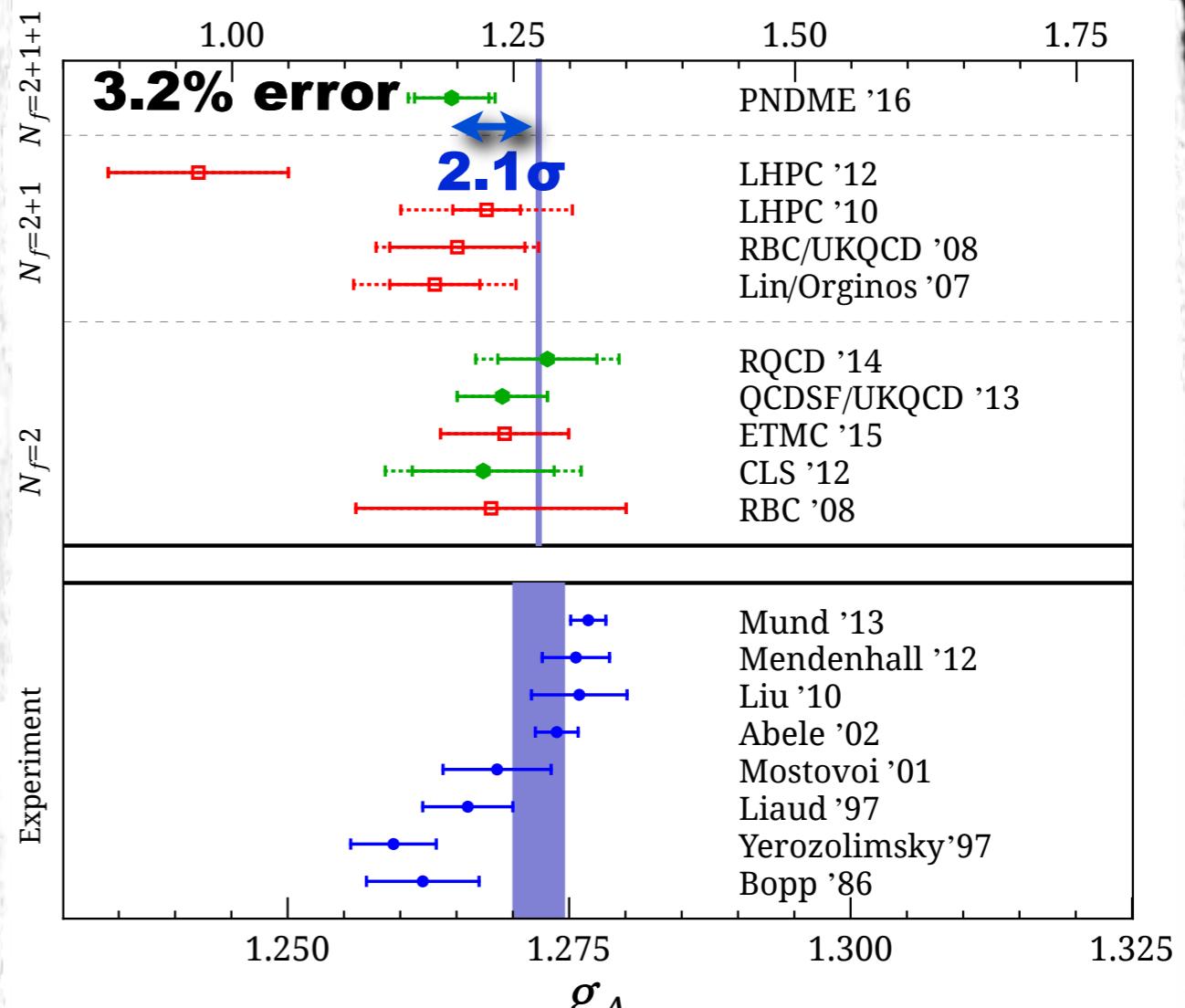
matrix elements from lattice QCD can constrain nuclear EFT parameters

- ◆ First LQCD calculations begun this year
[[Savage @ ICHEP 2016](#)]

Nucleon axial-vector coupling from lattice QCD

- ◆ Nucleon axial charge $g_A = -F_A(q^2=0)$ benchmark for accuracy of lattice-QCD nucleon matrix-element calculations and critical milestone on path to form factors over full kinematic range
- ◆ Present LQCD uncertainties over 10x larger than experimental errors
 - ❖ Central values mostly lie below experimental determinations
- ◆ In next few years, improved algorithms + simulations with larger lattice volumes, finer lattice spacings, and physical-mass pions will improve precision
- ◆ In ~5 years, expect percent-level g_A & few-percent vector and axial-vector form factors, with independent calculations providing checks

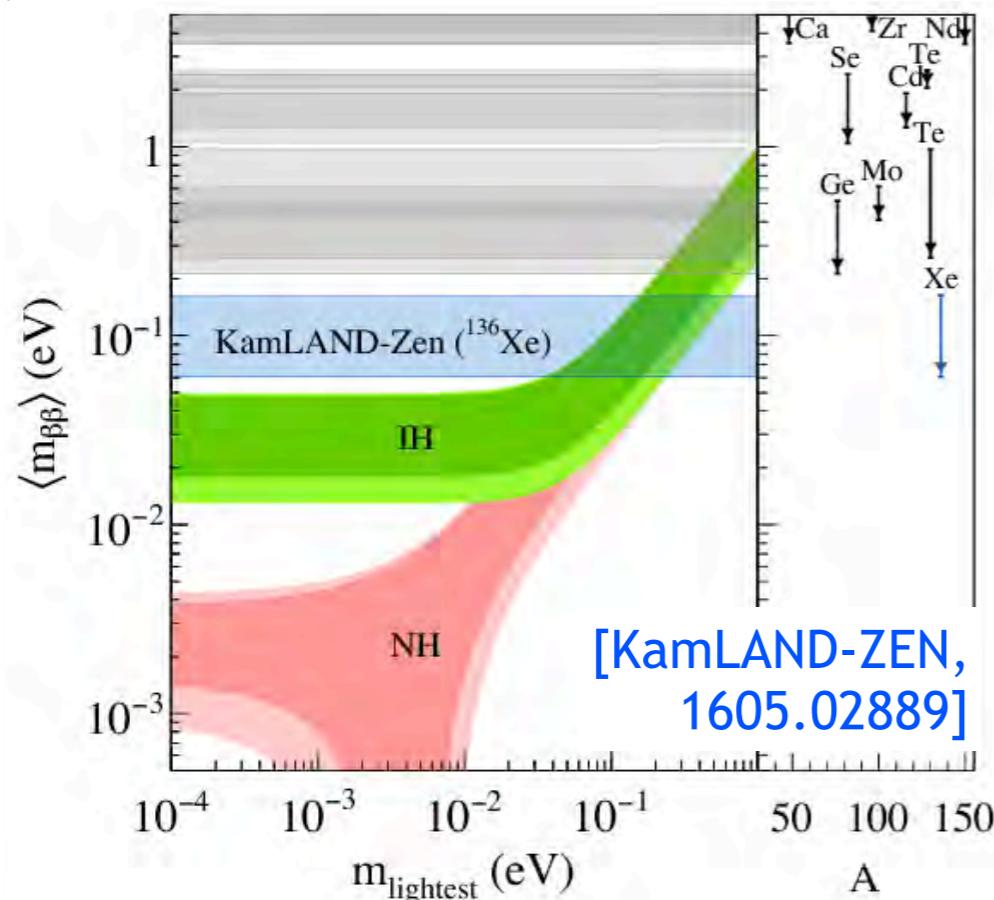
First g_A calculation with physical-mass pions & all systematics controlled this year



[PNDME, arXiv:1606.07049]

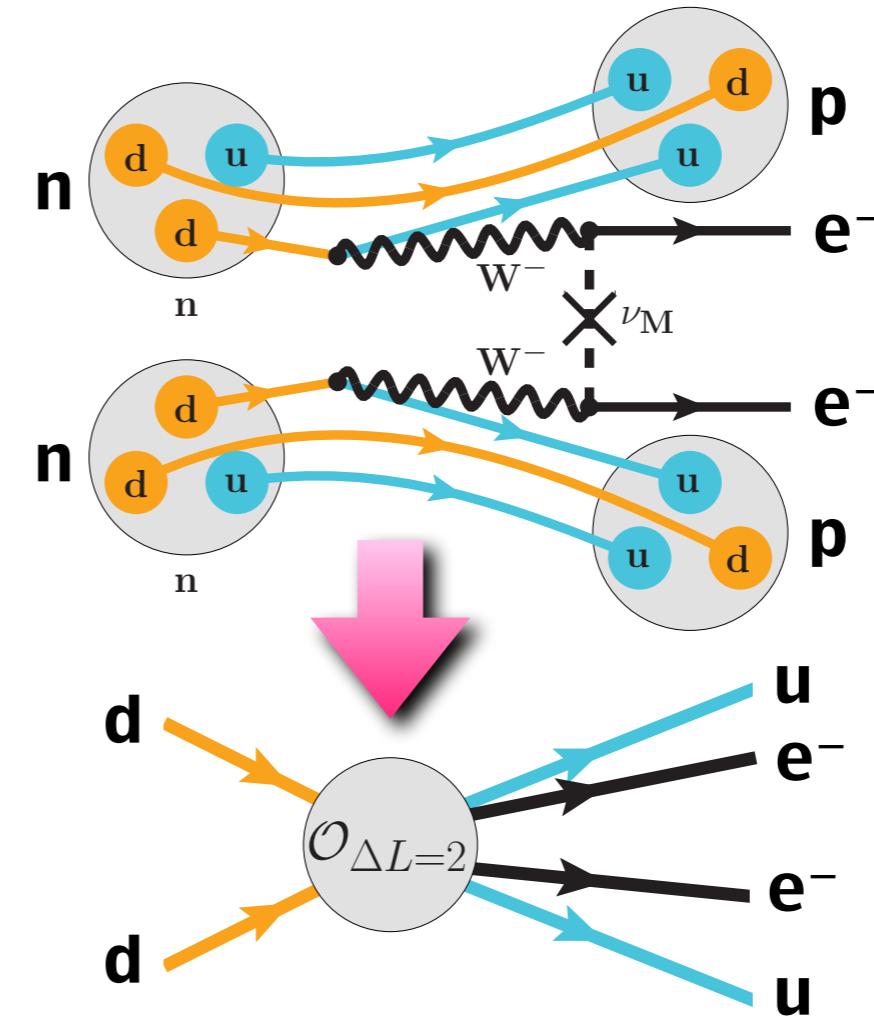
Neutrinoless double β -decay experiments

- ♦ Searching for Majorana neutrinos via **nuclear emissions of two electrons without neutrinos**



Bounds on Majorana mass limited by nuclear matrix-element uncertainties

- ♦ If Majorana ν 's heavy, $0\nu\beta\beta$ decay rate parameterized by matrix elements of six-fermion effective operators $O_{\Delta L=2}$



HIGH-SCALE THEORY

LOW-ENERGY THEORY

- ♦ First LQCD calculation of hadronic matrix elements begun this year [Nicholson @ Lattice 2016]



Conclusions

"Progress in science is based on the interplay between theory and experiment, between having an idea about nature and testing that idea in the laboratory. Neither can move forward without the other." – **Snowmass 2013 Executive Summary**

"Lattice QCD has [already] become an important tool in flavor physics. ...*The full exploitation of the experimental program requires continued support of theoretical developments.*" – **Snowmass 2013 Quark-flavor WG report**

Summary & outlook

- ◆ In the quark-flavor sector, lattice QCD already providing reliable computations of simplest hadronic matrix elements needed to extract CKM quark-mixing matrix elements and for many Standard-Model tests
 - ➡ Recent lattice-QCD results + experimental measurements imply several 2σ “tensions” with Standard-Model CKM framework
- ◆ Ambitious lattice-QCD program in place to meet theory needs of current & future worldwide high-energy-physics experimental program by
 - (1) Increasing precision of present calculations of simplest quark flavor-changing matrix elements, quark masses & strong coupling, scalar quark-content of nucleon, ...
 - (2) Providing first reliable QCD calculations of more challenging quantities such as quark-flavor-changing decays to resonances or multi-hadron final states, hadronic contributions to muon g-2, ν-nucleon scattering form factors & matrix elements, ...
- ◆ Future measurements combined with anticipated lattice-QCD improvements will sharpen precision tests of the Standard Model
 - ❖ If deviations from Standard-Model expectations are seen, correlations between measurements will provide information on new particle masses and couplings!

QCD is everywhere

- ♦ Hadronic physics is ubiquitous throughout high-energy physics experiments
- ♦ Reliable theoretical predictions are needed on same time scale as measurements with commensurate uncertainties

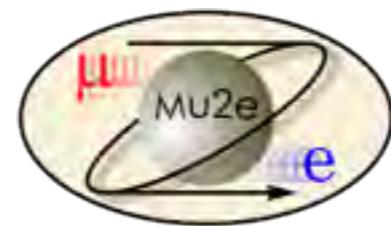
neutrino-nucleus
cross-sections



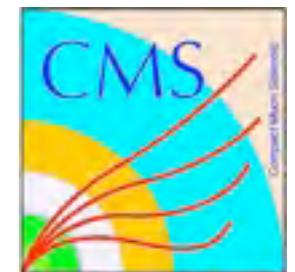
Decay constants, form factors,
& mixing parameters



muon-nucleus
cross sections



Parton distribution functions



dark-matter-nucleus
cross sections



hadronic vacuum
polarization &
light-by-light scattering



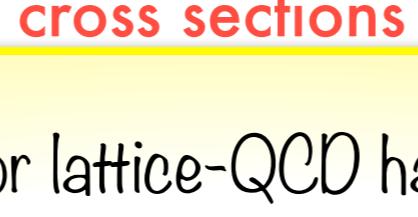
... lattice QCD is everywhere!

- ♦ Hadronic physics is ubiquitous throughout high-energy physics experiments
- ♦ Reliable theoretical predictions are needed on same time scale as measurements with commensurate uncertainties

neutrino-nucleus
cross-sections



muon-nucleus
cross sections



Parton distribution functions



Continued support for lattice-QCD hardware & software is essential to achieve scientific goals and fully capitalize on enormous investments in the HEP (and NP) experimental programs!

Decay constants
& mixing parameters



cross sections



polarization &
light-by-light scattering



See [Lattice 2016 website](#) for more hot-off-the-press results!



Further reading

Particle Data Group reviews:

- ♣ “[Lattice Quantum Chromodynamics](#)” (Laiho, Sharpe, Hashimoto)
- ♣ “[Leptonic Decays of Charged Pseudoscalar Mesons: 2015](#)” (Rosner, Stone, RV)

Flavor Lattice Averaging Group report:

- ♣ 2016 “[Review of lattice results concerning low-energy particle physics](#)”

Snowmass working-group reports:

- ♣ “[Charged Leptons](#)”
- ♣ “[Higgs Working Group Report ...](#)”
- ♣ “[Lattice field theory ... Scientific goals and computing needs](#)”
- ♣ “[Report of ... QCD Working Group](#)”
- ♣ “[Report of ... Quark Flavor Physics Working Group](#)”



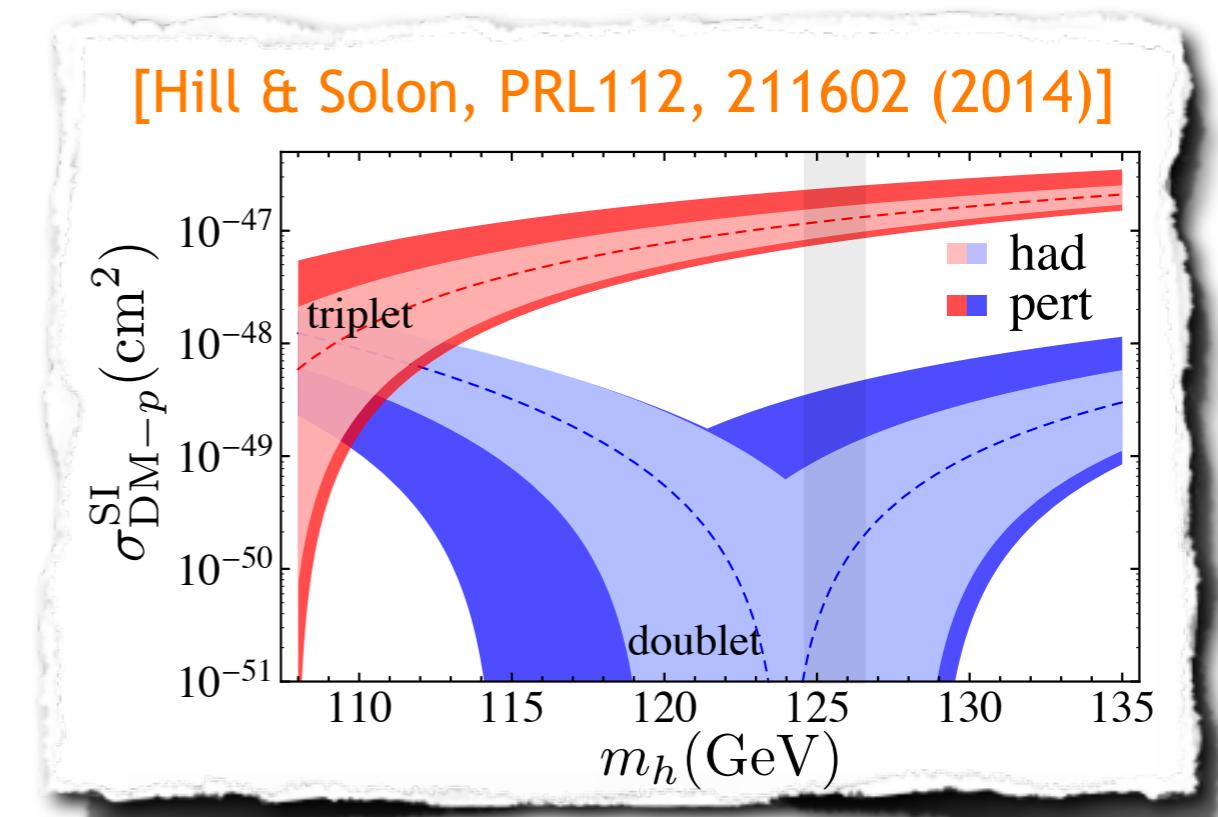
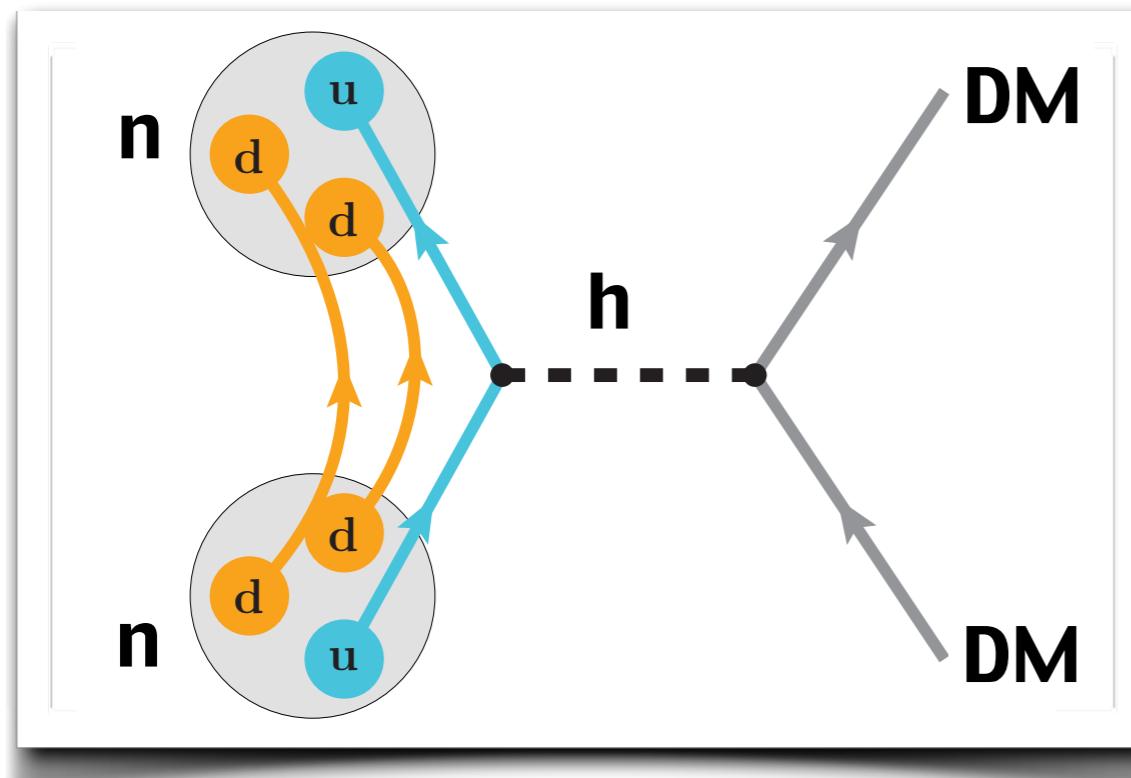
Lattice **QCD** for the cosmic frontier

- Identify the new physics of **dark matter**
- Understand **cosmic acceleration**:
dark energy and inflation



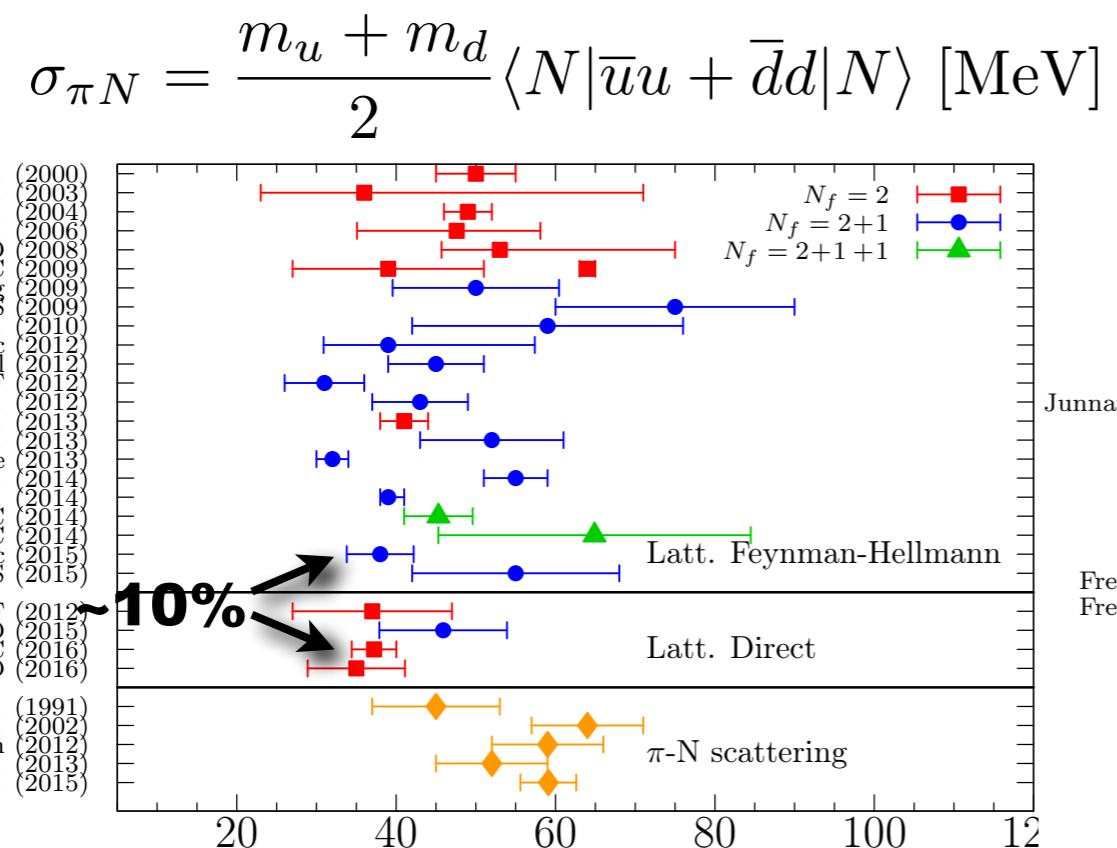
Direct dark-matter detection

- ◆ Experiments searching for dark-matter particles scattering off detector nuclei via exchange of a new or Standard-Model particle
- ◆ Next-generation experiments will improve X-section limits & hope to make measurements!
- ◆ Errors on DM X-section predictions limited by QCD uncertainties → must reduce theory errors to fully exploit measurements
- ◆ If mediator is scalar particle, cross-sections depend upon scalar matrix elements of quark contents of proton & neutron: $\langle N | \bar{q}q | N \rangle$ with q=u, d, s, (& c)
- ❖ Hadronic matrix elements can be calculated with controlled uncertainties in lattice QCD

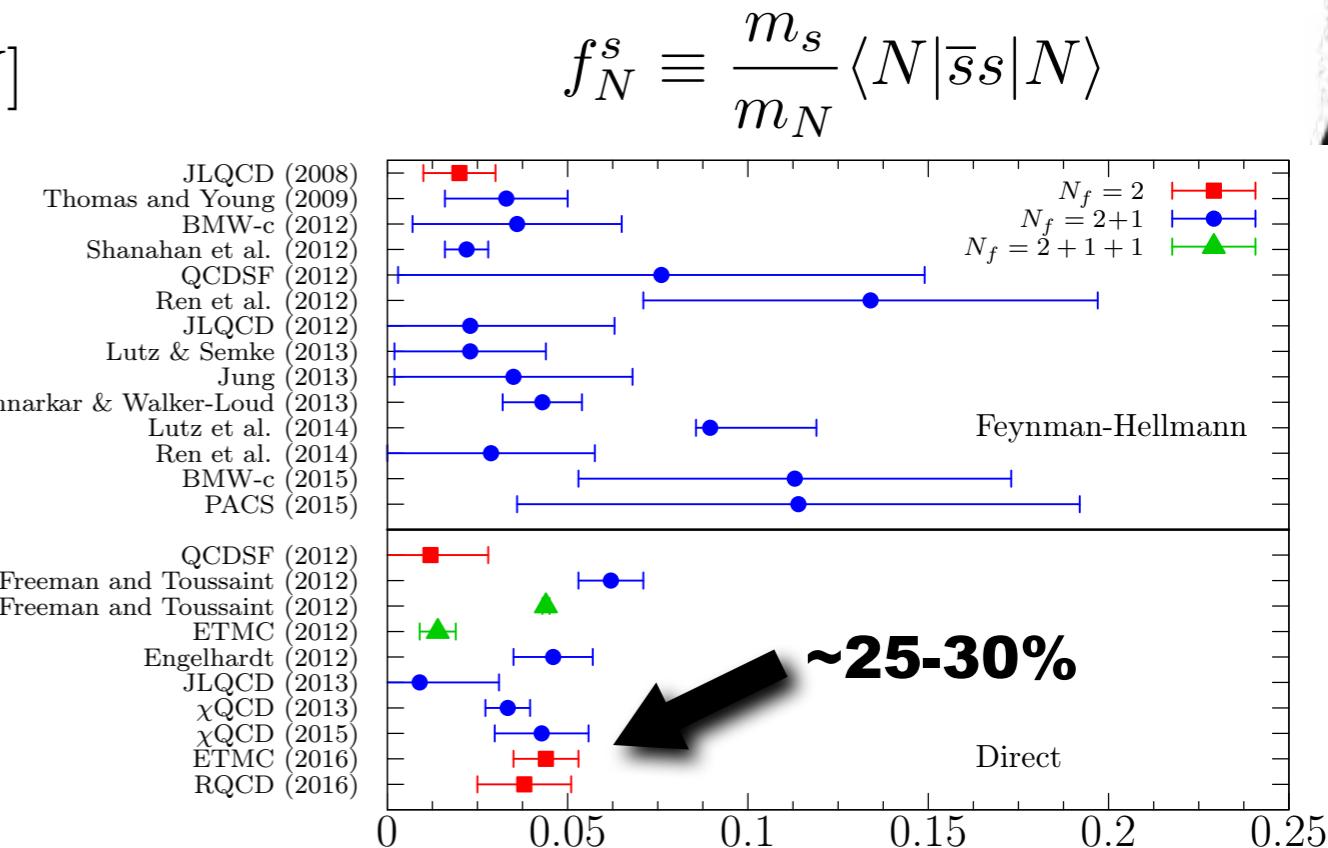


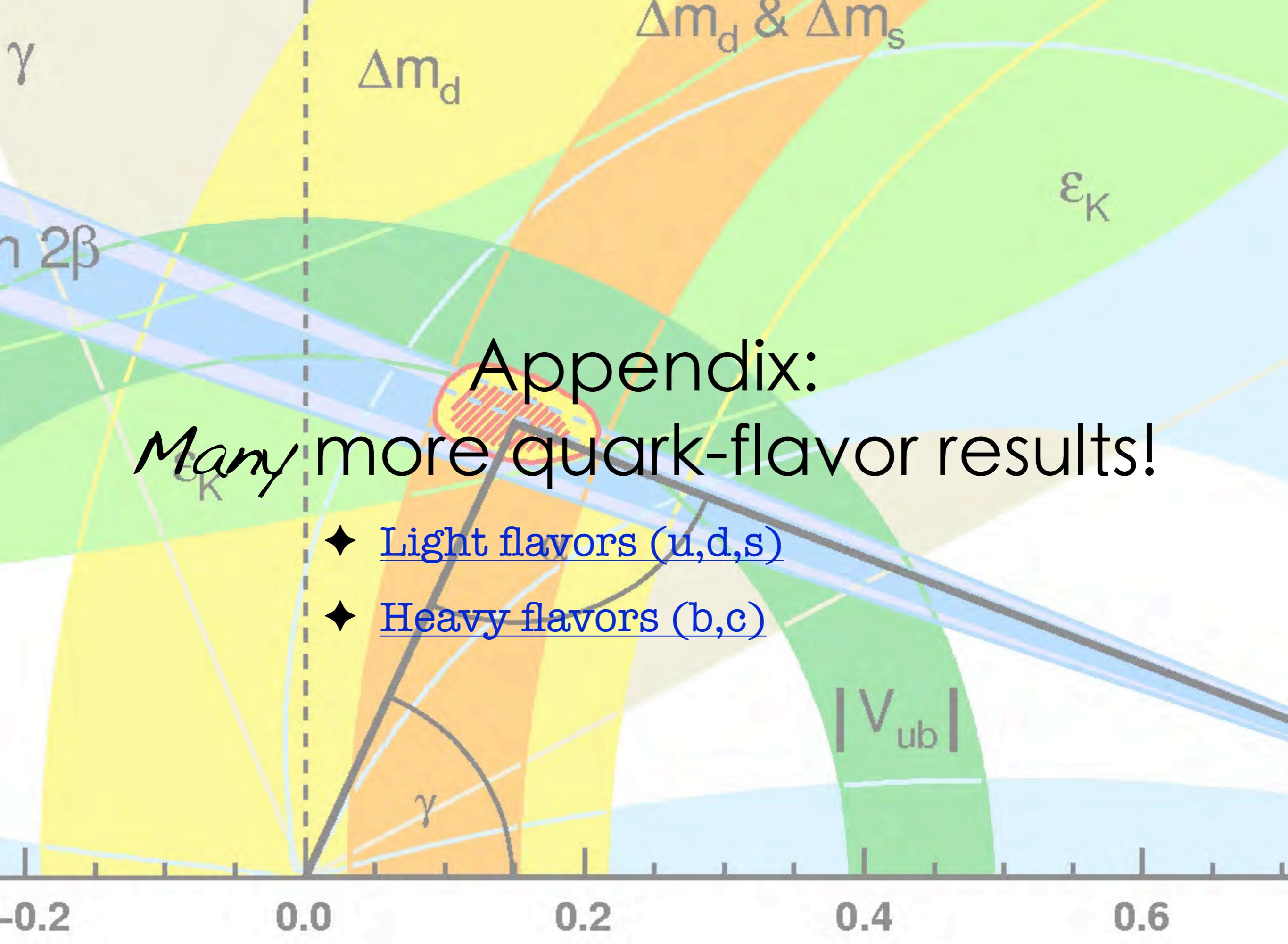
Quark content of nucleon from lattice QCD

- ◆ Several new results since ICHEP 2016 including first calculations at the physical pion mass [BMW, PRD116, 172001 (2016); χ QCD, 1511.09089]
- ◆ In next few years, increased computing resources + improved algorithms will continue to improve precision
- ◆ Increased corroboration from independent calculations will make determinations robust



[S. Collins @ Lattice 2016]





Appendix:

Many more quark-flavor results!

- ◆ Light flavors (u,d,s)
- ◆ Heavy flavors (b,c)

Lattice-QCD flavor-physics references

General reviews:

- ♣ “[Lattice Quantum Chromodynamics](#)” (Laiho, Sharpe, Hashimoto, PDG 2015)
- ♣ [2016 “Review of lattice results concerning low-energy particle physics”](#)
(Flavor Lattice Averaging Group)
- ♣ [“Leptonic Decays of Charged Pseudoscalar Mesons: 2015”](#)
(Rosner, Stone, RV, PDG 2015)

Pion & kaon physics:

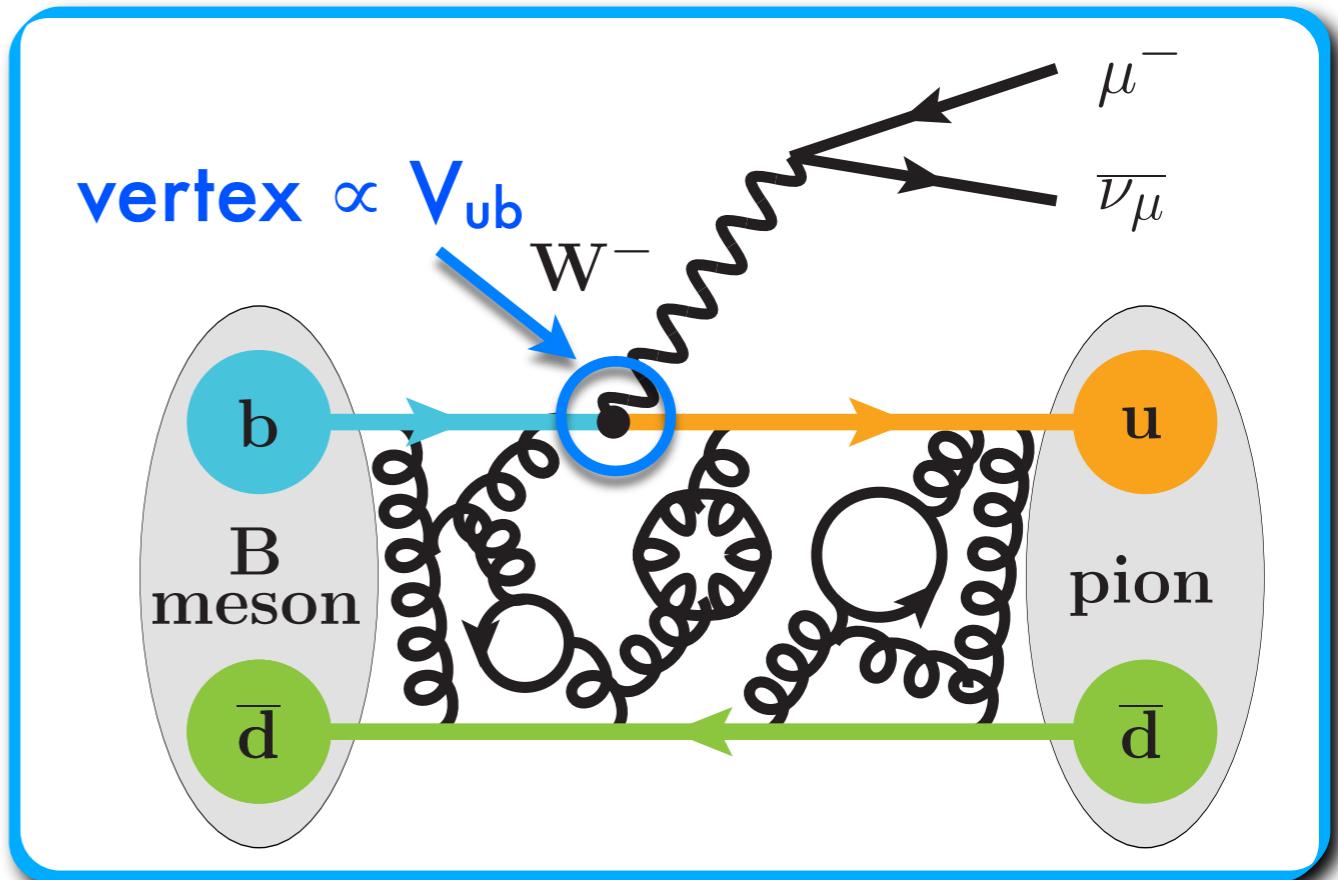
- ♣ [“Light flavour physics”](#) (Jüttner, Lattice 2015)
- ♣ [“CP violation & Kaon weak matrix elements from LQCD”](#)
(Garron, Chiral Dyn. 2015)

Heavy flavor physics:

- ♣ [“Progress and prospects for heavy flavour physics on the lattice”](#)
(Pena, Lat. 2015)
- ♣ [“Testing the Standard Model under the weight of heavy flavors”](#)
(Bouchard, Lat. 2015)

Determination of CKM elements

- ◆ Measure flavor-changing processes involving hadrons
- ◆ Infer CKM elements within Standard Model by comparing experimental measurements of flavor-changing interactions with theory predictions
 - ❖ Absorb nonperturbative QCD dynamics into hadronic parameters



$$\Delta m_{(d,s)}, \frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2}, \frac{d\Gamma(B \rightarrow D^{(*)} \ell \nu)}{dw}, \dots$$



(Experiment) = (known) \times (CKM factors) \times (Hadronic Matrix Element)

Compute nonperturbative QCD parameters
(decay constants, form factors, B-parameters,...)
numerically with LATTICE QCD



★ New since
ICHEP 2014!

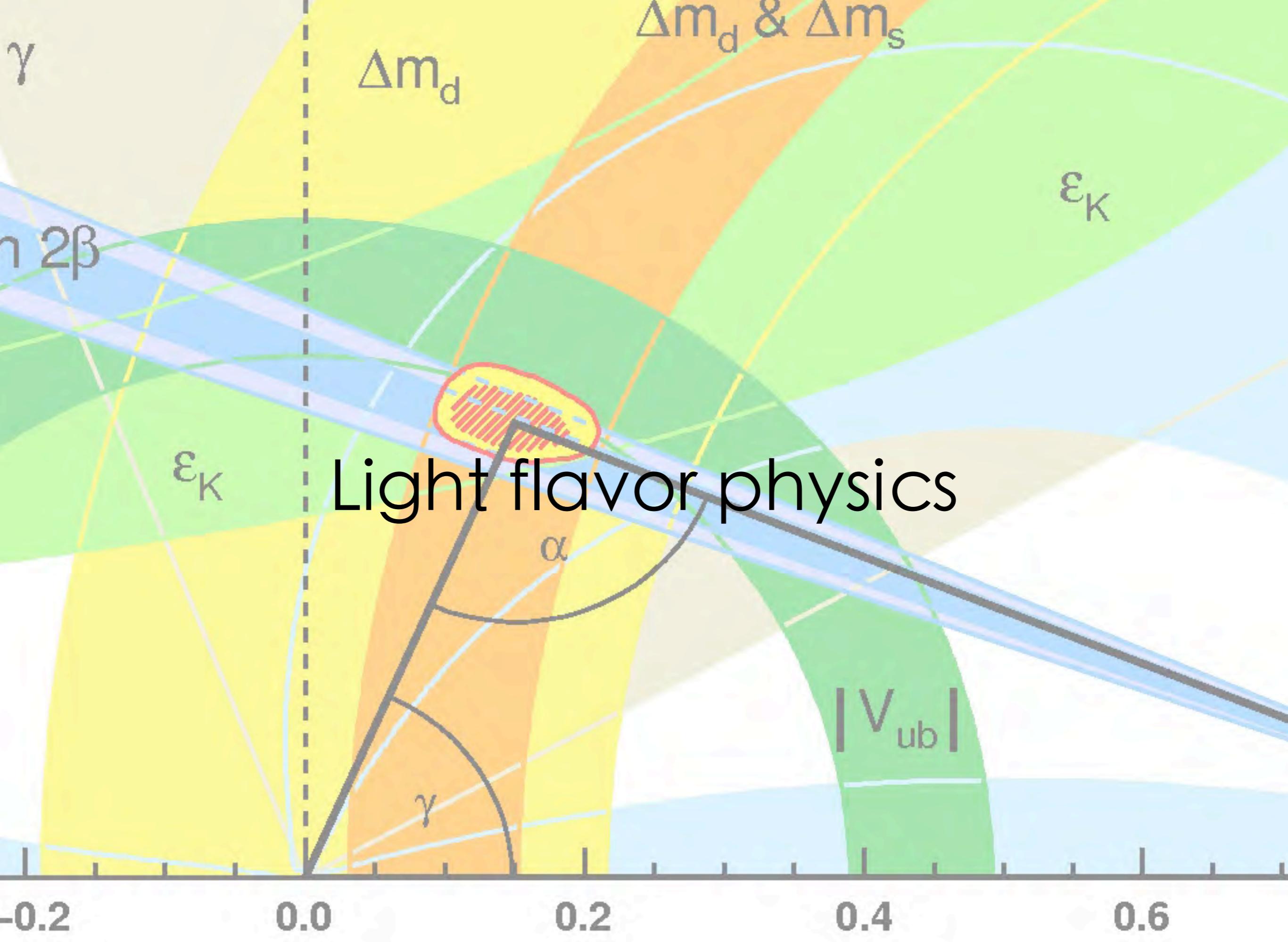
CKM summary

CKM element	process	LQCD input	$ V_{q_1 q_2} $	% error
$ V_{ud} $	$\pi \rightarrow \ell\nu$	f_π	0.9764	(1.3) _{LQCD} (0.0) _{exp} (0.1) _{EM}
$ V_{us} $	$K \rightarrow \ell\nu$	f_K	0.2255	(0.3) _{LQCD} (0.1) _{exp} (0.1) _{EM}
	$K \rightarrow \pi\ell\nu$	$f_+^{K\pi}(0)$	0.22310	(0.3) _{LQCD} (0.2) _{exp}
$ V_{us} / V_{ud} $	$K \rightarrow \ell\nu/\pi \rightarrow \ell\nu$	f_K/f_π	0.2314	(0.2) _{LQCD} (0.1) _{exp} (0.1) _{EM}
$ \mathbf{V}_{ub} $	$\mathbf{B} \rightarrow \pi\ell\nu$	$\mathbf{f}_+^{\mathbf{B}\pi}(\mathbf{q}^2)^\star$	3.72×10^{-3}	(~ 3.4) _{LQCD} (~ 2.8) _{exp}
	$B \rightarrow \ell\nu$	f_B	4.12×10^{-3}	(2.2) _{LQCD} (9.0) _{exp}
$ V_{cd} $	$D \rightarrow \ell\nu$	f_D	0.217	(0.5) _{LQCD} (2.3) _{exp}
	$D \rightarrow \pi\ell\nu$	$f_+^{D\pi}(0)$	0.2140	(4.4) _{LQCD} (1.3) _{exp}
$ V_{cs} $	$D_s \rightarrow \ell\nu$	f_{D_s}	1.007	(0.5) _{LQCD} (1.6) _{exp}
	$D_s \rightarrow K\ell\nu$	$f_+^{DK}(0)$	0.9746	(2.5) _{LQCD} (0.7) _{exp}
$ \mathbf{V}_{cb} $	$B \rightarrow D^*\ell\nu$	$F(1)$	38.9×10^{-3}	(1.3) _{LQCD} (1.3) _{exp} (0.5) _{QED}
	$\mathbf{B} \rightarrow \mathbf{D}\ell\nu$	$\mathbf{f}_+^{\mathbf{BD}}(\mathbf{w})^\star$	40.7×10^{-3}	(~ 1.2) _{LQCD} (~ 2.1) _{exp} (0.5) _{QED}
$ \mathbf{V}_{td} $	ΔM_d	$\mathbf{f}_{B_d}(\hat{\mathbf{B}}_{B_d})^{-1/2}^\star$	8.00×10^{-3}	(4.1) _{LQCD} (0.3) _{exp} (0.4) _{other}
$ \mathbf{V}_{ts} $	ΔM_s	$\mathbf{f}_{B_s}(\hat{\mathbf{B}}_{B_s})^{-1/2}^\star$	39.0×10^{-3}	(3.1) _{LQCD} (0.0) _{exp} (0.5) _{other}
$ \mathbf{V}_{td} / \mathbf{V}_{ts} $	$\Delta M_d/\Delta M_s$	ξ^\star	0.2052	(1.5) _{LQCD} (0.2) _{exp} (0.0) _{other}
$(\bar{\rho}, \bar{\eta})$	ϵ	\hat{B}_K	—	(1.3) _{B_K} [cf. (0.7) _{ϵ, exp}]

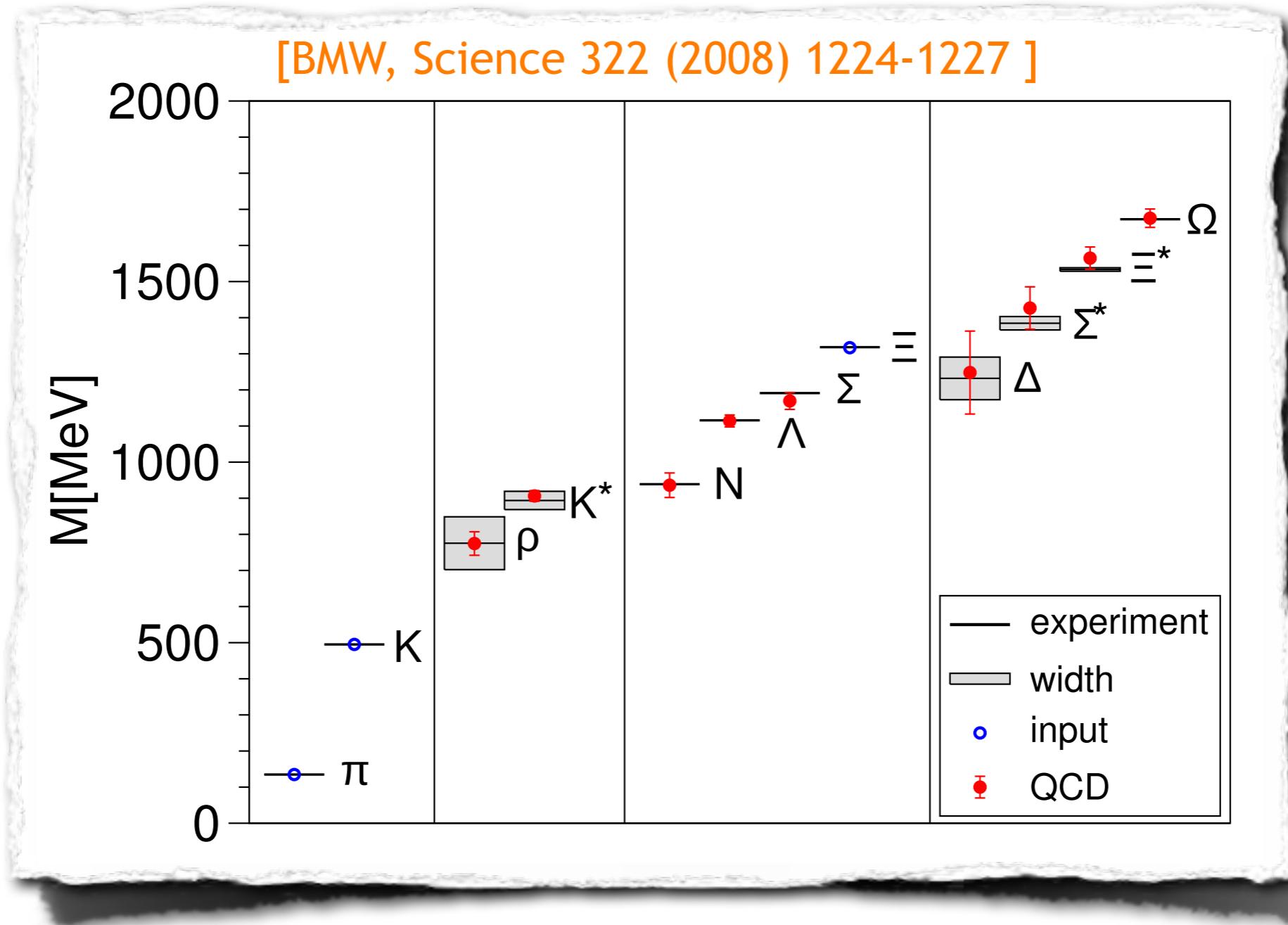
★ New since
ICHEP 2014!

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$ \mathbf{V}_{td} $	ΔM_d	$f_{B_d}(\hat{B}_{B_d})^{-1/2}^\star$	8.00×10^{-3}	$(4.1)_{LQCD}(0.3)_{exp}(0.4)_{other}$
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$ \mathbf{V}_{td} / \mathbf{V}_{ts} $	$\Delta M_d/\Delta M_s$	ξ^\star	0.2052	(1.5) _{LQCD} (0.2) _{exp} (0.0) _{other}
$(\bar{\rho}, \bar{\eta})$	ϵ	\hat{B}_K	—	$(1.3)_{B_K} [cf. (0.7)_{\epsilon, exp}]$



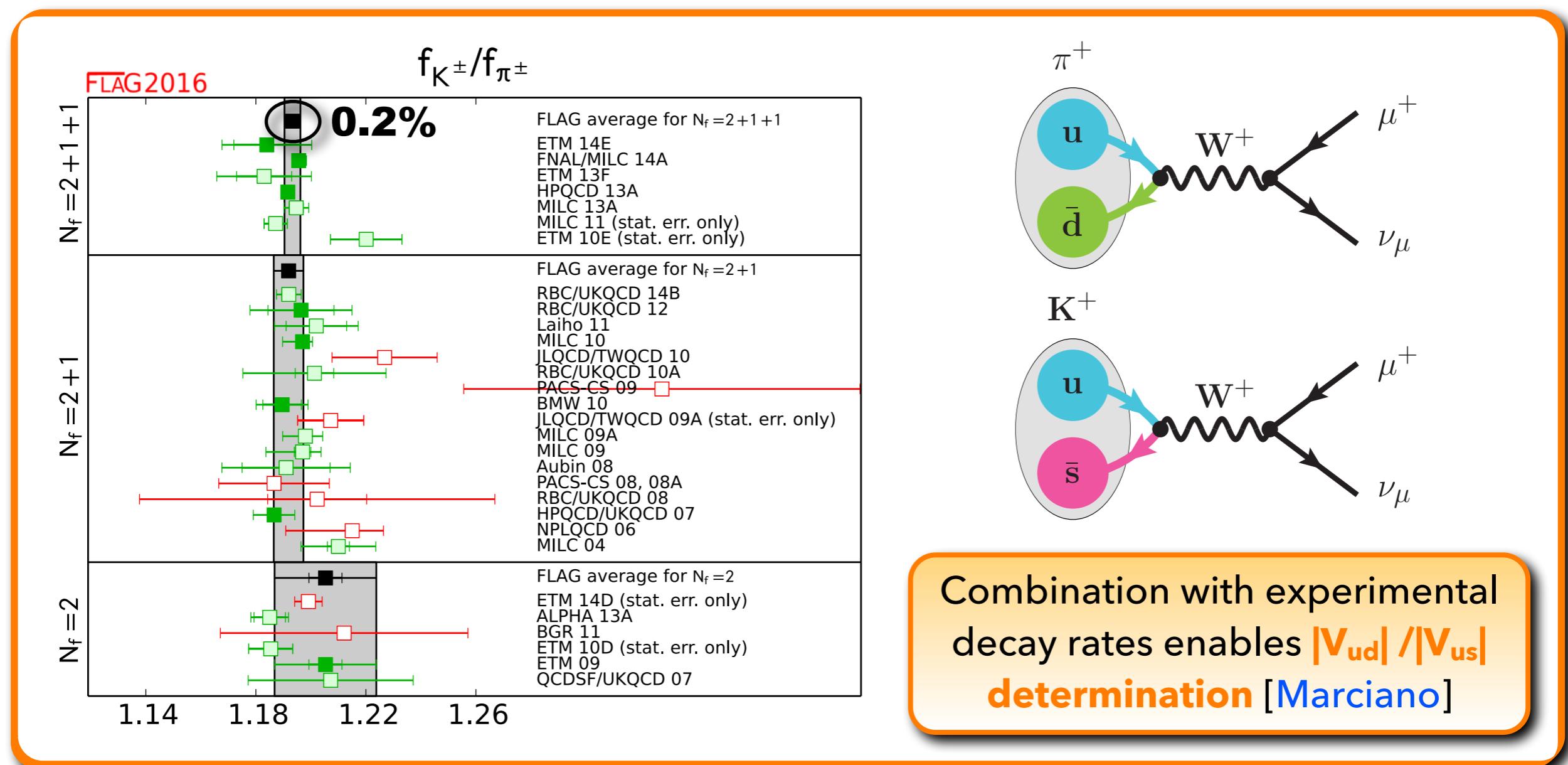
Light hadron spectrum



- ◆ Light hadron masses primarily due to energy stored in gluon field and to quarks' kinetic energy → **tests nonperturbative QCD dynamics**
- ◆ Also calculation of neutron-proton mass difference [BMW, Science 347, 1452, 2015])

Pion & kaon decay constants

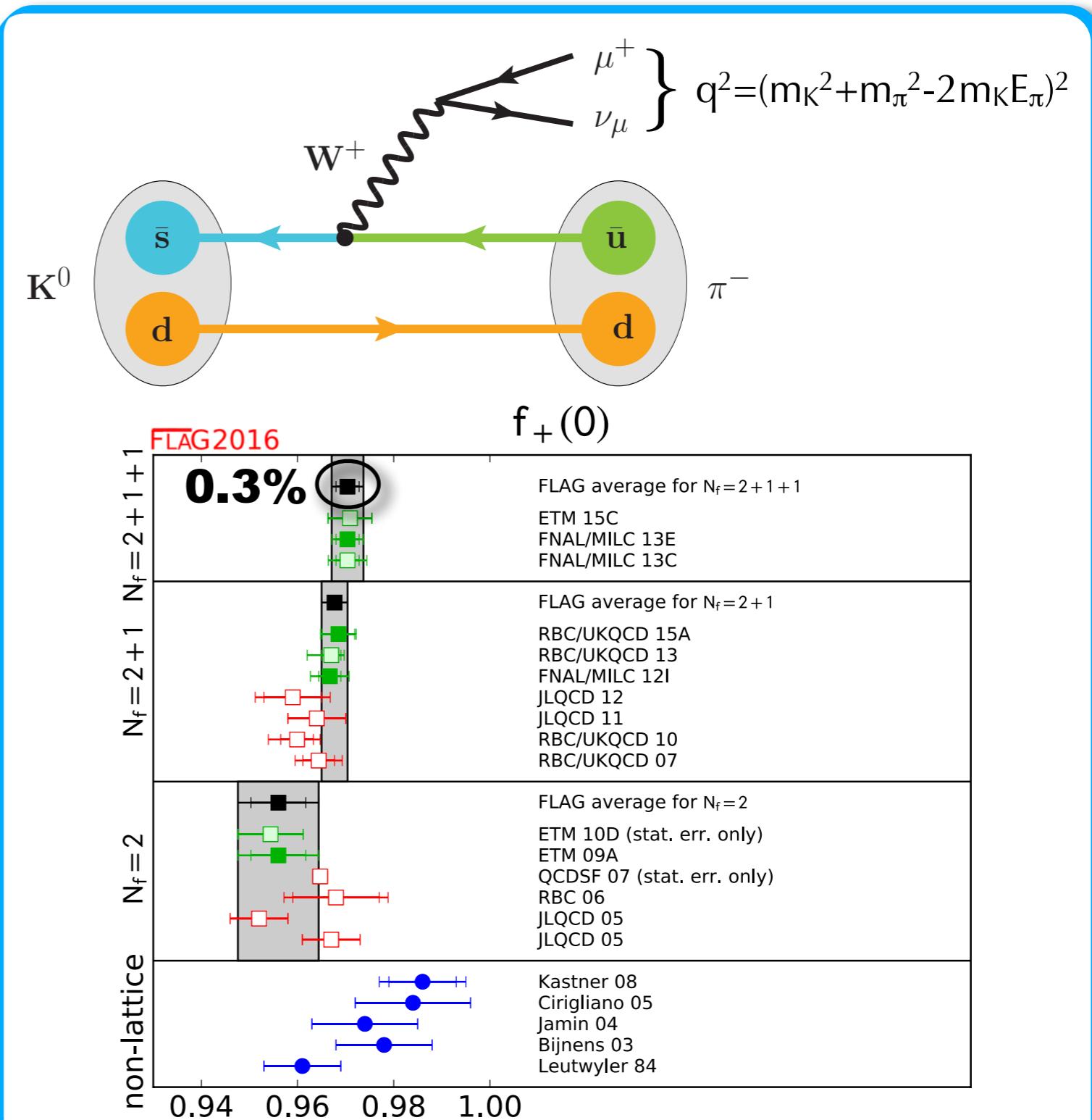
- ♦ Decay-constant ratio f_K/f_π can be computed to sub-% precision with lattice QCD:
 - ❖ Statistical fluctuations correlated between numerator & denominator
 - ❖ $f_K=f_\pi$ in SU(3) limit $m_s=m_{ud}$, so some systematics suppressed by $(m_s-m_{ud})/\Lambda_{\text{QCD}}$



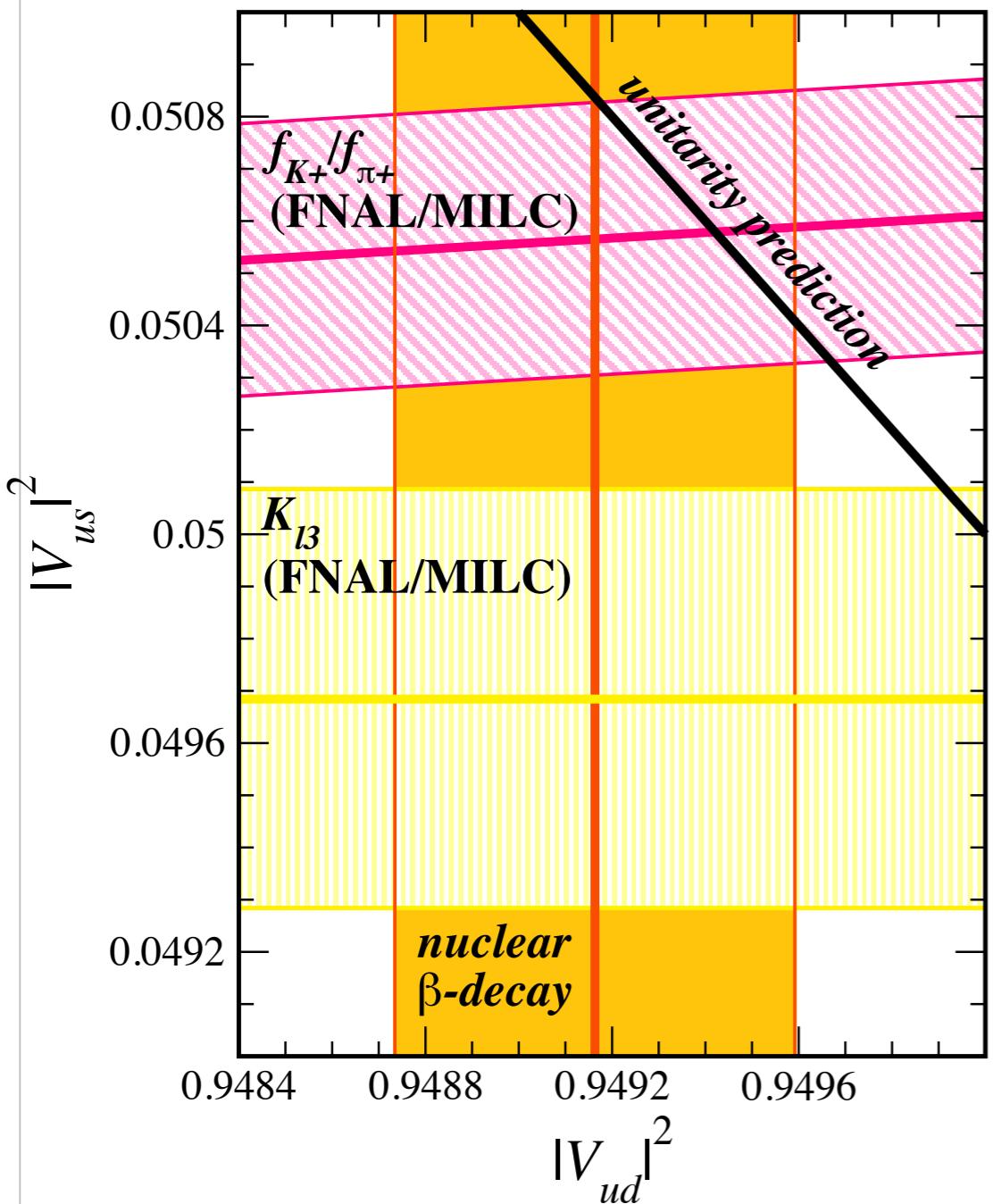
$K \rightarrow \pi \ell \bar{\nu}$ semileptonic form factor

- ♦ **Zero-recoil form factor $f_+(q^2=0)$ highly constrained by SU(3)_f and chiral symmetries:**
 - ❖ $f_+(0) = 1$ in SU(3) limit $m_s=m_{ud}$
 - ❖ Leading-order correction to unity is known function of $\{m_\pi, m_K, f_\pi\}$ [Leutwyler & Roos]
 - ❖ $f_2 = -0.023$ numerically small because second-order in $(m_K^2 - m_\pi^2)$ [Ademollo-Gatto]
- ♦ **Lattice-QCD calculation does not require renormalization**

Combination with experimental rate enables $|V_{us}|$ determination



Implications for 1st-row CKM unitarity



- ◆ ~2 σ tension between leptonic π & K decays, semileptonic K decays, β -decay, and CKM unitarity [see e.g. Rosner, Stone, & RV (PDG), 1509.02220]
- ◆ Unitarity test using $|V_{us}|$ from K_{l3} decay [FLAG 2015 prel. + Moulson, 1411.5252] and $|V_{ud}|$ from superallowed β -decay [Hardy & Towner, 1411.5987] limited by error on: $|V_{ud}|^2$:

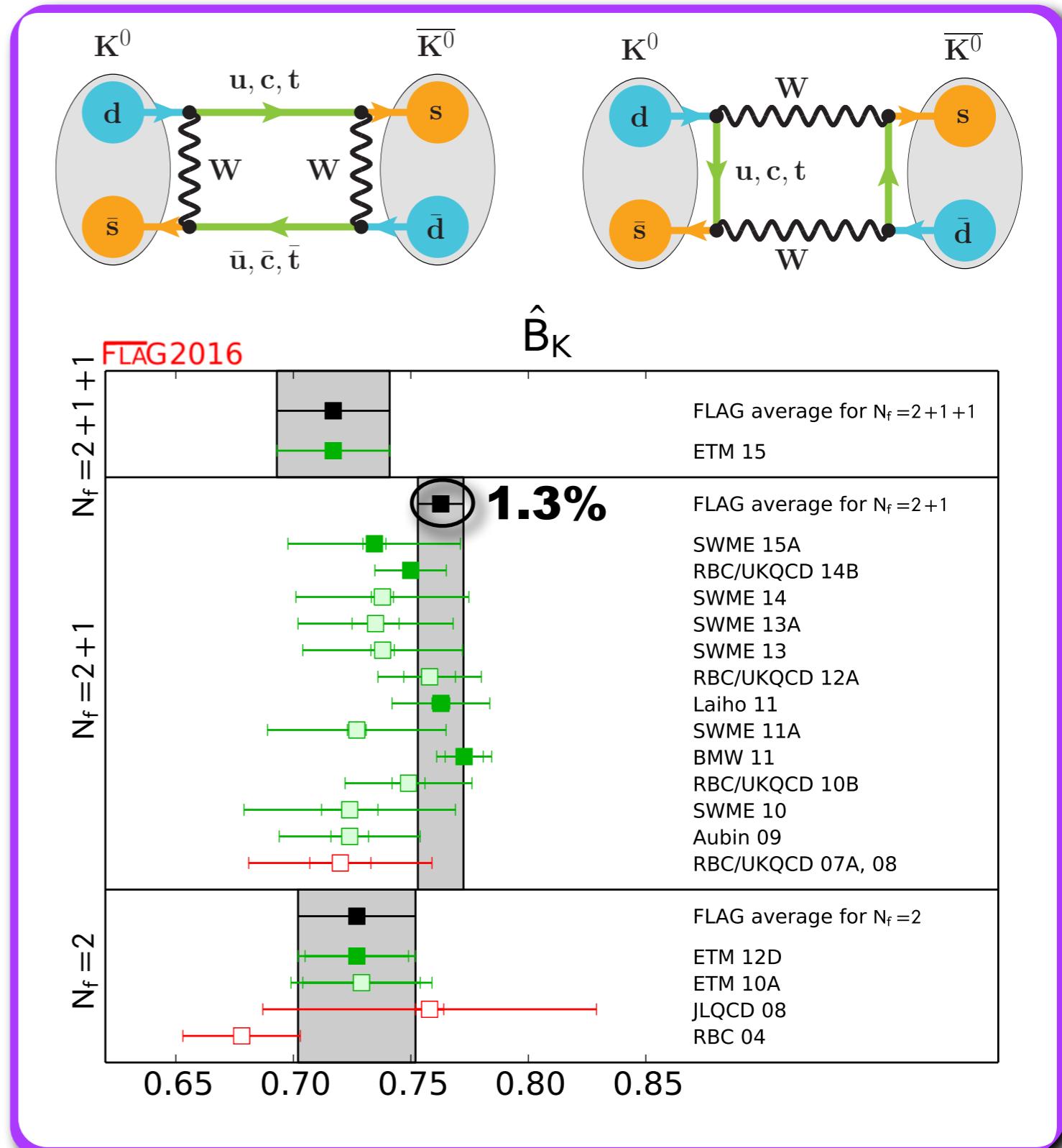
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = \\ -0.0021(29)|V_{us}(41)|V_{ud}$$

→ can $|V_{ud}|$ from β -decay be improved?

Neutral kaon mixing parameter

- ♦ Percent-level lattice-QCD calculation of B_K enabled by:
 - ❖ Chiral fermion actions
 - ❖ Nonperturbative renormalization
- ♦ Kaon mixing constrains scale of new physics with generic $O(1)$ flavor couplings to $\gtrsim 10,000$ TeV
[\[Isidori, Nir, Perez \(2010\)\]](#)
 - ❖ Lattice-QCD results for matrix elements of all possible $\Delta S=2$ BSM operators available for model building

Combination with measurement of indirect CP violation in kaon system (ε_K) **constrains CKM phase**



Status of the $|\epsilon_K|$ band

- ◆ Brod & Gorbahn [PRL108 (2012) 121801] give following error breakdown for $|\epsilon_K|$ in the Standard Model:

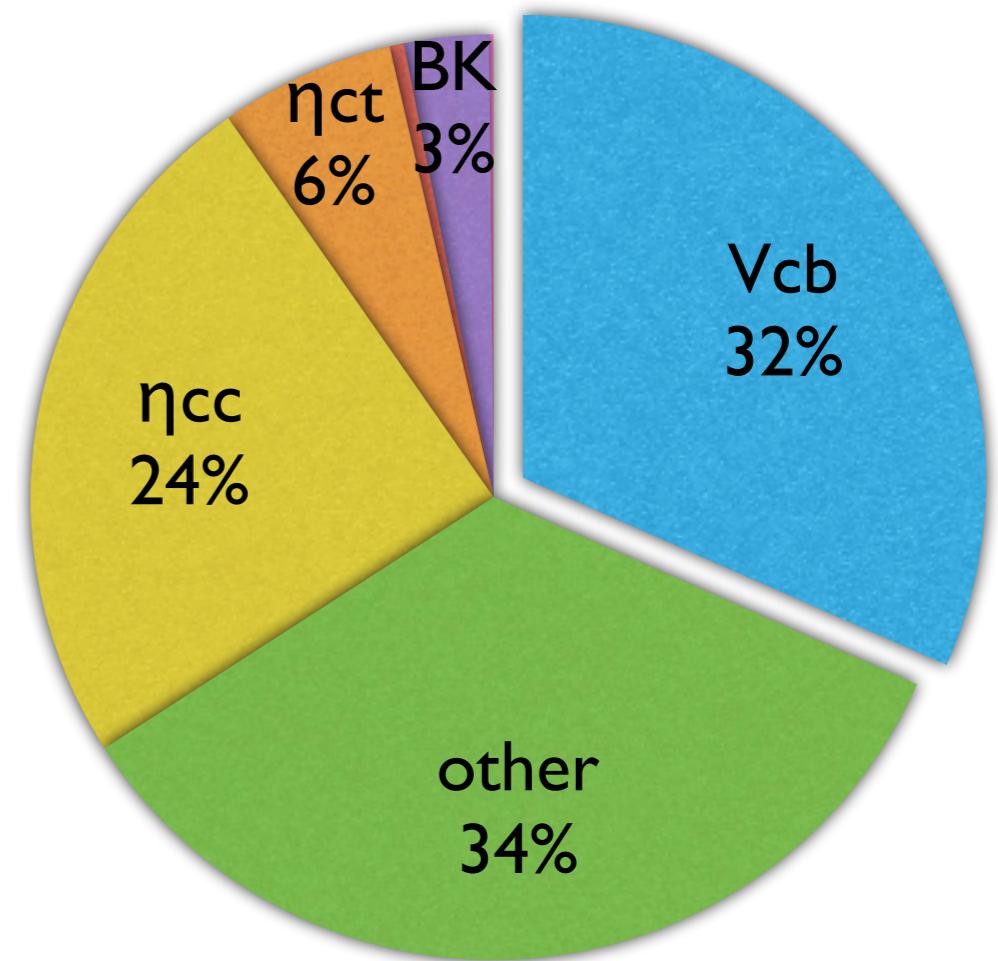
$$|\epsilon_K| = (1.81 \pm 0.14_{\eta_{cc}} \pm 0.02_{\eta_{tt}} \pm 0.07_{\eta_{ct}} \pm 0.05_{\text{LD}} \pm 0.23_{\text{parametric}}) \times 10^{-3}$$

(1) Largest individual uncertainty is from ~10% parametric error in $\propto |V_{cb}|^4$

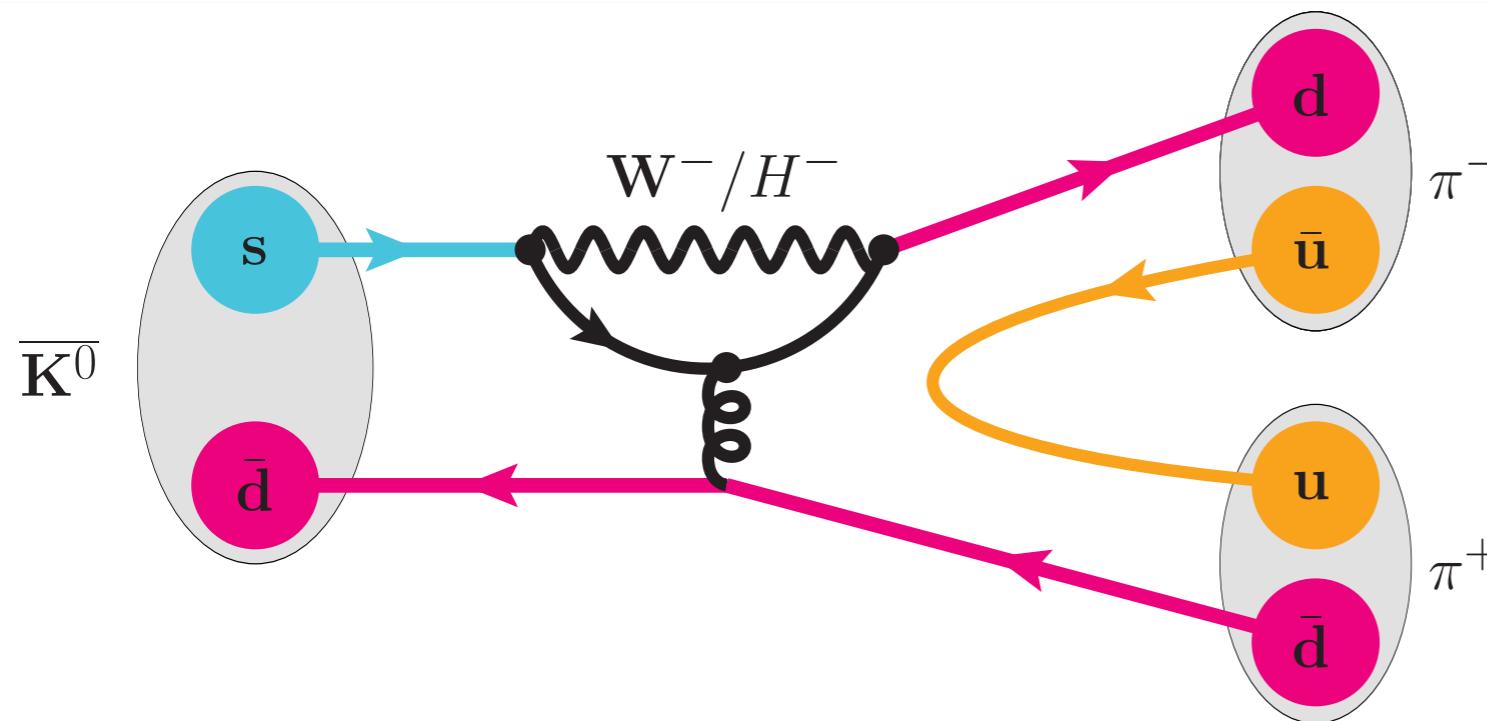
(2) η_{cc} and η_{ct} are both known to 3-loops (NNLO)

(3) Error from B_K only fourth-largest individual contribution

- ◆ Lattice community moving on to other more challenging kaon-physics quantities such as $K \rightarrow \pi\pi$ and $\Delta(M_K)$...



$K \rightarrow \pi\pi$ decays



- Sensitive to new particles, interactions, sources of CP violation
- “ $\Delta l=1/2$ rule”: empirically observe enhancement $\text{Re}A_0/\text{Re}A_2 \approx 22.5$

- ◆ Describe $\Delta S=1$ FCNC transitions with **effective Hamiltonian**
 - ❖ Short-distance effects factorized in Wilson coefficients \rightarrow continuum perturbation theory
 - ❖ Long-distance effects factorized in **matrix elements** $\langle \pi\pi | Q_i | K \rangle \rightarrow$ **lattice QCD**
- ◆ New physics above EW scale modifies Wilson coefficients, but hadronic matrix elements remain the same

$$\mathcal{H}_{\text{eff}}(\Delta S = 1) = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} \left(V_{us}^* V_{ud} z_i(\mu) - V_{ts}^* V_{td} y_i(\mu) \right) Q_i(\mu)$$

New! $K \rightarrow \pi\pi$ matrix elements (2015)

- * **Lattice complication:** additional Lüscher formalism needed to relate amplitudes calculated in Euclidean box to physical observables in Minkowski space [Briceño review, PoS LATTICE2014 (2015) 008]
- ◆ First complete three-flavor $K \rightarrow \pi\pi$ amplitudes with controlled errors using domain-wall (chiral) fermions
- ◆ Also first Wilson-fermion results from Ishizuka et al. [PRD92, 074503 (2015)] with heavy, zero-momentum pions

$\Delta I=3/2$ amplitude (A_2)

[RBC/UKQCD, PRD91, 074502 (2015)]

- ◆ Physical-mass pions, continuum limit, and approximately physical kinematics
- ◆ → ~10% errors on $\text{Re}(A_2)$, $\text{Im}(A_2)$
- ◆ Dominant uncertainty from perturbative truncation error in continuum Wilson coefficients

$\Delta I=1/2$ amplitude (A_0)

[RBC/UKQCD, PRL115, 212001 (2015)]

- ◆ 170 MeV pions with physical kinematics
- ◆ Small spatial volume $m_\pi L \approx 3.2$ and single lattice spacing $a \sim 0.14$ fm
- ◆ → ~35% error on $\text{Re}(A_0)$
- ◆ Will be reduced with higher statistics, larger volumes, continuum limit, ...

Implications for $\Delta l=1/2$ rule and $\text{Re}(\varepsilon'/\varepsilon)$

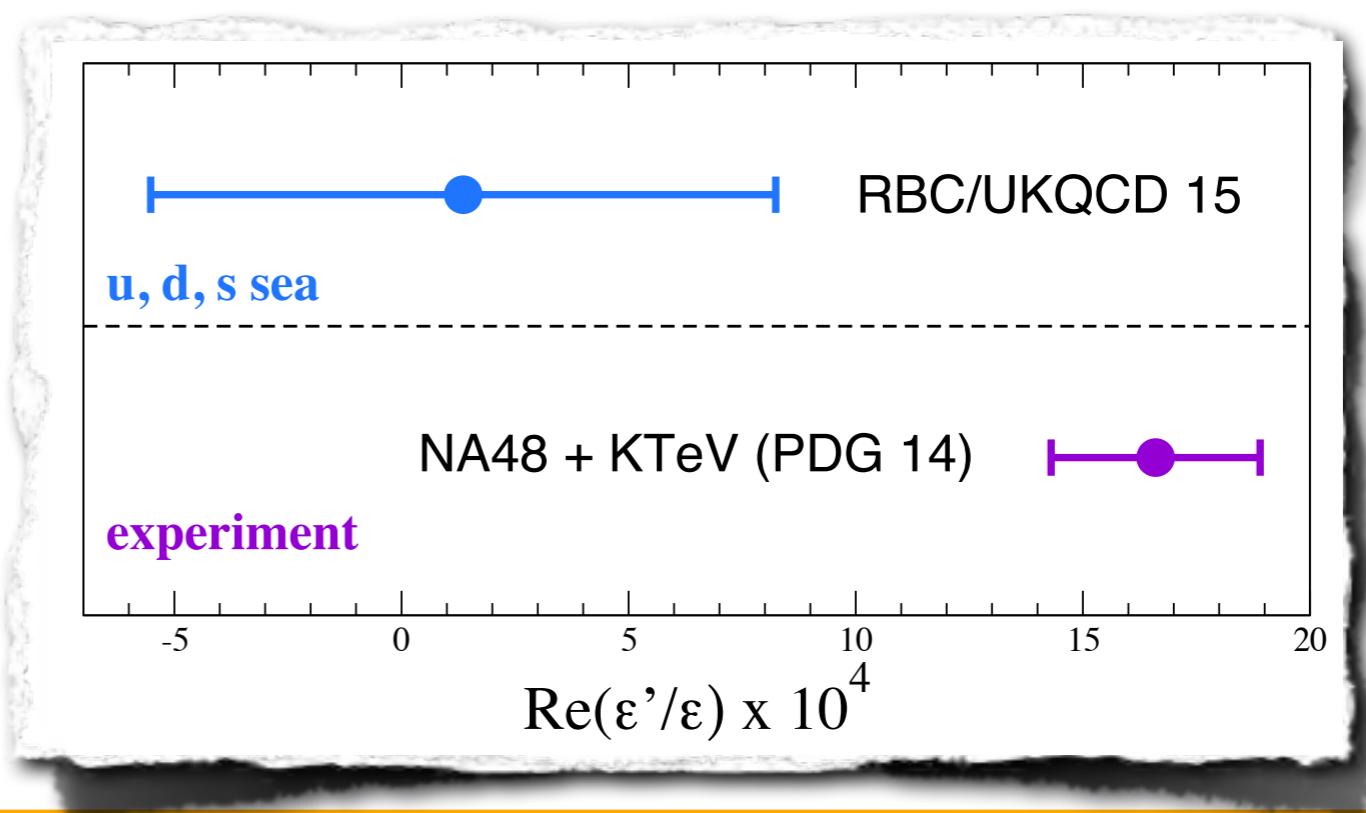
$\Delta l=1/2$ rule

- Empirically observe $\Delta l=1/2$ amplitude (A_0) $\sim 20\times$ larger than $\Delta l=3/2$ amplitude (A_2)
- Emerging understanding from lattice QCD:
significant cancellation between dominant contributions to $\text{Re}(A_2)$ not present for $\text{Re}(A_0)$

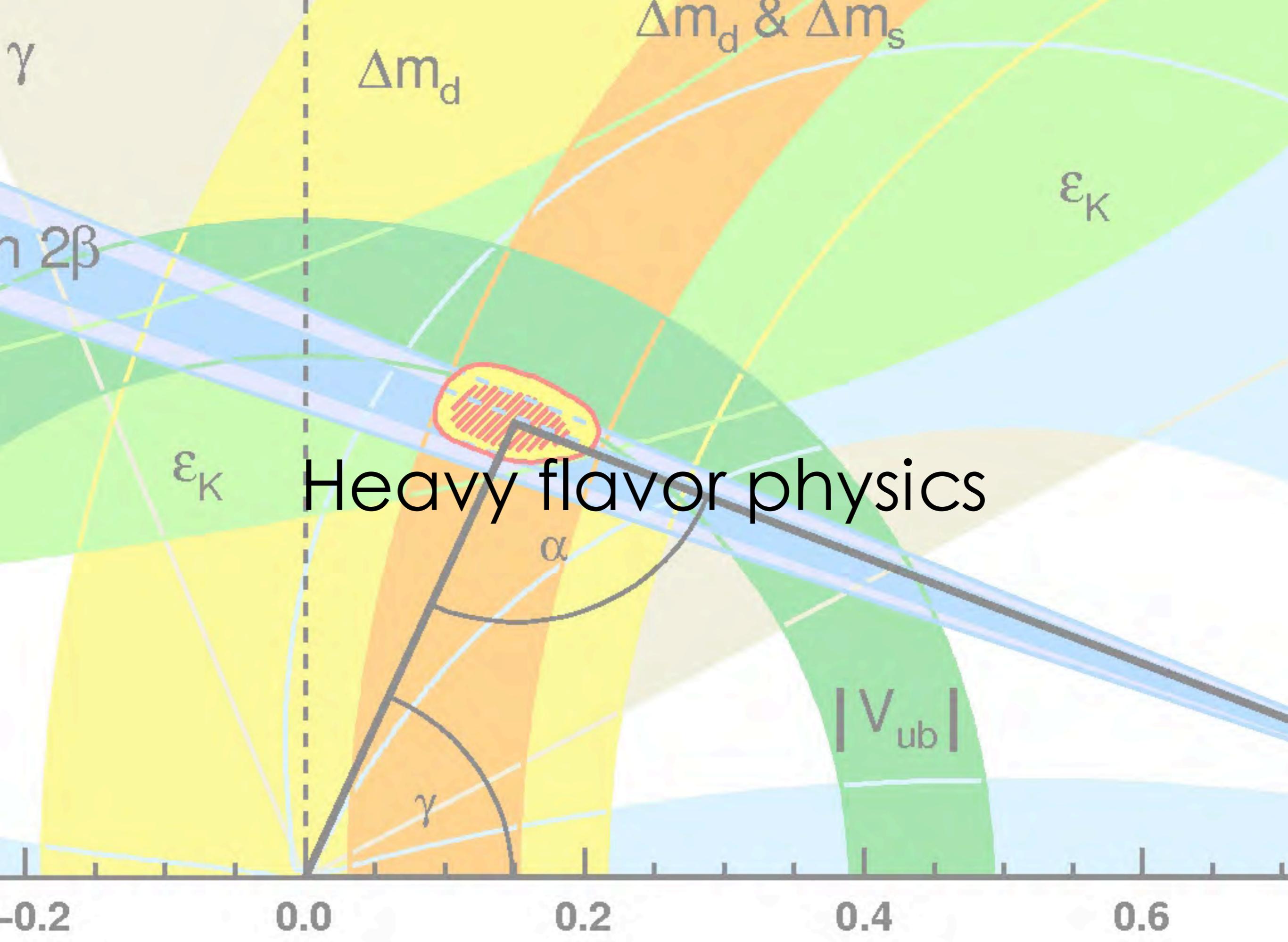
not present for $\text{Re}(A^0)$
contributions to $\text{Re}(A^3)$
dominated by gluons

$\text{Re}(\varepsilon'/\varepsilon)$

- First complete lattice-QCD calculation of direct CP violation in $K \rightarrow \pi\pi$ decays
- **2.1 σ tension with Standard Model**

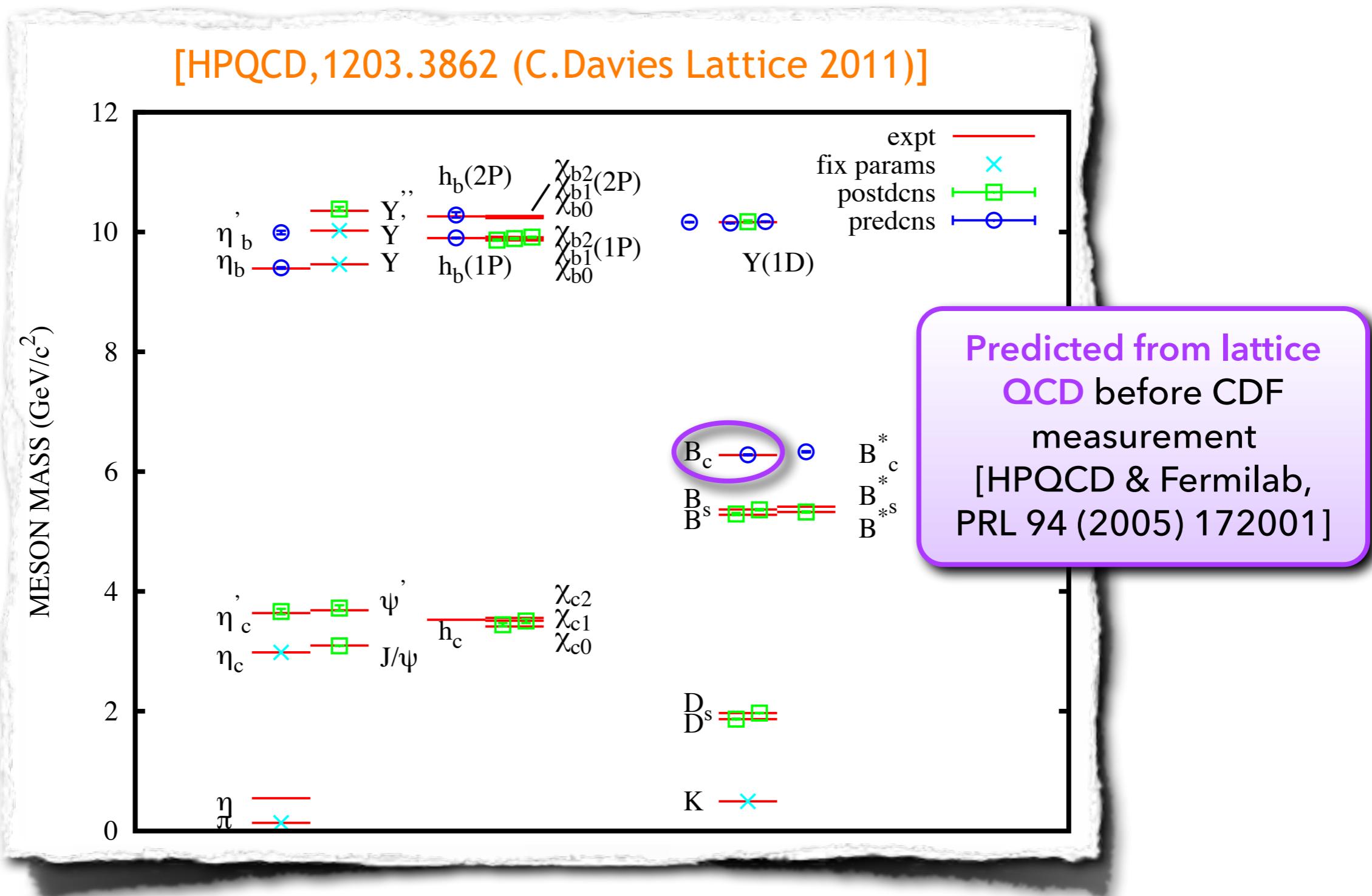


Anticipate reduction of QCD error to experimental size within few years with improved operator matching & data at finer lattice spacing (C. Kelly @ Lattice 2016)



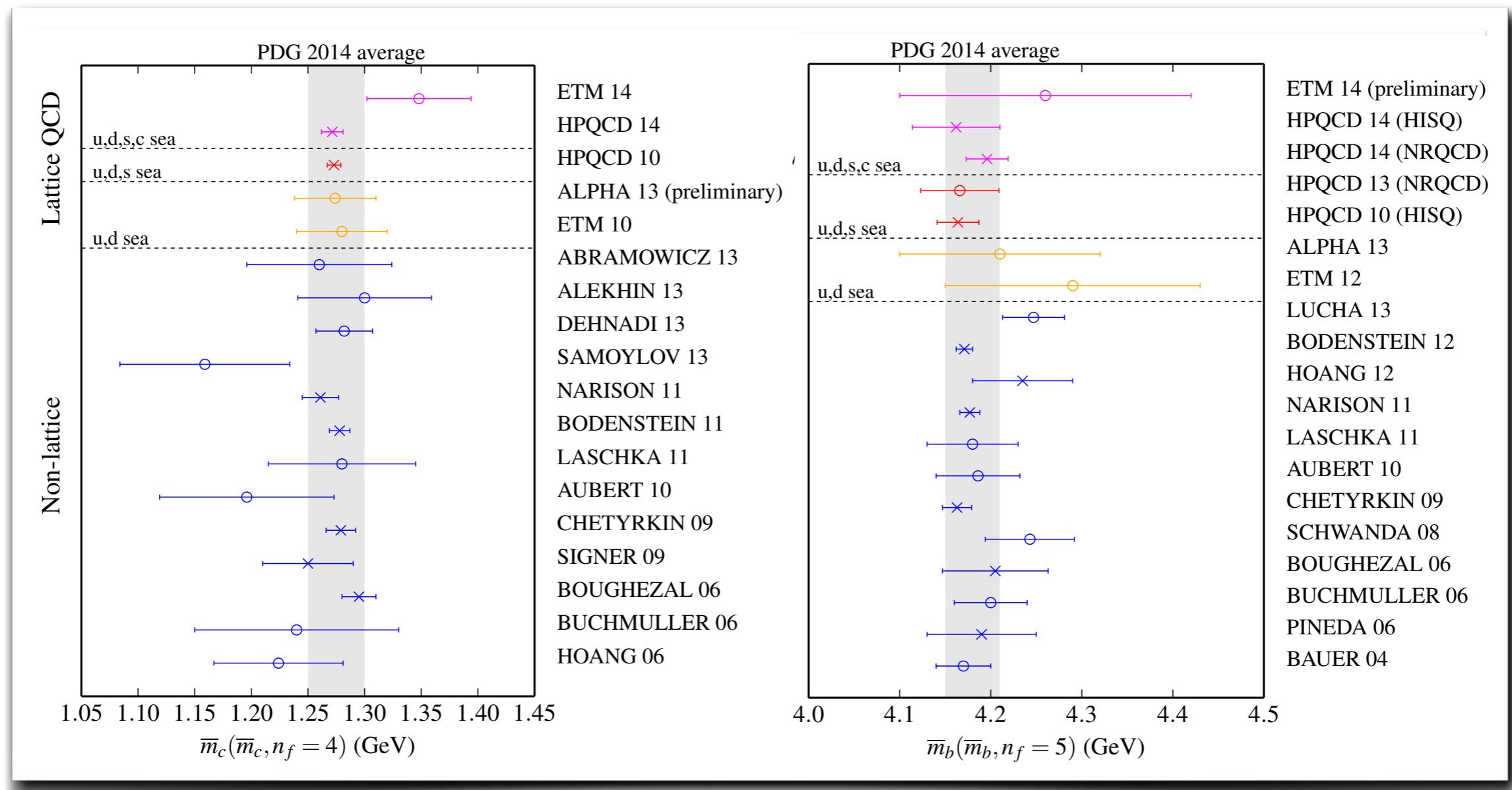
Heavy-hadron spectrum

- ◆ Tests lattice methods for charm & bottom quarks, which often rely on effective theories



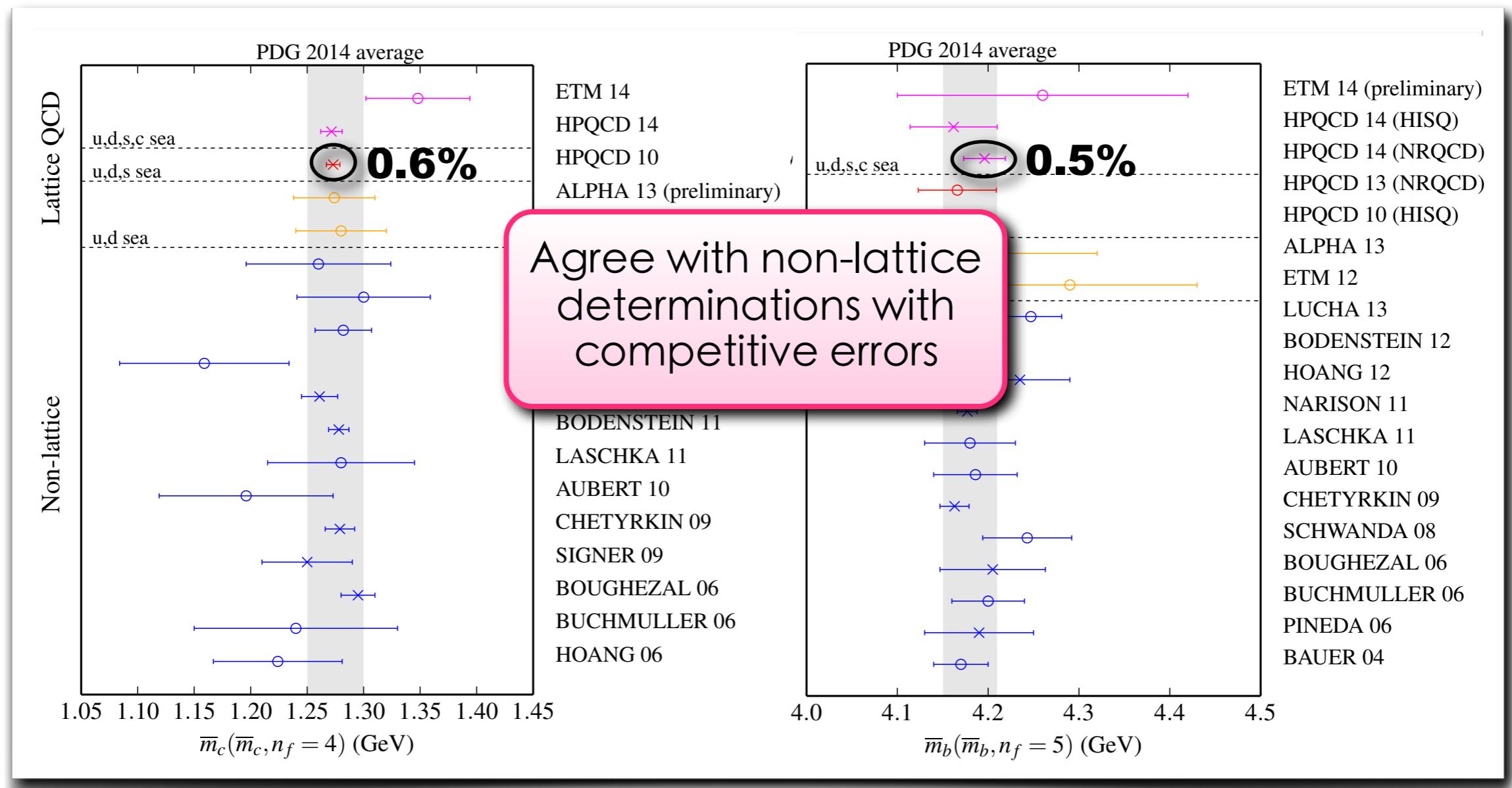
Heavy-quark masses

- ♦ Most precise m_c and m_b obtained by fitting moments of correlation functions of the quarks' electromagnetic current to $O(\alpha_s^3)$ perturbative expressions
- ♦ Moments can be obtained from experimental e^+e^- annihilation data, and also **computed numerically with lattice-QCD simulations with negligible statistical uncertainties**



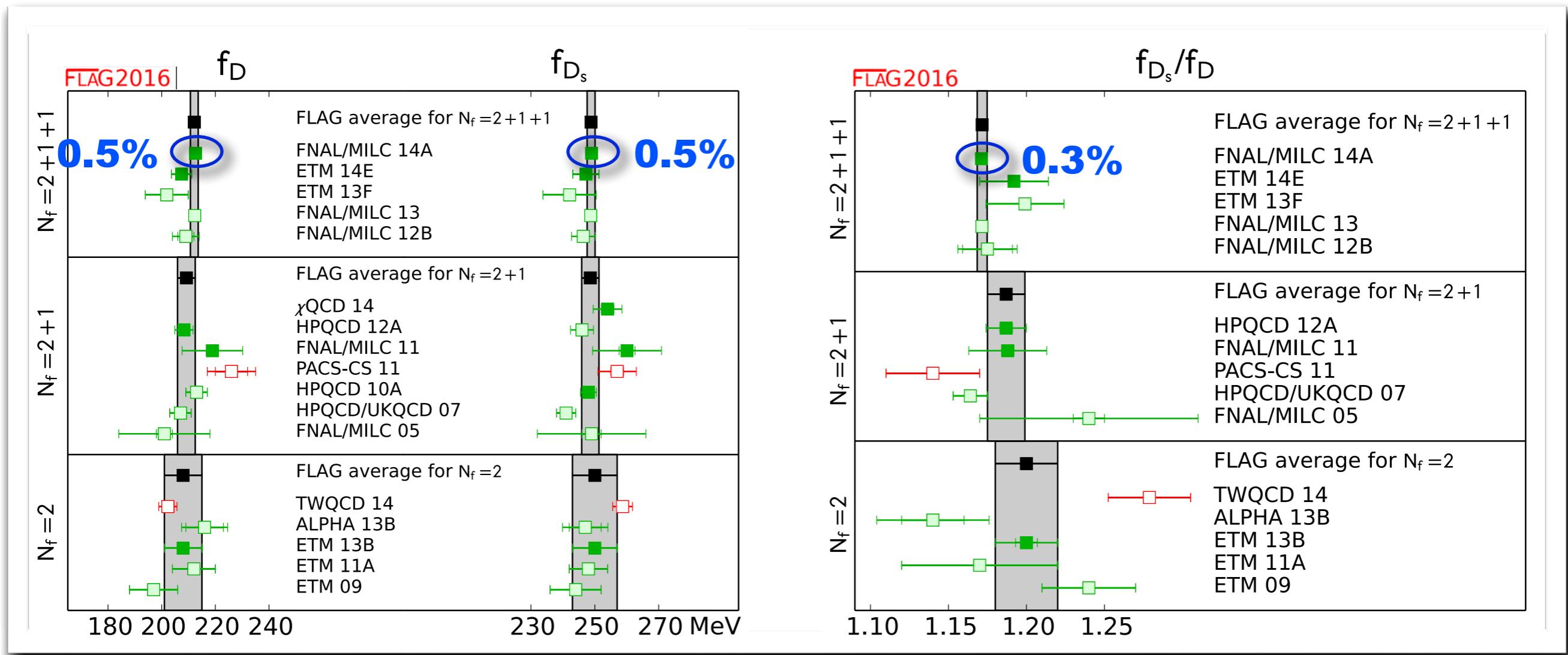
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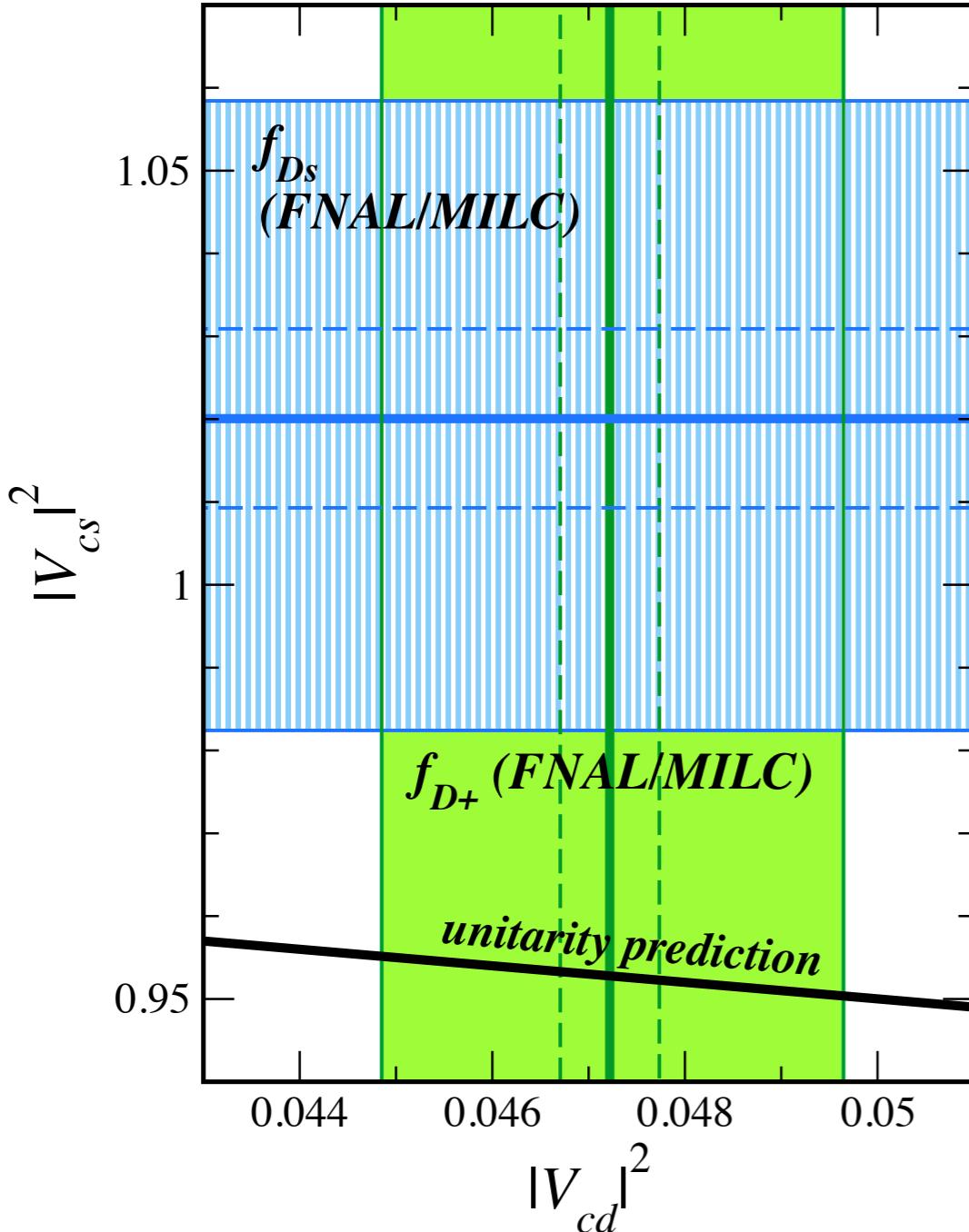


D_(s)-meson decay constants (2014)

- ◆ Fermilab/MILC [PRD90 (2014) 7, 074509] recently obtained **first four-flavor results for f_D & f_{D_s} with physical pions**
 - ❖ HISQ action for u,d,s, and c quarks and fine lattice spacings eliminates renormalization error and leads to small discretization errors
 - ❖ Errors $\sim 2\text{--}4\times$ smaller than previous best results



Implications for 2nd-row CKM unitarity



- Errors on $|V_{cd}|$ & $|V_{cs}|$ from leptonic $D_{(s)}$ decays limited by experimental branching fractions [Rosner, Stone, & RV (PDG), 1509.02220]

$$|V_{cd}| = 0.217(1)_{\text{LQCD}}(5)_{\text{expt}}$$

$$|V_{cs}| = 1.007(4)_{\text{LQCD}}(16)_{\text{expt}}$$

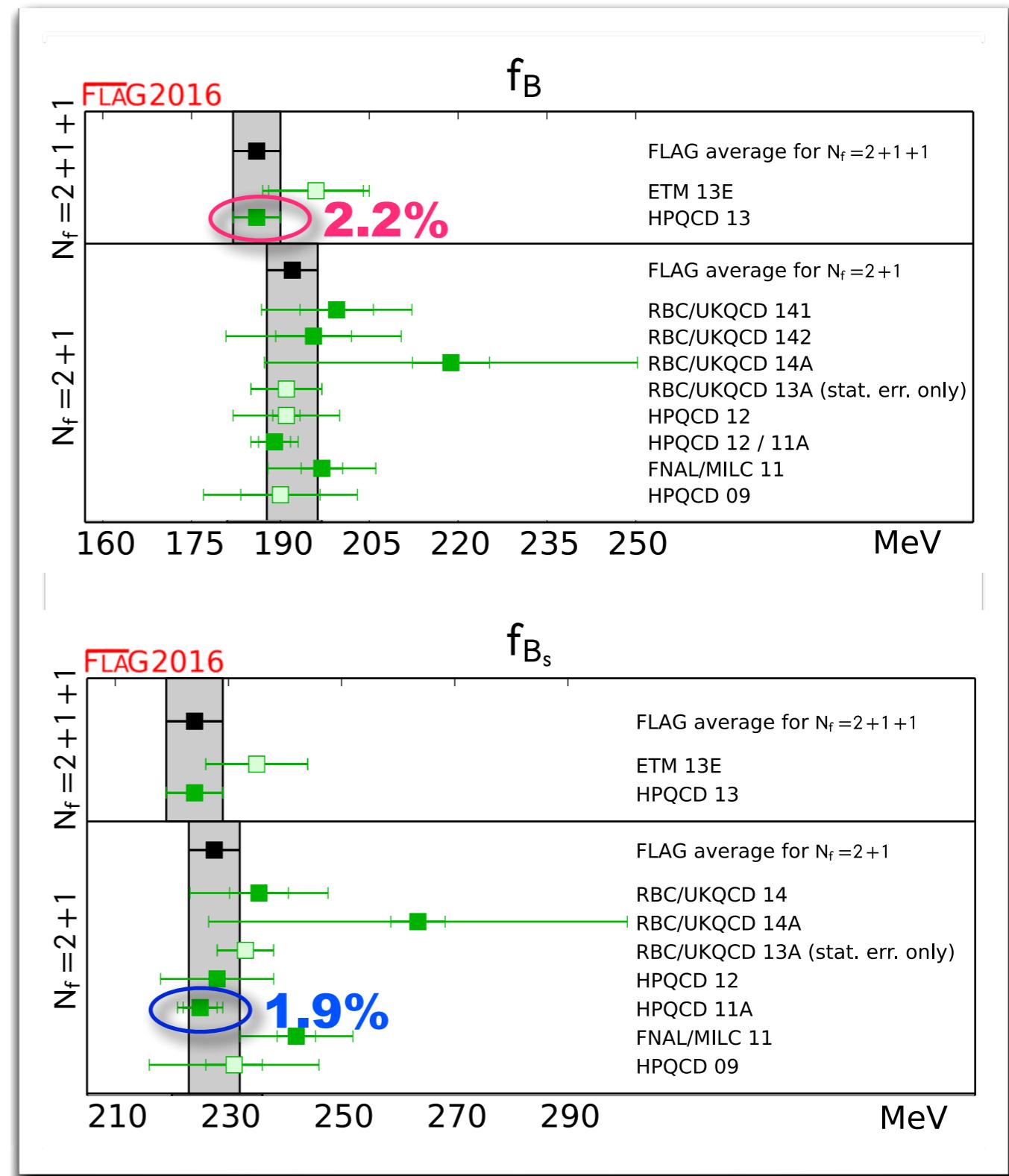
- Some tension with CKM unitarity

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.064(36)$$

→ Reaching precision where need estimate of hadronic structure-dependent EM contributions to $D_{(s)}$ leptonic decay rates

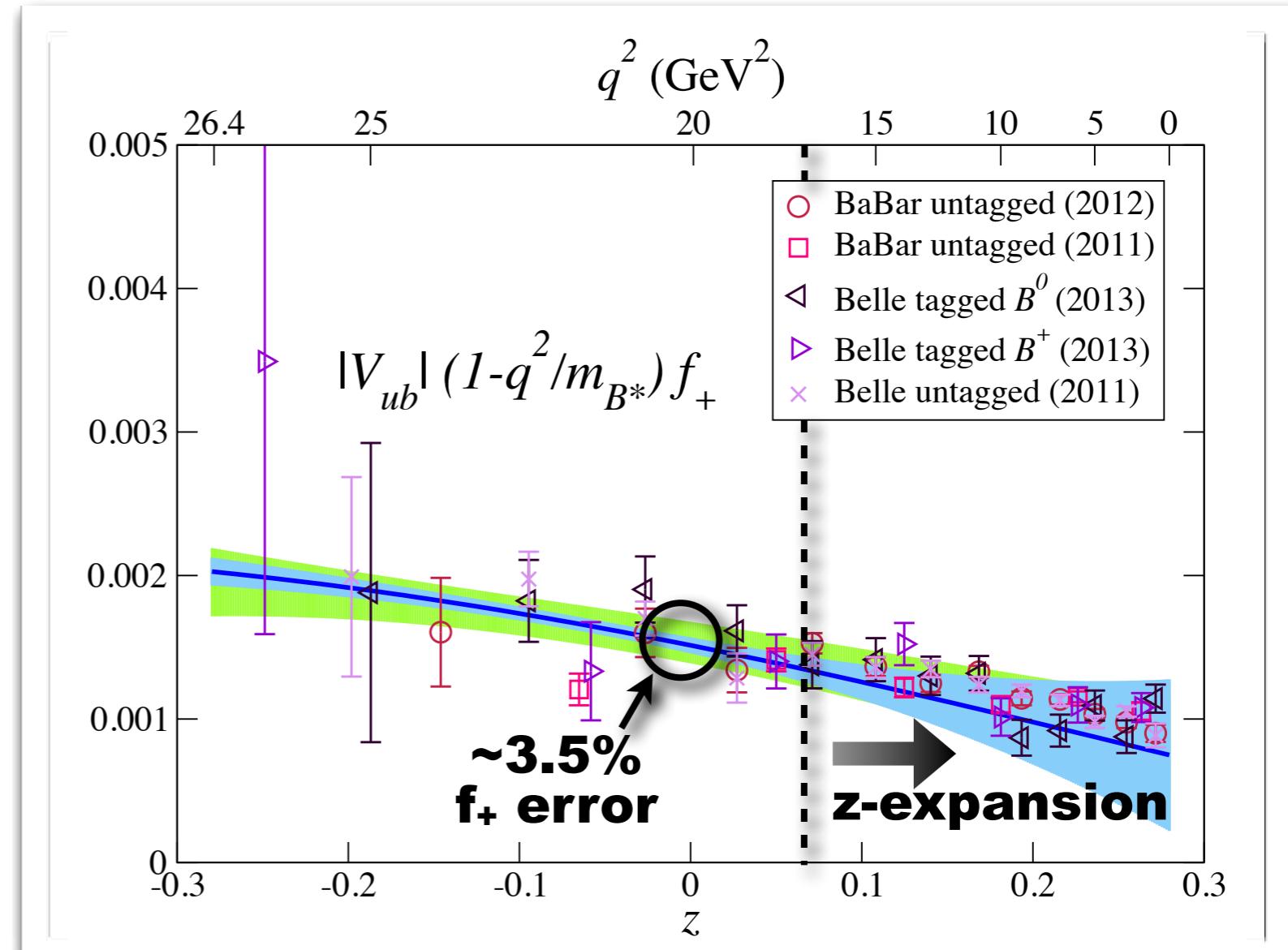
$B_{(s)}$ -meson decay constants

- ◆ Enter rates for leptonic decays $B \rightarrow \tau \nu$ and $B_{d,s} \rightarrow \mu^+ \mu^-$
- ◆ Most precise f_{B_s} calculation employs **physical-mass pions** [HPQCD, PRL110, 222003 (2013)]
 - **No chiral-extrapolation error**
- ◆ Most precise f_{B_s} uses **highly-improved staggered (HISQ) b-quark action** [HPQCD, PRD85, 031503 (2012)]
 - ❖ Lattice axial current absolutely normalized → **no renormalization error**
- ◆ **Confirmation from several independent calculations** using different gauge-field configurations, light-, and b-quark actions [see PDG review by Rosner, Stone, & RV, arXiv:1509.02220]



New! $B \rightarrow \pi l \nu$ form factors (2015)

- ♦ Two independent calculations last year!
 - ❖ First **RBC/UKQCD** f_+ & f_0 [**PRDD91 (2015) 7, 074510**]
 - ❖ New **FNAL/MILC** f_+ with more statistics & finer lattice spacings, and first f_0 [**PRDD92 (2015) 1, 014024**]
 - ❖ Extend lattice results to $q^2 \leq 16 \text{ GeV}^2$ using model-independent z-expansion
- ♦ $f_+(q^2)$ shape consistent with measured $B \rightarrow \pi l \nu$ dB/dq^2
- ♦ Obtain $|V_{ub}|$ from joint z-fit to lattice + experimental data



Use of all experimental & theoretical information minimizes error on $|V_{ub}|$

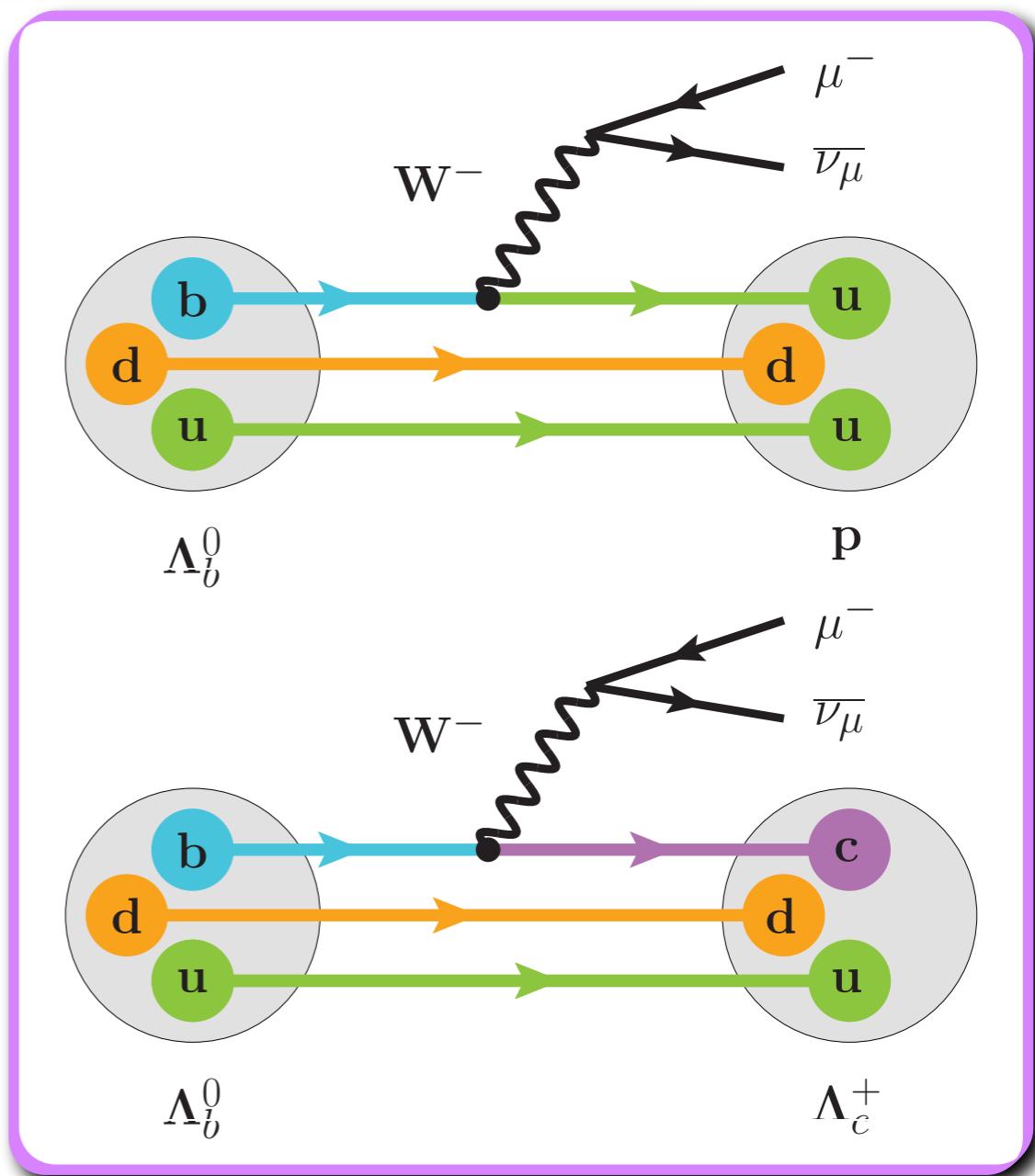
New! $\Lambda_b \rightarrow p \bar{v}$ & $\Lambda_c \bar{v}$ form factors (2015)

Combination with ratio of experimental rates **enables determination of $|V_{ub}| / |V_{cb}|$** → first from baryon decay!

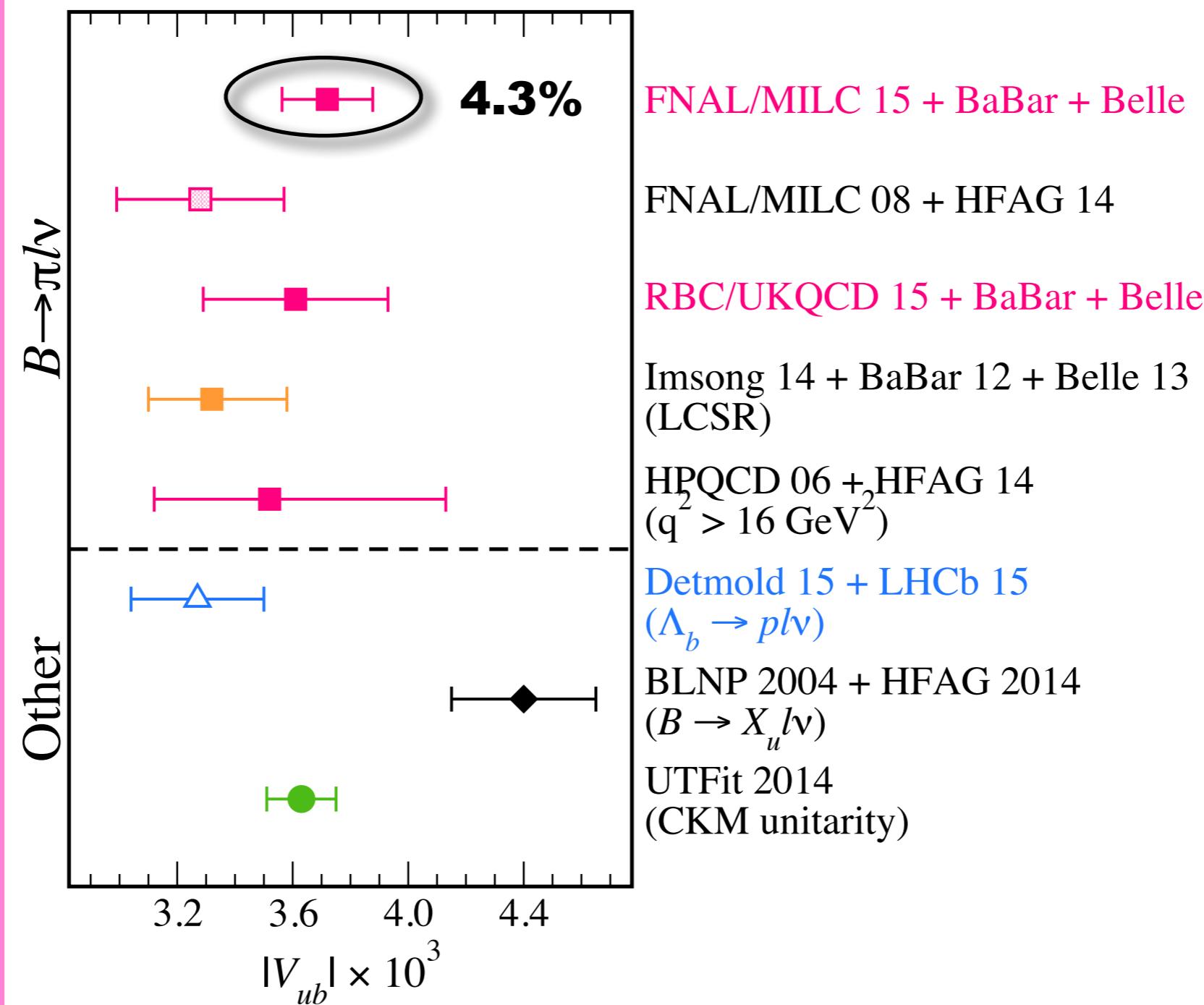
- ♦ First three-flavor $\Lambda_b \rightarrow p$ form factors with relativistic b-quark at physical mass; first three-flavor $\Lambda_b \rightarrow \Lambda_c$ form factors
[Detmold, Lehner, & Meinel, PRD92, 034503 (2015)]
- ♦ Combine chiral-continuum extrapolation with q^2 fit via modified z-expansion

$$\frac{|V_{cb}|^2}{|V_{ub}|^2} \frac{\int_{15 \text{ GeV}^2}^{q_{\max}^2} \frac{d\Gamma(\Lambda_b \rightarrow p \mu^- \bar{\nu}_\mu)}{dq^2} dq^2}{\int_{7 \text{ GeV}^2}^{q_{\max}^2} \frac{d\Gamma(\Lambda_b \rightarrow \Lambda_c \mu^- \bar{\nu}_\mu)}{dq^2} dq^2} = 1.470 \pm 0.115 \pm 0.104$$

→ 5.3% LQCD error on $|V_{ub}| / |V_{cb}|$

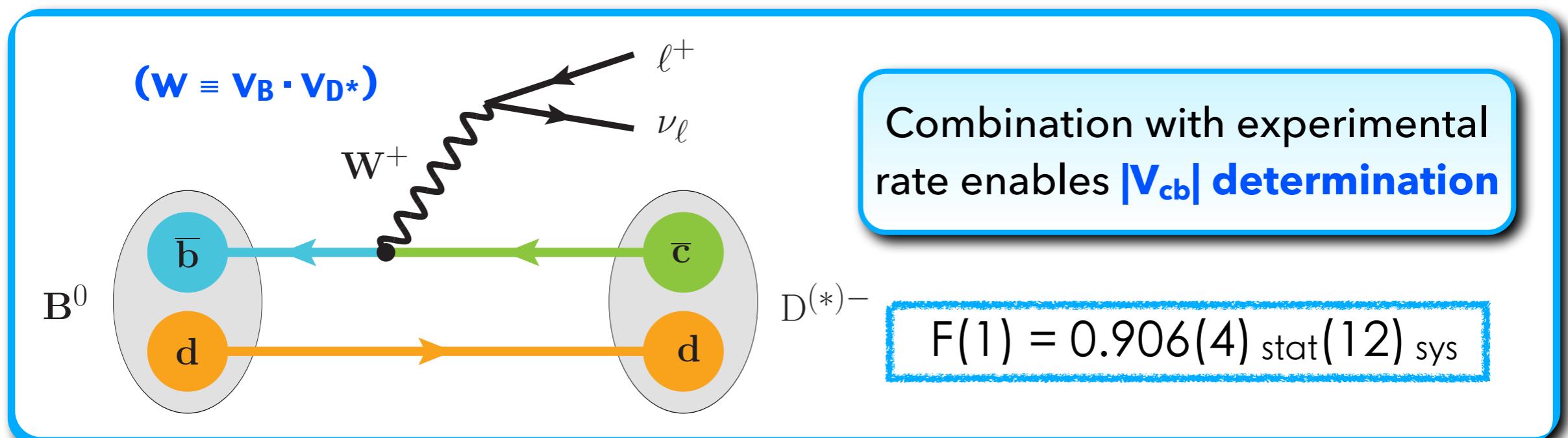


Comparison of $|V_{ub}|$ determinations

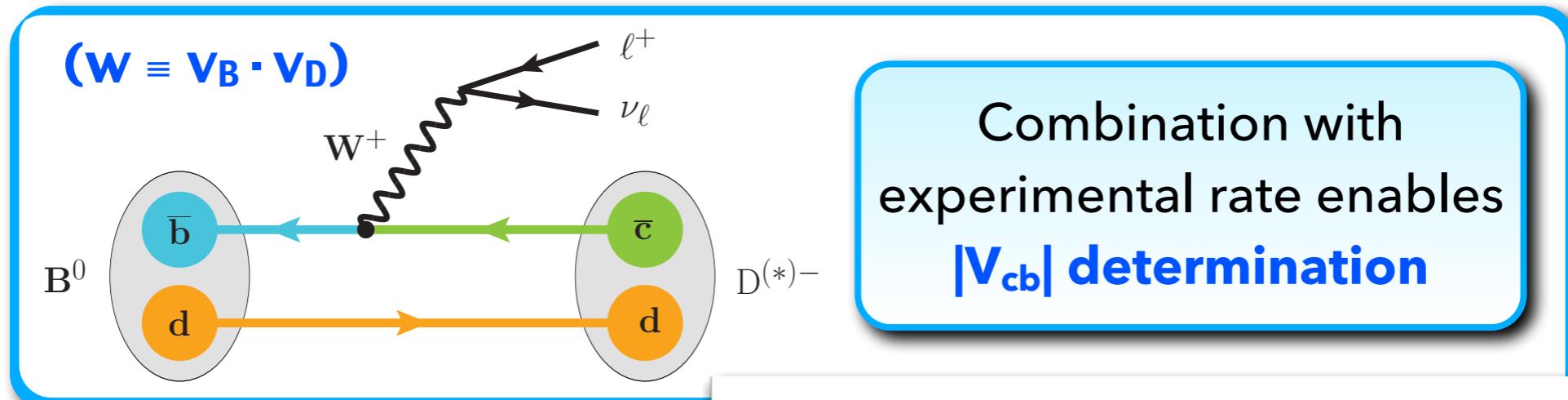


$B \rightarrow D^* l \bar{\nu}$ form factor @ zero recoil (2014)

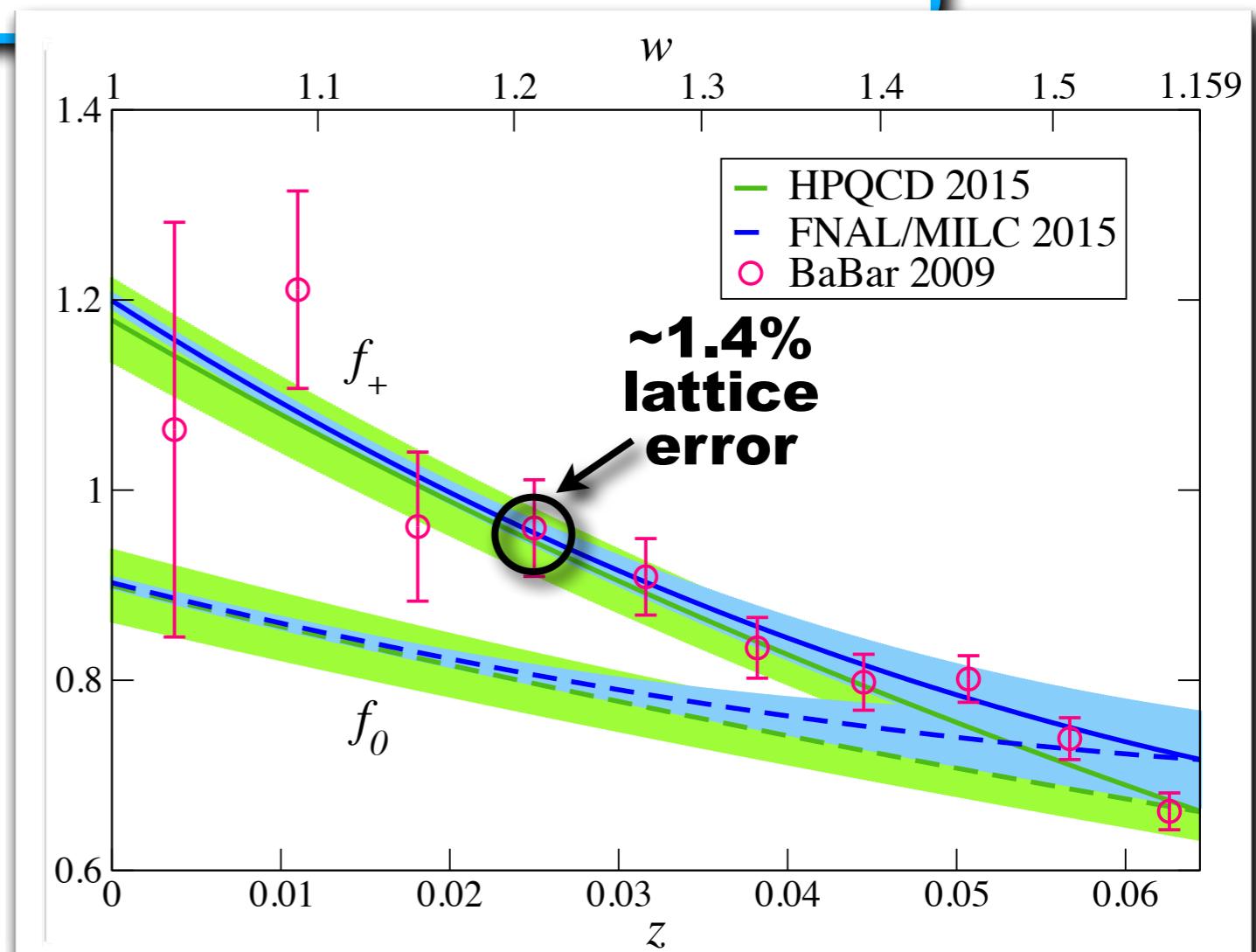
- ◆ Only need one normalization point from lattice QCD → **choose zero recoil ($w=1$) where it can be computed most precisely**
 - ❖ $F(1) \rightarrow 1$ in the static limit ($m_b = m_c \rightarrow \infty$) [**Isgur & Wise**], and **Luke's theorem** ensures that the leading heavy-quark corrections to $F(1)$ are of $\mathcal{O}(1/m_b^2, 1/m_c^2)$
 - ❖ Can compute form factor using double ratio of lattice three-point correlation functions in which statistical and systematic errors largely cancel
- ◆ New FNAL/MILC calculation with increased statistics, lighter quark masses, & finer lattice spacings → **1.4% precision on $F(1)$** [**FNAL/MILC, PRD89 (2014) 11, 114504**]



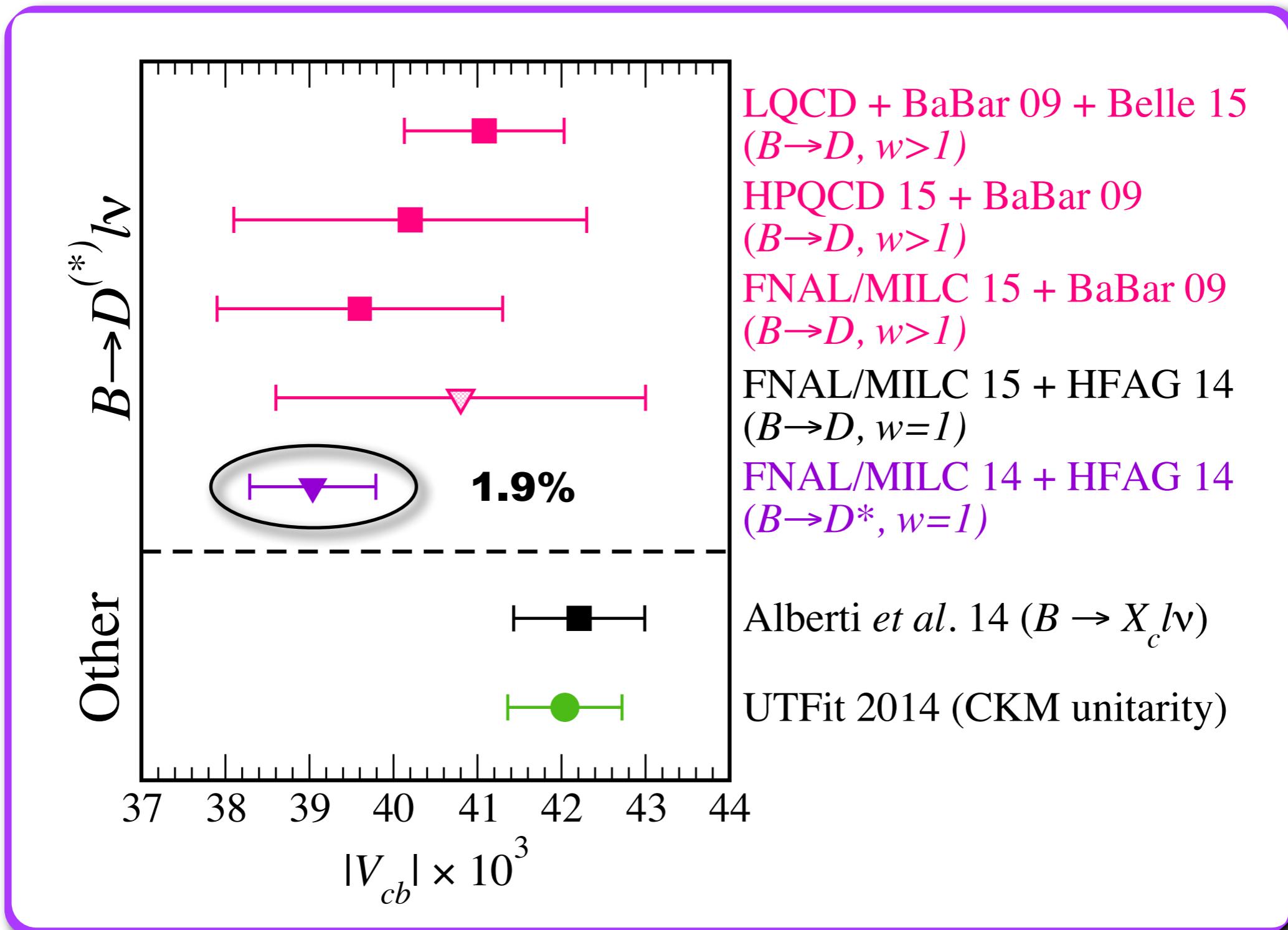
New! $B \rightarrow D l \bar{\nu}$ form factors (2015)



- Comparing theory & experiment at $w=1 \rightarrow$ large experimental errors in $|V_{cb}|$ because decay rate suppressed
- First three-flavor form-factor results over full kinematic range** [Fermilab/MILC, PRD92, 034506 (2015); HPQCD, PRD92, 054510 (2015)]
 - Independent calculations agree
 - Shapes consistent with experiment
- Joint lattice + experiment fit using $w>1$ data reduces error on $|V_{cb}|$**



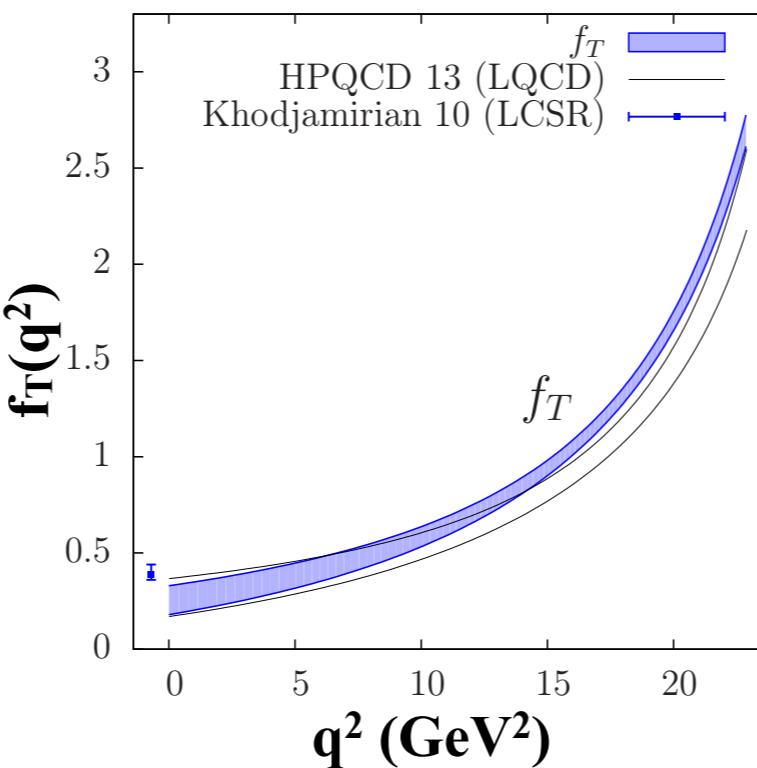
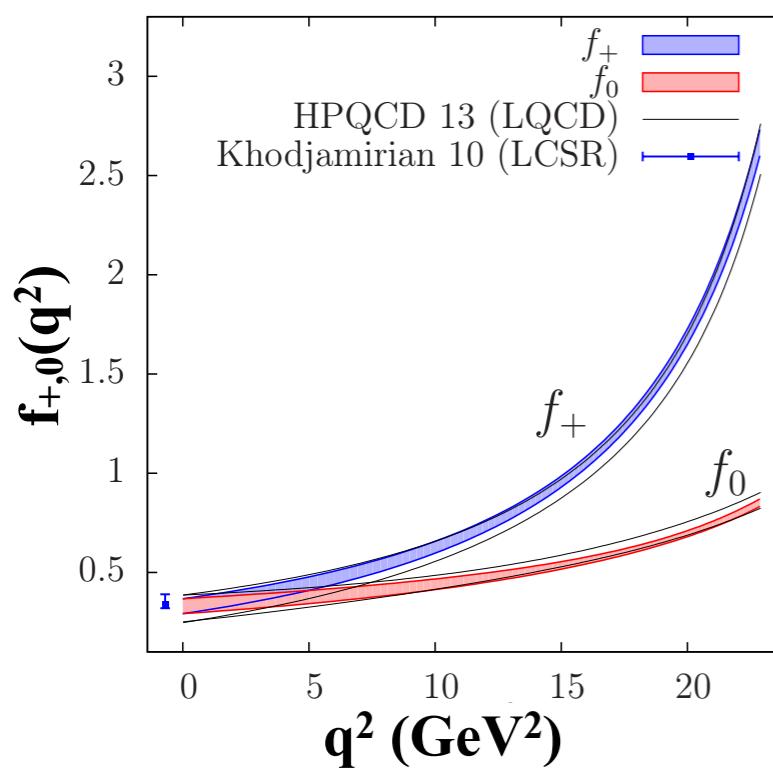
Comparison of $|V_{cb}|$ determinations



New! $B \rightarrow \pi \parallel$ & $B \rightarrow K \parallel$ form factors (2015)

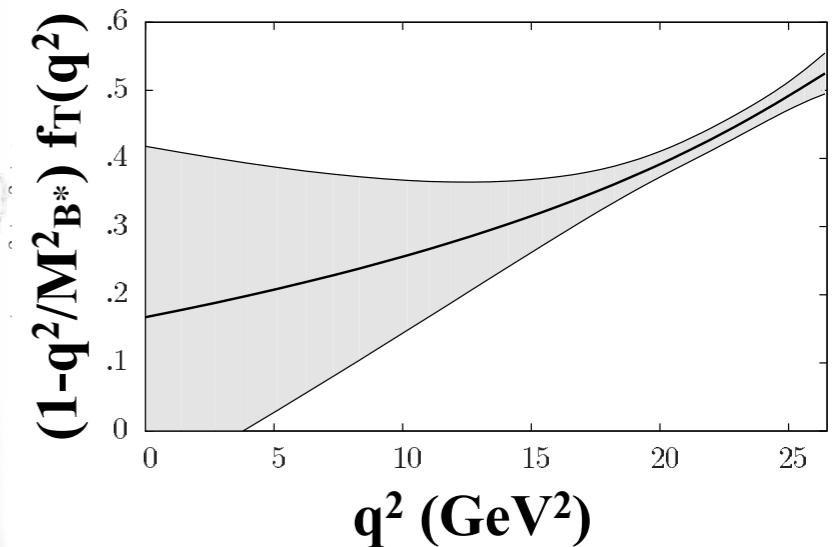
$B \rightarrow K$ form factors

[Fermilab/MILC, PRD 93, 025026 (2016)]



$B \rightarrow \pi$ tensor form factor

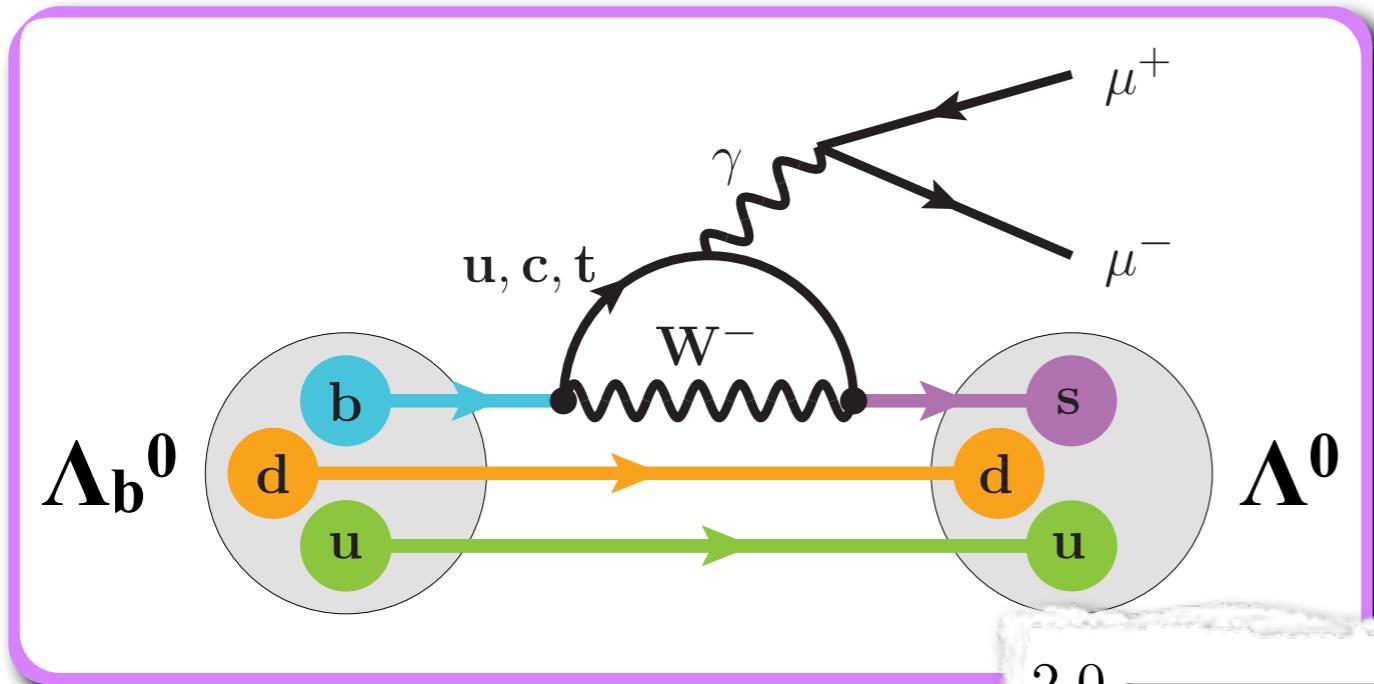
[PRL 115, 152002 (2015)]



First lattice result for $B \rightarrow \pi$ tensor form factor!

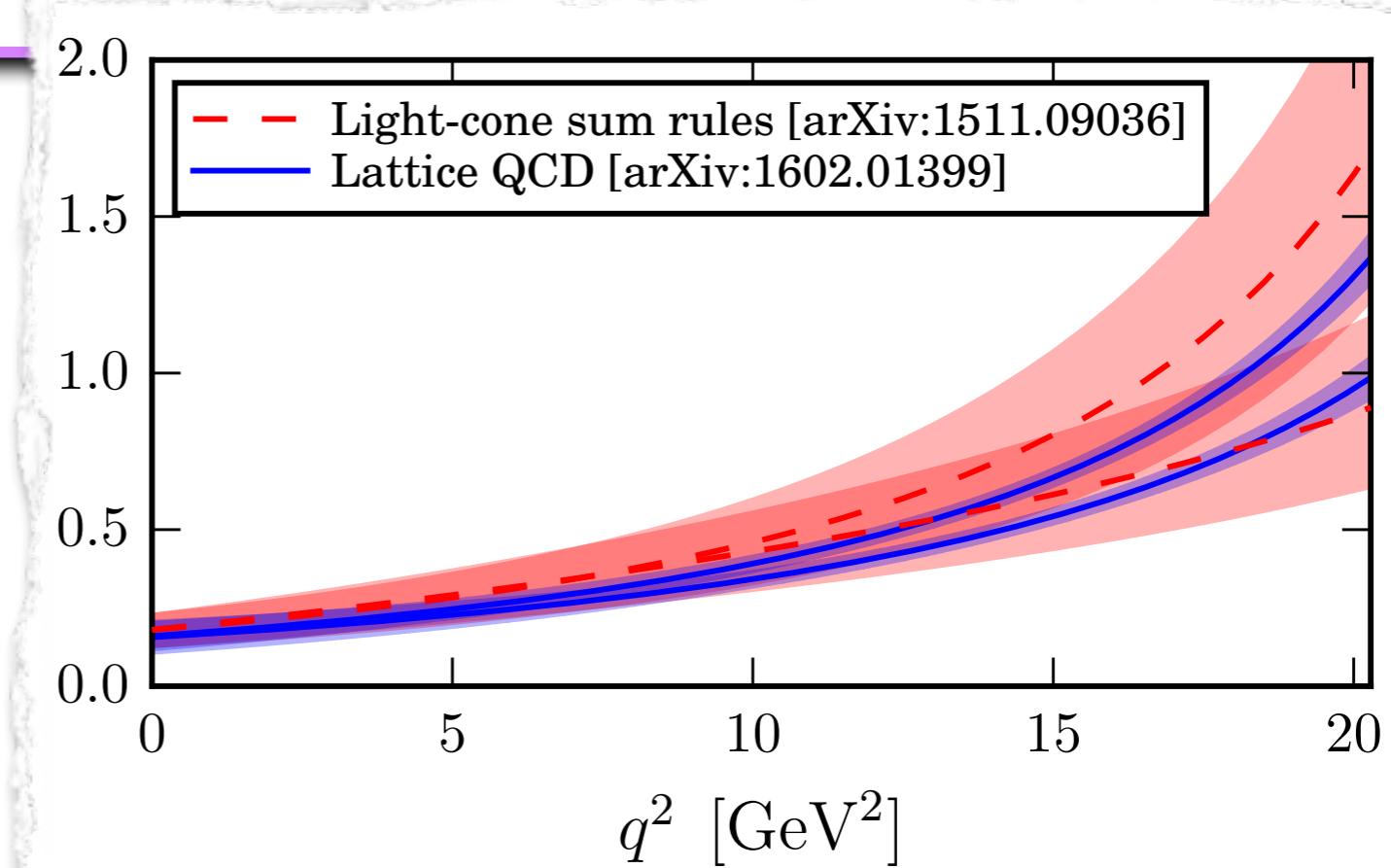
- ◆ $B \rightarrow K$ form factors consistent with HPQCD results obtained with different b-quark action [PRD88, 054509 (2013)] and LCSR at $q^2=0$ [JHEP 1009, 089 (2010)]
- ★ New lattice-QCD $B \rightarrow \pi$ and $B \rightarrow K$ form factors enable calculations of $B \rightarrow \pi(K)l^+l^-$, $B \rightarrow \pi(K)v\bar{v}$, and $B \rightarrow \pi \zeta v$ observables with fewer assumptions than previously possible!

New! $\Lambda_b \rightarrow \Lambda l^+ l^-$ form factors (2016)

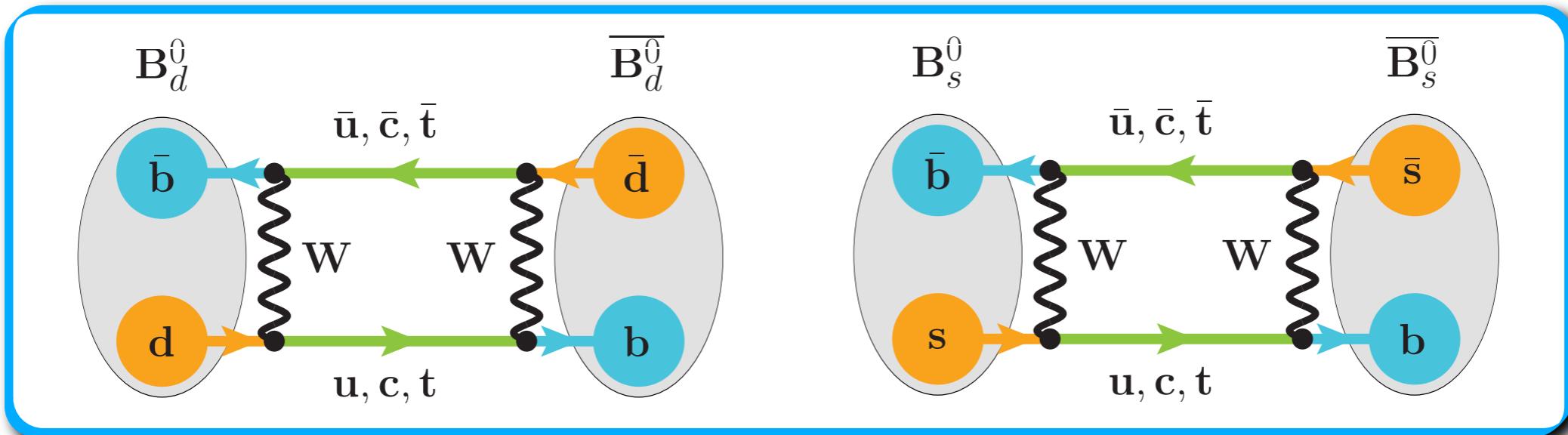


Enable **calculations of $\Lambda_b \rightarrow \Lambda l^+ l^-$ observables** in the Standard Model and beyond

- ♦ First $\Lambda_b \rightarrow \Lambda$ form factors with relativistic, physical-mass b-quark [Detmold & Meinel, PRD93, 074501(2016)]
- ♦ Joint chiral-continuum extrapolation and q^2 fit via modified z-expansion
- ♦ More precise than light-cone-sum-rule determinations [Wang & Shen, JHEP 1602, 179 (2016)]



Neutral B-meson mixing

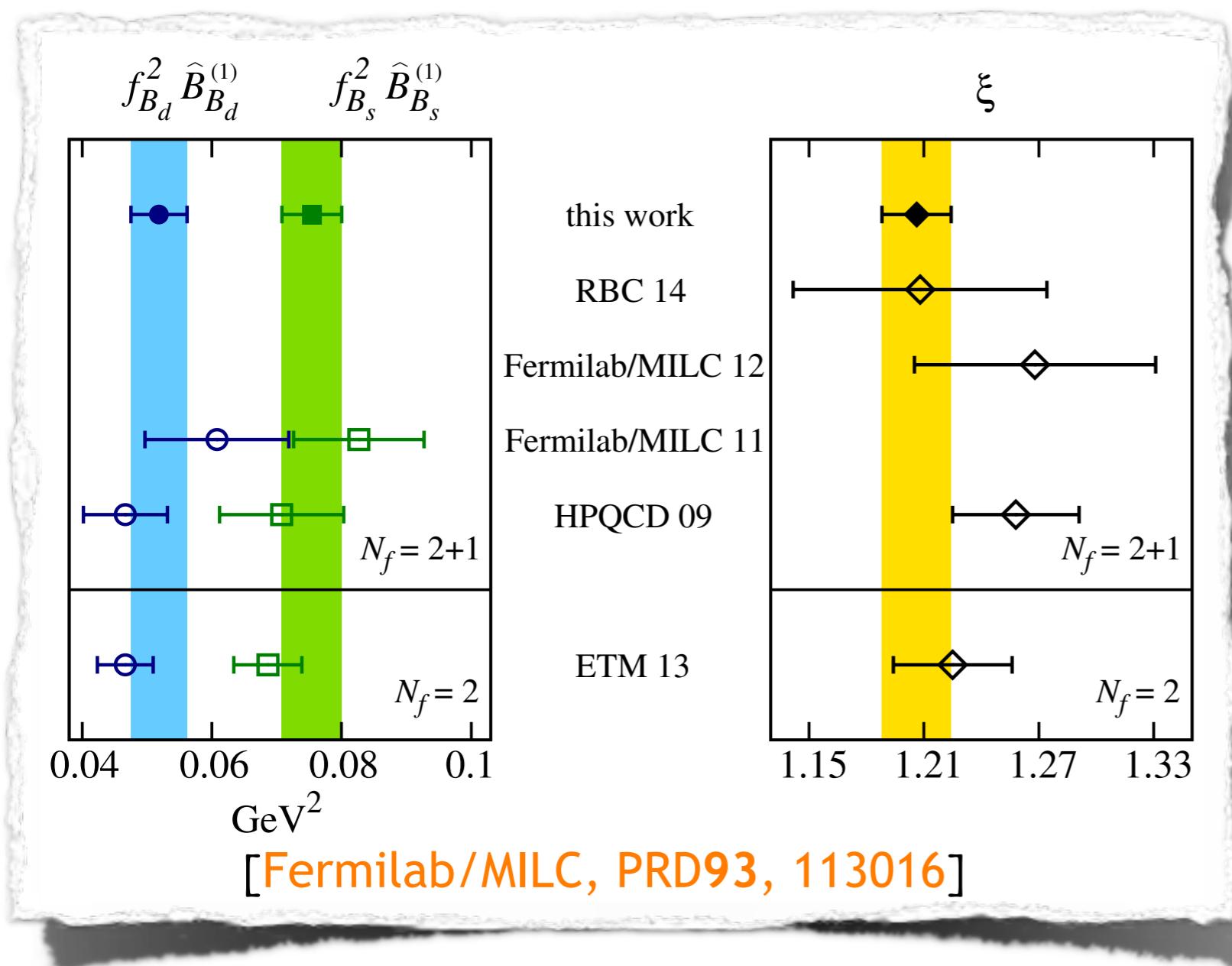


- ♦ Ratio of B_d to B_s oscillation frequencies (Δm_q) determines apex of UT ($\bar{\rho}, \bar{\eta}$) via

$$\begin{aligned} \frac{\Delta m_d}{\Delta m_s} &= \left(\frac{f_{B_d} \sqrt{\hat{B}_{B_d}}}{f_{B_s} \sqrt{\hat{B}_{B_s}}} \right)^2 \frac{m_{B_d}}{m_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2} \\ &= \xi^2 \frac{m_{B_d}}{m_{B_s}} \left(\frac{\lambda}{1 - \lambda^2/2} \right)^2 \frac{((1 - \bar{\rho})^2 + \bar{\eta}^2)}{\left(1 + \frac{\lambda^2}{1 - \lambda^2/2} \bar{\rho} \right) + \lambda^4 \bar{\eta}^2} \end{aligned}$$

- ♦ **Ratio of B_d - to B_s -mixing hadronic matrix elements can be computed precisely in lattice QCD because:**
 - ❖ Statistical fluctuations correlated
 - ❖ Matrix elements equal in SU(3) limit $m_s = m_{ud}$, so some systematic errors suppressed by $(m_s - m_{ud})/\Lambda_{QCD}$

New! B-mixing matrix elements (2016)



- ◆ Significant Fermilab/MILC update of 2012 ξ calculation with increased statistics, finer lattice spacings, better treatment of chiral extrapolation

→ Error on ξ reduced from ~5% to 1.6%!
- ◆ Also present individual B_d - and B_s -mixing matrix elements
- ◆ ... and first three-flavor results for all five local operators that contribute to neutral B-meson mixing in and beyond the Standard Model