Recent Progress on Renormalization Group Flow

Clay Córdova

School of Natural Sciences
Institute for Advanced Study

August 9, 2016

Quantum Field Theory is Organized by Scale

- Particle physics is organized by energy scale: $UV \longrightarrow IR$

Quantum Field Theory is Organized by Scale

- Particle physics is organized by energy scale: $UV \longrightarrow IR$
- At longest distances (IR) only massless particles propagate. This leads to macroscopic forces, (e.g. photon and electromagetism)

Quantum Field Theory is Organized by Scale

- Particle physics is organized by energy scale: $UV \longrightarrow IR$
- At longest distances (IR) only massless particles propagate. This leads to macroscopic forces, (e.g. photon and electromagetism)
- As energies increase (UV) more massive fields can be seen.

Quantum Field Theory is Organized by Scale

- Particle physics is organized by energy scale: $UV \longrightarrow IR$
- At longest distances (IR) only massless particles propagate. This leads to macroscopic forces, (e.g. photon and electromagetism)
- As energies increase (UV) more massive fields can be seen.
    - First indirectly through their small effects on light particles

Quantum Field Theory is Organized by Scale

- Particle physics is organized by energy scale: $UV \longrightarrow IR$
- At longest distances (IR) only massless particles propagate. This leads to macroscopic forces, (e.g. photon and electromagetism)
- As energies increase (UV) more massive fields can be seen.
  - First indirectly through their small effects on light particles
  - Then, at sufficient energies, massive particles are produced and (hopefully!!!) directly detected in experiments

Quantum Field Theory is Organized by Scale

- Particle physics is organized by energy scale: $UV \longrightarrow IR$
- At longest distances (IR) only massless particles propagate. This leads to macroscopic forces, (e.g. photon and electromagetism)
- As energies increase (UV) more massive fields can be seen.
  - First indirectly through their small effects on light particles
  - Then, at sufficient energies, massive particles are produced and (hopefully!!!) directly detected in experiments

Upshot:

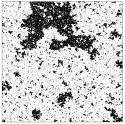
At high energies there are more particle species, more interactions possible, **more information**

Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)

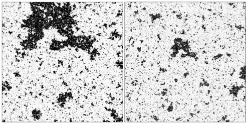
Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)



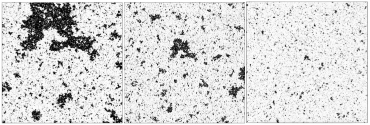
Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)



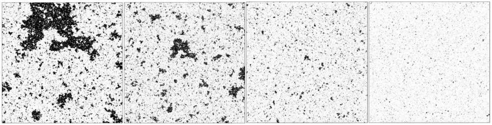
Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)



Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)



Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)

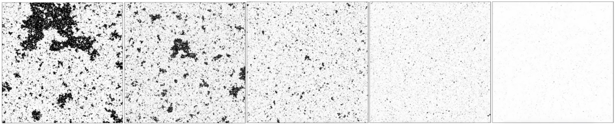


Figure: $T < T_c$: spins uniform

Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)

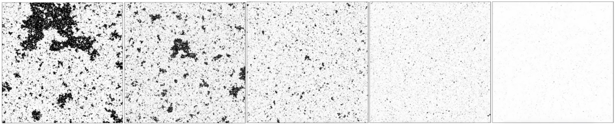
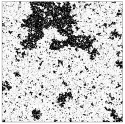


Figure: $T < T_c$: spins uniform



Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)

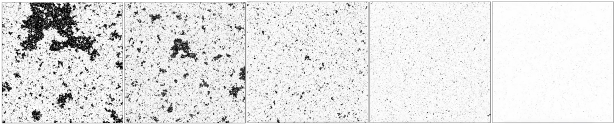
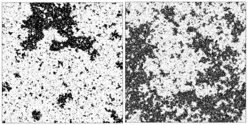


Figure: $T < T_c$: spins uniform



Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)

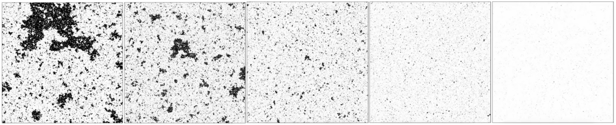
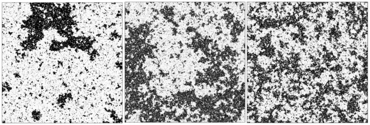


Figure: $T < T_c$: spins uniform



Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)

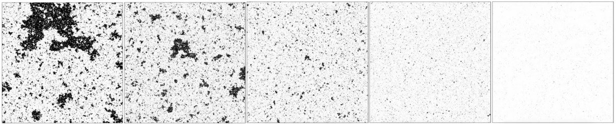
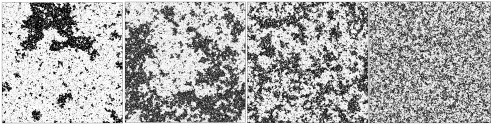


Figure: $T < T_c$: spins uniform



Quantum Field Theory is Organized by Scale

- The same paradigm often works in condensed matter physics. For example: block spin renormalization of the Ising model (Nearest neighbor interactions of spins: black vs white)

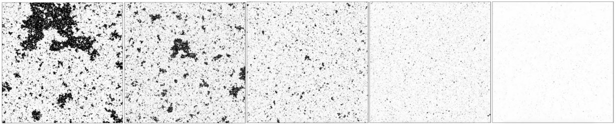


Figure: $T < T_c$: spins uniform

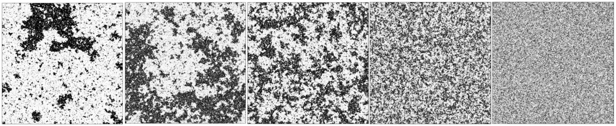


Figure: $T > T_c$: spins uncorrelated

Major Theme in Formal Theory

- The idea that particle physics at high energies is more complicated then at low energies is a unifying theme

Major Theme in Formal Theory

- The idea that particle physics at high energies is more complicated then at low energies is a unifying theme
- The task of formal theorists is to make this intuition precise and quantitative

Major Theme in Formal Theory

- The idea that particle physics at high energies is more complicated then at low energies is a unifying theme
- The task of formal theorists is to make this intuition precise and quantitative

The *a*-Theorem:

Major Theme in Formal Theory

- The idea that particle physics at high energies is more complicated then at low energies is a unifying theme
- The task of formal theorists is to make this intuition precise and quantitative

The *a*-Theorem:

- Define a quantity "*a*" for each quantum field theory

Major Theme in Formal Theory

- The idea that particle physics at high energies is more complicated then at low energies is a unifying theme
- The task of formal theorists is to make this intuition precise and quantitative

The *a*-Theorem:

- Define a quantity "a" for each quantum field theory
- $a \geq 0$ is a measure of the number of degrees of freedom that can be excited. It is a function of energy scale

Major Theme in Formal Theory

- The idea that particle physics at high energies is more complicated then at low energies is a unifying theme
- The task of formal theorists is to make this intuition precise and quantitative

The *a*-Theorem:

- Define a quantity "a" for each quantum field theory
- $a \geq 0$ is a measure of the number of degrees of freedom that can be excited. It is a function of energy scale
- Prove that a is a monotonic function: $a_{UV} > a_{IR}$

Major Theme in Formal Theory

- The idea that particle physics at high energies is more complicated then at low energies is a unifying theme
- The task of formal theorists is to make this intuition precise and quantitative

The *a*-Theorem:

- Define a quantity "*a*" for each quantum field theory
- $a \geq 0$ is a measure of the number of degrees of freedom that can be excited. It is a function of energy scale
- Prove that a is a monotonic function: $a_{UV} > a_{IR}$

Recent significant progress: Cardy, Intriligator, Komargodski-Schwimmer, Cordova-Dumitrescu-Intriligator, $\cdots$

The Challenge

- Why is this problem hard? Can't we just count the number of fields with masses less than a given energy scale?

The Challenge

- Why is this problem hard? Can't we just count the number of fields with masses less than a given energy scale?
- This is problematic because of strong coupling phenomena

The Challenge

- Why is this problem hard? Can't we just count the number of fields with masses less than a given energy scale?
- This is problematic because of strong coupling phenomena
- Consider for instance QCD:

The Challenge

- Why is this problem hard? Can't we just count the number of fields with masses less than a given energy scale?
- This is problematic because of strong coupling phenomena
- Consider for instance QCD:
  - At high energies: quarks, gluons

The Challenge

- Why is this problem hard? Can't we just count the number of fields with masses less than a given energy scale?
- This is problematic because of strong coupling phenomena
- Consider for instance QCD:
  - At high energies: quarks, gluons
  - At low energies: mesons, baryons

The Challenge

- Why is this problem hard? Can't we just count the number of fields with masses less than a given energy scale?
- This is problematic because of strong coupling phenomena
- Consider for instance QCD:
  - At high energies: quarks, gluons
  - At low energies: mesons, baryons

  In what sense is the IR simpler?!

The Challenge

- Why is this problem hard? Can't we just count the number of fields with masses less than a given energy scale?
- This is problematic because of strong coupling phenomena
- Consider for instance QCD:
  - At high energies: quarks, gluons
  - At low energies: mesons, baryons

  In what sense is the IR simpler?!
- Need our measure of degrees of freedom a to be sufficiently refined that it can handle this kind of example

How to Define a

- Use the one operator that every quantum field theory possesses: the energy momentum tensor $T_{\mu\nu}(x)$ (conserved currents from translational symmetry)

How to Define a

- Use the one operator that every quantum field theory possesses: the energy momentum tensor $T_{\mu\nu}(x)$ (conserved currents from translational symmetry)
- For instance in Yang-Mills theory:

$$T_{\mu\nu} = F^c_{\mu\alpha} F^c_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu} F^c_{\alpha\beta} F^c_{\alpha\beta}$$

Note the sum over species c. So $T_{\mu\nu}$ can be used to count the number of colors. Similar expressions for other fields

How to Define a

- Use the one operator that every quantum field theory possesses: the energy momentum tensor $T_{\mu\nu}(x)$ (conserved currents from translational symmetry)
- For instance in Yang-Mills theory:

$$T_{\mu\nu} = F^c_{\mu\alpha} F^c_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu} F^c_{\alpha\beta} F^c_{\alpha\beta}$$

  Note the sum over species c. So $T_{\mu\nu}$ can be used to count the number of colors. Similar expressions for other fields
- To compute a for any theory we look at correlation functions of $T_{\mu\nu}$'s

$$\langle T_{\mu_1\nu_1(x)} T_{\mu_2\nu_2(y)} T_{\mu_3\nu_3(z)} \rangle \sim a$$

Defining a Using Scattering Amplitudes

- Alternative definition, look at scattering amplitudes. These amplitudes involve gravitons, as well as the rest of the theory.

Defining a Using Scattering Amplitudes

- Alternative definition, look at scattering amplitudes. These amplitudes involve gravitons, as well as the rest of the theory. For instance in Yang-Mills theory a can be determined from the amplitude

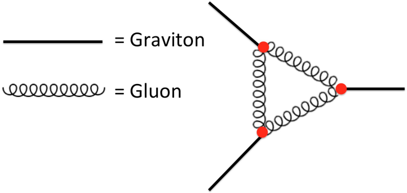


Figure: Defining a from scattering amplitudes

Defining a Using Scattering Amplitudes

- Alternative definition, look at scattering amplitudes. These amplitudes involve gravitons, as well as the rest of the theory. For instance in Yang-Mills theory a can be determined from the amplitude

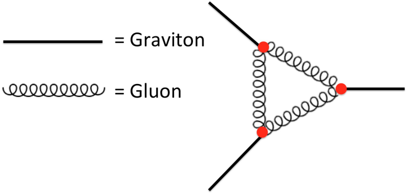


Figure: Defining a from scattering amplitudes

- Of course, gravitons have not been experimentally detected but they can be used as formal probes of the theory. This also makes sense: gravity couples to everything, so its a good for measuring degrees of freedom.

Monotonacity of a from Scattering Amplitudes

- Remarkably, the same quantity a also shows up in a 4-graviton scattering amplitude

Monotonacity of a from Scattering Amplitudes

- Remarkably, the same quantity a also shows up in a 4-graviton scattering amplitude

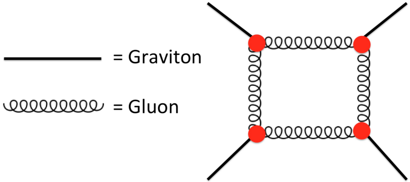


Figure: a as a function of energy scale

Monotonacity of a from Scattering Amplitudes

- Remarkably, the same quantity a also shows up in a 4-graviton scattering amplitude

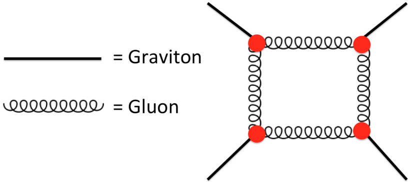


Figure: a as a function of energy scale

- This scattering amplitude depends on a center of mass energy E of the gravitons. Thus it gives us a function $a(E)$.

Monotonacity of a from Scattering Amplitudes

- Remarkably, the same quantity a also shows up in a 4-graviton scattering amplitude

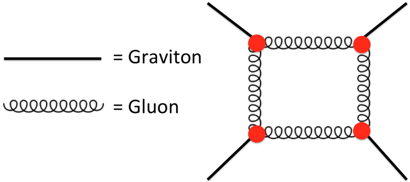


Figure: a as a function of energy scale

- This scattering amplitude depends on a center of mass energy E of the gravitons. Thus it gives us a function $a(E)$.
- We want to show that this function is monotonic:

$$E_1 > E_2 \Longrightarrow a(E_1) > a(E_2)$$

Main Results

- The function $a(E)$ can be investigated using dispersion relations.

Main Results

- The function $a(E)$ can be investigated using dispersion relations. From the optical theorem:

$$a(E_1) - a(E_2) = \int_{s=E_2}^{s=E_1} ds \, \frac{\sigma(s)}{s^2} \ ,$$

Where σ is the total cross section for two gravitons to anything.

Main Results

- The function $a(E)$ can be investigated using dispersion relations. From the optical theorem:

$$a(E_1) - a(E_2) = \int_{s=E_2}^{s=E_1} ds \, \frac{\sigma(s)}{s^2} \; ,$$

  Where σ is the total cross section for two gravitons to anything.

- This is positive definite and establishes the a-Theorem! Komargodski-Schwimmer

Main Results

- The function $a(E)$ can be investigated using dispersion relations. From the optical theorem:

$$a(E_1) - a(E_2) = \int_{s=E_2}^{s=E_1} ds \, \frac{\sigma(s)}{s^2} \ ,$$

  Where σ is the total cross section for two gravitons to anything.
- This is positive definite and establishes the a-Theorem! Komargodski-Schwimmer
- This line of investigation has been generalized in numerous directions (e.g different spacetime dimensions, or condensed matter applications)

Main Results

- The function $a(E)$ can be investigated using dispersion relations. From the optical theorem:

$$a(E_1) - a(E_2) = \int_{s=E_2}^{s=E_1} ds \, \frac{\sigma(s)}{s^2} \; ,$$

  Where σ is the total cross section for two gravitons to anything.
- This is positive definite and establishes the a-Theorem! Komargodski-Schwimmer
- This line of investigation has been generalized in numerous directions (e.g different spacetime dimensions, or condensed matter applications)

Conclusions:

These ideas make rigorous our basic intuition that complexity in physics grows with the energy scale!