Recent Progress on Renormalization Group Flow

Clay Córdova

School of Natural Sciences
Institute for Advanced Study

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Quantum Field Theory is Organized by Scale

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Upshot:
At high energies there are more particle species, more interactions possible, more information
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The $a$-Theorem:

- Define a quantity $a$ for each quantum field theory.
- $a \geq 0$ is a measure of the number of degrees of freedom that can be excited. It is a function of energy scale.
- Prove that $a$ is a monotonic function: $a_{UV} > a_{IR}$.

Recent significant progress: Cardy, Intriligator, Komargodski-Schwimmer, Cordova-Dumitrescu-Intriligator, ···
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- Need our measure of degrees of freedom $a$ to be sufficiently refined that it can handle this kind of example
How to Define $a$

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• For instance in Yang-Mills theory:

$$T_{\mu\nu} = F_{\mu\alpha}^c F_{\nu\alpha}^c - \frac{1}{4} \delta_{\mu\nu} F_{\alpha\beta}^c F_{\alpha\beta}^c$$

Note the sum over species $c$. So $T_{\mu\nu}$ can be used to count the number of colors. Similar expressions for other fields
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- To compute $a$ for any theory we look at correlation functions of $T_{\mu \nu}$’s

$$ \langle T_{\mu_1 \nu_1}(x) T_{\mu_2 \nu_2}(y) T_{\mu_3 \nu_3}(z) \rangle \sim a $$
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![Diagram](image)

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**Figure:** Defining \( a \) from scattering amplitudes
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![Diagram showing a 4-graviton scattering amplitude with a center of mass energy $E$ and a function $a(E)$.]

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- This scattering amplitude depends on a center of mass energy $E$ of the gravitons. Thus it gives us a function $a(E)$.
- We want to show that this function is monotonic:

\[ E_1 > E_2 \implies a(E_1) > a(E_2) \]
Main Results

- The function \( a(E) \) can be investigated using dispersion relations.

\[
\int_{s = E_1}^{s = E_2} \sigma(s) \, ds
\]

Where \( \sigma \) is the total cross section for two gravitons to anything.

This is positive definite and establishes the \( a \)-Theorem!

Komargodski-Schwimmer

This line of investigation has been generalized in numerous directions (e.g. different spacetime dimensions, or condensed matter applications).

Conclusions:

These ideas make rigorous our basic intuition that complexity in physics grows with the energy scale!
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$$a(E_1) - a(E_2) = \int_{s=E_2}^{s=E_1} ds \frac{\sigma(s)}{s^2},$$

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