Towards Higher Precision Parton Showers

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ICHEP

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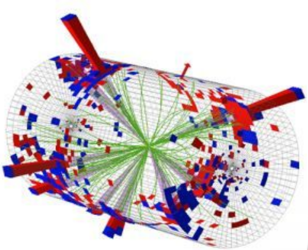
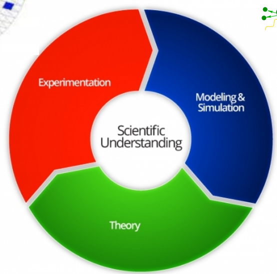
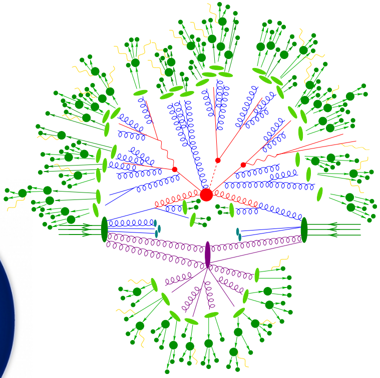
How to make sense of hadron collider data







$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\Psi} \not{D} \psi + h.c.$$

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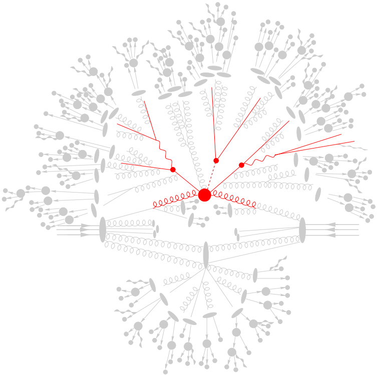

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays

Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \to X} = \sum_{i,j \in \{q,g\}} \int \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{f_{p_1,i}(x_1,\mu_F^2) f_{p_2,j}(x_2,\mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij\to X}(x_1 x_2,\mu_F^2)}_{\text{short distance}}$$

Factorization to the rescue

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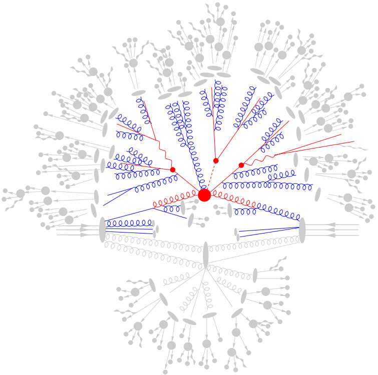

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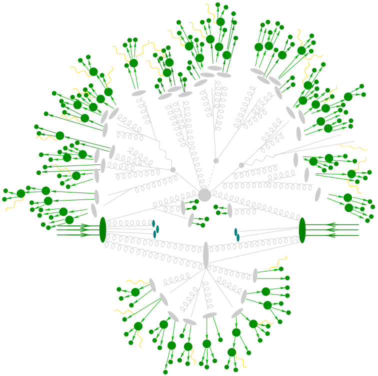

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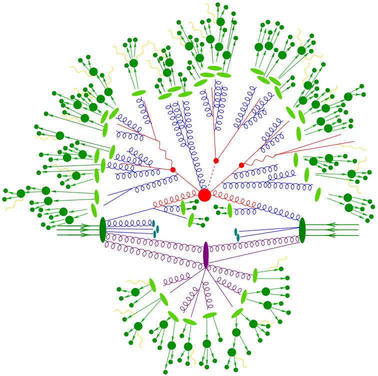

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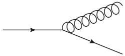
Parton showers in a nutshell

[Marchesini,Webber] NPB238(1984)1, [Sjöstrand] PLB157(1985)321

- Parton "decay" can occur in two ways:



\- observed

\- unobserved

- Probability conservation $\Rightarrow$ all observed + all unobserved = 1
  Splitting governed by Poisson statistics $\rightarrow$ survival probability $\Delta(t,t')$

$$\Delta(t,t') = \exp\left\{-\int_t^{t'} \mathrm{d}\bar{t}\,\Gamma(\bar{t})\right\}, \quad \Gamma(t) = \sum_b \frac{\alpha_s}{2\pi\,t}\int\frac{\mathrm{d}z}{z}\,P_{ba}(z)\,\frac{f_b(x/z,t)}{f_a(x,t)}$$

- Key to Markovian Monte-Carlo simulation of DGLAP equations
- Open questions
  - Implementation of local four-momentum conservation
  - Functional form of evolution "time"
  - Choice of renormalization scale

Scale uncertainties

[Les Houches SM WG] arXiv:1605.04692

- Choice of renormalization scale in parton showers
  - k_T [Amati,Bassetto,Ciafaloni,Marchesini,Veneziano] NPB173(1980)429
  - CMW rescaling [Catani,Marchesini,Webber] NPB349(1991)635
  - potentially additional factor to be tuned to data
- Scale variations typically not considered
  First attempt during LesHouches '15
- Participating projects
  - Deductor [Nagy,Soper] arXiv:1401.6364
  - Herwig [Bellm,Plätzer,Richardson,Siódmok,Webster] arXiv:1605.08256
    - $\tilde{q}$-shower [Gieseke,Stephens,Webber] hep-ph/0310083
    - Dipole shower [Plätzer,Gieseke] arXiv:0909.5593
  - Sherpa [Bothmann,Schönherr,Schumann] arXiv:1606.08753
    - Ants [Krauss,Zapp] in preparation
    - CSS [Schumann,Krauss] arXiv:0709.1027
    - Dire [Prestel,SH] arXiv:1506.05057
  - Pythia [Mrenna,Skands] arXiv:1605.08352

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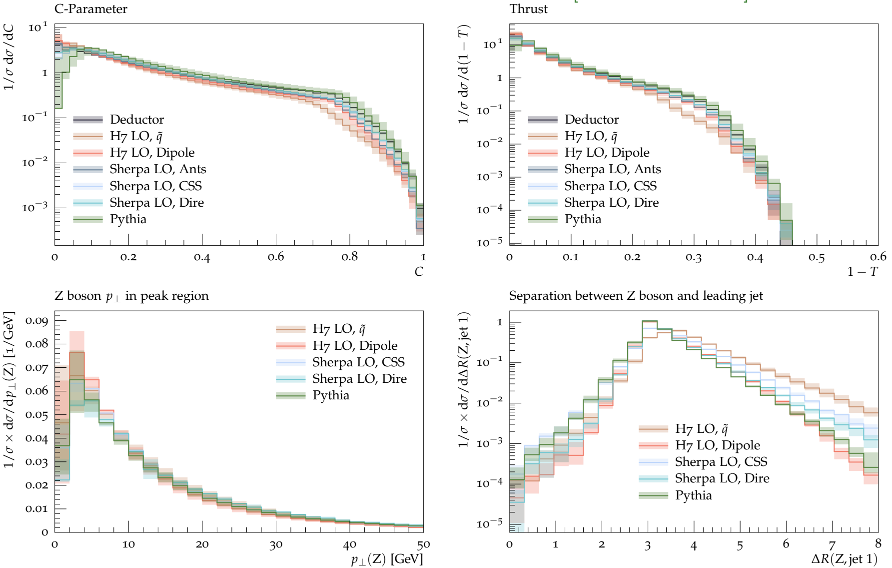
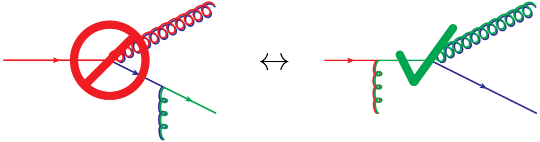


Color coherence and the dipole picture

[Marchesini,Webber] NPB310(1988)461

- Individual color charges inside a color dipole cannot be resolved by gluons of wavelength larger than the dipole size
  → emission off combined mother parton instead



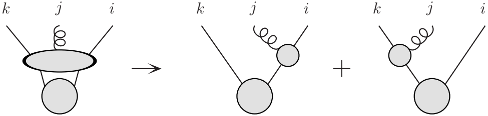
- Net effect is destructive interference outside cone with opening angle defined by emitting color dipole
  → Soft anomalous dimension halved due to reduced phase space
- Formerly implemented by angular ordering / angular veto
- Alternative description in terms of color dipoles

  [Gustafsson,Pettersson] NPB306(1988)746, [Kharraziha,Lönnblad] hep-ph/9709424
  [Winter,Krauss] arXiv:0712.3913

The midpoint between dipole and parton showers

- Angular ordered / vetoed parton shower does not fill full phase space
  Dipole shower lacks parton interpretation $\to$ prefer alternative to both
- Can preserve parton picture by partial fractioning soft eikonal
  $\leftrightarrow$ soft enhanced part of splitting function [Catani,Seymour] hep-ph/9605323

$$\frac{p_ip_k}{(p_ip_j)(p_jp_k)} \to \frac{1}{p_ip_j}\,\frac{p_ip_k}{(p_i+p_k)p_j} + \frac{1}{p_kp_j}\,\frac{p_ip_k}{(p_i+p_k)p_j}$$



- "Spectator"-dependent kernels, singular in soft-collinear region only
  $\to$ capture dominant coherence effects (3-parton correlations)

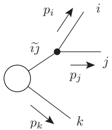
$$\frac{1}{1-z} \to \frac{1-z}{(1-z)^2+\kappa^2} \qquad \kappa^2 = \frac{k_\perp^2}{Q^2}$$

- For correct soft evolution, ordering variable must be identical
  at both "dipole ends" ($\to$ recover soft eikonal at integrand level)

The midpoint between dipole and parton showers

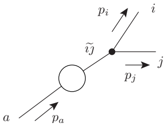
Choose parametrization such that soft term is $\frac{1-z}{(1-z)^2+\kappa^2}$ in all dipole types

(1) FF



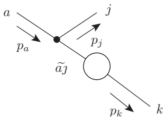
$$\kappa^2 = \frac{p_i p_j \, p_j p_k}{(p_{\widetilde{ij}} p_{\tilde{k}})^2} \qquad z_j = \frac{p_j p_k}{p_{\widetilde{ij}} p_{\tilde{k}}}$$

(2) FI



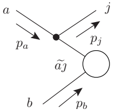
$$\kappa^2 = \frac{p_i p_j \, p_j p_a}{(p_{ij} p_a)^2} \qquad z_j = \frac{p_j p_a}{p_{ij} p_a}$$

(3) IF



$$\kappa^2 = \frac{p_a p_j \, p_j p_k}{(p_{jk} p_a)^2} \qquad z_j = \frac{p_j p_k}{p_{jk} p_a}$$

(4) II



$$\kappa^2 = \frac{p_a p_j \, p_j p_b}{(p_a p_b)^2} \qquad z_j = \frac{p_j p_b}{p_a p_b}$$

Preserve collinear anomalous dimensions & sum rules $\rightarrow$ splitting functions fixed

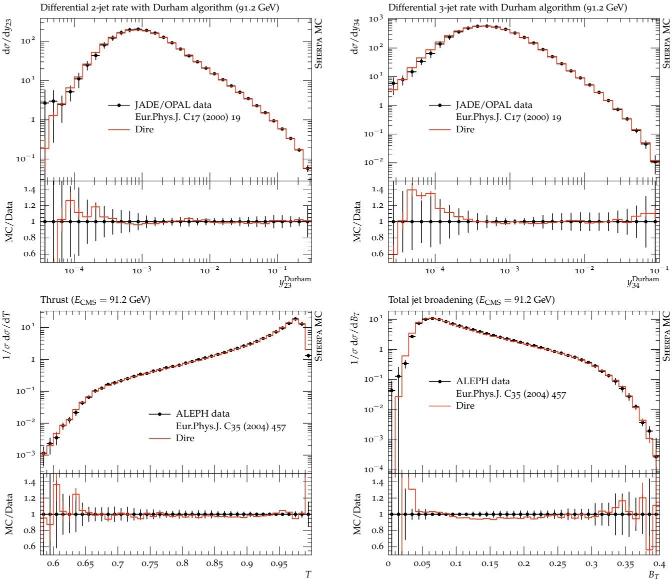
$$P_{qq}(z,\kappa^2) = 2\,C_F\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ - \frac{1+z}{2}\right] + \gamma_q\,\delta(1-z)$$

$$P_{gg}(z,\kappa^2) = 2\,C_A\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ + \frac{z}{z^2+\kappa^2} - 2 + z(1-z)\right] + \gamma_g\,\delta(1-z)$$

$$P_{qg}(z,\kappa^2) = 2\,C_F\left[\frac{z}{z^2+\kappa^2} - \frac{2-z}{2}\right] \qquad P_{gq}(z,\kappa^2) = T_R\left[z^2+(1-z)^2\right]$$

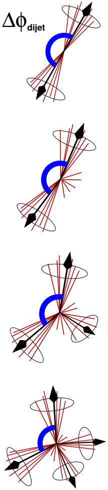
Predictions for LEP

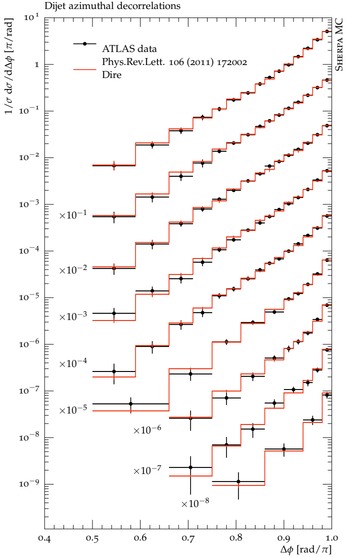
[Prestel,SH] arXiv:1506.05057

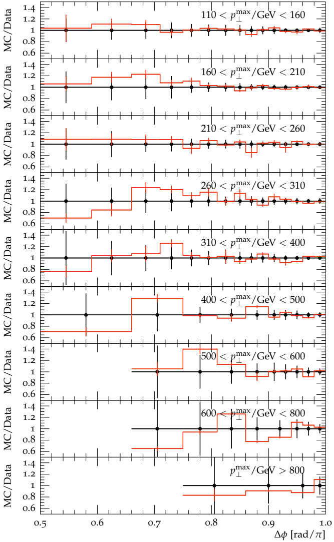


Predictions for the LHC

[Prestel,SH] arXiv:1506.05057







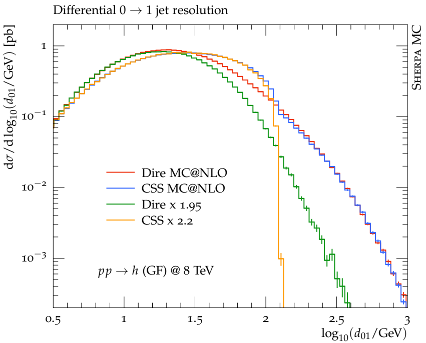
NLO counterterms and MC@NLO matching

[SH]

- Can view new shower model as modification of CS subtraction
- IR-finite counterterms computed, implemented in event generator Sherpa (improved cancellation in $pp \to h + j$ due to regulated $1/z$ terms)

- Sherpa MC@NLO based on exponentiation of CS dipole subtraction terms

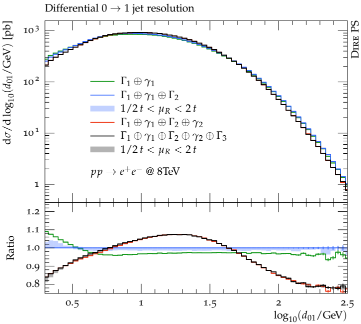
  [Krauss,Siegert,Schönherr,SH]
  arXiv:1111.1220, arXiv:1208.2815
- Dire modified CS subtraction automatically available for MC@NLO matching
- Interesting differences due to evolution variables and kernels

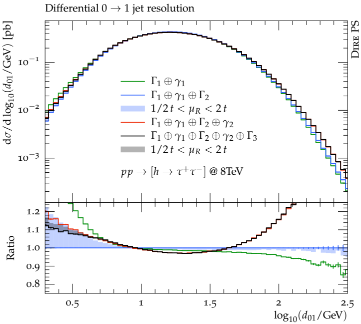


Towards higher logarithmic accuracy

[Catani,Krauss,Prestel,SH] soon

- Big drawback of parton showers is lack of higher-order kernels
- Start improving with integrated NLO splitting functions
  [Curci,Furmanski,Petronzio] NPB175(1980)27, PLB97(1980)437
- 2-loop cusp term subtracted & combined with LO soft contribution (similar to CMW rescaling [Catani,Marchesini,Webber] NPB349(1991)635)
- Implemented using weighting algorithms [Schumann,Siegert,SH] arXiv:0912.3501





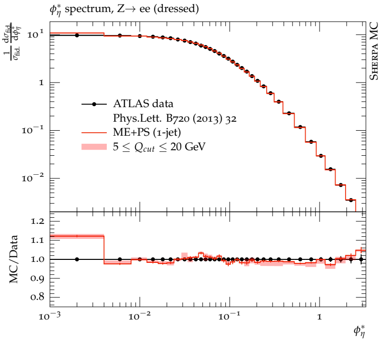
Outlook

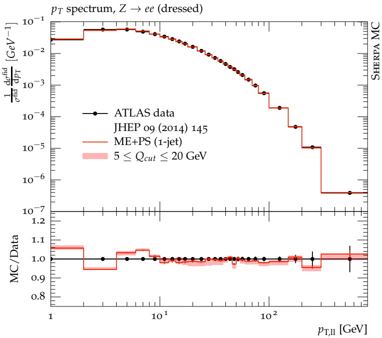
- Parton showers are indispensable tools for
  - phenomenology
  - experimental analysis
  - experiment design
- Proper treatment of soft gluon radiation is essential
- Matching at (N)NLO & merging at (N)LO
  improves PS approximation at fixed jet multiplicity
  $\rightarrow$ focus area of development during past decade
- Reduction of uncertainties in intra-jet region
  or for jet multiplicites beyond reach at fixed order
  requires improved resummation $\rightarrow$ NLO kernels
- This development just started ... stay tuned!

Thank you for your attention!

The midpoint between dipole and parton showers

[Prestel,SH] arXiv:1506.05057





- Parton shower merged with 1-jet tree-level ME using CKKW-L

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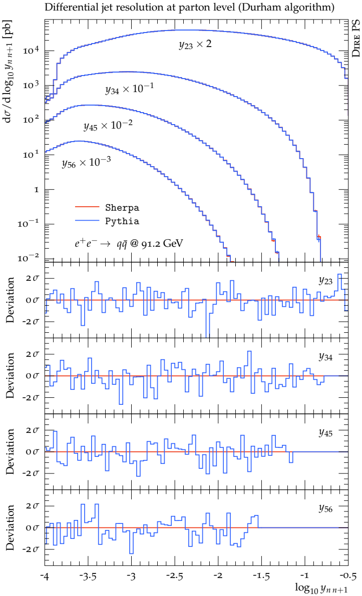

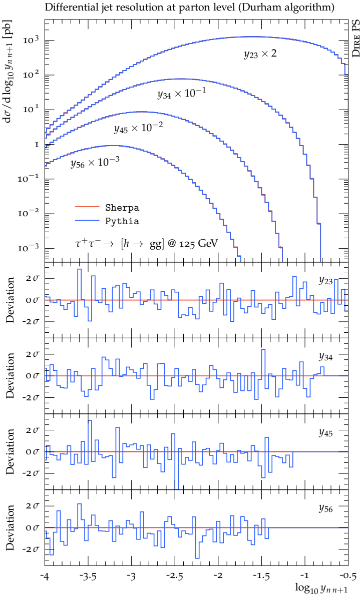

Negative "probabilities" in parton showers

- Standard parton shower: $\Delta(t,t') = \exp\{F(t) - F(t')\}$

  Exact MC solution $t = F^{-1}\big[\,F(t') + \log R\,\big]$, R – random number

  But don't want to compute $F(t) = -\int_t \mathrm{d}\bar{t} f(\bar{t})$, as $f(t) = \sum_b \int \mathrm{d}z \frac{\alpha_s}{2\pi\,t} \mathrm{P}_{ab}(t,z)$

Solution in veto algorithm (hit-or-miss for Poisson distributions)

- Find overestimate $g(t) > f(t)$ with simple integral $G(t)$
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Probability for one acceptance

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} \mathrm{d}\bar{t}\, g(\bar{t})\right\}$$

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Probability for one acceptance with one rejection

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} \mathrm{d}\bar{t}\, g(\bar{t})\right\} \left[\int_t^{t'} \mathrm{d}t_1 \left(1 - \frac{f(t_1)}{g(t_1)}\right) g(t_1)\, \exp\left\{-\int_{t_1}^{t'} \mathrm{d}\bar{t}\, g(\bar{t})\right\}\right]$$

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Probability for one acceptance with two rejections

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} \mathrm{d}\bar{t}\, g(\bar{t})\right\} \left[\int_t^{t'} \mathrm{d}t_1 \left(1 - \frac{f(t_1)}{g(t_1)}\right) g(t_1)\, \exp\left\{-\int_{t_1}^{t_2} \mathrm{d}\bar{t}\, g(\bar{t})\right\}\right]$$
$$\times \left[\int_{t_1}^{t'} \mathrm{d}t_2 \left(1 - \frac{f(t_2)}{g(t_2)}\right) g(t_2)\, \exp\left\{-\int_{t_2}^{t'} \mathrm{d}\bar{t}\, g(\bar{t})\right\}\right]$$

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Probability for one acceptance with n rejections

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} \mathrm{d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} \mathrm{d}t_i \left(1 - \frac{f(t_i)}{g(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} \mathrm{d}\bar{t}\, g(\bar{t})\right\}\right]$$

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Disentangle nested integrals:

$$f(t)\, \exp\left\{-\int_t^{t'} \mathrm{d}\bar{t}\, g(\bar{t})\right\} \frac{1}{n!} \left[\int_t^{t'} \mathrm{d}\bar{t}\, \big(g(\bar{t}) - f(\bar{t})\big)\right]^n$$

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- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Probability for one acceptance with n rejections

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} \mathrm{d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} \mathrm{d}t_i \left(1 - \frac{f(t_i)}{g(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} \mathrm{d}\bar{t}\, g(\bar{t})\right\}\right]$$

Disentangle nested integrals and sum over n:

$$f(t)\, \exp\left\{-\int_t^{t'} \mathrm{d}\bar{t}\, g(\bar{t})\right\} \frac{1}{n!} \left[\int_t^{t'} \mathrm{d}\bar{t}\, (g(\bar{t}) - f(\bar{t}))\right]^n \to f(t)\, \exp\left\{-\int_t^{t'} \mathrm{d}\bar{t}\, f(\bar{t})\right\}$$

Negative "probabilities" in parton showers

- Standard parton shower: $\Delta(t,t') = \exp\{F(t) - F(t')\}$

  Exact MC solution $t = F^{-1}\big[\,F(t') + \log R\,\big]$, R – random number

  But don't want to compute $F(t) = -\int_t \mathrm{d}\bar{t} f(\bar{t})$, as $f(t) = \sum_b \int \mathrm{d}z \frac{\alpha_s}{2\pi\, t} \mathrm{P}_{ab}(t,z)$

Solution in veto algorithm (hit-or-miss for Poisson distributions)

- Find overestimate $g(t) > f(t)$ with simple integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Standard probability for one acceptance with n rejections

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} \mathrm{d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} \mathrm{d}t_i \left(1 - \frac{f(t_i)}{g(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} \mathrm{d}\bar{t}\, g(\bar{t})\right\}\right]$$

Split weight into MC and analytic part using auxiliary function $h(t)$

$$\frac{f(t)}{h(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} \mathrm{d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t'} \mathrm{d}t_i \left(1 - \frac{f(t_i)}{h(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} \mathrm{d}\bar{t}\, g(\bar{t})\right\}\right]$$

$$w(t, t_1, \ldots, t_n) = \frac{h(t)}{g(t)} \prod_{i=1}^{n} \frac{h(t_i)}{g(t_i)} \frac{g(t_i) - f(t_i)}{h(t_i) - f(t_i)}$$

Negative “probabilities” in parton showers

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Weighted veto algorithm

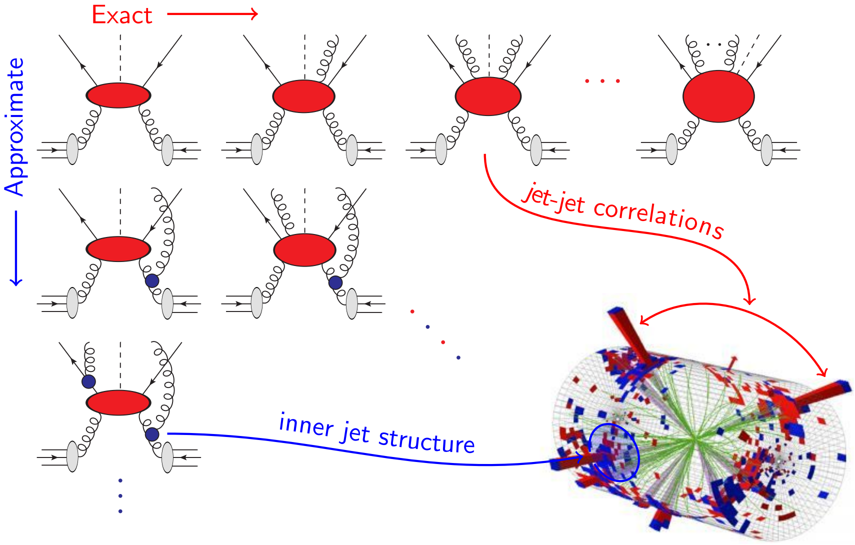
$$\frac{f(t)}{h(t)}\,g(t)\,\exp\left\{-\int_t^{t_1}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t'}\mathrm{d}t_i\left(1-\frac{f(t_i)}{h(t_i)}\right)g(t_i)\,\exp\left\{-\int_{t_i}^{t_{i+1}}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\right]$$

$$w(t,t_1,\ldots,t_n)=\frac{h(t)}{g(t)}\prod_{i=1}^{n}\frac{h(t_i)}{g(t_i)}\frac{g(t_i)-f(t_i)}{h(t_i)-f(t_i)}$$

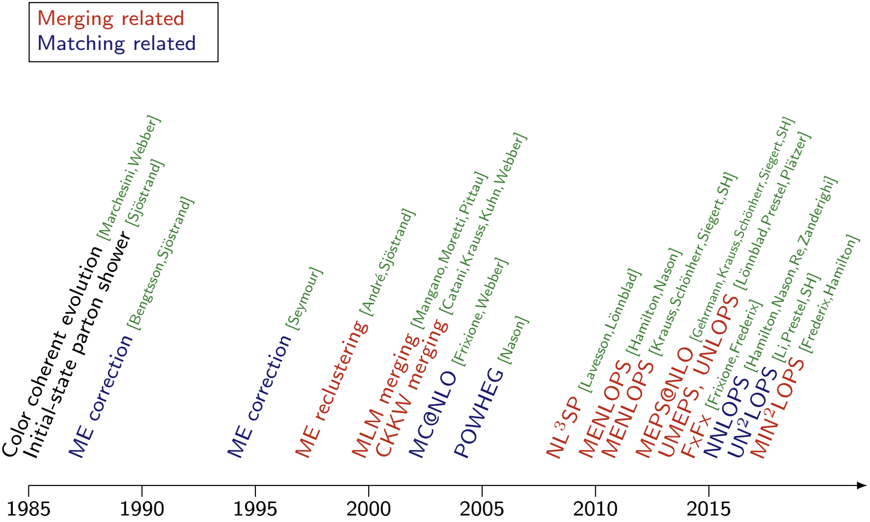
Looks trivial, surprisingly it’s not: It allows to

- Resum sub-leading color terms in MC@NLO and POWHEG
  [Krauss,Schönherr,Siegert,SH] arXiv:1111.1220
- Implement higher-order splitting functions in parton showers
  [Catani,Krauss,Prestel,SH] in preparation
- Use PDFs with negative values in parton showers
  [Prestel,SH] arXiv:1506.05057
- Enhance branching probabilities in parton showers
  [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
- Reweight parton showers [Bellm,Plätzer,Richardson,Siódmok,Webster] arXiv:1605.08256
  [Mrenna,Skands] arXiv:1605.08352, [Bothmann,Schönherr,Schumann] arXiv:1606.08753

Parton-shower matching & merging



The long road to precision simulations



Matrix-element corrections

[Sjöstrand] PLB185(1987)435

- Parton shower evolution kernels similar to NLO subtraction terms
  Equivalent at leading color and w/o spin correlations:

$$\mathrm{D}_{ij,k}(\Phi_\mathrm{R}) = \frac{8\pi\alpha_s}{2p_ip_j}\mathrm{B}(b_{ij,k}(\Phi_\mathrm{R}))\mathrm{P}_{ij,k}(t_{ij,k}, z_{ij,k}, \phi_{ij,k})$$

  $b_{ij,k}$ maps real kinematics to Born, Catani-Seymour style

- Can project real-emission term onto singular regions in PS
  $\rightarrow$ no "leftover" singularities (full color & spin $\nearrow$ next slide)

$$\mathrm{R}_{ij,k}(\Phi_\mathrm{R}) = \rho_{ij,k}(\Phi_R)\mathrm{R}(\Phi_\mathrm{R})\;, \qquad \rho_{ij,k} = \frac{\mathrm{D}_{ij,k}(\Phi_\mathrm{R})}{\sum_{mn,l}\mathrm{D}_{mn,l}(\Phi_\mathrm{R})}$$

- Now replace PS kernels by full real-emission corrections using weight

$$w(\Phi_R) = \left[\sum_{mn,l}\frac{8\pi\alpha_s}{2p_np_m}\frac{\mathrm{B}(b_{mn,l}(\Phi_\mathrm{R}))\mathrm{P}_{mn,l}(t', z', \phi')}{\mathrm{R}(\phi_R)}\right]^{-1}$$

  $\rightarrow$ generic form of a matrix-element correction
  Note splitter-spectator independence, i.e. $w_{ij,k} = w$ for all ij, k

NLO+PS matching – Basics

SLAC

- Leading-order calculation for observable O

$$\langle O\rangle = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B)$$

- NLO calculation for same observable

$$\langle O\rangle = \int \mathrm{d}\Phi_B \left\{\mathrm{B}(\Phi_B) + \tilde{V}(\Phi_B)\right\} O(\Phi_B) + \int \mathrm{d}\Phi_R\, \mathrm{R}(\Phi_R)\, O(\Phi_R)$$

- Parton-shower result (zero and one emission)

$$\langle O\rangle = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B) \left[\Delta^{(\mathrm{K})}(t_c)\, O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1\, \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t(\Phi_1))\, O(\Phi_R)\right]$$

$$\overset{\mathcal{O}(\alpha_s)}{\to} \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B) \left\{1 - \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\right\} O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_B \mathrm{d}\Phi_1\, \mathrm{B}(\Phi_B)\, \mathrm{K}(\Phi_1)\, O(\Phi_R)$$

Phase space: $\mathrm{d}\Phi_1 = \mathrm{d}t\, \mathrm{d}z\, \mathrm{d}\phi\, J(t,z,\phi)$
Splitting functions: $\mathrm{K}(t,z) \to \alpha_s/(2\pi t) \sum \mathrm{P}(z)\, \Theta(\mu_Q^2 - t)$
Sudakov factors: $\Delta^{(\mathrm{K})}(t) = \exp\left\{ -\int_t \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\right\}$

NLO+PS matching – MC@NLO

[Frixione,Webber] hep-ph/0204244

- Subtract $\mathcal{O}(\alpha_s)$ PS terms from subtracted NLO result ($t_c \to 0$)
  $1/N_c$ corrections faded out in soft region by smoothing function

$$\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \left[\mathrm{S}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) \right] f(\Phi_1)$$

$$\mathrm{H}^{(\mathrm{K})}(\Phi_R) = \left[\mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1) \right] f(\Phi_1)$$

- Add parton shower, described by generating functional $\mathcal{F}_{\mathrm{MC}}$

$$\langle O \rangle = \int \mathrm{d}\Phi_B\, \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\, \mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2, O) + \int \mathrm{d}\Phi_R\, \mathrm{H}^{(\mathrm{K})}(\Phi_R)\, \mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R), O)$$

- Expansion of matched result up to first emission

$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) \Bigg[\Delta^{(\mathrm{K})}(t_c)\, O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1) \Delta^{(\mathrm{K})}(t(\Phi_1))\, O(\Phi_r) \Bigg] + \int \mathrm{d}\Phi_R\, \mathrm{H}^{(\mathrm{K})}(\Phi_{n+1})\, O(\Phi_R)$$

NLO+PS matching – POWHEG

[Nason] hep-ph/0409146
[Frixione,Nason,Oleari] arXiv:0709.2092

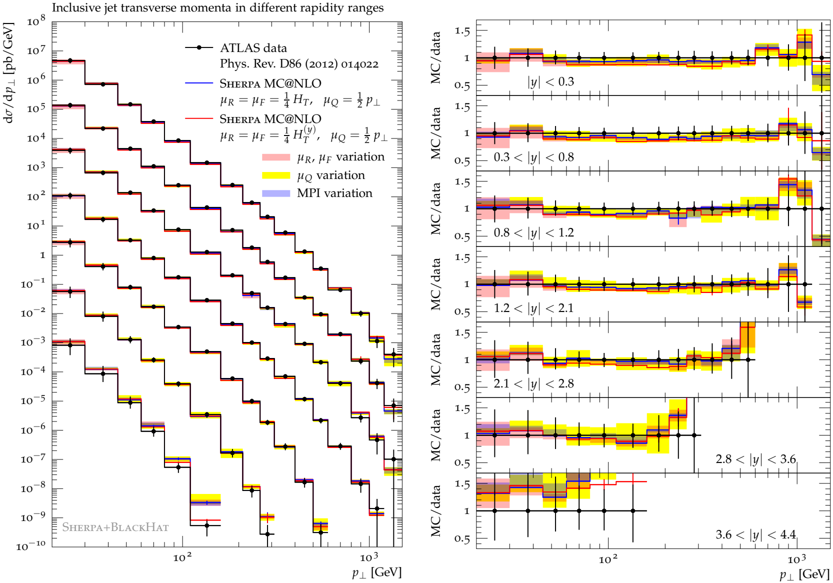
- Replace $\mathrm{BK} \to \mathrm{R} \Rightarrow \mathrm{H}^{(\mathrm{R})}$ zero, $\bar{\mathrm{B}}^{(\mathrm{R})}$ positive in physical region

$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R})}(\Phi_B) \Bigg[\Delta^{(\mathrm{R})}(t_c, s_{\mathrm{had}})\, O(\Phi_B) + \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R}(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R})}(t(\Phi_1), s_{\mathrm{had}})\, O(\Phi_R) \Bigg]$$

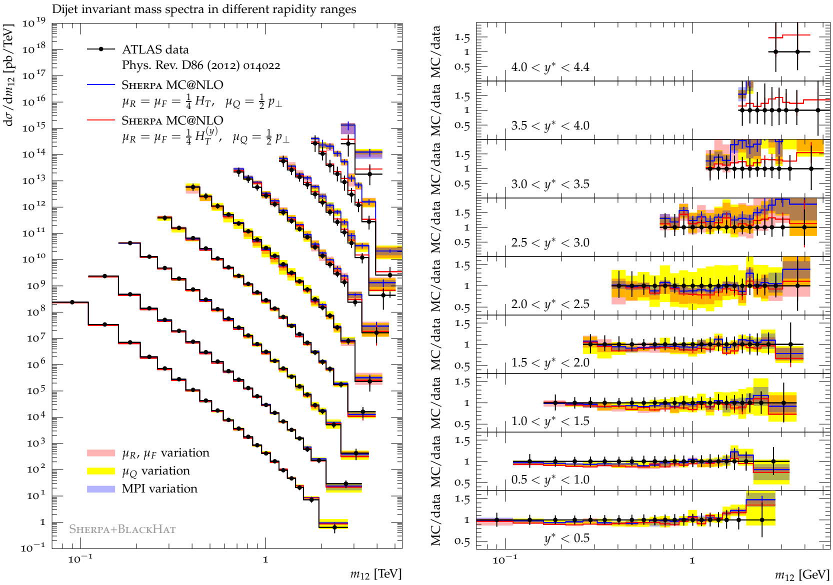
- μ_Q^2 changed to hadronic centre-of-mass energy squared, s_{had}, to cover full phase space for real-emission correction
- Absence of hard events $\to$ enhanced high-p_T region ($K = \bar{\mathrm{B}}/\mathrm{B}$)
  Formally beyond NLO, but often sizeable $\to$ Avoid by split $\mathrm{R} \to \mathrm{R}^s + \mathrm{R}^f$

$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R^s})}(\Phi_B) \Bigg[\Delta^{(\mathrm{R^s})}(t_c, s_{\mathrm{had}})\, O(\Phi_B) + \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R^s}(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R^s})}(t(\Phi_1), s_{\mathrm{had}})\, O(\Phi_R) \Bigg] + \int \mathrm{d}\Phi_R\, \mathrm{R^f}(\Phi_R)$$

Example: Inclusive jet production at the LHC



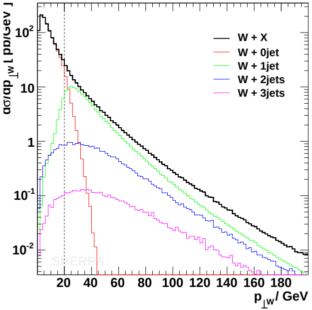
Example: Inclusive jet production at the LHC



Basic idea of ME+PS merging

SLAC

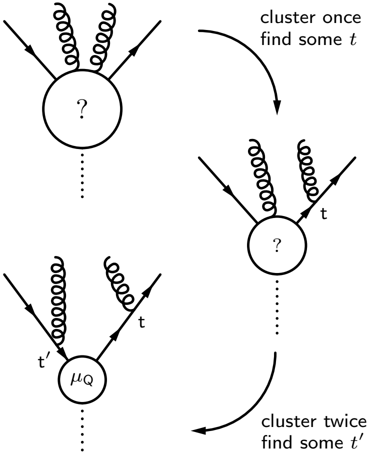
- Separate phase space into "hard" and "soft" region
- Matrix elements populate hard domain
- Parton shower populates soft domain
- Need criterion to define "hard" & "soft" $\rightarrow$ jet measure Q and corresponding cut, $Q_{\rm cut}$



Parton-shower histories

[André,Sjöstrand] hep-ph/9708390

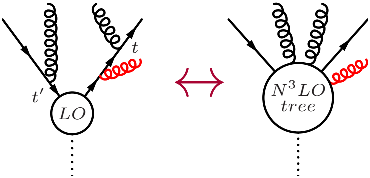
- Start with some "core" process
  for example $e^+e^- \to q\bar{q}$
- This process is considered inclusive
  It sets the resummation scale μ_Q^2
- Higher-multiplicity ME can be reduced to core by clustering
- Clustering algorithm uniquely defined by requiring exact correspondence between ME & PS
  - Identify most likely splitting according to PS emission probability
  - Combine partons into mother according to PS kinematics
  - Continue until no clustering possible



Truncated & vetoed parton showers

[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
[Lönnblad] hep-ph/0112284, arXiv:1211.7204

- Higher-multiplicity MEs that can be reduced to core process are included in core's inclusive cross section (unitarity of PS)
- Sudakov suppression factors needed to make inclusive MEs exclusive
- Most efficiently computed with pseudo-showers
  - Start PS from core process
  - Evolve until predefined branching $\leftrightarrow$ truncated parton shower
  - Emissions producing additional hard jets lead to event veto/weight



$$\Delta^{(\mathrm{K})}(t; >Q_{\mathrm{cut}}) = \exp\left\{ -\int_t \mathrm{d}\Phi_1\, \mathrm{K}(\Phi_1)\, \Theta(Q - Q_{cut}) \right\}$$

ME+PS merging

- ME+PS for 0+1-jet in MC@NLO notation

$$\langle O\rangle = \int \mathrm{d}\Phi_B \mathrm{B}(\Phi_B)\bigg[\Delta^{(\mathrm{K})}(t_c)\,O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\,\Delta^{(\mathrm{K})}(t)\,\Theta(Q_{\mathrm{cut}} - Q)\,O(\Phi_R)\bigg]$$
$$+ \int \mathrm{d}\Phi_R\,\mathrm{R}(\Phi_R)\,\Delta^{(\mathrm{K})}(t(\Phi_R); > Q_{\mathrm{cut}})\,\Theta(Q - Q_{\mathrm{cut}})\,O(\Phi_R) + \ldots$$

- Reorder by parton multiplicity k, change notation $\mathrm{R}_k \to \mathrm{B}_{k+1}$
- Analyze exclusive contribution from k hard partons only $(t_0 = \mu_Q^2)$

$$\langle O\rangle_k^{\mathrm{excl}} = \int \mathrm{d}\Phi_k\,\mathrm{B}_k\,\prod_{i=0}^{k-1}\Delta_i^{(\mathrm{K})}(t_{i+1}, t_i; > Q_{\mathrm{cut}})\,\Theta(Q_k - Q_{\mathrm{cut}})$$
$$\times \bigg[\Delta_k^{(\mathrm{K})}(t_c, t_k)\,O_k + \int_{t_c}^{t_k} \mathrm{d}\Phi_1\;\mathrm{K}_k\,\Delta_k^{(\mathrm{K})}(t_{k+1}, t_k)\,\Theta(Q_{\mathrm{cut}} - Q_{k+1})\,O_{k+1}\bigg]$$

ME+PS merging at NLO

SLAC

- Analyze exclusive contribution from k hard partons

$$
\begin{aligned}
\langle O\rangle_k^{\rm excl} \;=\; & \int {\rm d}\Phi_k\, \bar{\rm B}_k^{\rm (K)} \prod_{i=0}^{k-1} \Delta_i^{\rm (K)}(t_{i+1},t_i;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut}) \\
& \times\left(1+\frac{{\rm B}_k}{\bar{\rm B}_k^{\rm (K)}}\sum_{i=0}^{k-1}\int_{t_{i+1}}^{t_i}{\rm d}\Phi_1{\rm K}_i\,\Theta(Q_i-Q_{\rm cut})\right) \\
& \times\left[\Delta_k^{\rm (K)}(t_c,t_k)\,O_k+\int_{t_c}^{t_k}{\rm d}\Phi_1\;{\rm K}_k\,\Delta_k^{\rm (K)}(t_{k+1},t_k)\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}\right] \\
& +\int {\rm d}\Phi_{k+1}\,{\rm H}_k^{\rm (K)}\,\Delta_k^{\rm (K)}(t_k;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut})\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}
\end{aligned}
$$

- Born matrix element $\rightarrow$ NLO-weighted Born
- Add hard remainder function
- Subtract $\mathcal{O}(\alpha_s)$ terms contained in truncated PS

ME+PS merging – Unitarization

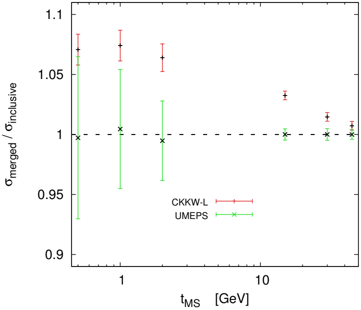
[Lönnblad,Prestel] arXiv:1211.4827, [Plätzer] arXiv:1211.5467

- Unitarity condition of PS:

$$1 = \Delta^{(K)}(t_c) + \int_{t_c} d\Phi_1 \, K(\Phi_1) \, \Delta^{(K)}(t)$$

- ME+PS(@NLO) violates PS unitarity as ME ratio replaces splitting kernels in emission terms, but not in Sudakovs

$$K(\Phi_1) \to \frac{R(\Phi_1, \Phi_B)}{B(\Phi_B)}$$



- Can be corrected by explicit subtraction

$$1 = \underbrace{\left\{ \Delta^{(K)}(t_c) + \int_{t_c} d\Phi_1 \left[K(\Phi_1) - \frac{R(\Phi_1, \Phi_B)}{B(\Phi_B)} \right] \Theta(Q - Q_{\rm cut}) \, \Delta^{(K)}(t) \right\}}_{\text{unresolved emission / virtual correction}}$$

$$+ \underbrace{\int_{t_c} d\Phi_1 \left[K(\Phi_1) \Theta(Q_{\rm cut} - Q) + \frac{R(\Phi_1, \Phi_B)}{B(\Phi_B)} \Theta(Q - Q_{\rm cut}) \right] \Delta^{(K)}(t)}_{\text{resolved emission}}$$

ME+PS merging – Practical implementations

- LO schemes

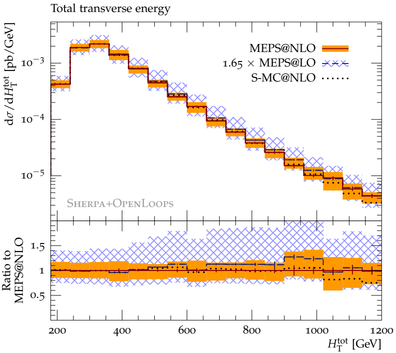
Method	Shower Generator	Unitary	References
MLM	Herwig/Pythia	No	[Mangano,Moretti,Pittau] hep-ph/0108069 [Alwall et al.] arXiv:0706.2569
CKKW	Apacic	No	[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
CKKW-L	Ariadne/Pythia	No	[Lönnblad] hep-ph/0112284 [Lönnblad,Prestel] arXiv:1109.4829
METS	Sherpa CSS	No	[Krauss,Schumann,Siegert,SH] arXiv:0903.1219
CKKW'	Herwig++	No	[Hamilton,Richardson,Tully] arXiv:0905.3072
UMEPS	Pythia/Herwig++	Yes	[Lönnblad,Prestel] arXiv:1211.4827 [Plätzer] arXiv:1211.5467

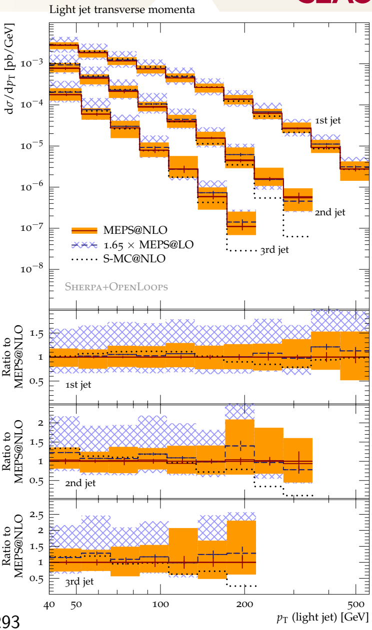
- NLO schemes

Method	Shower Generator	Unitary	References
NL^3	Ariadne/Pythia	No	[Lavesson,Lönnblad] arXiv:0811.2912
MEPS@NLO	Sherpa CSS	No	[Krauss,Schönherr,Siegert,SH] arXiv:1207.5030 [Gehrmann,Krauss,Schönherr,Siegert,SH] 5031
FxFx	Herwig(++)/Pythia	No	[Frederix,Frixione] arXiv:1209.6215
UNLOPS	Pythia/Herwig++	Yes	[Lönnblad,Prestel] arXiv:1211.7278

Example: Top quark pair production

- First matched/merged sim for $t\bar{t}$+2j full result has $t\bar{t}$+0,1,2j@NLO, 3j@LO
- Largely reduced theory uncertainty for both for measurement (p_T, $N_{\rm jet}$) and BSM search (H_T) observables



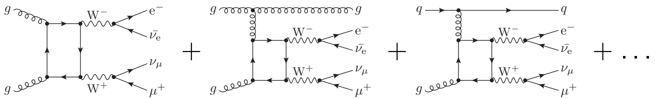


[Krauss,Maierhöfer,Pozzorini,Schönherr,Siegert,SH] arXiv:1402.6293

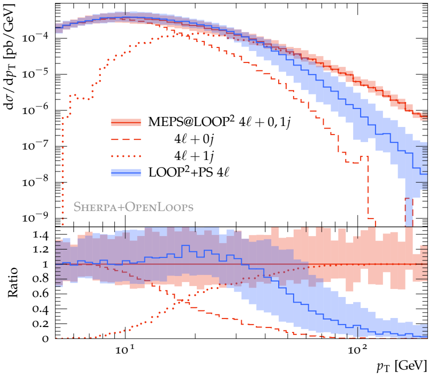
Example: Squared-loop ME+PS merging

[Cascioli,Krauss,Maierhöfer,Pozzorini,Siegert,SH] arXiv:1309.0500

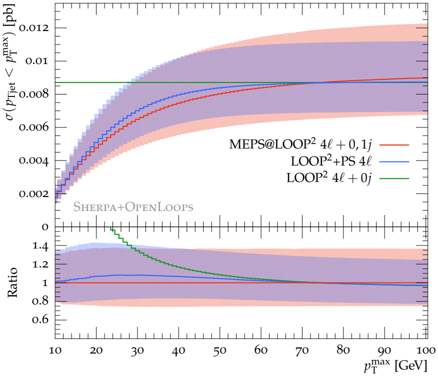
- Combine



Transverse momentum of leading jet



Integrated cross section in the exclusive 0-jet bin



NNLO matching – Precision frontier at particle level

SLAC

[Lönnblad,Prestel] arXiv:1211.4827

- PS expression for infrared safe observable, O

$$\langle O \rangle = \int \mathrm{d}\Phi_0\, \mathrm{B}_0\, \mathcal{F}_0(\mu_Q^2, O)$$

$$\mathcal{F}_n(t,O) = \Delta_n(t_c,t)\, O(\Phi_n) + \int_{t_c}^{t} \mathrm{d}\hat{\Phi}_1\, \mathrm{K}_n\, \Delta_n(\hat{t},t)\, \mathcal{F}_{n+1}(\hat{t},O)$$

- Add ME correction to first emission $(\mathrm{B}_0\mathrm{K}_0 \to \mathrm{B}_1)$ & unitarize

$$+ \int_{t_c} \mathrm{d}\Phi_1\, \Delta_0(t_1,\mu_Q^2)\, \mathrm{B}_1\, \mathcal{F}_1(t_1,O) - \int_{t_c} \mathrm{d}\Phi_1\, \Delta_0(t_1,\mu_Q^2)\, \mathrm{B}_1\, O(\Phi_0)$$

- ME evaluated at fixed scales $\mu_{R/F} \to$ need to adjust to PS

$$w_1 = \frac{\alpha_s(b\,t_1)}{\alpha_s(\mu_R^2)}\, \frac{f_a(x_a,t_1)}{f_a(x_a,\mu_F^2)}\, \frac{f_{a'}(x_{a'},\mu_F^2)}{f_{a'}(x_{a'},t_1)}$$

- Replace B_0 by vetoed xs $\bar{\mathrm{B}}_0^{t_c} = \mathrm{B}_0 - \int_{t_c} \mathrm{d}\Phi_1 \mathrm{B}_1$

$$\langle O \rangle = \left\{ \int \mathrm{d}\Phi_0\, \bar{\mathrm{B}}_0^{t_c} + \int_{t_c} \mathrm{d}\Phi_1 \left[1 - \Delta_0(t_1,\mu_Q^2)\, w_1\right] \mathrm{B}_1 \right\} O(\Phi_0)$$

$$+ \int_{t_c} \mathrm{d}\Phi_1\, \Delta_0(t_1,\mu_Q^2)\, w_1\, \mathrm{B}_1\, \mathcal{F}_1(t_1,O)$$

NNLO matching – Precision frontier at particle level

SLAC

[Li,Prestel,SH] arXiv:1405.3607

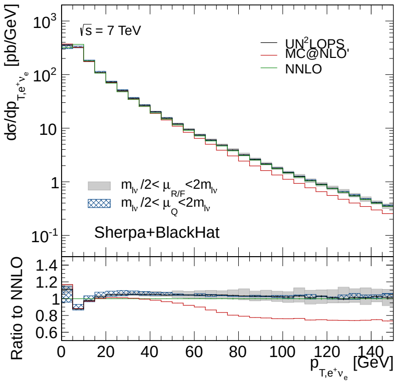
- Promote vetoed cross section to NNLO
- Add NLO corrections to B_1 using MC@NLO
- Subtract $\mathcal{O}(\alpha_s)$ term of w_1 and Δ_0

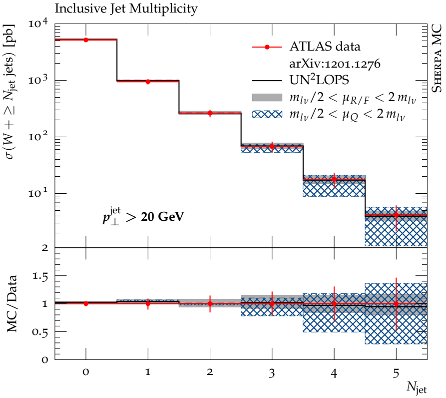
$$
\begin{aligned}
\langle O\rangle = & \int \mathrm{d}\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{t_c}\, O(\Phi_0) \\
& + \int_{t_c} \mathrm{d}\Phi_1 \left[1 - \Delta_0(t_1,\mu_Q^2)\, w_1 \left(1 + w_1^{(1)} + \Delta_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1\, O(\Phi_0) \\
& + \int_{t_c} \mathrm{d}\Phi_1\, \Delta_0(t_1,\mu_Q^2)\, w_1 \left(1 + w_1^{(1)} + \Delta_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{t_c} \mathrm{d}\Phi_1 \left[1 - \Delta_0(t_1,\mu_Q^2)\, w_1\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}\, O(\Phi_0) + \int_{t_c} \mathrm{d}\Phi_1 \Delta_0(t_1,\mu_Q^2)\, w_1\, \tilde{\mathrm{B}}_1^{\mathrm{R}}\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{t_c} \mathrm{d}\Phi_2 \left[1 - \Delta_0(t_1,\mu_Q^2)\, w_1\right] \mathrm{H}_1^{\mathrm{R}}\, O(\Phi_0) + \int_{t_c} \mathrm{d}\Phi_2\, \Delta_0(t_1,\mu_Q^2)\, w_1\, \mathrm{H}_1^{\mathrm{R}}\, \mathcal{F}_2(t_2,O) \\
& + \int_{t_c} \mathrm{d}\Phi_2\, \mathrm{H}_1^{\mathrm{E}}\, \mathcal{F}_2(t_2,O)
\end{aligned}
$$

- $\tilde{\mathrm{B}}_1^{\mathrm{R}} = \bar{\mathrm{B}}_1 - \mathrm{B}_1 = \tilde{\mathrm{V}}_1 + \mathrm{I}_1 + \int \mathrm{d}\Phi_{+1}\mathrm{S}_1\Theta(t_2 - t_1)$
  $\mathrm{H}_1^{\mathrm{R}}$ ($\mathrm{H}_1^{\mathrm{E}}$) $\to$ regular (exceptional) double real configurations

NNLO matching – Precision frontier at particle level

[Li,Prestel,SH] arXiv:1405.3607

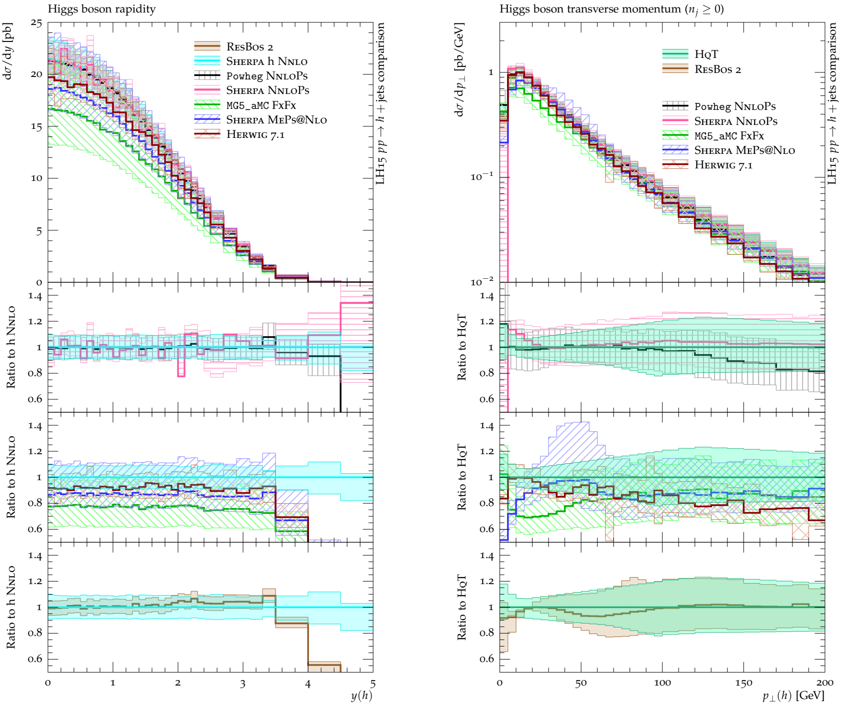




Comparison of approaches

[Les Houches SM WG] arXiv:1605.04692

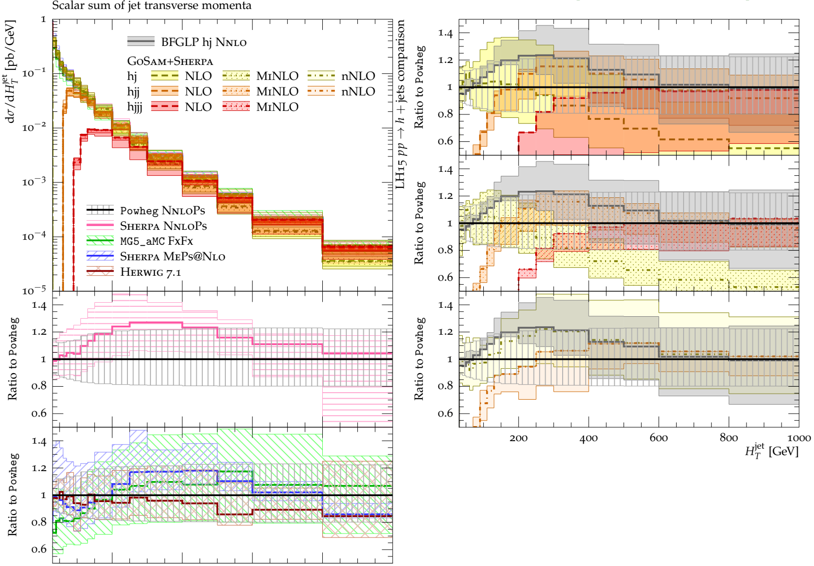
- Setup
  - Stable Higgs
  - anti-k_T jets, R =0.4, $p_{T,j}$ >30 GeV $|\eta_j|$ <4.4
- Calculations & tools in the comparison
  - Fixed-order NLO for $h+\leq 3$ jets, $H'_T/2$ & MINLO
  - LoopSim
  - NNLO for $pp \to h$ (Sherpa), $pp \to h+j$ (BFGLP)
  - Resummed h-p_T (HqT & ResBos)
  - Resummed jet veto cross section (STWZ)
  - NNLO+PS (POWHEG & Sherpa)
  - Multi-jet merging up to 2 jets at NLO (Madgraph5_aMC@NLO, Herwig 7.1)
  - Multi-jet merging up to 3 jets at NLO (Sherpa)
  - High-energy resummation (HEJ)



Higgs boson rapidity
dσ/dy [pb]
ResBos 2
Sherpa h Nnlo
Powheg NnloPs
Sherpa NnloPs
MG5_aMC FxFx
Sherpa MePs@Nlo
Herwig 7.1
LH15 pp → h + jets comparison
Ratio to h Nnlo
y(h)
Higgs boson transverse momentum ($n_j \geq 0$)
dσ/dp⊥ [pb/GeV]
HqT
ResBos 2
Powheg NnloPs
Sherpa NnloPs
MG5_aMC FxFx
Sherpa MePs@Nlo
Herwig 7.1
LH15 pp → h + jets comparison
Ratio to HqT
p⊥(h) [GeV]

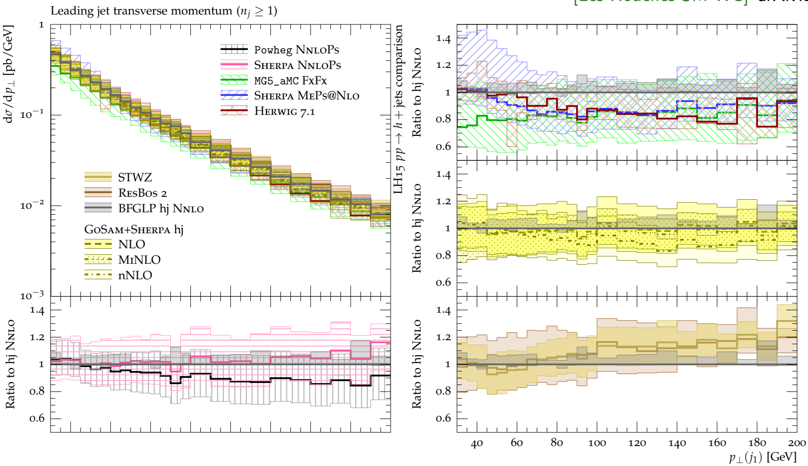
Comparison of approaches

[Les Houches SM WG] arXiv:1605.04692



Comparison of approaches

[Les Houches SM WG] arXiv:1605.04692



Comparison of approaches

[Les Houches SM WG] arXiv:1605.04692