Towards Higher Precision Parton Showers

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How to make sense of hadron collider data

\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \psi \gamma^\mu \psi + \text{h.c.} \]
Factorization to the rescue

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays

Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

\[
\sigma_{p_1p_2 \to X} = \sum_{i,j \in \{q,g\}} \int dx_1 dx_2 \int f_{p_1,i}(x_1, \mu_F^2) f_{p_2,j}(x_2, \mu_F^2) \hat{\sigma}_{ij} \to X(x_1 x_2, \mu_F^2)
\]

\[\text{long distance} \quad \text{short distance}\]
Factorization to the rescue

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Divide and Conquer

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\sigma_{p_1 p_2 \rightarrow X} = \sum_{i,j \in \{q,g\}} \int dx_1 dx_2 \left( f_{p_1,i}(x_1, \mu_F^2) f_{p_2,j}(x_2, \mu_F^2) \hat{\sigma}_{ij} \right) X(x_1 x_2, \mu_F^2)
\]
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\]

\(\hat{\sigma}_{ij}\)

long distance

short distance
Factorization to the rescue

- Short distance interactions
  - Signal process
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Divide and Conquer

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\]
Parton showers in a nutshell

- Parton “decay” can occur in two ways:
  - observed
  - unobserved

- Probability conservation $\Rightarrow$ all observed + all unobserved $= 1$

Splitting governed by Poisson statistics $\Rightarrow$ survival probability $\Delta(t, t')$

$$\Delta(t, t') = \exp \left\{ - \int_{t}^{t'} d\bar{t} \Gamma(\bar{t}) \right\}, \quad \Gamma(t) = \sum_b \frac{\alpha_s}{2\pi t} \int \frac{dz}{z} P_{ba}(z) \frac{f_{b}(x/z, t)}{f_{a}(x, t)}$$

- Key to Markovian Monte-Carlo simulation of DGLAP equations

- Open questions
  - Implementation of local four-momentum conservation
  - Functional form of evolution “time”
  - Choice of renormalization scale
Scale uncertainties

Choice of renormalization scale in parton showers

- CMW rescaling [Catani,Marchesini,Webber] NPB349(1991)635
- potentially additional factor to be tuned to data

Scale variations typically not considered
First attempt during LesHouches ’15

Participating projects

  - $\tilde{q}$-shower [Gieseke,Stephens,Webber] hep-ph/0310083
Scale uncertainties

C-Parameter

1/σ × dσ/dC

10^1

10^-1

10^-2

10^-3

Deducator
H7 LO, ̇q
H7 LO, Dipole
Sherpa LO, Ants
Sherpa LO, CSS
Sherpa LO, Dire
Pythia

0 0.2 0.4 0.6 0.8 1

Z boson p_⊥ in peak region

1/σ × dσ/dp_⊥(Z) [1/GeV]

0 10 20 30 40 50

0

0.01

0.02

0.03

0.04

0.05

0.06

0.07

0.08

0.09

H7 LO, ̇q
H7 LO, Dipole
Sherpa LO, CSS
Sherpa LO, Dire
Pythia

Separation between Z boson and leading jet

1/σ × dσ/dΔR(Z, jet 1)

0 1 2 3 4 5 6 7 8

10^-5

10^-4

10^-3

10^-2

10^-1

1

0 1 2 3 4 5 6 7 8

ΔR(Z, jet 1)

10^-5

10^-4

10^-3

10^-2

10^-1

1

H7 LO, ̇q
H7 LO, Dipole
Sherpa LO, CSS
Sherpa LO, Dire
Pythia

Thrust

1/σ × dσ/d(1 − T)

10^1

10^-1

10^-2

10^-3

10^-4

10^-5

Deducator
H7 LO, ̇q
H7 LO, Dipole
Sherpa LO, Ants
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Pythia

0 0.1 0.2 0.3 0.4 0.5 0.6

Z boson p_⊥ in peak region

p_⊥(Z) [GeV]

1/σ × dσ/dp_⊥(Z) [1/GeV]

0 10 20 30 40 50

0

0.01

0.02

0.03

0.04

0.05

0.06

0.07

0.08

0.09

H7 LO, ̇q
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Separation between Z boson and leading jet

ΔR(Z, jet 1)

0 1 2 3 4 5 6 7 8

10^-5

10^-4

10^-3

10^-2

10^-1

1

H7 LO, ̇q
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Pythia
Individual color charges inside a color dipole cannot be resolved by gluons of wavelength larger than the dipole size → emission off combined mother parton instead

Net effect is destructive interference outside cone with opening angle defined by emitting color dipole → Soft anomalous dimension halved due to reduced phase space

Formerly implemented by angular ordering / angular veto

Alternative description in terms of color dipoles


The midpoint between dipole and parton showers

- Angular ordered / vetoed parton shower does not fill full phase space
  Dipole shower lacks parton interpretation \(\rightarrow\) prefer alternative to both

- Can preserve parton picture by partial fractioning soft eikonal
  \(\leftrightarrow\) soft enhanced part of splitting function [Catani, Seymour] hep-ph/9605323

\[
\frac{p_ip_k}{(p_ip_j)(p_jp_k)} \rightarrow \frac{1}{p_ip_j} \frac{p_ip_k}{(p_i + p_k)p_j} + \frac{1}{p_kp_j} \frac{p_ip_k}{(p_i + p_k)p_j}
\]

- “Spectator”-dependent kernels, singular in soft-collinear region only
  \(\rightarrow\) capture dominant coherence effects (3-parton correlations)

\[
\frac{1}{1 - z} \rightarrow \frac{1 - z}{(1 - z)^2 + \kappa^2}
\]

- For correct soft evolution, ordering variable must be identical at both “dipole ends” (\(\rightarrow\) recover soft eikonal at integrand level)
The midpoint between dipole and parton showers

Choose parametrization such that soft term is \( \frac{1-z}{(1-z)^2 + \kappa^2} \) in all dipole types

(1) FF

\[ \kappa^2 = \frac{p_i p_j p_j p_k}{(p_i p_j p_k)^2} \]
\[ z_j = \frac{p_j p_k}{p_i p_j p_k} \]

(2) FI

\[ \kappa^2 = \frac{p_i p_j p_j p_a}{(p_i p_j p_a)^2} \]
\[ z_j = \frac{p_j p_a}{p_i p_j p_a} \]

(3) IF

\[ \kappa^2 = \frac{p_a p_j p_j p_k}{(p_j p_k p_a)^2} \]
\[ z_j = \frac{p_j p_k}{p_a p_j p_k} \]

(4) II

\[ \kappa^2 = \frac{p_a p_j p_j p_b}{(p_a p_j p_b)^2} \]
\[ z_j = \frac{p_j p_b}{p_a p_j p_b} \]

Preserve collinear anomalous dimensions & sum rules \( \rightarrow \) splitting functions fixed

\[ P_{qq}(z, \kappa^2) = 2 C_F \left[ \left( \frac{1-z}{(1-z)^2 + \kappa^2} \right) + \frac{1+z}{2} \right] + \gamma_q \delta(1-z) \]

\[ P_{gg}(z, \kappa^2) = 2 C_A \left[ \left( \frac{1-z}{(1-z)^2 + \kappa^2} \right) + \frac{z}{z^2 + \kappa^2} - \frac{2+z(1-z)}{2} \right] + \gamma_g \delta(1-z) \]

\[ P_{qg}(z, \kappa^2) = 2 C_F \left[ \frac{z}{z^2 + \kappa^2} - \frac{2-z}{2} \right] \]

\[ P_{gq}(z, \kappa^2) = T_R \left[ z^2 + (1-z)^2 \right] \]
Predictions for LEP

Differential 2-jet rate with Durham algorithm (91.2 GeV)

Differential 3-jet rate with Durham algorithm (91.2 GeV)

Thrust ($E_{CMS} = 91.2$ GeV)

Total jet broadening ($E_{CMS} = 91.2$ GeV)

Predictions for the LHC

\[ \frac{1}{\sigma} \frac{d\sigma}{d\Delta \phi} [\pi/\text{rad}] \]

\[ \Delta \phi_{\text{dijet}} \]

\[ p_{\text{max}}/\text{GeV} > 800 \]

\[ 600 < p_{\text{max}}/\text{GeV} < 800 \]

\[ 400 < p_{\text{max}}/\text{GeV} < 500 \]

\[ 310 < p_{\text{max}}/\text{GeV} < 400 \]

\[ 210 < p_{\text{max}}/\text{GeV} < 260 \]

\[ 160 < p_{\text{max}}/\text{GeV} < 210 \]

\[ 110 < p_{\text{max}}/\text{GeV} < 160 \]


[Dire]

\[ \sim \]

[arXiv:1506.05057]

\[ \perp \]

\[ \phi [\text{rad}/\pi] \]

\[ \Delta \phi [\text{rad}/\pi] \]

\[ \sigma \]

\[ \text{MC/Data} \]

\[ \text{ATLAS data} \]

SHERPA MC
NLO counterterms and MC@NLO matching

- Can view new shower model as modification of CS subtraction
- IR-finite counterterms computed, implemented in event generator Sherpa (improved cancellation in $pp \rightarrow h + j$ due to regulated $1/z$ terms)

- Sherpa MC@NLO based on exponentiation of CS dipole subtraction terms
  [Krauss,Siegert, Schönherr,SH]

- Dire modified CS subtraction automatically available for MC@NLO matching
- Interesting differences due to evolution variables and kernels

Differential $0 \rightarrow 1$ jet resolution

$$d\sigma / d \log_{10}(d_{01}/\text{GeV}) \ [\text{pb}]$$

$pp \rightarrow h \ (\text{GF}) @ 8 \text{ TeV}$
Towards higher logarithmic accuracy

- Big drawback of parton showers is lack of higher-order kernels
- Start improving with integrated NLO splitting functions
  \( [\text{Curci,Furmanski,Petronzio}] \) NPB175(1980)27, PLB97(1980)437
- 2-loop cusp term subtracted & combined with LO soft contribution (similar to CMW rescaling \( [\text{Catani,Marchesini,Webber}] \) NPB349(1991)635
- Implemented using weighting algorithms \( [\text{Schumann,Siegert,SH}] \) arXiv:0912.3501
Parton showers are indispensable tools for
  - phenomenology
  - experimental analysis
  - experiment design

Proper treatment of soft gluon radiation is essential

Matching at (N)NLO & merging at (N)LO improves PS approximation at fixed jet multiplicity → focus area of development during past decade

Reduction of uncertainties in intra-jet region or for jet multiplicities beyond reach at fixed order requires improved resummation → NLO kernels

This development just started ... stay tuned!
Thank you for your attention!
The midpoint between dipole and parton showers

\[ \phi_{\eta} \] spectrum, \( Z \rightarrow ee \) (dressed)

- Black line: ATLAS data
- Red line: ME+PS (1-jet)
- Pink line: 5 \( \leq Q_{cut} \leq 20 \) GeV

\( \frac{d \sigma}{d \phi_{\eta}} \) [\( \phi_{\eta} \approx \eta \)]

- MC/Data

\( p_T \) spectrum, \( Z \rightarrow ee \) (dressed)

- Black line: ATLAS data
- Red line: ME+PS (1-jet)
- Pink line: 5 \( \leq Q_{cut} \leq 20 \) GeV

\( \frac{d \sigma}{d p_T} \) [\( p_T \approx p_{T,\ell\ell} \) [GeV]]

- MC/Data

- Parton shower merged with 1-jet tree-level ME using CKKW-L
The midpoint between dipole and parton showers

Differential jet resolution at parton level (Durham algorithm)

\[ \frac{d\sigma}{d \log_{10} y_{n n+1}} \text{[pb]} \]

\[ e^+ e^- \rightarrow q\bar{q} @ 91.2 \text{ GeV} \]

\[ \tau^+ \tau^- \rightarrow [h \rightarrow gg] @ 125 \text{ GeV} \]
Negative “probabilities” in parton showers

- Standard parton shower: \( \Delta(t, t') = \exp\{F(t) - F(t')\} \)

  Exact MC solution \( t = F^{-1}[F(t') + \log R] \), \( R \) – random number

  But don’t want to compute \( F(t) = -\int_t^\infty d\bar{t} f(\bar{t}) \), as \( f(t) = \sum_b \int dz \frac{\alpha_s}{2\pi t} P_{ab}(t, z) \)

**Solution in veto algorithm** (hit-or-miss for Poisson distributions)

- Find overestimate \( g(t) \geq f(t) \) with simple integral \( G(t) \)
- Generate points according to \( g(t) \) and accept with \( f(t) / g(t) \)
Negative “probabilities” in parton showers

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Exact MC solution $t = F^{-1}[F(t') + \log R]$, $R$ – random number

But don’t want to compute $F(t) = -\int_t^1 dt' f(t')$, as $f(t) = \sum_b \int dz \frac{\alpha_s}{2\pi t} P_{ab}(t, z)$

**Solution in veto algorithm** (hit-or-miss for Poisson distributions)

- Find overestimate $g(t) > f(t)$ with simple integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Probability for one acceptance

$$\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\}$$
Negative “probabilities” in parton showers

- Standard parton shower: $\Delta(t, t') = \exp\{F(t) - F(t')\}$

  Exact MC solution $t = F^{-1}\left[F(t') + \log R\right]$, $R$ – random number

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- Find overestimate $g(t) > f(t)$ with simple integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Probability for one acceptance with one rejection

$$\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \left[ \int_t^{t'} dt_1 \left(1 - \frac{f(t_1)}{g(t_1)}\right) g(t_1) \exp \left\{ - \int_{t_1}^{t'} d\bar{t} g(\bar{t}) \right\} \right]$$
Negative “probabilities” in parton showers

- Standard parton shower: \( \Delta(t, t') = \exp\{F(t) - F(t')\} \)

  Exact MC solution \( t = F^{-1}[F(t') + \log R] \), \( R \) – random number

  But don’t want to compute \( F(t) = -\int_t^t d\bar{t} f(\bar{t}) \), as \( f(t) = \sum_b \int d\bar{z} \frac{\alpha_s}{2\pi} P_{ab}(t, z) \)

Solution in veto algorithm (hit-or-miss for Poisson distributions)

- Find overestimate \( g(t) > f(t) \) with simple integral \( G(t) \)
- Generate points according to \( g(t) \) and accept with \( f(t)/g(t) \)

Probability for one acceptance with two rejections

\[
\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \left[ \int_t^{t'} dt_1 \left( 1 - \frac{f(t_1)}{g(t_1)} \right) g(t_1) \exp \left\{ - \int_{t_1}^{t_2} d\bar{t} g(\bar{t}) \right\} \right] \\
\times \left[ \int_{t_1}^{t'} dt_2 \left( 1 - \frac{f(t_2)}{g(t_2)} \right) g(t_2) \exp \left\{ - \int_{t_2}^{t'} d\bar{t} g(\bar{t}) \right\} \right]
\]
Negative “probabilities” in parton showers

- Standard parton shower: \( \Delta(t, t') = \exp\{F(t) - F(t')\} \)

  Exact MC solution \( t = F^{-1}\left[ F(t') + \log R \right], \ R \text{ random number} \)

  But don’t want to compute \( F(t) = -\int_t^\infty d\bar{t} f(\bar{t}) \), as \( f(t) = \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ab}(t, z) \)

Solution in veto algorithm (hit-or-miss for Poisson distributions)

- Find overestimate \( g(t) > f(t) \) with simple integral \( G(t) \)
- Generate points according to \( g(t) \) and accept with \( f(t)/g(t) \)

Probability for one acceptance with \( n \) rejections

\[
\frac{f(t)}{g(t)} g(t) \exp \left\{-\int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^{n} \left[ \int_{t_{i-1}}^{t_i} dt_i \left( 1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp \left\{ -\int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]
\]
Negative “probabilities” in parton showers

- Standard parton shower: \( \Delta(t, t') = \exp\{F(t) - F(t')\} \)

  Exact MC solution \( t = F^{-1}[F(t') + \log R], \ R \) – random number

  But don’t want to compute \( F(t) = - \int_t d\bar{t} f(t), \) as \( f(t) = \sum_b \int dz \frac{\alpha_s}{2\pi t} P_{ab}(t, z) \)

Solution in veto algorithm (hit-or-miss for Poisson distributions)

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Probability for one acceptance with \( n \) rejections

\[
\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^n \left[ \int_{t_{i-1}}^{t_i} dt_i \left( 1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]
\]

Disentangle nested integrals:

\[
f(t) \exp \left\{ - \int_t^{t'} d\bar{t} g(\bar{t}) \right\} \frac{1}{n!} \left[ \int_t^{t'} d\bar{t} (g(\bar{t}) - f(\bar{t})) \right]^n
\]
Negative “probabilities” in parton showers

- Standard parton shower: \( \Delta(t, t') = \exp\{F(t) - F(t')\} \)
  
  Exact MC solution \( t = F^{-1}[F(t') + \log R], \quad R - \text{random number} \)
  
  But don’t want to compute \( F(t) = -\int_t \tilde{t} f(\tilde{t}) \), as \( f(t) = \sum_b \int dz \frac{\alpha_s}{2\pi} P_{ab}(t, z) \)

**Solution in veto algorithm** (hit-or-miss for Poisson distributions)

- Find overestimate \( g(t) > f(t) \) with simple integral \( G(t) \)
- Generate points according to \( g(t) \) and accept with \( f(t)/g(t) \)

Probability for one acceptance with \( n \) rejections

\[
\frac{f(t)}{g(t)} g(t) \exp \left\{ - \int_t^{t_1} \tilde{t} g(\tilde{t}) \right\} \prod_{i=1}^n \left[ \int_{t_{i-1}}^{t_i} dt_i \left( 1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} \tilde{t} g(\tilde{t}) \right\} \right]
\]

Disentangle nested integrals and sum over \( n \):

\[
f(t) \exp \left\{ - \int_t^{t'} \tilde{t} g(\tilde{t}) \right\} \frac{1}{n!} \left[ \int_t^{t'} \tilde{t} (g(\tilde{t}) - f(\tilde{t})) \right]^n \rightarrow f(t) \exp \left\{ - \int_t^{t'} \tilde{t} f(\tilde{t}) \right\}
\]
Negative “probabilities” in parton showers

- Standard parton shower: $\Delta(t, t') = \exp\{F(t) - F(t')\}$

  Exact MC solution $t = F^{-1}[F(t') + \log R], \ R \sim \text{random number}$

  But don’t want to compute $F(t) = -\int_t^1 d\bar{t} f(\bar{t}), \text{as} \ f(t) = \sum b \int dz \frac{\alpha_s}{2\pi} P_{ab}(t, z)$

Solution in veto algorithm (hit-or-miss for Poisson distributions)

- Find overestimate $g(t) > f(t)$ with simple integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Standard probability for one acceptance with $n$ rejections

$$\frac{f(t)}{g(t)} g(t) \exp\left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^{n} \left[ \int_{t_{i-1}}^{t_i} dt_i \left( 1 - \frac{f(t_i)}{g(t_i)} \right) g(t_i) \exp\left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]$$

Split weight into MC and analytic part using auxiliary function $h(t)$

$$\frac{f(t)}{h(t)} g(t) \exp\left\{ - \int_t^{t_1} d\bar{t} g(\bar{t}) \right\} \prod_{i=1}^{n} \left[ \int_{t_{i-1}}^{t_i} dt_i \left( 1 - \frac{f(t_i)}{h(t_i)} \right) g(t_i) \exp\left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} g(\bar{t}) \right\} \right]$$

$$w(t, t_1, \ldots, t_n) = \frac{h(t)}{g(t)} \prod_{i=1}^{n} \frac{h(t_i) g(t_i) - f(t_i)}{g(t_i) h(t_i) - f(t_i)}$$
Negative “probabilities” in parton showers

Weighted veto algorithm

\[
\frac{f(t)}{h(t)} g(t) \exp \left\{ - \int_t^{t_1} d\bar{t} \, g(\bar{t}) \right\} \prod_{i=1}^n \left[ \int_{t_{i-1}}^{t_i} dt_i \left( 1 - \frac{f(t_i)}{h(t_i)} \right) g(t_i) \exp \left\{ - \int_{t_i}^{t_{i+1}} d\bar{t} \, g(\bar{t}) \right\} \right]
\]

\[
w(t, t_1, \ldots, t_n) = \frac{h(t)}{g(t)} \prod_{i=1}^n \frac{h(t_i) \, g(t_i) - f(t_i)}{g(t_i) \, h(t_i) - f(t_i)}
\]

Looks trivial, surprisingly it’s not: It allows to

- Resum sub-leading color terms in MC@NLO and POWHEG
- Implement higher-order splitting functions in parton showers
  [Catani,Krauss,Prestel,SH] in preparation
- Use PDFs with negative values in parton showers
- Enhance branching probabilities in parton showers
Parton-shower matching & merging

Exact

Approximate

jet-jet correlations

inner jet structure
The long road to precision simulations

Merging related
Matching related

Color coherent evolution
Initial-state parton shower
ME correction
Initial-state parton shower
ME correction
ME re-clustering
MLM merging
MC@NLO
POWHEG

Matrix-element corrections

Parton shower evolution kernels similar to NLO subtraction terms
Equivalent at leading color and w/o spin correlations:

\[ D_{ij,k}(\Phi_R) = \frac{8\pi\alpha_s}{2p_ip_j} B(b_{ij,k}(\Phi_R))P_{ij,k}(t_{ij,k},z_{ij,k},\phi_{ij,k}) \]

\( b_{ij,k} \) maps real kinematics to Born, Catani-Seymour style

Can project real-emission term onto singular regions in PS
\( \rightarrow \) no “leftover” singularities (full color & spin \( \uparrow \) next slide)

\[ R_{ij,k}(\Phi_R) = \rho_{ij,k}(\Phi_R)R(\Phi_R), \quad \rho_{ij,k} = \frac{D_{ij,k}(\Phi_R)}{\sum_{mn,l} D_{mn,l}(\Phi_R)} \]

Now replace PS kernels by full real-emission corrections using weight

\[ w(\Phi_R) = \left[ \sum_{mn,l} \frac{8\pi\alpha_s}{2p_ip_j} \frac{B(b_{mn,l}(\Phi_R))P_{mn,l}(t',z',\phi')}{R(\phi_R)} \right]^{-1} \]

\( \rightarrow \) generic form of a matrix-element correction
Note splitter-spectator independence, i.e. \( w_{ij,k} = w \) for all \( ij,k \)
Leading-order calculation for observable $O$

$$\langle O \rangle = \int d\Phi_B B(\Phi_B) O(\Phi_B)$$

NLO calculation for same observable

$$\langle O \rangle = \int d\Phi_B \left\{ B(\Phi_B) + \tilde{V}(\Phi_B) \right\} O(\Phi_B) + \int d\Phi_R R(\Phi_R) O(\Phi_R)$$

Parton-shower result (zero and one emission)

$$\langle O \rangle = \int d\Phi_B B(\Phi_B) \left[ \Delta^{(K)}(t_c) O(\Phi_B) + \int_{t_c} O(\Phi_B) + \int d\Phi_B d\Phi_1 B(\Phi_B) K(\Phi_1) O(\Phi_R) \right]$$

$$\mathcal{O}(\alpha_s) \int d\Phi_B B(\Phi_B) \left\{ 1 - \int_{t_c} d\Phi_1 K(\Phi_1) \right\} O(\Phi_B) + \int_{t_c} d\Phi_B d\Phi_1 B(\Phi_B) K(\Phi_1) O(\Phi_R)$$

Phase space: $d\Phi_1 = dt dz d\phi J(t, z, \phi)$

Splitting functions: $K(t, z) \to \alpha_s/(2\pi t) \sum P(z) \Theta(\mu_Q^2 - t)$

Sudakov factors: $\Delta^{(K)}(t) = \exp \left\{ - \int_t d\Phi_1 K(\Phi_1) \right\}$
NLO+PS matching – MC@NLO

- Subtract $\mathcal{O}(\alpha_s)$ PS terms from subtracted NLO result ($t_c \to 0$)
  
  $1/N_c$ corrections faded out in soft region by smoothing function

  $$\bar{B}^{(K)}(\Phi_B) = B(\Phi_B) + \tilde{V}(\Phi_B) + I(\Phi_B) + \int d\Phi_1 \left[ S(\Phi_R) - B(\Phi_B) K(\Phi_1) \right] f(\Phi_1)$$

  $$H^{(K)}(\Phi_R) = \left[ R(\Phi_R) - B(\Phi_B) K(\Phi_1) \right] f(\Phi_1)$$

- Add parton shower, described by generating functional $\mathcal{F}_{MC}$

  $$\langle O \rangle = \int d\Phi_B \bar{B}^{(K)}(\Phi_B) \mathcal{F}_{MC}^{(0)}(\mu_Q^2, O) + \int d\Phi_R H^{(K)}(\Phi_R) \mathcal{F}_{MC}^{(1)}(t(\Phi_R), O)$$

- Expansion of matched result up to first emission

  $$\langle O \rangle = \int d\Phi_B \bar{B}^{(K)}(\Phi_B) \left[ \Delta^{(K)}(t_c) O(\Phi_B) \right.$$  
  
  $$+ \int_{t_c} d\Phi_1 K(\Phi_1) \Delta^{(K)}(t(\Phi_1)) O(\Phi_R) \bigg] + \int d\Phi_R H^{(K)}(\Phi_{n+1}) O(\Phi_R)$$
NLO+PS matching – POWHEG

- Replace $BK \to R \Rightarrow H^{(R)}$ zero, $\bar{B}^{(R)}$ positive in physical region

\[
\langle O \rangle = \int d\Phi_B \bar{B}^{(R)}(\Phi_B) \left[ \Delta^{(R)}(t_c, s_{\text{had}}) O(\Phi_B) \right. \\
\left. + \int_{t_c}^{s_{\text{had}}} d\Phi_1 \frac{R(\Phi_R)}{B(\Phi_B)} \Delta^{(R)}(t(\Phi_1), s_{\text{had}}) O(\Phi_R) \right]
\]

- $\mu_Q^2$ changed to hadronic centre-of-mass energy squared, $s_{\text{had}}$, to cover full phase space for real-emission correction

- Absence of hard events $\rightarrow$ enhanced high-$p_T$ region ($K = \bar{B}/B$)

  Formally beyond NLO, but often sizeable $\rightarrow$ Avoid by split $R \rightarrow R^s + R^f$

\[
\langle O \rangle = \int d\Phi_B \bar{B}^{(R^s)}(\Phi_B) \left[ \Delta^{(R^s)}(t_c, s_{\text{had}}) O(\Phi_B) \right. \\
\left. + \int_{t_c}^{s_{\text{had}}} d\Phi_1 \frac{R^s(\Phi_R)}{B(\Phi_B)} \Delta^{(R^s)}(t(\Phi_1), s_{\text{had}}) O(\Phi_R) \right] + \int d\Phi_R R^f(\Phi_R)
\]
Example: Inclusive jet production at the LHC

Inclusive jet transverse momenta in different rapidity ranges

\[ \frac{d^2 \sigma}{dp_T^2} \text{ [GeV]} \]

\[ p_T \text{ [GeV]} \]

- ATLAS data
- SHERPA MC@NLO
  \( \mu_R = \mu_F = \frac{1}{3} \, H_T, \quad \mu_Q = \frac{1}{2} \, p_T \)
- SHERPA MC@NLO
  \( \mu_R = \mu_F = \frac{1}{3} \, H_T^{(y)}, \quad \mu_Q = \frac{1}{2} \, p_T \)

\( \mu_R, \mu_F \) variation
\( \mu_Q \) variation
MPI variation

MC/data

\( \frac{1}{r} \) dependence on rapidity:
- \( |y| < 0.3 \)
- \( 0.3 < |y| < 0.8 \)
- \( 0.8 < |y| < 1.2 \)
- \( 1.2 < |y| < 2.1 \)
- \( 2.1 < |y| < 2.8 \)
- \( 2.8 < |y| < 3.6 \)
- \( 3.6 < |y| < 4.4 \)
Example: Inclusive jet production at the LHC

Dijet invariant mass spectra in different rapidity ranges

\[ m_{12} \text{ [GeV]} \]

\[ \frac{d\sigma}{d m_{12}} \text{ [pb/TeV]} \]

**Math**

\[ \mu_R = \mu_F = \frac{1}{2} H_T, \quad \mu_Q = \frac{1}{2} p_\perp \]

**Graphs**

- ATLAS data
- Sherpa MC@NLO
  - \( \mu_R = \mu_F = \frac{1}{2} H_T^{(y)} \), \( \mu_Q = \frac{1}{2} p_\perp \)

**Legend**

- \( \mu_R, \mu_F \) variation
- \( \mu_Q \) variation
- MPI variation

**Figure**

- 4.0 \( < y^* < 4.4 \)
- 3.5 \( < y^* < 4.0 \)
- 3.0 \( < y^* < 3.5 \)
- 2.5 \( < y^* < 3.0 \)
- 2.0 \( < y^* < 2.5 \)
- 1.5 \( < y^* < 2.0 \)
- 1.0 \( < y^* < 1.5 \)
- 0.5 \( < y^* < 1.0 \)
- \( y^* < 0.5 \)
Basic idea of ME+PS merging

- Separate phase space into “hard” and “soft” region
- Matrix elements populate hard domain
- Parton shower populates soft domain
- Need criterion to define “hard” & “soft” → jet measure $Q$ and corresponding cut, $Q_{\text{cut}}$
Parton-shower histories

- Start with some “core” process for example $e^+e^- \rightarrow q\bar{q}$
- This process is considered inclusive. It sets the resummation scale $\mu^2_Q$
- Higher-multiplicity ME can be reduced to core by clustering
- Clustering algorithm uniquely defined by requiring exact correspondence between ME & PS
  - Identify most likely splitting according to PS emission probability
  - Combine partons into mother according to PS kinematics
  - Continue until no clustering possible

[André, Sjöstrand] hep-ph/9708390
Truncated & vetoed parton showers

- Higher-multiplicity MEs that can be reduced to core process are included in core’s inclusive cross section (unitarity of PS)
- Sudakov suppression factors needed to make inclusive MEs exclusive
- Most efficiently computed with pseudo-showers

  - Start PS from core process
  - Evolve until predefined branching ↔ truncated parton shower
  - Emissions producing additional hard jets lead to event veto/weight

\[
\Delta^{(K)}(t; > Q_{\text{cut}}) = \exp \left\{ - \int_t d\Phi_1 K(\Phi_1) \Theta(Q - Q_{\text{cut}}) \right\}
\]

[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
ME+PS merging

- ME+PS for 0+1-jet in MC@NLO notation

\[
\langle O \rangle = \int d\Phi_B B(\Phi_B) \Delta^{(K)}(t_c) O(\Phi_B) + \int_{t_c} d\Phi_1 K(\Phi_1) \Delta^{(K)}(t) \Theta(Q_{\text{cut}} - Q) O(\Phi_R)
\]

\[
+ \int d\Phi_R R(\Phi_R) \Delta^{(K)}(t(\Phi_R); > Q_{\text{cut}}) \Theta(Q - Q_{\text{cut}}) O(\Phi_R) + \ldots
\]

- Reorder by parton multiplicity \( k \), change notation \( R_k \rightarrow B_{k+1} \)

- Analyze exclusive contribution from \( k \) hard partons only \( (t_0 = \mu^2_Q) \)

\[
\langle O \rangle_{k}^{\text{excl}} = \int d\Phi_k B_k \prod_{i=0}^{k-1} \Delta^{(K)}_i(t_{i+1}; t_i; > Q_{\text{cut}}) \Theta(Q_k - Q_{\text{cut}})
\]

\[
\times \left[ \Delta^{(K)}_k(t_c, t_k) O_k + \int_{t_c}^{t_k} d\Phi_1 K_k \Delta^{(K)}_k(t_{k+1}, t_k) \Theta(Q_{\text{cut}} - Q_{k+1}) O_{k+1} \right]
\]
ME+PS merging at NLO

- Analyze exclusive contribution from $k$ hard partons

$$
\langle O \rangle_{k}^{\text{excl}} = \int d\Phi_k \bar{B}^{(K)}_k \prod_{i=0}^{k-1} \Delta^{(K)}_i (t_{i+1}, t_i; > Q_{\text{cut}}) \Theta(Q_k - Q_{\text{cut}}) \\
\times \left( 1 + \frac{B_k}{\bar{B}^{(K)}_k} \sum_{i=0}^{k-1} \int_{t_{i+1}}^{t_i} d\Phi K_i \Theta(Q_i - Q_{\text{cut}}) \right) \\
\times \left[ \Delta^{(K)}_k (t_c, t_k) O_k + \int_{t_c}^{t_k} d\Phi_1 K_k \Delta^{(K)}_k (t_{k+1}, t_k) \Theta(Q_{\text{cut}} - Q_{k+1}) O_{k+1} \right] \\
+ \int d\Phi_{k+1} H^{(K)}_k \Delta^{(K)}_k (t_k; > Q_{\text{cut}}) \Theta(Q_k - Q_{\text{cut}}) \Theta(Q_{\text{cut}} - Q_{k+1}) O_{k+1}
$$

- Born matrix element $\rightarrow$ NLO-weighted Born

- Add hard remainder function

- Subtract $O(\alpha_s)$ terms contained in truncated PS
Unitarity condition of PS:

\[ 1 = \Delta^{(K)}(t_c) + \int_{t_c} d\Phi_1 K(\Phi_1) \Delta^{(K)}(t) \]

ME+PS(\@NLO) violates PS unitarity as ME ratio replaces splitting kernels in emission terms, but not in Sudakovs

\[ K(\Phi_1) \rightarrow \frac{R(\Phi_1, \Phi_B)}{B(\Phi_B)} \]

Can be corrected by explicit subtraction

\[ 1 = \left\{ \Delta^{(K)}(t_c) + \int_{t_c} d\Phi_1 \left[ K(\Phi_1) - \frac{R(\Phi_1, \Phi_B)}{B(\Phi_B)} \right] \Theta(Q - Q_{\text{cut}}) \Delta^{(K)}(t) \right\} \]

unresolved emission / virtual correction

\[ + \int_{t_c} d\Phi_1 \left[ K(\Phi_1) \Theta(Q_{\text{cut}} - Q) + \frac{R(\Phi_1, \Phi_B)}{B(\Phi_B)} \Theta(Q - Q_{\text{cut}}) \right] \Delta^{(K)}(t) \]

resolved emission
## ME+PS merging – Practical implementations

### LO schemes

<table>
<thead>
<tr>
<th>Method</th>
<th>Shower Generator</th>
<th>Unitary</th>
<th>References</th>
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<tbody>
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<td>Herwig/Pythia</td>
<td>No</td>
<td>[Mangano, Moretti, Pittau] hep-ph/0108069</td>
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<td>[Alwall et al.] arXiv:0706.2569</td>
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### NLO schemes

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<td>[Lavesson, Lönnblad] arXiv:0811.2912</td>
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<td>MEPS@NLO</td>
<td>Sherpa CSS</td>
<td>No</td>
<td>[Krauss, Schönherr, Siegert, SH] arXiv:1207.5030</td>
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<td>[Gehrmann, Krauss, Schönherr, Siegert, SH] 5031</td>
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<td>FxFx</td>
<td>Herwig(++)/Pythia</td>
<td>No</td>
<td>[Frederix, Frixione] arXiv:1209.6215</td>
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Example: Top quark pair production

- First matched/merged sim for $t\bar{t}+2j$
  full result has $t\bar{t}+0,1,2j@NLO, 3j@LO$
- Largely reduced theory uncertainty
  for both for measurement ($p_T, N_{jet}$)
  and BSM search ($H_T$) observables

Example: Squared-loop ME+PS merging


▶ Combine

Transverse momentum of leading jet

Integrated cross section in the exclusive 0-jet bin
PS expression for infrared safe observable, $O$

$$\langle O \rangle = \int d\Phi_0 \, B_0 \, F_0(\mu_Q^2, O)$$

$$F_n(t, O) = \Delta_n(t_c, t) \, O(\Phi_n) + \int_{t_c}^t d\Phi_1 \, K_n \, \Delta_n(\hat{t}, t) \, F_{n+1}(\hat{t}, O)$$

Add ME correction to first emission ($B_0 K_0 \rightarrow B_1$) & unitarize

$$+ \int_{t_c} d\Phi_1 \, \Delta_0(t_1, \mu_Q^2) \, B_1 \, F_1(t_1, O) - \int_{t_c} d\Phi_1 \, \Delta_0(t_1, \mu_Q^2) \, B_1 \, O(\Phi_0)$$

ME evaluated at fixed scales $\mu_{R/F} \rightarrow$ need to adjust to PS

$$w_1 = \frac{\alpha_s(b \, t_1)}{\alpha_s(\mu_R^2)} \frac{f_a(x_a, t_1)}{f_a(x_a, \mu_F^2)} \frac{f_a'(x_a', \mu_F^2)}{f_a'(x_a', t_1)}$$

Replace $B_0$ by vetoed xs $\bar{B}_0^{t_c} = B_0 - \int_{t_c} d\Phi_1 B_1$

$$\langle O \rangle = \left\{ \int d\Phi_0 \, \bar{B}_0^{t_c} + \int_{t_c} d\Phi_1 \left[ 1 - \Delta_0(t_1, \mu_Q^2) \, w_1 \right] B_1 \right\} O(\Phi_0)$$

$$+ \int_{t_c} d\Phi_1 \, \Delta_0(t_1, \mu_Q^2) \, w_1 \, B_1 \, F_1(t_1, O)$$
NNLO matching – Precision frontier at particle level


- Promote vetoed cross section to **NNLO**
- Add NLO corrections to $B_1$ using **MC@NLO**
- Subtract $O(\alpha_s)$ term of $w_1$ and $\Delta_0$

\[
\langle O \rangle = \int d\Phi_0 \tilde{B}_0^{tc} O(\Phi_0) \\
+ \int_{t_c} d\Phi_1 \left[ 1 - \Delta_0(t_1, \mu_Q^2) w_1 \left( 1 + w_1^{(1)} + \Delta_0^{(1)}(t_1, \mu_Q^2) \right) \right] B_1 O(\Phi_0) \\
+ \int_{t_c} d\Phi_1 \Delta_0(t_1, \mu_Q^2) w_1 \left( 1 + w_1^{(1)} + \Delta_0^{(1)}(t_1, \mu_Q^2) \right) B_1 \tilde{F}_1(t_1, O) \\
+ \int_{t_c} d\Phi_1 \left[ 1 - \Delta_0(t_1, \mu_Q^2) w_1 \right] \tilde{B}_1^R O(\Phi_0) + \int_{t_c} d\Phi_1 \Delta_0(t_1, \mu_Q^2) w_1 \tilde{B}_1^R \tilde{F}_1(t_1, O) \\
+ \int_{t_c} d\Phi_2 \left[ 1 - \Delta_0(t_1, \mu_Q^2) w_1 \right] H_1^R O(\Phi_0) + \int_{t_c} d\Phi_2 \Delta_0(t_1, \mu_Q^2) w_1 H_1^R F_2(t_2, O) \\
+ \int_{t_c} d\Phi_2 \ H_1^E F_2(t_2, O)
\]

- $\tilde{B}_1^R = \tilde{B}_1 - B_1 = \tilde{V}_1 + I_1 + \int d\Phi_1 S_1 \Theta(t_2 - t_1)$
- $H_1^R (H_1^E) \rightarrow$ regular (exceptional) double real configurations
NNLO matching – Precision frontier at particle level

\[ \sigma(W + \geq N_{\text{jet}}) \] [pb]

**Sherpa+BlackHat**

\[ \frac{d\sigma}{dp_{\nu e}} \] [pb/GeV]

**MC@NLO’ NNLO** = 7 TeV

\[ \nu_l < 2m \]

**Ratio to NNLO**

0.6

0.8

1

1.2

1.4

\[ \text{GeV} \]

\[ \frac{1}{\text{GeV}} \]

**Inclusive Jet Multiplicity**

**Sherpa MC**

**p_{\perp} > 20 GeV**

Comparison of approaches


- Setup
  - Stable Higgs
  - anti-$k_T$ jets, $R=0.4$, $p_{T,j}>30$ GeV $|\eta_j| < 4.4$

- Calculations & tools in the comparison
  - Fixed-order NLO for $h+ \leq 3$ jets, $H_T'/2$ & MINLO
  - LoopSim
  - NNLO for $pp \rightarrow h$ (Sherpa), $pp \rightarrow h + j$ (BFGLP)
  - Resummed $h-p_T$ (HqT & ResBos)
  - Resummed jet veto cross section (STWZ)
  - NNLO+PS (POWHEG & Sherpa)
  - Multi-jet merging up to 2 jets at NLO (Madgraph5_aMC@NLO, Herwig 7.1)
  - Multi-jet merging up to 3 jets at NLO (Sherpa)
  - High-energy resummation (HEJ)
Comparison of approaches

Scalar sum of jet transverse momenta

\[ \frac{d\sigma}{dH_{\text{jet}}^T} \text{[pb/GeV]} \]

- BFGLP hj NNLO
- GoSam+Sherpa
- Powheg NNLOPs
- Sherpa NNLOPs
- MG5_aMC FxFx
- Sherpa MePs@Nlo
- Herwig 7.1

Ratio to Powheg

\begin{align*}
\text{H}_\text{jet}^\text{T} \text{[GeV]} & \\
200 & 400 & 600 & 800 & 1000
\end{align*}
Leading jet transverse momentum ($n_j \geq 1$)

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<th>ResBos 2</th>
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<td>1.3</td>
<td>1.5</td>
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</table>

Comparison of approaches

Comparison of approaches

LH15 pp → h + jets comparison

Jet veto cross section

Jet veto cross section

Ratio to STWZ

Ratio to Powheg

$F_{V(\perp),1}^\dag$ (jet veto, $p_T^{\perp}$) [pb]

$F_{V(\perp),1}^\dag$ (jet veto, $p_T^{\perp}$) [pb]

STWZ
Powheg NnloPs
Sherpa NnloPs
MG5_aMC FxFx
Sherpa MePs@Nlo
Herwig 7.1

Ratio to STWZ

Ratio to Powheg

$p_T^{\perp} > 200$ GeV

$p_T^{\perp} > 200$ GeV