HGS-HIRe for FAIR

Helmholtz Graduate School for Hadron and Ion Research

CERN

Module 1

Why linear accelerators
Basic linac structure
Acceleration in periodic structures

Definitions

Linear accelerator: a device where charged particles acquire energy moving on a linear path

RF linear accelerator: acceleration is provided by time-varying electric fields (i.e. excludes electrostatic accelerators)

A few definitions:

- CW (Continuous wave) linacs when the beam comes out continously;
- Pulsed linac when the beam is produced in pulses: τ pulse length, f_r repetition frequency, beam duty cycle $\tau \times f_r$ (%)
- Main parameters: *E* kinetic energy of the particles coming out of the linac [MeV]

  I average current during the beam pulse [mA] (different from *average current* and from *bunch current* !)

  P beam power = electrical power transferred to the beam during acceleration

  P [W] = $V_{tot} \times I = E$ [eV] $\times I$ [A] $\times$ duty cycle

Variety of linacs

- The first and the smallest: Rolf Widerøe thesis (1923)

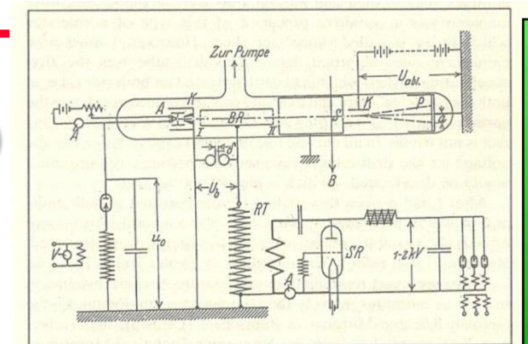
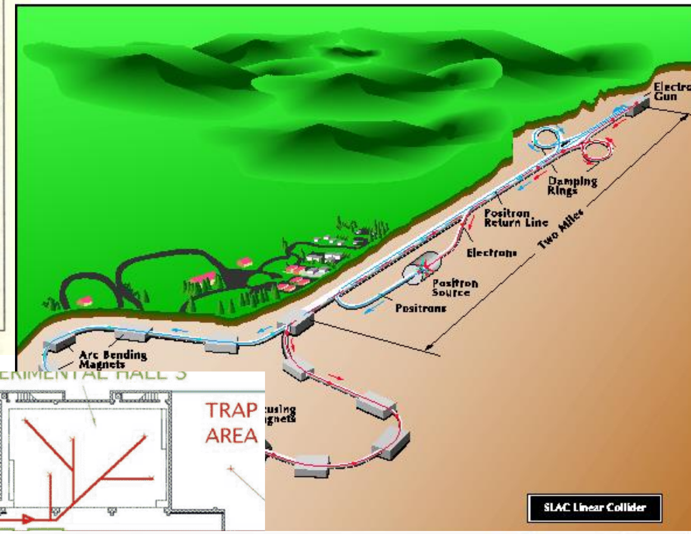
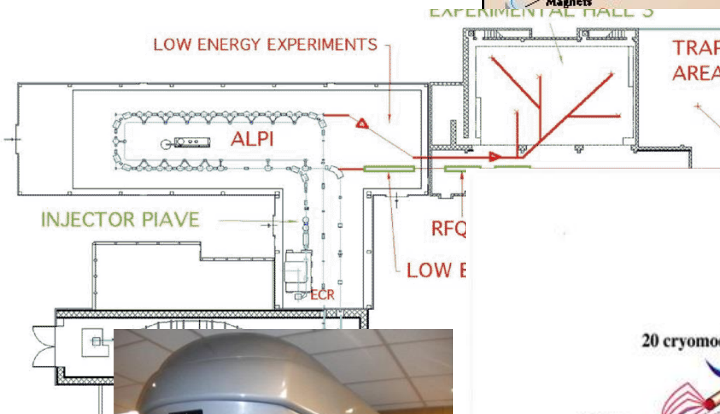


Fig. 3.6: Acceleration tube and switching circuits [Wi28].

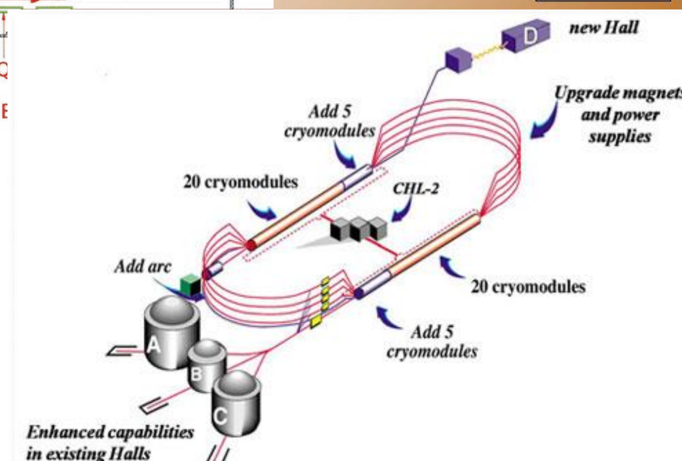
- The largest: Stanford Linear Collider (2 miles = 3.2 km) (but CLIC design goes to 48.3 km !)



- One of the less linear: ALPI at LNL (Italy)



- A limit case, multi-pass linacs: CEBAF at JLAB



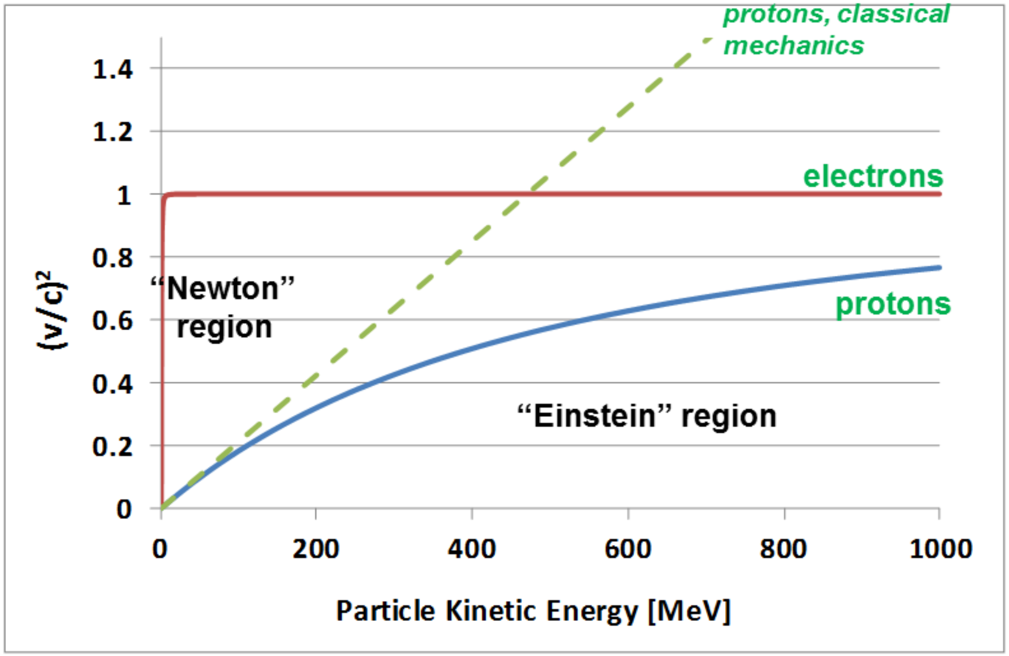
- The most common: medical electron linac (more than 7'000 in operation around the world!)

Why Linear Accelerators

	LINACS		SYNCHROTRONS
	Low Energy	**High Energy**	**High Energy**
Protons, Ions	Injectors to synchrotrons, stand alone applications *synchronous with the RF fields in the range where velocity increases with energy.* *Protons : β = v/c =0.51 at 150 MeV, 0.95 at 2 GeV.*	Production of secondary beams (n, ν, RIB, …) *higher cost/MeV than synchrotron but:* *- high average beam current (high repetition rate, less resonances, easier beam loss).* *- higher linac energy allows for higher intensity in the synchrotron.*	*very efficient when velocity is ~constant, (multiple crossings of the RF gaps).* *limited mean current (limited repetition frequency, instabilities)*
Electrons	Conventional e- linac *simple and compact*	Linear colliders *do not lose energy because of synchrotron radiation – only option for high energy!*	Light sources, factories *can accumulate high beam intensities*

« traditional » linac range ⇨ **« new » linac range**

Proton and Electron Velocity



$\beta^2=(v/c)^2$ as function of kinetic energy T for protons and electrons.

Classic (Newton) relation:

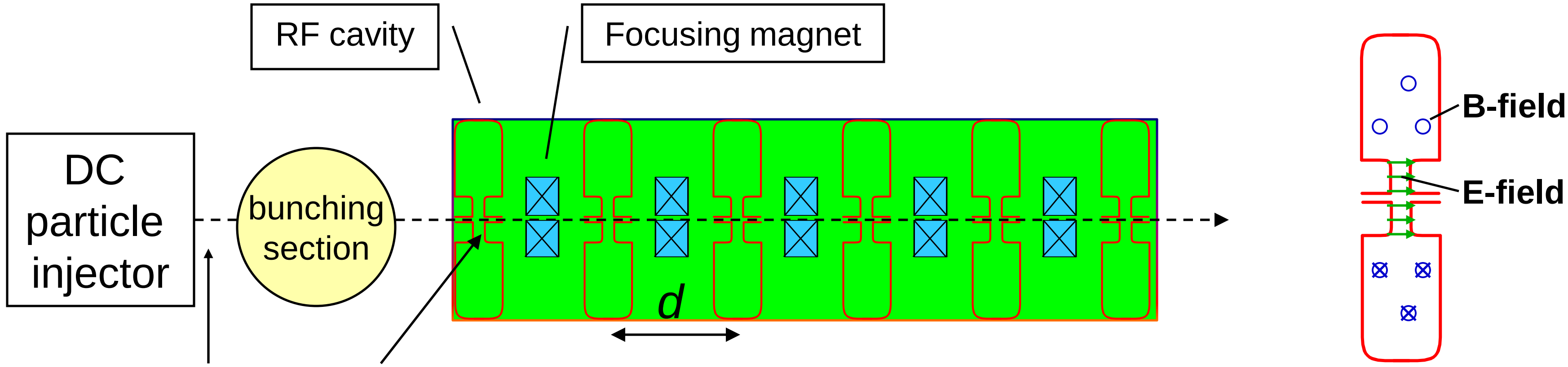
$$T = m_0 \frac{v^2}{2}, \quad \frac{v^2}{c^2} = \frac{2T}{m_0 c^2}$$

Relativistic (Einstein) relation:

$$\frac{v^2}{c^2} = 1 - \frac{1}{\sqrt{1 + T/m_0 c^2}}$$

- → **Protons** (rest energy 938.3 MeV): follow "Newton" mechanics up to some tens of MeV ($\Delta v/v < 1\%$ for W < 15 MeV) then slowly become relativistic ("Einstein"). From the GeV range velocity is nearly constant (*v*~0.95c at 2 GeV) → linacs can cope with the increasing particle velocity, synchrotrons cover the range where *v* nearly constant.
- → **Electrons** (rest energy 511 keV, 1/1836 of protons): relativistic from the keV range (v~0.1c at 2.5 keV) then increasing velocity up to the MeV range (v~0.95c at 1.1 MeV) → v~c after few meters of acceleration in a linac (typical gradient 10 MeV/m).

Basic linear accelerator structure

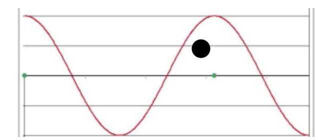


Protons: energy ~100 keV

β= v/c ~ 0.015

Accelerating gap: $E = E_0 \cos(\omega t + \phi)$

en. gain $\Delta W = eV_0 T\cos\phi$



Acceleration → the beam has to pass in each cavity on a phase ϕ near the crest of the wave

1. The beam must to be bunched at frequency ω
2. distance between cavities and phase of each cavity must be correlated

Phase change from cavity *i* to *i*+1 is $$\Delta\phi = \omega\tau = \omega\frac{d}{\beta c} = 2\pi\frac{d}{\beta\lambda}$$

For the beam to be synchronous with the RF wave ("ride on the crest") phase must be related to distance by the relation:

$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

… and on top of acceleration, we need to introduce in our "linac" some focusing elements

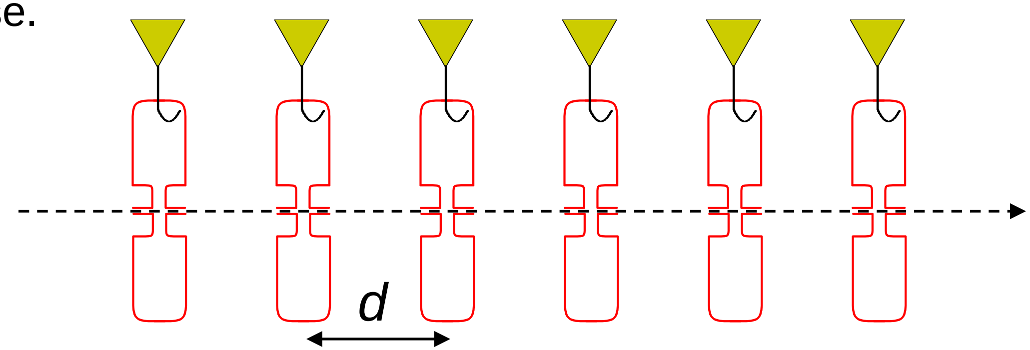
… and on top of that, we will couple a number of gaps in an "accelerating structure"

Accelerating structure architecture

When β increases during acceleration, either the phase difference between cavities $\Delta\phi$ must decrease or their distance *d* must increase.

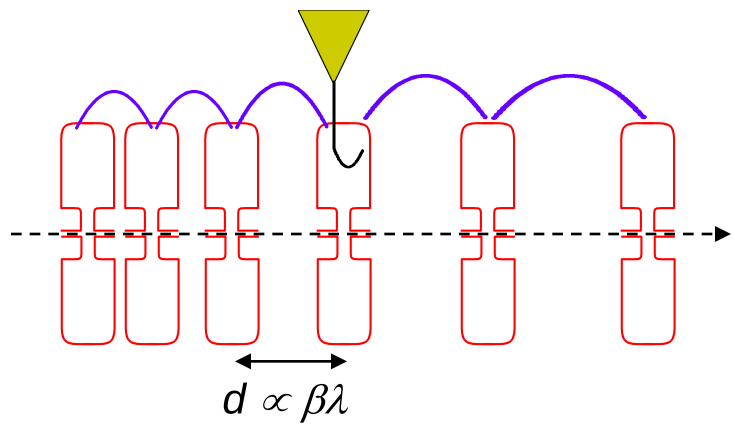
$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

***d* = const.**
***ϕ* variable**



Individual cavities – *distance between cavities constant, each cavity fed by an individual RF source, phase of each cavity adjusted to keep synchronism, used for linacs required to operate with different ions or at different energies. Flexible but expensive!*

***ϕ* = const.**
***d* variable**

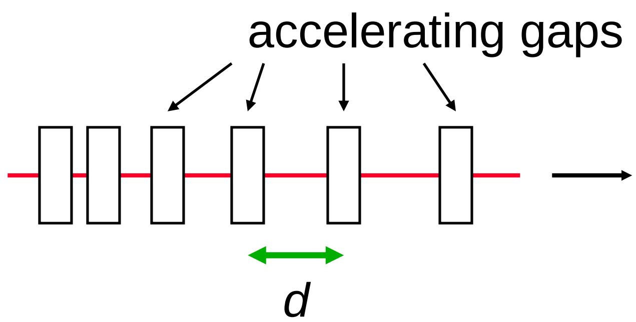


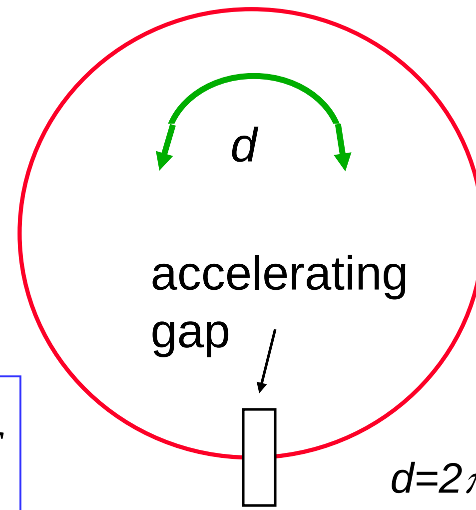
Better, but 2 problems:

1. *create a **"coupling"**;*
2. *create a mechanical and RF structure with increasing cell length.*

Coupled cell cavities - *a single RF source feeds a large number of cells (up to ~100!) - the phase between adjacent cells is defined by the coupling and the distance between cells increases to keep synchronism . Once the geometry is defined, it can accelerate only one type of ion for a given energy range. Effective but not flexible.*

Linear and circular accelerators





$d=\beta\lambda/2$=variable
f=constant

$$d=\frac{\beta c}{2f}=\frac{\beta\lambda}{2}\,,\ \beta c=2df$$

$d=2\pi R$=constant
$f=\beta c/2d$=variable

Linear accelerator:

Particles accelerated by a sequence of gaps (all at the same RF phase).

Distance between gaps increases proportionally to the particle velocity, to keep synchronicity.

Used in the range where β increases.
"Newton" machine

Circular accelerator:

Particles accelerated by one (or more) gaps at given positions in the ring.

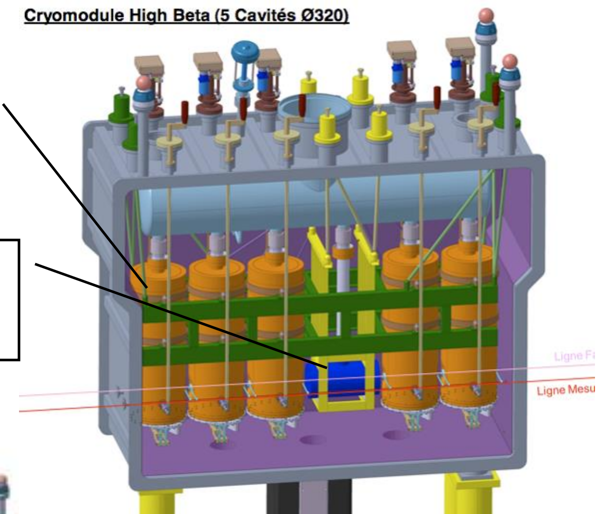
Distance between gaps is fixed. Synchronicity only for β~const, or varying (in a limited range!) the RF frequency.

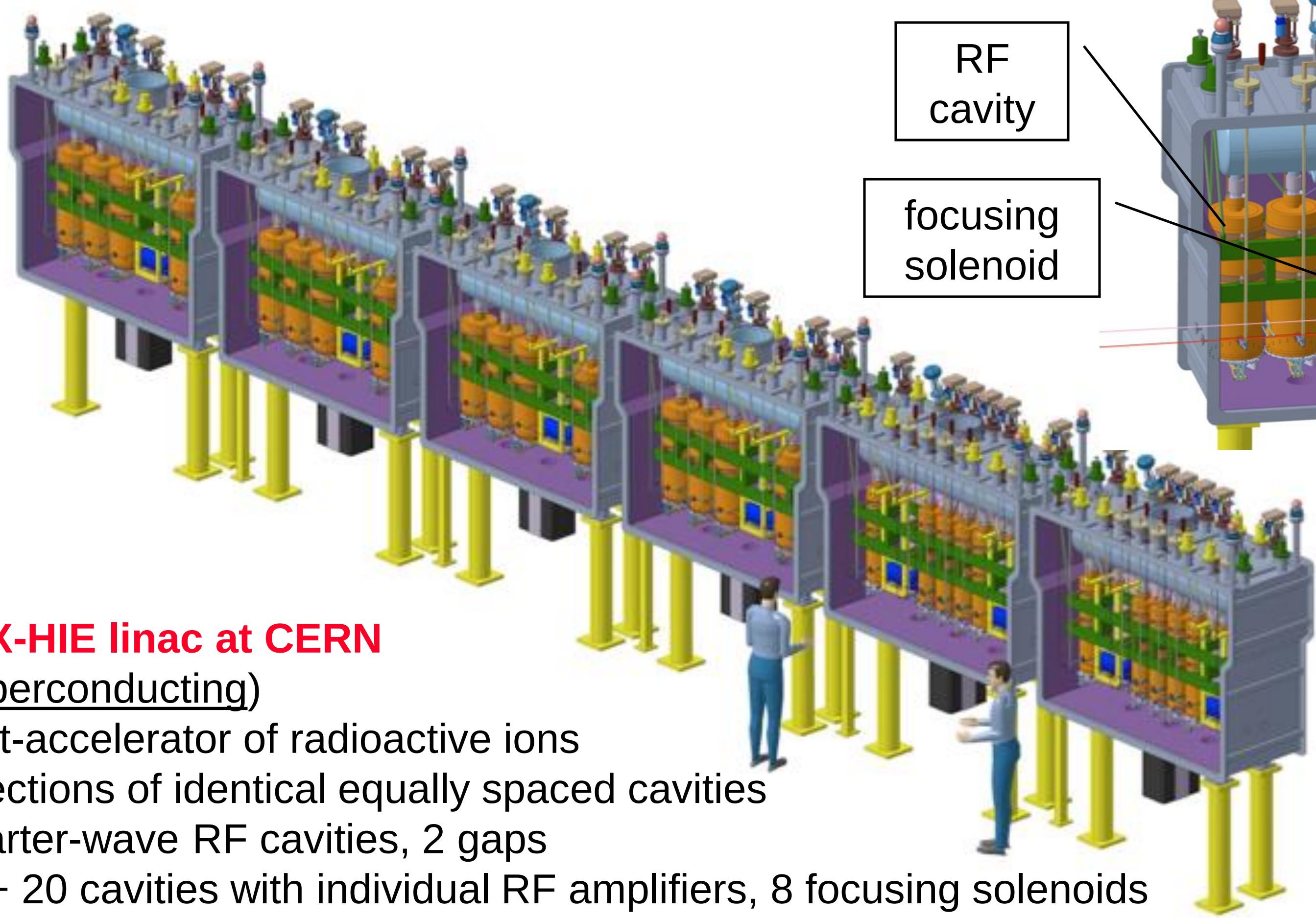
Used in the range where β is nearly constant.
"Einstein" machine

Note that only linacs are real «accelerators», synchrotrons are «mass increaser»!

Case 1: a single-cavity linac

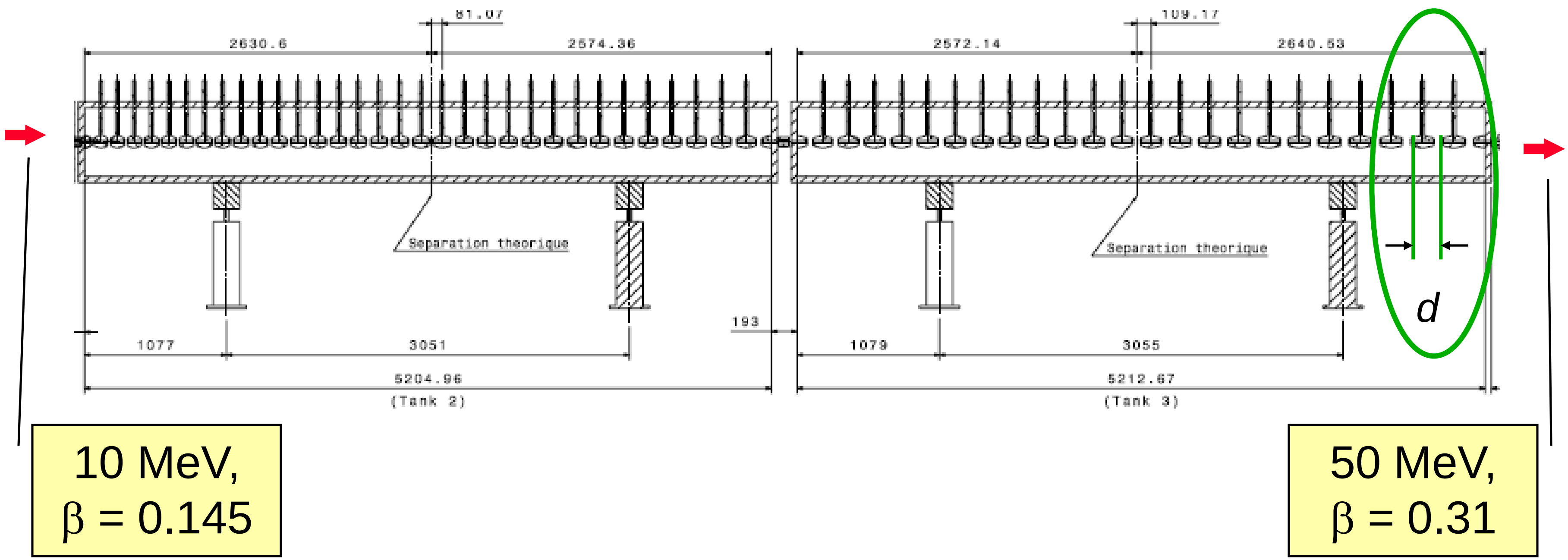
The goal is flexibility: acceleration of different ions (e/m) at different energies
→ need to change phase relation for each ion-energy



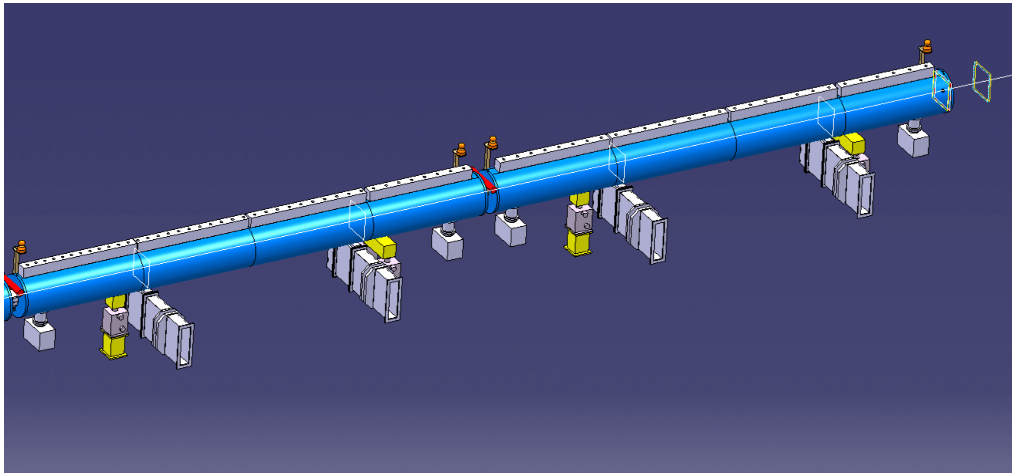


REX-HIE linac at CERN
(<u>superconducting</u>)
Post-accelerator of radioactive ions
2 sections of identical equally spaced cavities
Quarter-wave RF cavities, 2 gaps
12 + 20 cavities with individual RF amplifiers, 8 focusing solenoids
Energy 1.2 → 10 MeV/u, accelerates different A/m

Case 2 : a Drift Tube Linac



Tank 2 and 3 of the new Linac4 at CERN:
57 coupled accelerating gaps
Frequency 352.2 MHz, λ = 85 cm
Cell length ($d=\beta\lambda$) from 12.3 cm to 26.4 cm (factor 2 !).



Intermediate cases

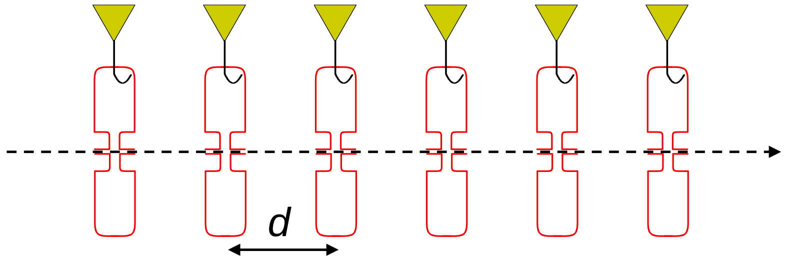
But:

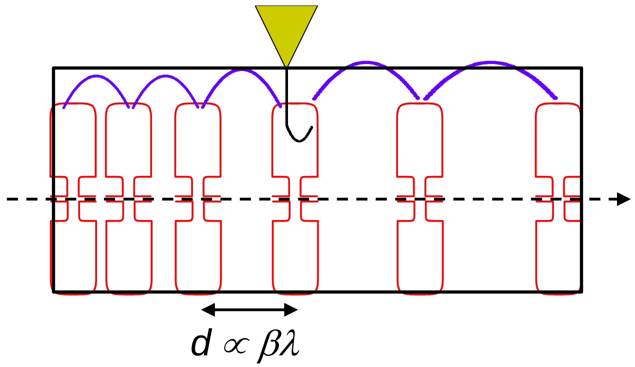
Between the 2 “extremes” there are many “intermediate” cases, because:

a. Single-gap cavities are expensive (both cavity and RF source!).
b. Structures with each cell matched to the beta profile are mechanically complicated and expensive.

→ as soon as the increase of beta with energy becomes small ($\Delta\beta/\Delta W$) we can accept a small error and:

1. Use multi-gap cavities with constant distance between gaps.
2. Use series of identical cavities (standardised design and construction).





Synchronism condition in a multicell cavity

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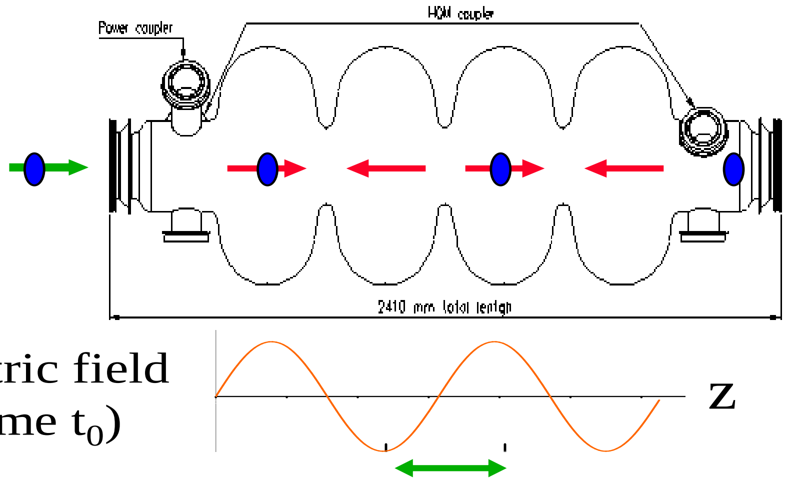
Typical linac case: multi-cell accelerating cavity with d=constant and phase difference between cells $\Delta\phi=\pi$ given by the electric field distribution.

Beam



Electric field (at time t_0)

Example: a linac superconducting 4-cell accelerating structure

Synchronism condition bw. particle and wave

t (travel between centers of cells) = *T/2*

$$\frac{d}{\beta c} = \frac{1}{2f} \Rightarrow d = \frac{\beta c}{2f} = \frac{\beta\lambda}{2}$$

d=distance between centres of consecutive cells

1. In an ion linac cell length has to increase (up to a factor 200 !) and the linac will be made of a sequence of different accelerating structures (changing cell length, frequency, operating mode, etc.) matched to the ion velocity.
2. For electron linacs, $\beta=1$, $d=\lambda/2$ $\rightarrow$ An electron linac will be made of an injector + a series of identical accelerating structures, with cells all the same length

Note that in the example above, we neglect the increase in particle velocity inside the cavity !

Sections of identical cavities: a superconducting linac (medium β)

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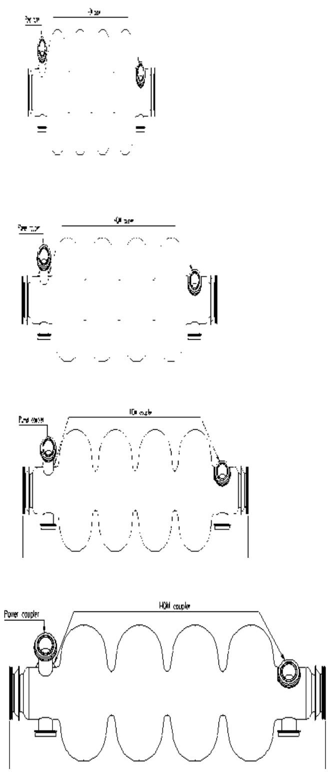
The same superconducting cavity design can be used for different proton velocities. The linac has different sections, each made of cavities with cell length matched to the average beta in that section. At "medium energy" (>150 MeV) we are not obliged to dimension every cell or every cavity for the particular particle beta at that position, and we can accept a slight "asynchronicity" → phase slippage + reduction in acceleration efficiency from the optimum one.

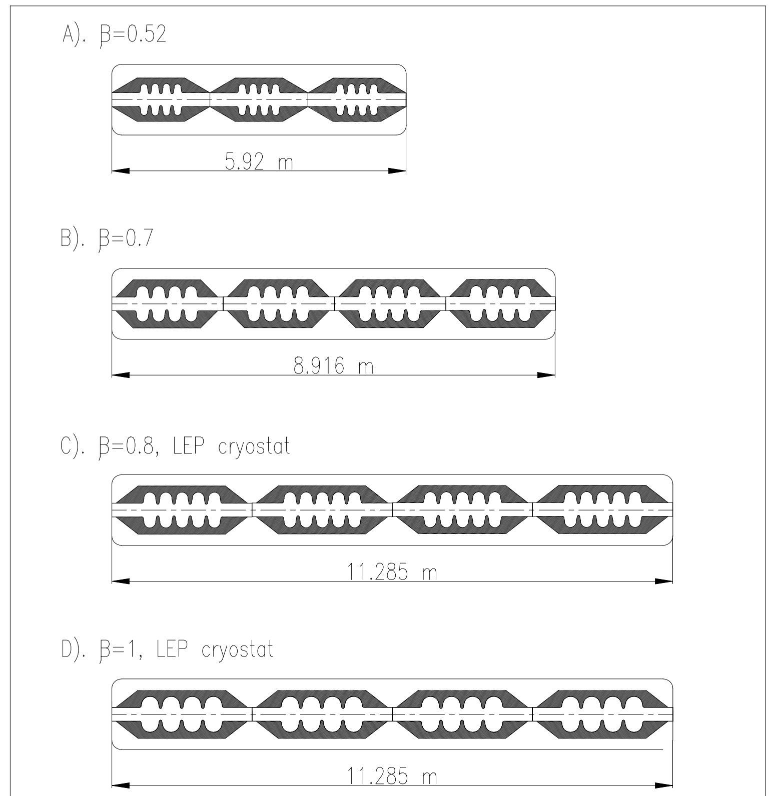


β=0.52

β=0.7

β=0.8

β=1



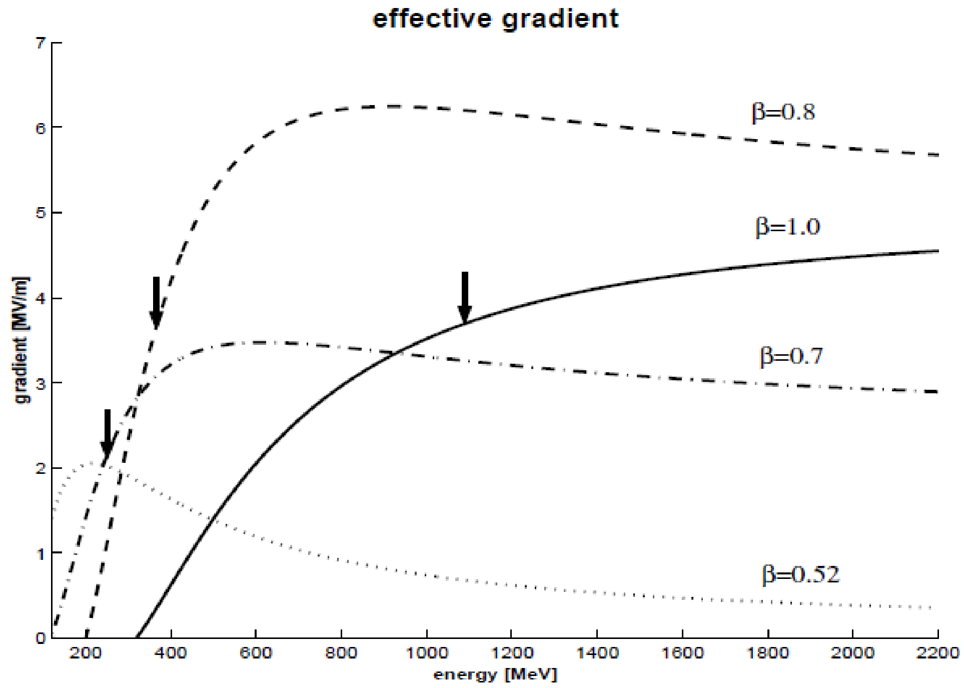
CERN (old) SPL design, SC linac 120 - 2200 MeV, 680 m length, 230 cavities

Effects of phase slippage

When sequences of cells are not matched to the particle beta → phase slippage

$$\Delta\phi = \omega\Delta t = \pi \frac{\Delta\beta}{\beta}$$

1. The effective gradient seen by the particle is lower.
2. The phase of the bunch centre moves away from the synchronous phase → can go (more) into the non-linear region, with possible longitudinal emittance growth and beam loss.

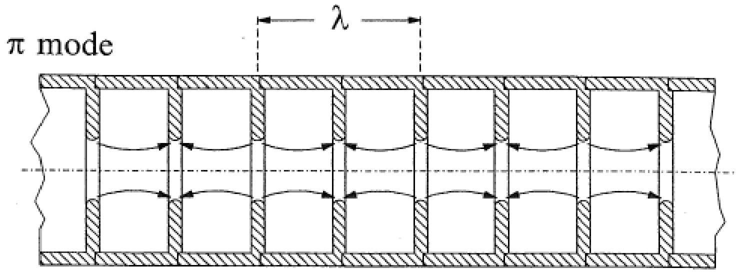


$$\Delta\phi = \pi \frac{\Delta\beta}{\beta} = \pi \frac{1}{\gamma(\gamma-1)} \frac{\Delta W}{W}$$

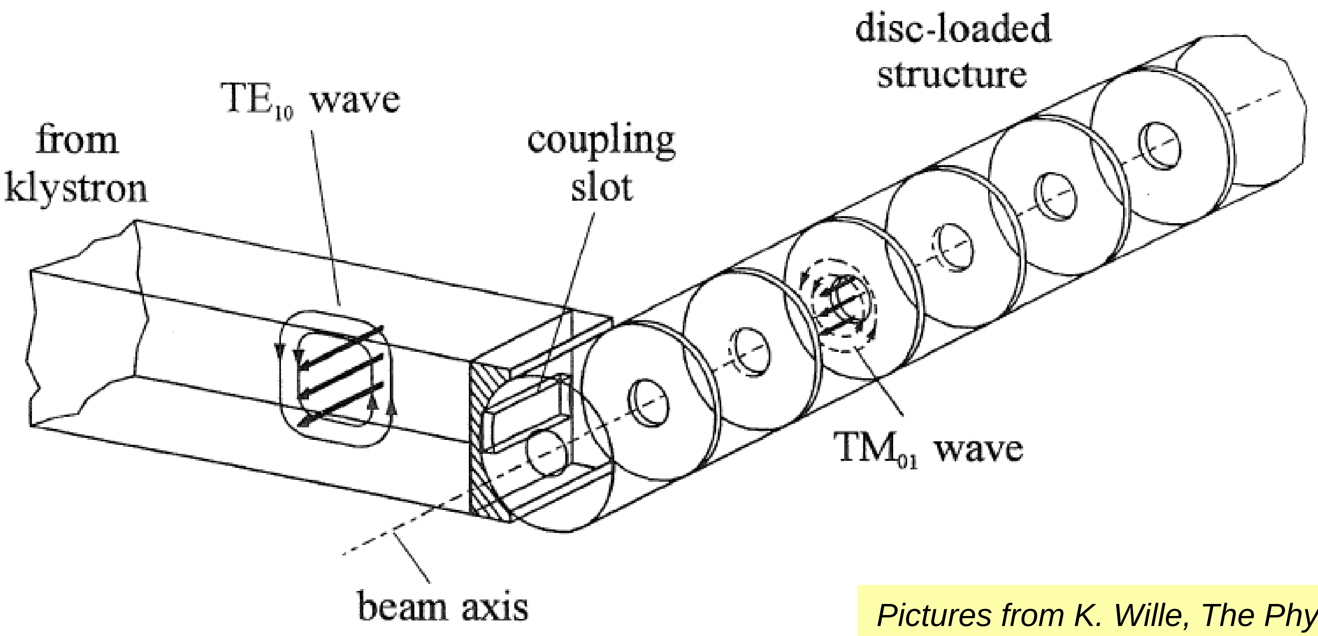
Very large at small energy (γ~1) becomes negligible at high energy (~2.5 °/m for γ~1.5, W=500 MeV).

Curves of effective gradient (gradient seen by the beam for a constant gradient in the cavity) for the previous case (4 sections of beta 0.52, 0.7, 0.8 and 1.0).

Electron linacs



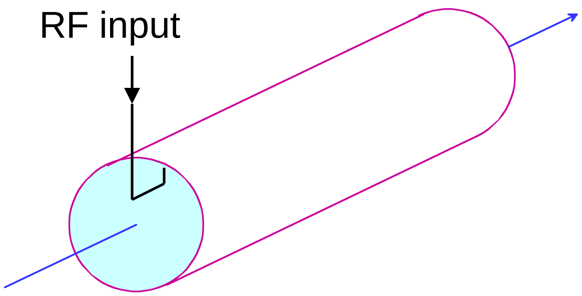
1. In an electron linac velocity is ~ constant. To use the fundamental accelerating mode cell length must be $d = \beta\lambda / 2$.
2. the linac structure will be made of a sequence of identical cells. Because of the limits of the RF source, the cells will be grouped in cavities operating in travelling wave mode.



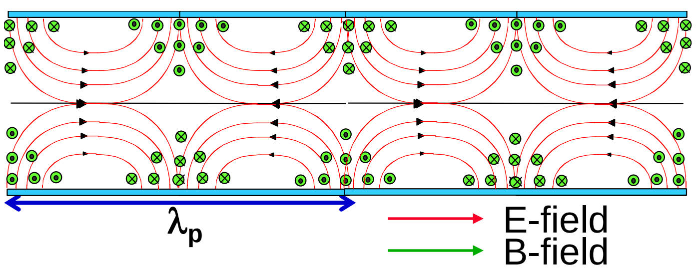
Pictures from K. Wille, The Physics of Particle Accelerators

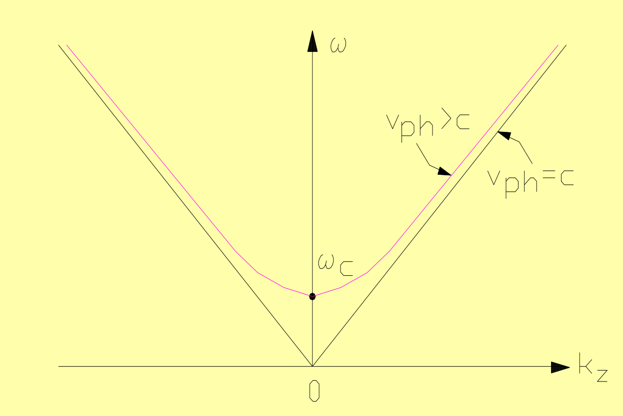
Acceleration in Periodic Structures

Wave propagation in a cylindrical pipe



TM01 field configuration

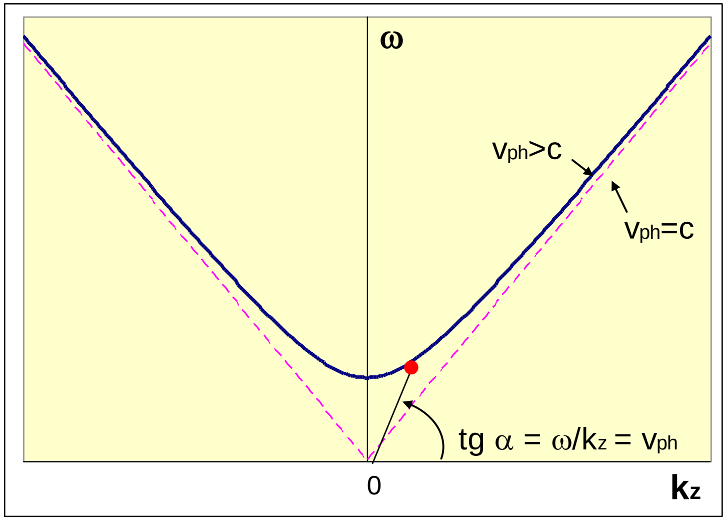




- In a cylindrical waveguide different modes can propagate (=Electromagnetic field distributions, transmitting power and/or information). The field is the superposition of waves reflected by the metallic walls of the pipe → velocity and wavelength of the modes will be different from free space (c, λ)
- To accelerate particles, we need a mode with longitudinal E-field component on axis: a TM mode (Transverse Magnetic, B_z=0). The simplest is TM01.
- We inject RF power at a frequency exciting the TM01 mode: sinusoidal E-field on axis, wavelength λ_p depending on frequency and on cylinder radius. Wave velocity (called "phase velocity") is $v_{ph} = \lambda_p/T = \lambda_p f = \omega/k_z$ with $k_z = 2\pi/\lambda_p$
- The relation between <u>frequency</u> ω and <u>propagation constant</u> k is the DISPERSION RELATION (red curve on plot), a fundamental property of waveguides.

Wave velocity: the dispersion relation

The dispersion relation $\omega(k)$ can be calculated from the theory of waveguides:
$\omega^2 = k^2c^2 + \omega_c^2$ Plotting this curve (hyperbola), we see that:



$k=2\pi/\lambda_p$

$v_{ph}=\omega/k = (c^2+\omega_c^2/k^2)^{1/2}$

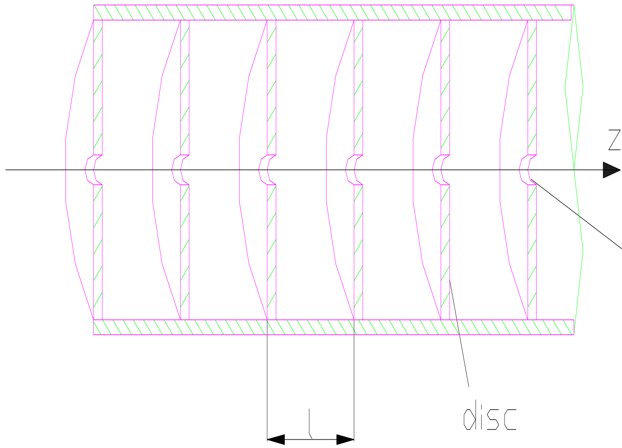
$v_g=d\omega/dk$

1) There is a "cut-off frequency", below which a wave will not propagate. It depends on dimensions (λ_c=2.61a for the cylindrical waveguide).
2) At each excitation frequency is associated a phase velocity, the velocity at which a certain phase travels in the waveguide. $v_p=\infty$ at k=0, $\omega=\omega_c$ and then decreases towards $v_p=c$ for $k,\omega\to\infty$.
3) To see at all times an accelerating E-field a particle traveling inside our cylinder has to travel at $v = v_{ph} \rightarrow$ **v > c** !!!

Are we violating relativity? No, energy (and information) travel at group velocity $d\omega/dk$, always between 0 and c.

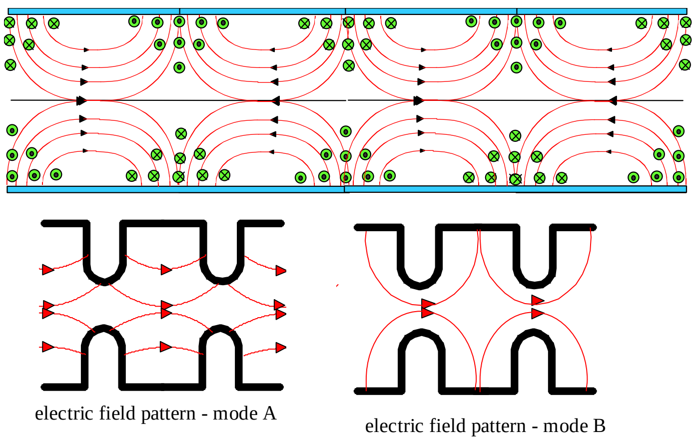
To use the waveguide to accelerate particles, we need a "trick" to slow down the wave.

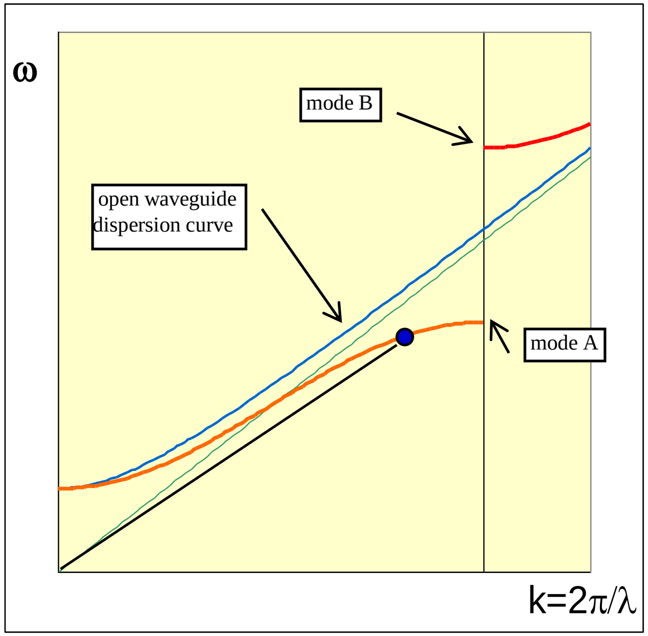
Slowing down waves: the disc-loaded waveguide



Discs inside the cylindrical waveguide, spaced by a distance ℓ, will induce multiple reflections between the discs.

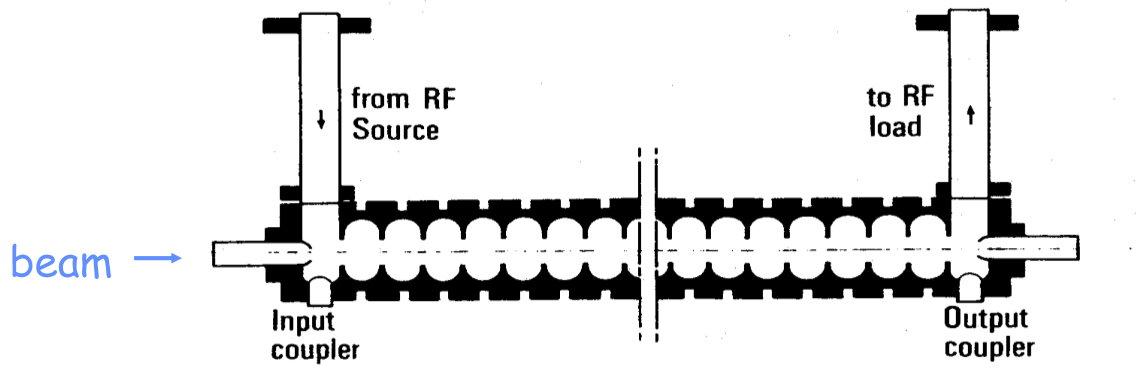
Dispersion relation for the disc-loaded waveguide





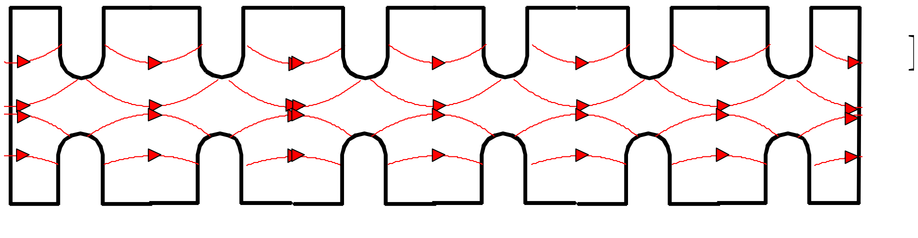
- Wavelengths with $\lambda_p/2 \sim \ell$ will be most affected by the discs. On the contrary, for $\lambda_p=0$ and $\lambda_p=\infty$ the wave does not see the discs → the dispersion curve remains that of the empty cylinder.
- At $\lambda_p/2=\ell$, the wave will be confined between the discs, and present 2 "polarizations" (mode A and B in the figure), 2 modes with same wavelength but different frequencies → the dispersion curve splits into 2 branches, separated by a stop band.
- In the disc-loaded waveguide, the lower branch of the dispersion curve is now "distorted" in such a way that we can find a range of frequencies with $v_{ph} = c$ → we can use it to accelerate a particle beam!
- We have built a linac for v~c → a TRAVELING WAVE (TW) ELECTRON LINAC

Traveling wave linac structures



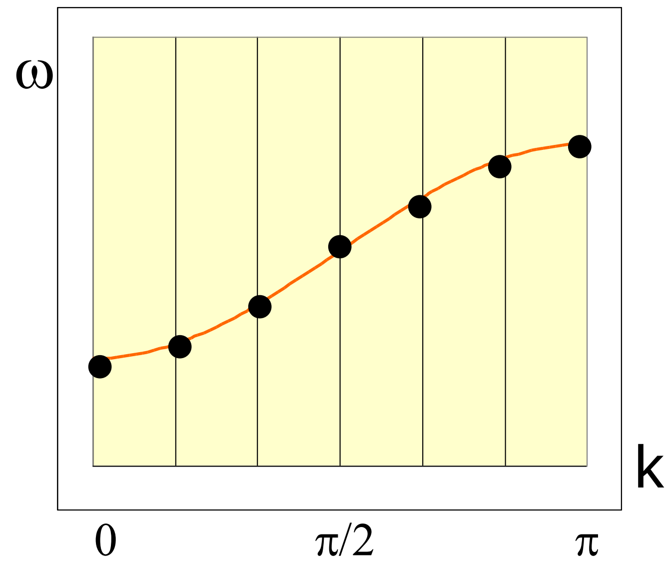
- Disc-loaded waveguide designed for v_{ph}=c at a given frequency, equipped with an input and an output coupler.
- RF power is introduced via the input coupler. Part of the power is dissipated in the structure, part is taken by the beam (beam loading) and the rest is absorbed in a matched load at the end of the structure. Usually, structure length is such that ~30% of power goes to the load.
- The "traveling wave" structure is the standard linac for electrons from β~1.
- Can not be used for protons at v<c:
  1. constant cell length does not allow synchronism
  2. structures are long, without space for transverse focusing

Standing wave linac structures



To obtain an accelerating structure for protons we close our disc-loaded structure at both ends with metallic walls → multiple reflections of the waves.

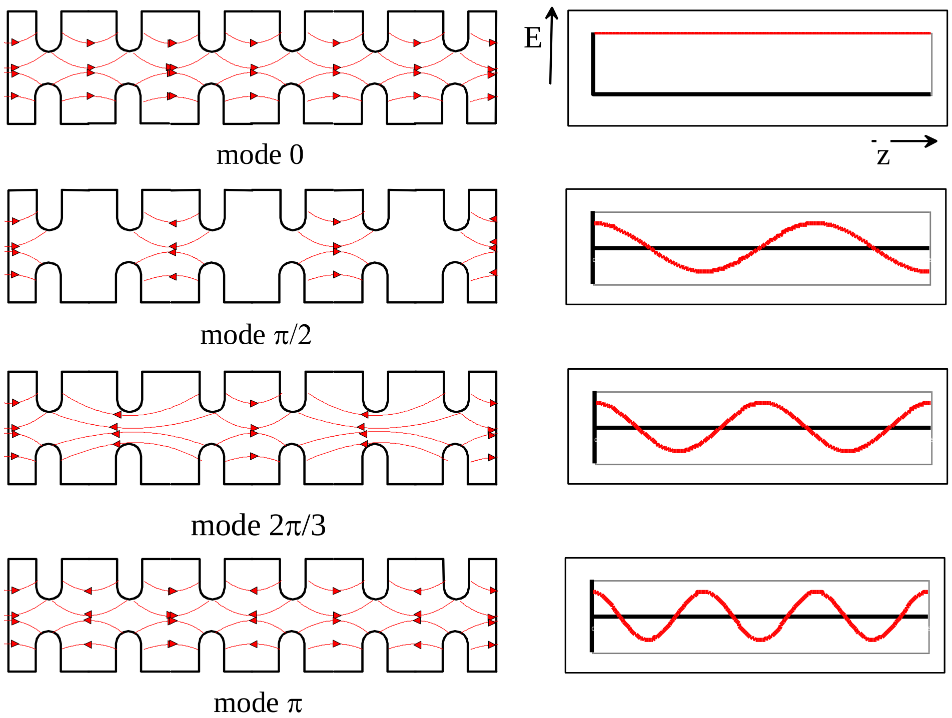
Boundary condition at both ends is that electric field must be perpendicular to the cover → Only some modes on the disc-loaded dispersion curve are allowed → only some frequencies on the dispersion curve are permitted.



In general:

1. the modes allowed will be equally spaced in k
2. The number of modes will be identical to the number of cells (N cells → N modes)
3. k represents the phase difference between the field in adjacent cells.

More on standing wave structures



Standing wave modes are named from the phase difference between adjacent cells: in the example above, mode 0, $\pi/2$, $2\pi/3$, π.

In standing wave structures, cell length can be matched to the particle velocity !

→ STANDING WAVE MODES are generated by the sum of 2 waves traveling in opposite directions, adding up in the different cells.

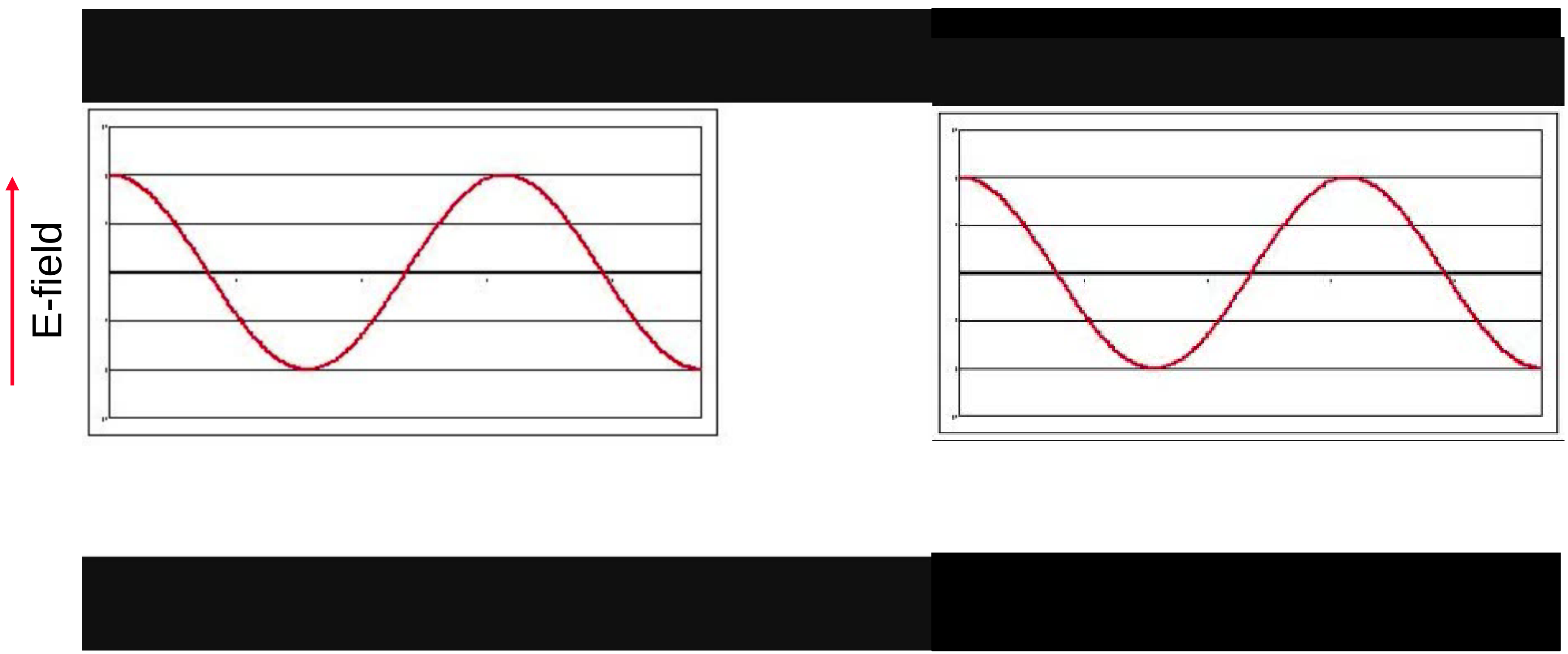
→ For acceleration, the particles must be in phase with the E-field on axis. We have already seen the π mode: synchronism condition for cell length $\ell = \beta\lambda/2$.

→ Standing wave structures can be used for any β (→ ions and electrons) and their cell length can increase, to follow the increase in β of the ions.

Synchronism conditions:
0-mode : $\ell = \beta\lambda$
$\pi/2$ mode: $2\ell = \beta\lambda/2$
π mode: $\ell = \beta\lambda/2$

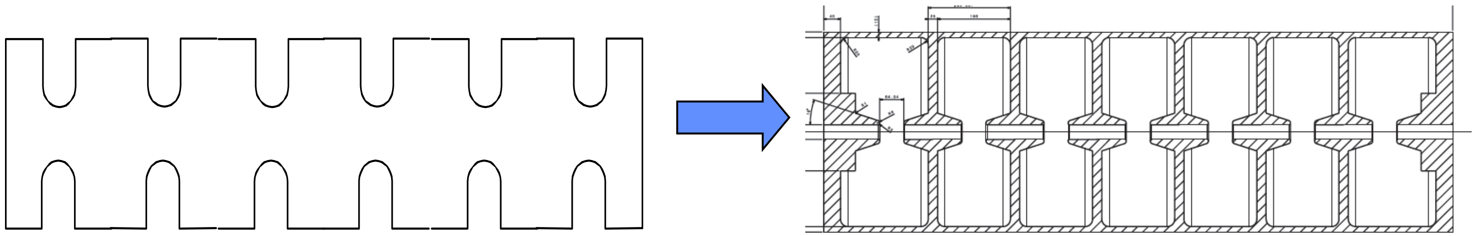
Acceleration on traveling and standing waves

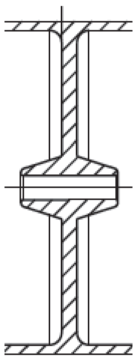
TRAVELING Wave ***STANDING Wave***



position z position z

Practical standing wave structures



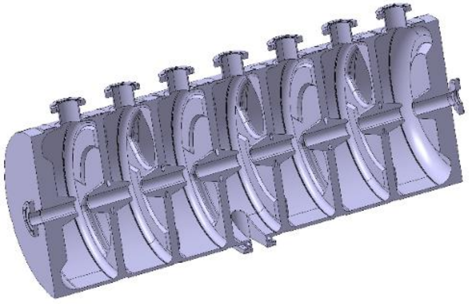


From disc-loaded structure to a real cavity (Linac4 PIMS, Pi-Mode Structure)

1. To increase acceleration efficiency (=shunt impedance ZT^2!) we need to concentrate electric field on axis (Z↑) and to shorten the gap (T↑) → introduction of "noses" on the openings.

2. The smaller opening would not allow the wave to propagate → introduction of "coupling slots" between cells.

3. The RF wave has to be coupled into the cavity from one point, usually in the center.

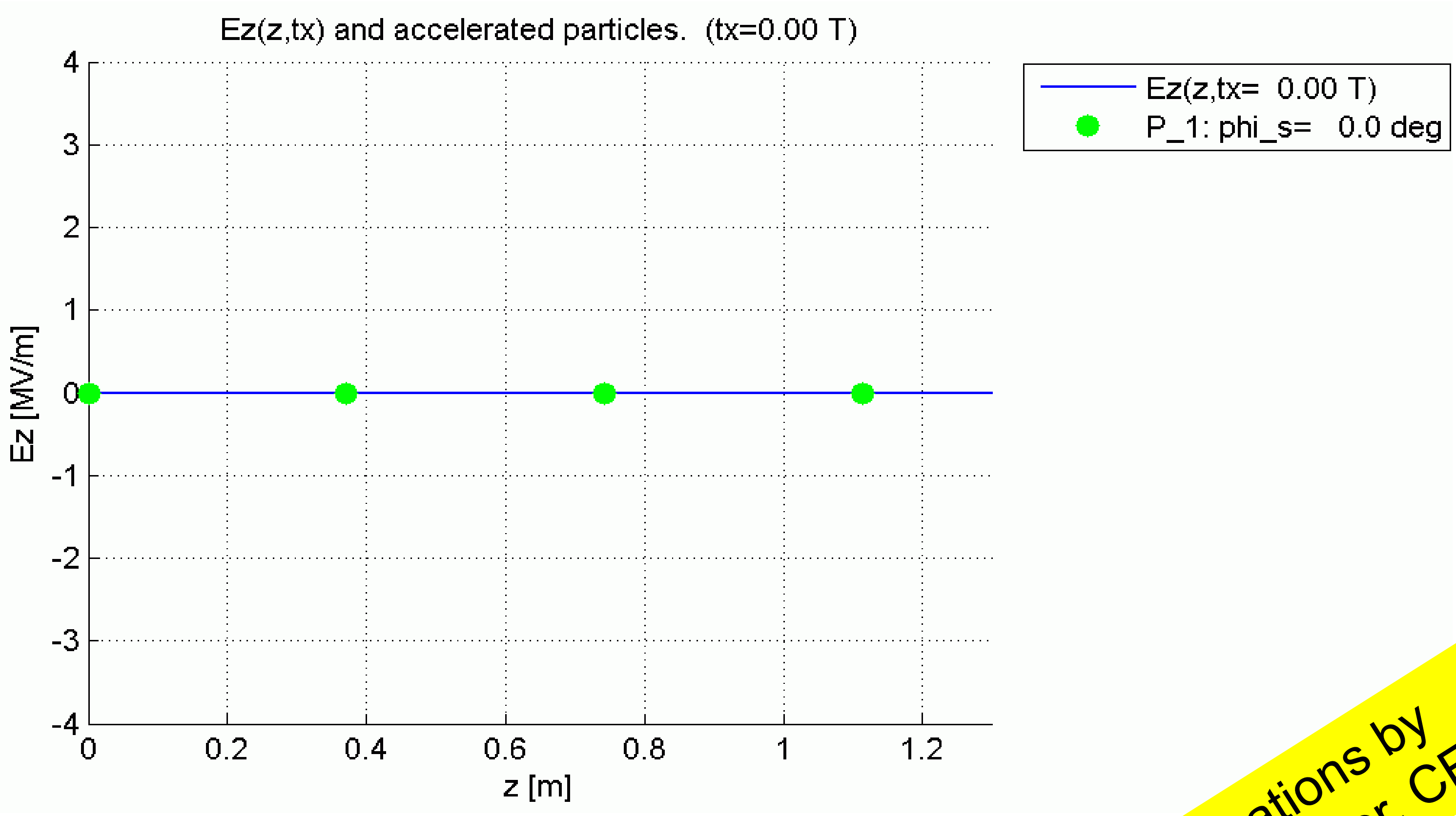


HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

PIMS Prototype

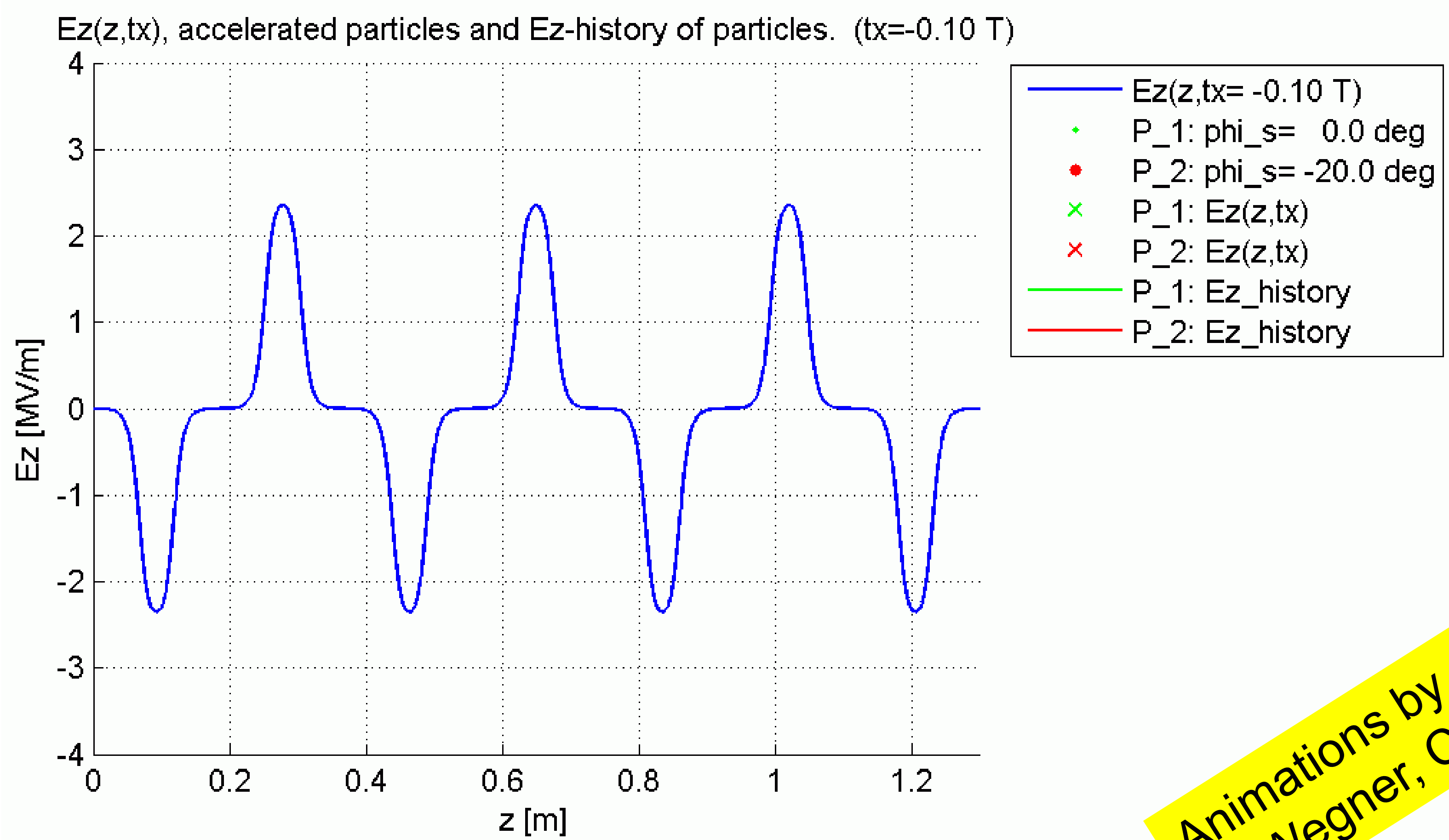
CERN

Synchronism in the PIMS



Animations by
R. Wegner, CERN

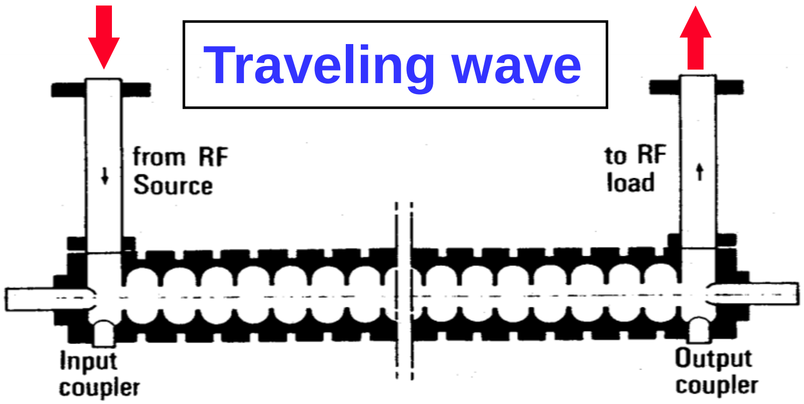
Acceleration in the PIMS



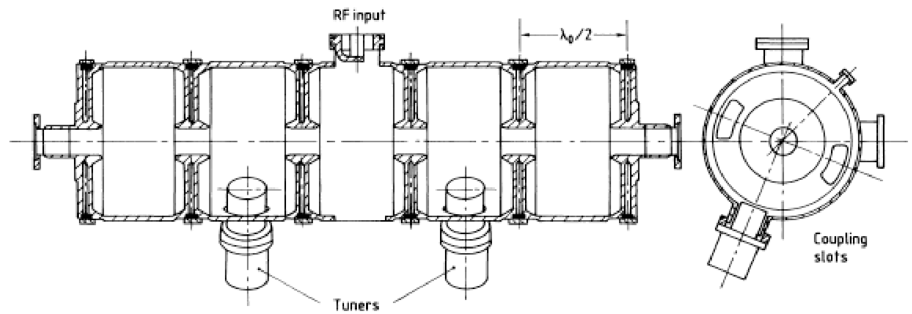
Animations by R. Wegner, CERN

Comparing traveling and standing wave structures

Traveling wave



Standing wave



Chain of coupled cells in TW mode
Coupling bw. cells from on-axis aperture.
RF power from input coupler at one end, dissipated in the structure and on a load.

Short pulses, High frequency (≥ 3 GHz).
Gradients 10-20 MeV/m

Used for Electrons at v~c

Chain of coupled cells in SW mode.
Coupling (bw. cells) by slots (or open). On-axis aperture reduced, higher E-field on axis and power efficiency.
RF power from a coupling port, dissipated in the structure (ohmic loss on walls).

Long pulses. Gradients 2-5 MeV/m

Used for Ions and electrons, all energies

Comparable RF efficiencies

Questions on Module 1 ?

- Types of linacs and domains of application
- Basic linac structure, synchronicity, single cell and multicell linacs
- Acceleration in a disc-loaded waveguide, standing-wave and traveling-wave structures