HGS-HIRe for FAIR

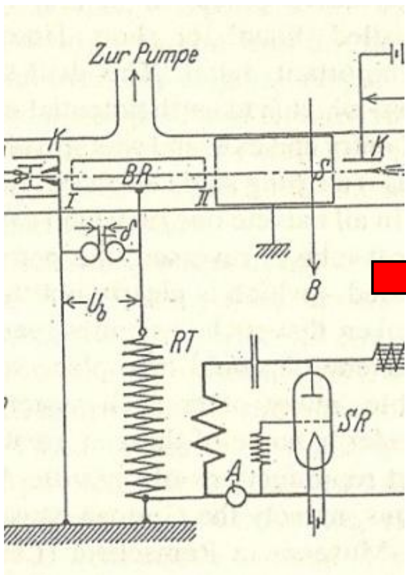
Helmholtz Graduate School for Hadron and Ion Research

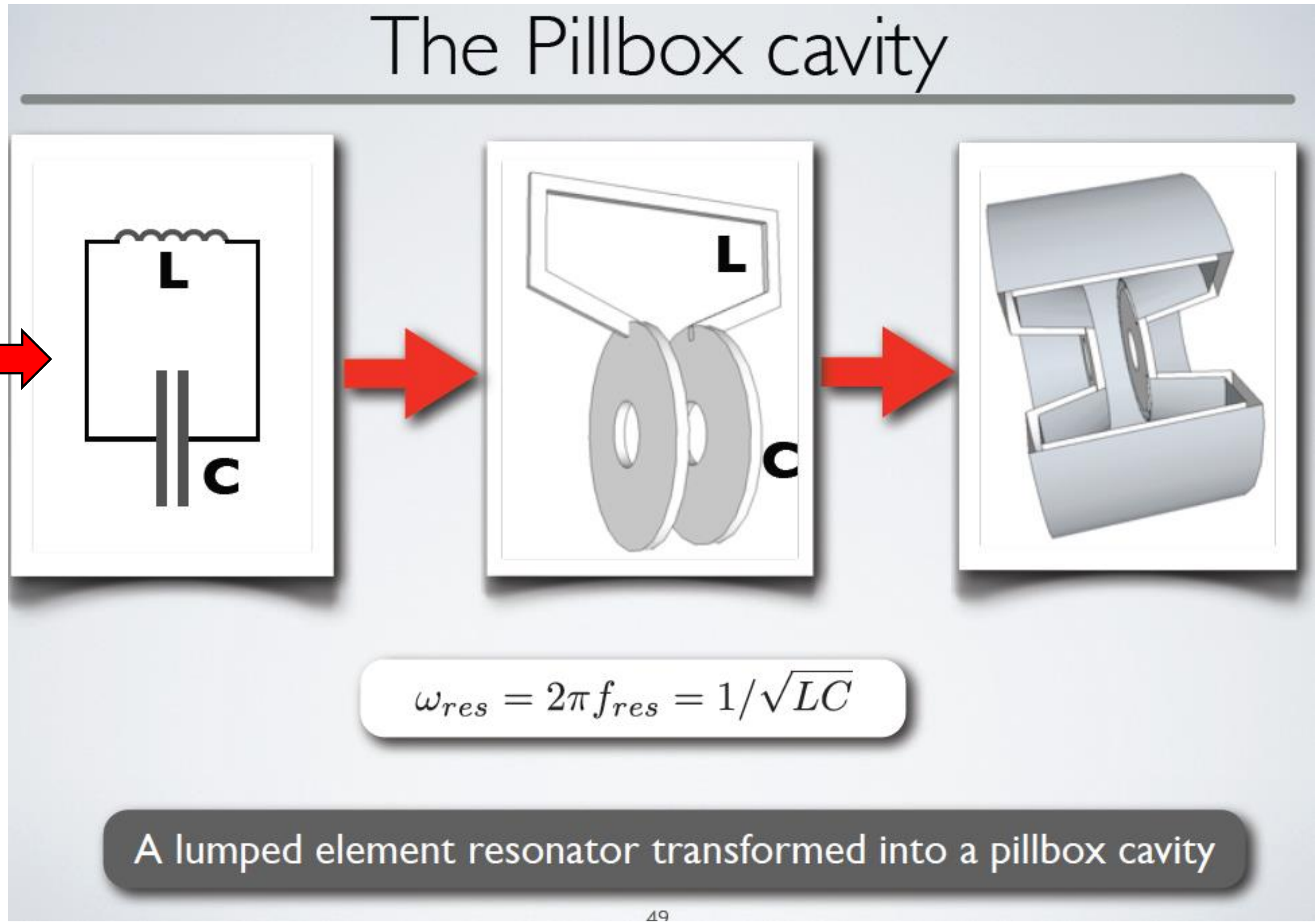
CERN

Module 3

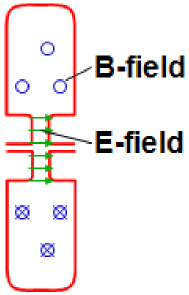
Coupled resonator chains
Stability and stabilization
Acceleration in periodic structures
Special accelerating structures
Superconducting linac structures

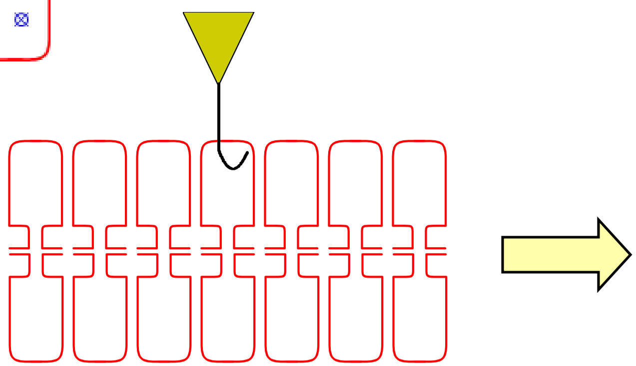
From the Wideröe gap to the linac cell

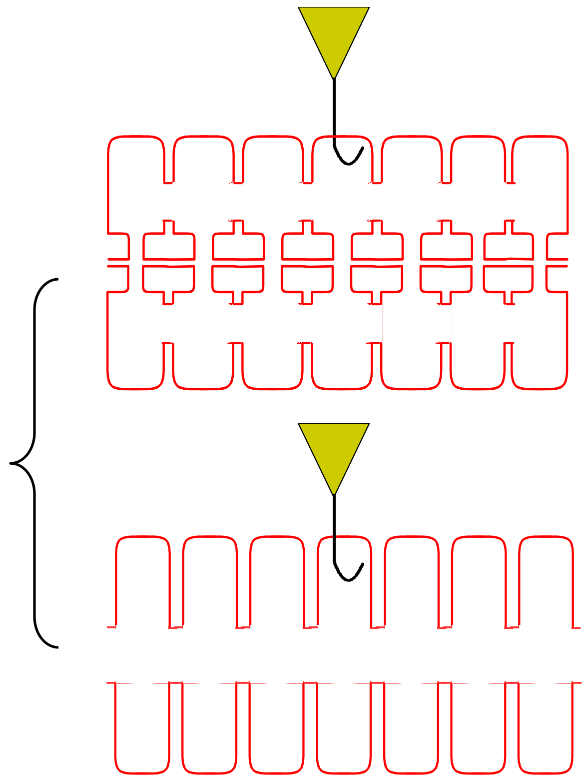




Coupling cavities







1. **Magnetic coupling:**

open “slots” in regions of high magnetic field → B-field can couple from one cell to the next

2. **Electric coupling**:

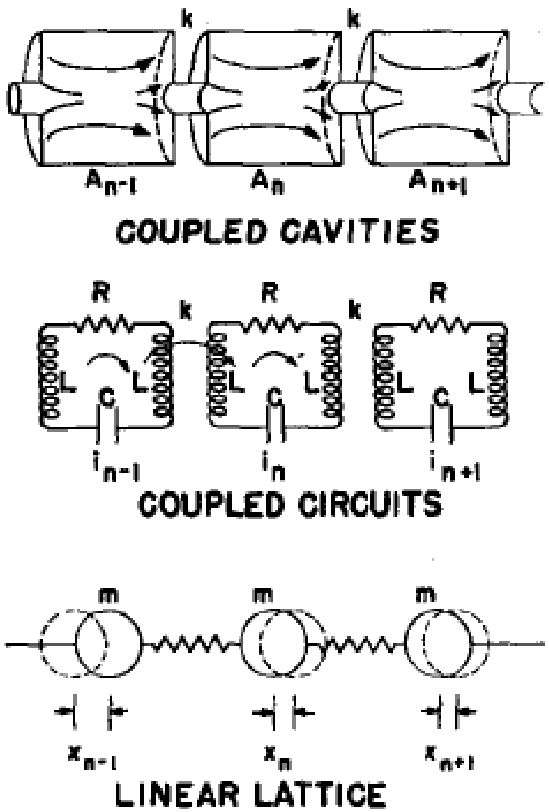
enlarge the beam aperture → E-field can couple from one cell to the next

How can we couple together a chain of n reentrant (=loaded pillbox) accelerating cavities ?

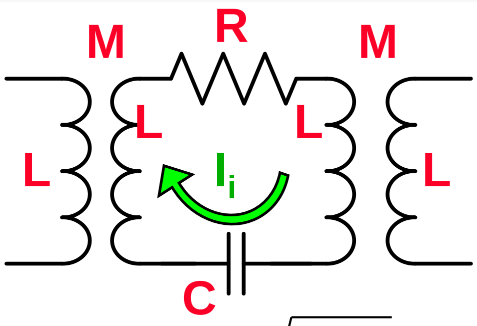
The effect of the coupling is that the cells no longer resonate independently, but will have common resonances with well defined field patterns.

Chains of coupled resonators

What is the relative phase and amplitude between cells in a chain of coupled cavities?



A linear chain of accelerating cells can be represented as a sequence of resonant circuits magnetically coupled.



$$\omega_0 = 1/\sqrt{2LC}$$

$$M = k\sqrt{L_1 L_2} = kL$$

Individual cavity resonating at $\omega_0 \rightarrow$ frequenci(es) of the coupled system ?

Resonant circuit equation for circuit *i* (neglecting the losses, R≅0):

$$I_i\left(2j\omega L + \frac{1}{j\omega C}\right) + j\omega kL\left(I_{i-1} + I_{i+1}\right) = 0$$

Dividing both terms by $2j\omega L$:

$$X_i\left(1 - \frac{\omega_0^2}{\omega^2}\right) + \frac{k}{2}\left(X_{i-1} + X_{i+1}\right) = 0$$

General response term, ∝ (stored energy)$^{1/2}$, can be voltage, E-field, B-field, etc.

General resonance term

Contribution from adjacent oscillators

The Coupled-system Matrix

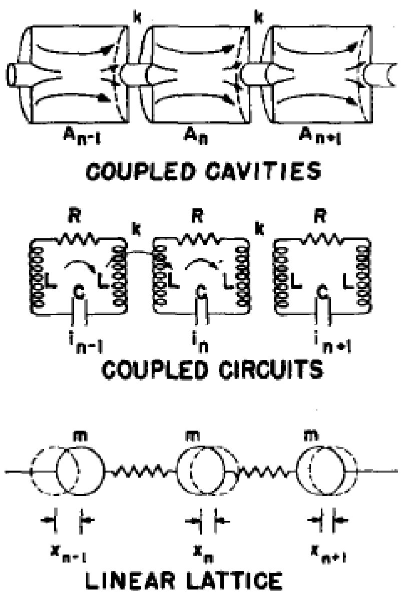
A chain of N+1 resonators is described by a (N+1)x(N+1) matrix:

$$X_i\left(1-\frac{\omega_0^2}{\omega^2}\right)+\frac{k}{2}\left(X_{i-1}+X_{i+1}\right)=0$$

$$i=0,..,N$$

$$\begin{vmatrix} 1-\frac{\omega_0^2}{\omega^2} & \frac{k}{2} & 0 & \dots \\ \frac{k}{2} & 1-\frac{\omega_0^2}{\omega^2} & \frac{k}{2} & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \frac{k}{2} & 1-\frac{\omega_0^2}{\omega^2} \end{vmatrix} \begin{vmatrix} X_0 \\ X_2 \\ \dots \\ X_N \end{vmatrix} = 0$$

or $M\,X=0$



This matrix equation has solutions only if $\det M = 0$

Eigenvalue problem!

1. System of order (N+1) in $\omega \rightarrow$ only N+1 frequencies will be solution of the problem ("eigenvalues", corresponding to the resonances) $\rightarrow$ a system of N coupled oscillators has N resonance frequencies $\rightarrow$ an *individual resonance opens up into a band of frequencies*.
2. At each frequency ω_i will correspond a set of relative amplitudes in the different cells (X_0, X_2, …, X_N): the "eigenmodes" or "modes".

Modes in a linear chain of oscillators

We can find an analytical expression for eigenvalues (frequencies) and eigenvectors (modes):

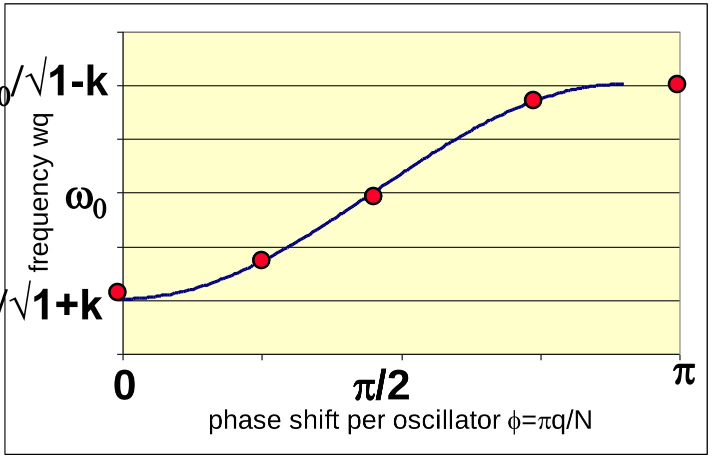
Frequencies of the coupled system :

$$\omega_q^2 = \frac{\omega_0^2}{1 + k\cos\frac{\pi q}{N}}, \qquad q = 0,..,N$$

the index q defines the number of the solution → is the "mode index"

→ Each mode is characterized by a phase **πq/N**. Frequency vs. phase of each mode can be plotted as a "dispersion curve" ω=f(ϕ):

1. each mode is a point on a sinusoidal curve.
2. modes are equally spaced in phase.

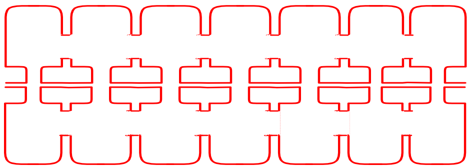


The "eigenvectors = relative amplitude of the field in the cells are:

$$X_i^{(q)} = (const)\cos\frac{\pi q i}{N} e^{j\omega_q t} \qquad q = 0,...,N$$

STANDING WAVE MODES, defined by a phase πq/N corresponding to the phase shift between an oscillator and the next one → πq/N=Φ is the phase difference between adjacent cells that we have introduces in the 1st part of the lecture.

Acceleration on the normal modes of a 7-cell structure

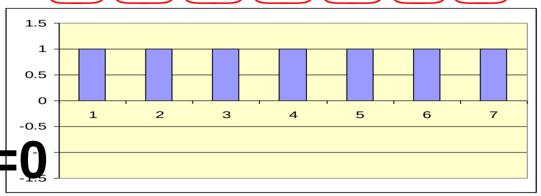


$$X_i^{(q)} = (const) \cos\frac{\pi q i}{N} e^{j\omega_q t} \qquad q = 0, \ldots, N$$

Remember the phase relation!

$$\Delta\phi = 2\pi \frac{d}{\beta\lambda}$$

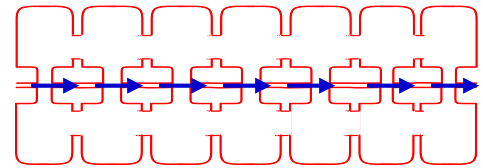
0
q=0

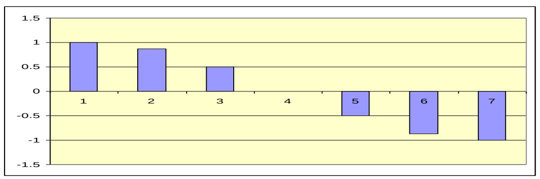


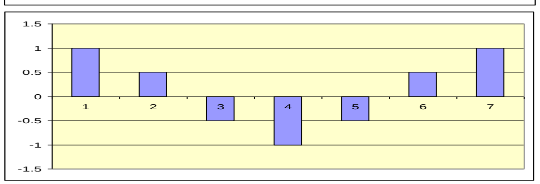
$$\Phi = 2\pi, \quad 2\pi\frac{d}{\beta\lambda} = 2\pi, \quad d = \beta\lambda$$

$\omega = \omega_0/\sqrt{1+k}$

0 (or 2π) mode, acceleration if **d = *βλ***

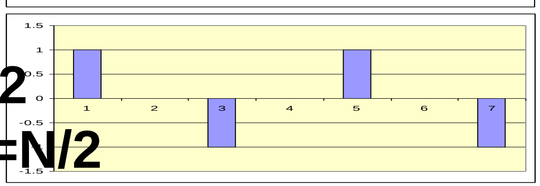






Intermediate modes

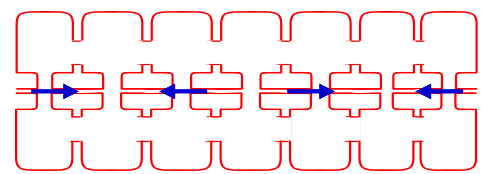
π/2
q=N/2



$$\Phi = \frac{\pi}{2}, \quad 2\pi\frac{d}{\beta\lambda} = \frac{\pi}{2}, \quad d = \frac{\beta\lambda}{4}$$

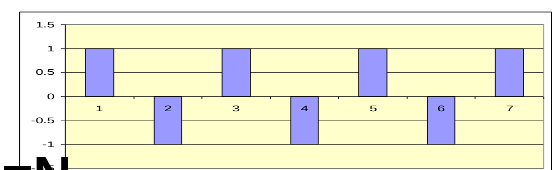
$\omega = \omega_0$

π/2 mode, acceleration if **d = *βλ/4***



…

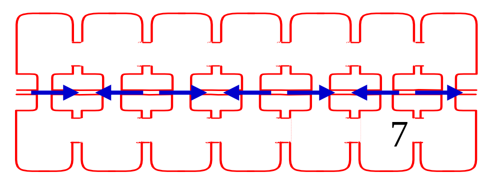
π
q=N



$$\Phi = \pi, \quad \pi\frac{d}{\beta\lambda} = 2\pi, \quad d = \frac{\beta\lambda}{2}$$

$\omega = \omega_0/\sqrt{1-k}$

π mode, acceleration if **d = *βλ/2,***



Note: Field always maximum in first and last cell!

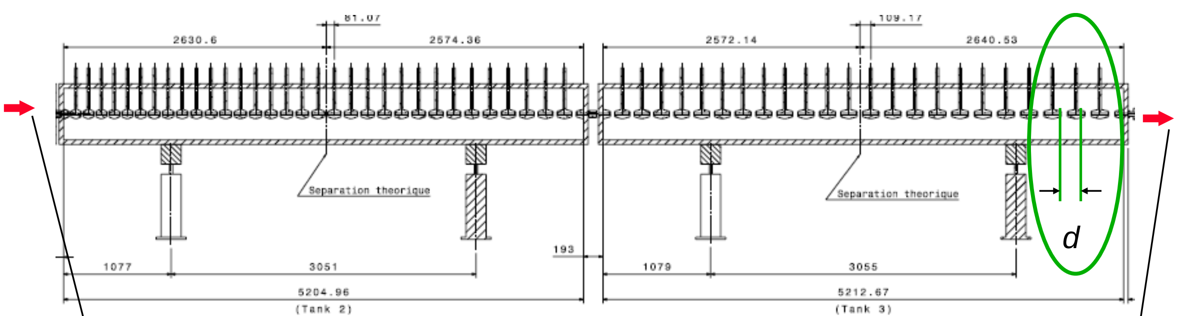
Practical linac accelerating structures

Note: our equations depend only on the cell frequency ω, not on the cell length d !!!

$$\omega_q^2 = \frac{\omega_0^2}{1 + k\cos\frac{\pi q}{N}}, \qquad q = 0,\ldots,N$$

$$X_n^{(q)} = (const)\cos\frac{\pi q n}{N}\, e^{j\omega_q t} \qquad q = 0,\ldots,N$$

→ As soon as we keep the frequency of each cell constant, we can change the cell length following any acceleration (β) profile!



10 MeV, $\beta = 0.145$

50 MeV, $\beta = 0.31$

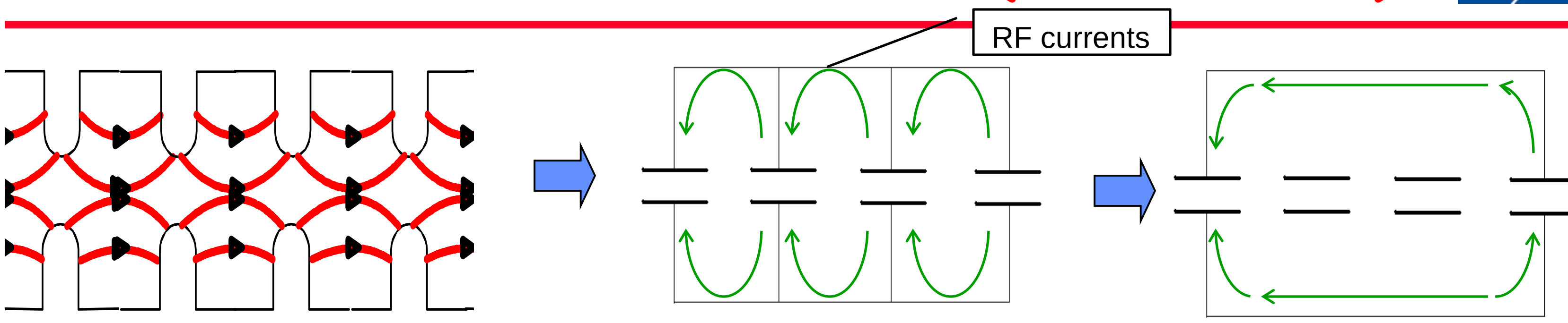
Example:
The Drift Tube Linac (DTL)

Chain of many (up to 100!) accelerating cells operating in the 0 mode. The ultimate coupling slot: no wall between the cells!

Each cell has a different length, but the cell frequency remains constant → *"the EM fields don't see that the cell length is changing!"*

d↑ → (L ↑ , C↓) → LC ~ const → ω ~ const

0-mode structures: the Drift Tube Linac ("Alvarez")



Disc-loaded structures operating in 0-mode

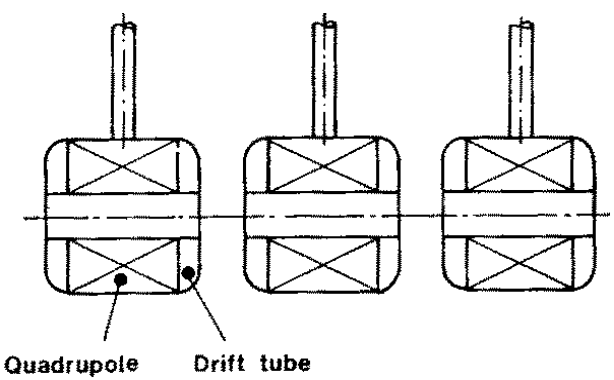
Add tubes for high shunt impedance

Maximize coupling between cells → remove completely the walls

2 advantages of the 0-mode:

1. the fields are such that if we eliminate the walls between cells the fields are not affected, but we have less RF currents and higher power efficiency ("shunt impedance").
2. The "drift tubes" are long (~0.75 $\beta\lambda$). The particles are inside the tubes when the electric field is decelerating, and we have space to introduce focusing elements (quadrupoles) inside the tubes.

Disadvantage (w.r.t. the π mode): half the number of gaps per unit length!



The Drift Tube Linac

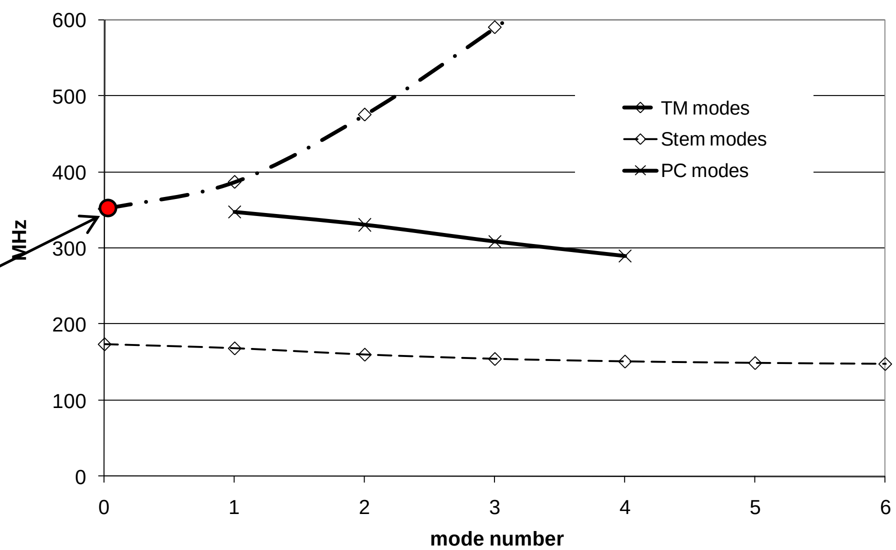
A DTL tank with N drift tubes will have N modes of oscillation.
For acceleration, we choose the 0-mode, the lowest of the band.
All cells (gaps) are in phase, then $\Delta\phi=2\pi$

$$\Delta\phi = 2\pi \frac{d}{\beta\lambda} = 2\pi \quad \Rightarrow \quad d = \beta\lambda$$

Distance between gaps must be $\beta\lambda$

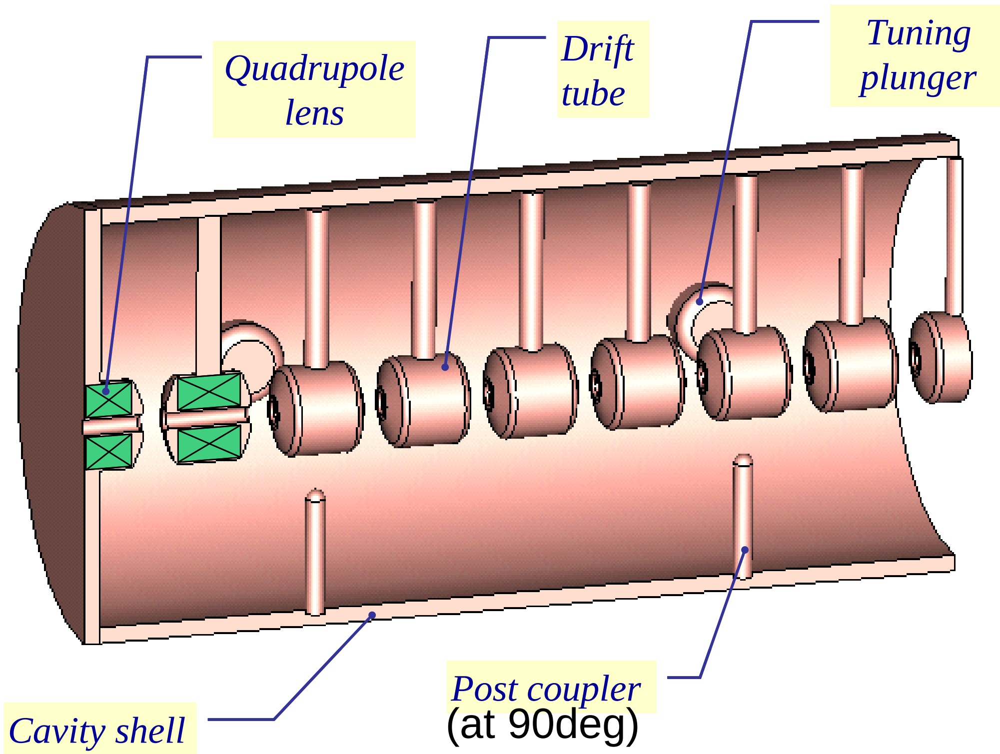
The other modes in the band (and many others!) are still present.
If mode separation >> bandwidth, they are not "visible" at the operating frequency, but they can come out in case of frequency errors between the cells (mechanical errors or others).

operating 0-mode



mode distribution in a DTL tank (operating frequency 352 MHz, are plotted all frequencies < 600 MHz)

DTL construction

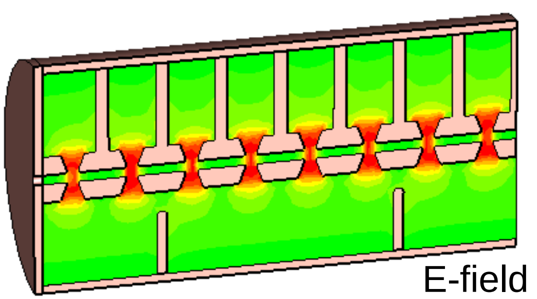


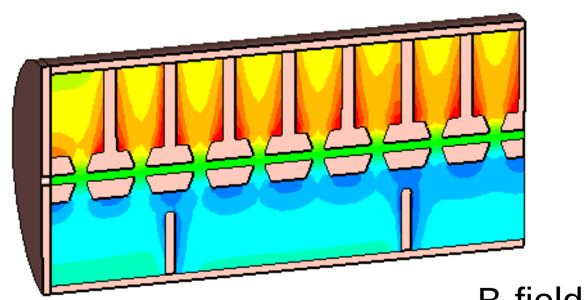
Standing wave linac structure for protons and ions, β=0.1-0.5, f=20-400 MHz

Drift tubes are suspended by stems (no net RF current on stem)

Coupling between cells is maximum (no slot, fully open !)

The 0-mode allows a long enough cell (d=βλ) to house <u>focusing quadrupoles inside the drift tubes</u>!



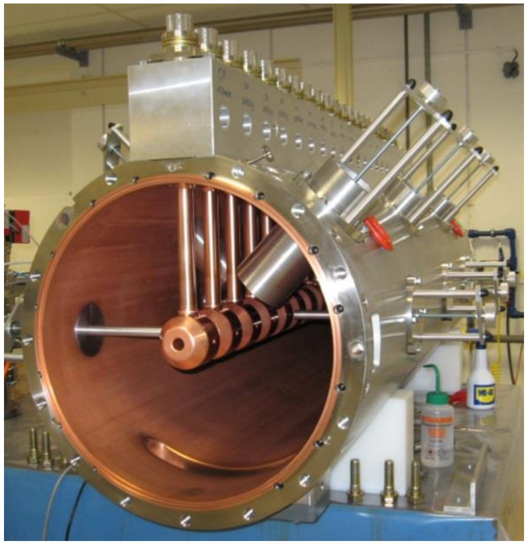


Examples of DTL

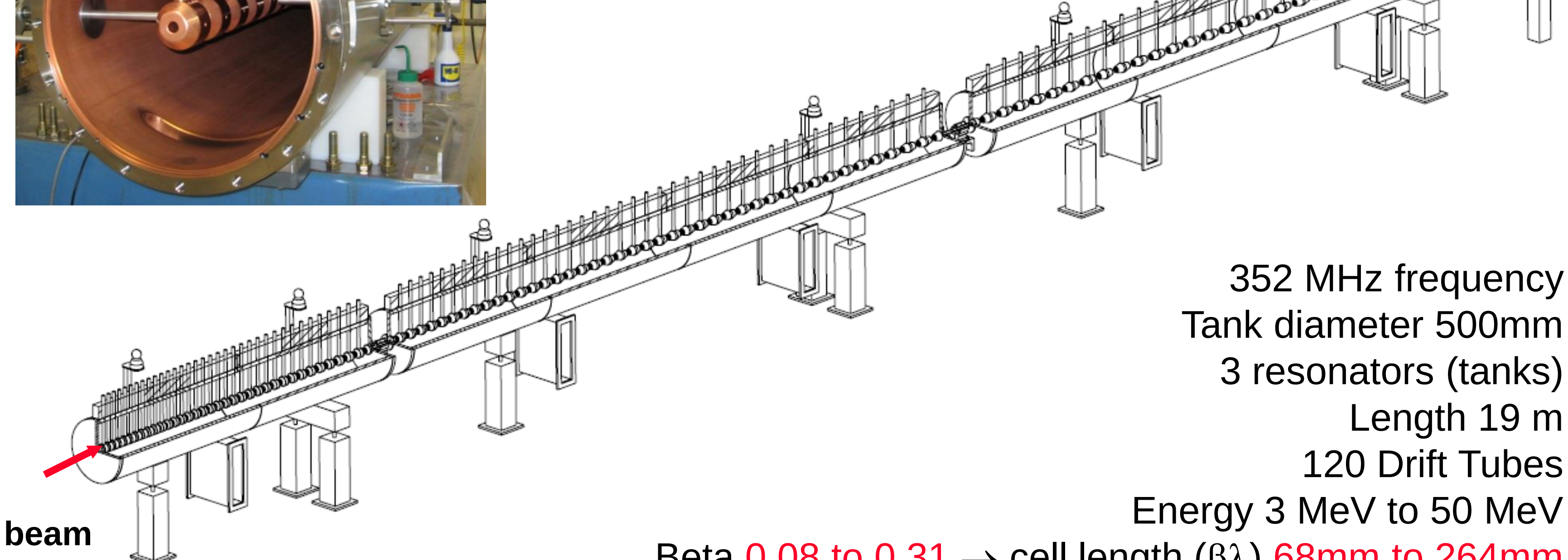
Top; CERN Linac2 Drift Tube Linac: 1978, 202.5 MHz, 3 tanks, final energy 50 MeV, tank diameter 1 meter.

Left: The Drift Tube Linac of the SNS at Oak Ridge (USA): 402.5 MHz, 6 tanks, final energy 87 MeV.

The Linac4 DTL



DTL tank 1 fully equipped: focusing by small permanent quadrupoles inside drift tubes.



352 MHz frequency

Tank diameter 500mm

3 resonators (tanks)

Length 19 m

120 Drift Tubes

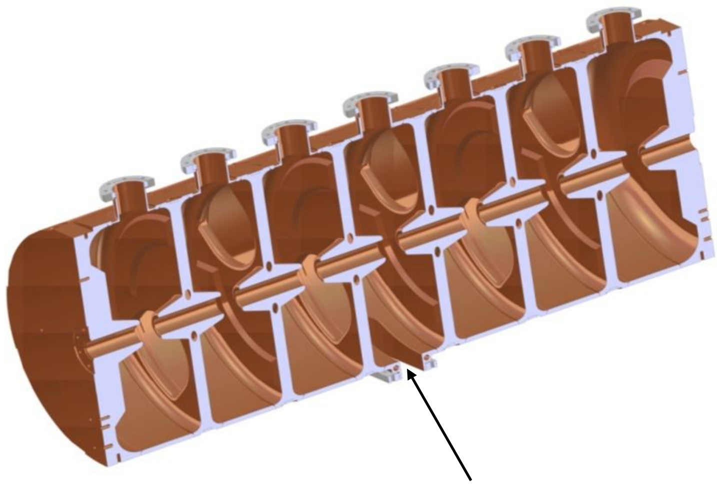
Energy 3 MeV to 50 MeV

Beta 0.08 to 0.31 → cell length ($\beta\lambda$) 68mm to 264mm

→ factor 3.9 increase in cell length

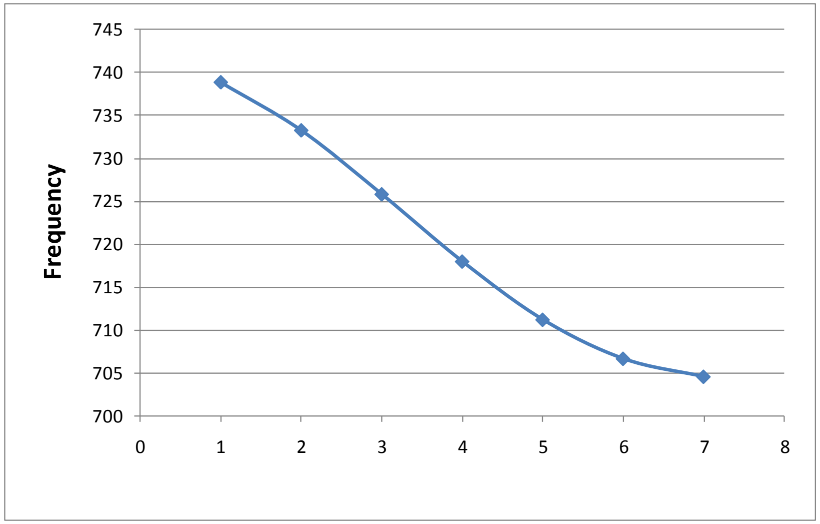
Pi-mode structures: the PIMS

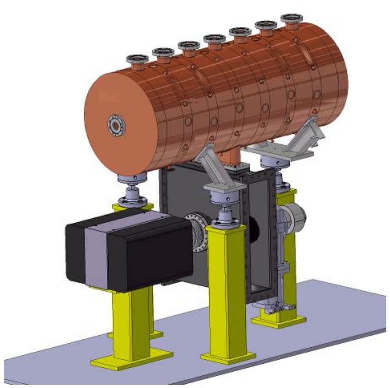
PIMS = Pi-Mode Structure, will be used in Linac4 at CERN to accelerate protons from 100 to 160 MeV (β > 0.4)



RF input

7 cells magnetically coupled, 352 MHz
Operating in π-mode, cell length $\beta\lambda/2$.

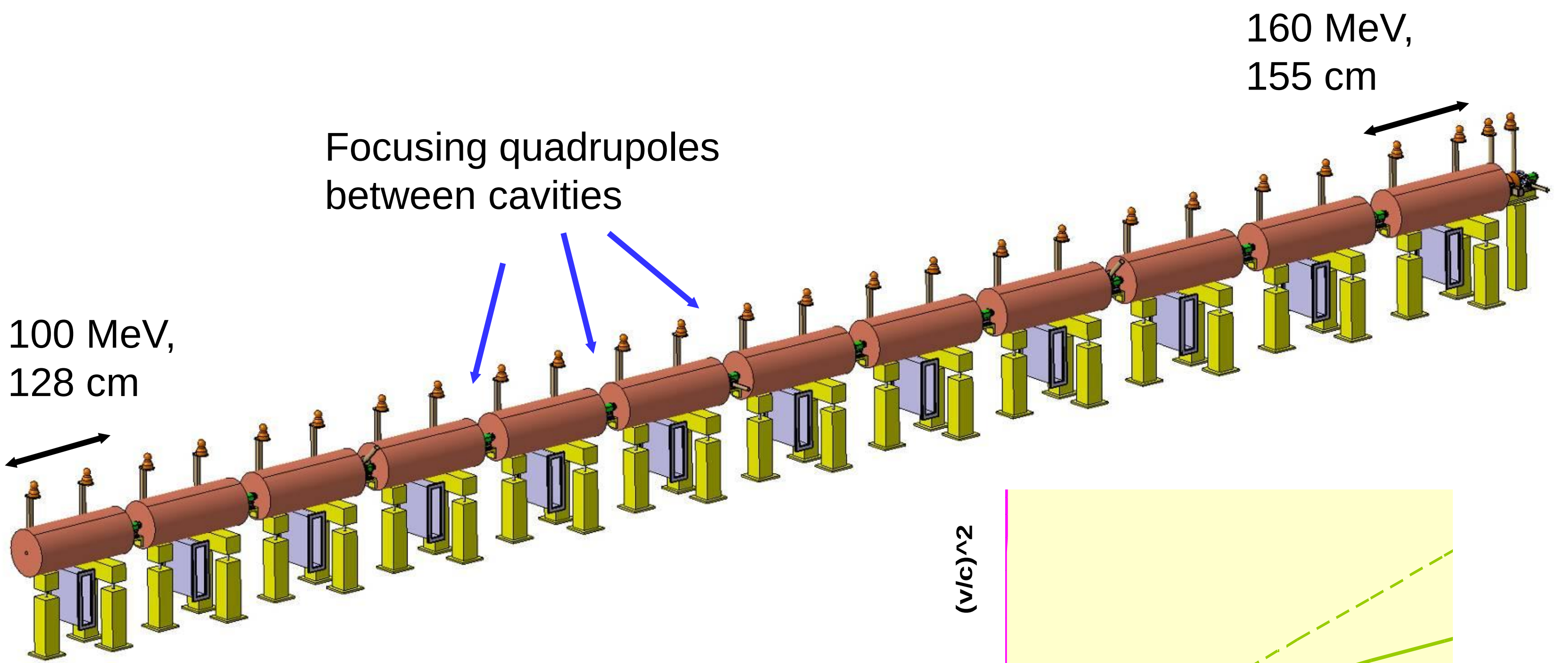


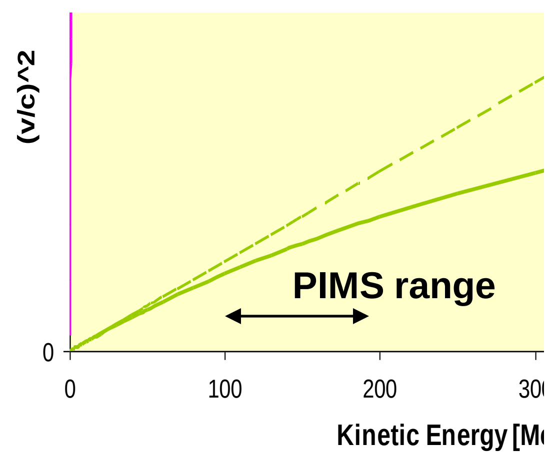


Cells in a cavity have the same length.
When more cavities are used for acceleration, the cells are longer from one cavity to the next, to follow the increase in beam velocity.

Sequence of PIMS cavities

Cells have same length inside a cavity (7 cells) but increase from one cavity to the next.
At high energy (>100 MeV) beta changes slowly and phase error ("phase slippage") is small.





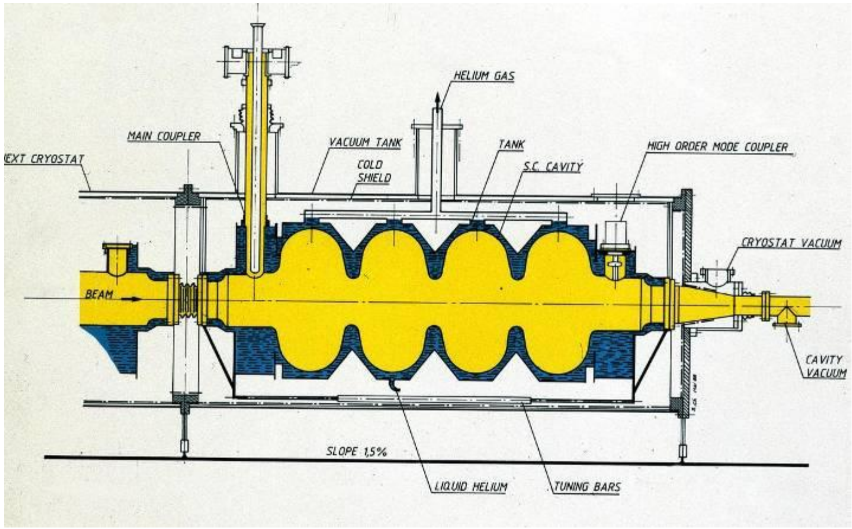
Pi-mode superconducting structures (elliptical)

Standing wave structures for particles at β>0.5-0.7, widely used for protons (SNS, etc.) and electrons (ILC, etc.) f=350-700 MHz (protons), f=350 MHz – 3 GHz (electrons)

Chain of cells electrically coupled, large apertures (ZT^2 not a concern).

Operating in π-mode, cell length $\beta\lambda/2$

Input coupler placed at one end.



Coupling between two cavities

In the PIMS, cells are coupled via a slot in the walls. But what is the meaning of coupling, and how can we achieve a given coupling?

Simplest case: **2 resonators coupled via a slot**

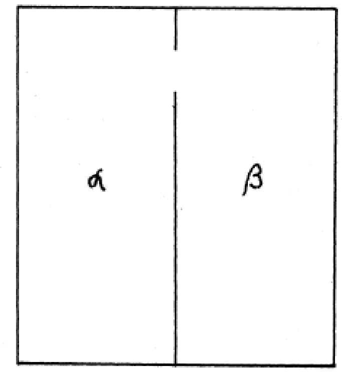


FIG. 1. Two cavities, α and β, coupled by a small hole.


E-field mode 1
E-field mode 2

Described by a system of 2 equations:

$$\begin{cases} X_1\left(1-\dfrac{\omega_1^2}{\omega^2}\right)+kX_2=0 \\ kX_1+X_2\left(1-\dfrac{\omega_2^2}{\omega^2}\right)=0 \end{cases}$$

or

$$\begin{vmatrix} 1-\dfrac{\omega_1^2}{\omega^2} & k \\ k & 1-\dfrac{\omega_2^2}{\omega^2} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = 0$$

If $\omega_1=\omega_2=\omega_0$, usual 2 solutions (mode 0 and mode π):

$$\omega_{c,1}=\frac{\omega_0}{\sqrt{1+k}} \quad \text{with} \quad \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \qquad \text{and} \qquad \omega_{c,2}=\frac{\omega_0}{\sqrt{1-k}} \quad \text{with} \quad \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} 1 \\ -1 \end{vmatrix}$$

Mode + + (field in phase in the 2 resonators) and mode + - (field with opposite phase)

Taking the difference between the 2 solutions (squared), approximated for k<<1

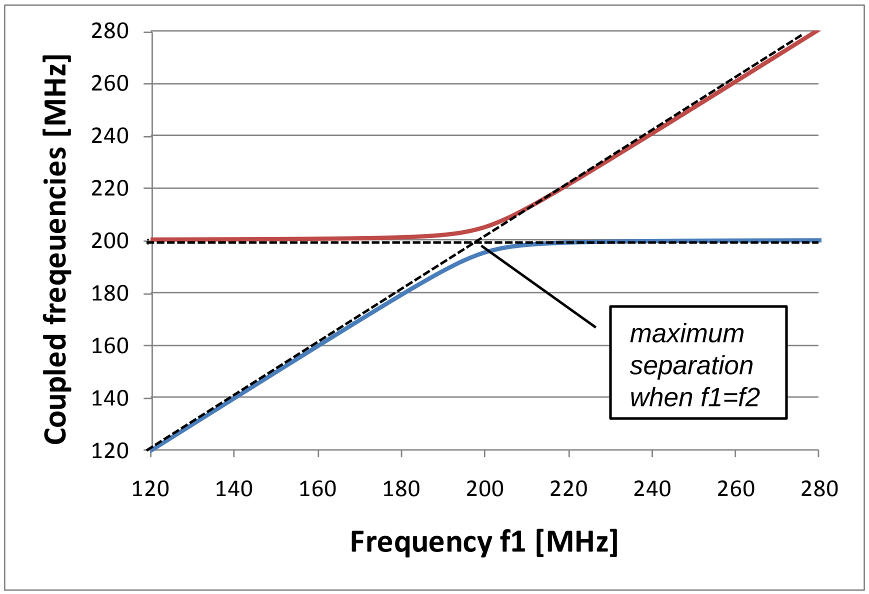
$$\frac{\omega_{c,2}^2-\omega_{c,1}^2}{\omega_0^2}=\frac{1}{1-k}-\frac{1}{1+k}\approx 2k \quad \text{or} \quad \boxed{\frac{\omega_{c,2}-\omega_{c,1}}{\omega_0}\approx k}$$

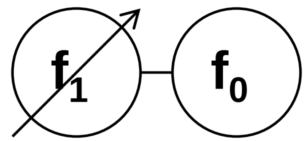
The coupling k is equal to the difference between highest and lowest frequencies.

$\longrightarrow$ k is the **bandwidth of the coupled system.**

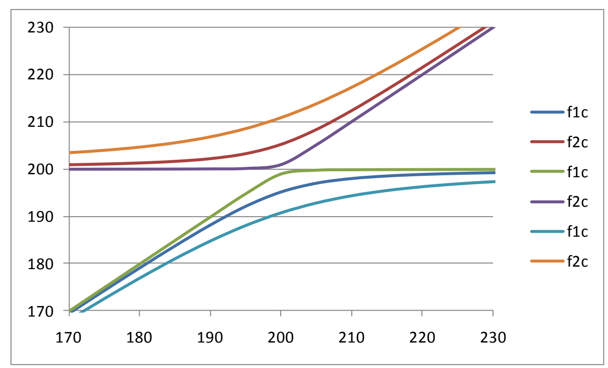
More on coupling

Solving the previous equations allowing a different frequency for each cell, we can plot the frequencies of the coupled system as a function of the frequency of the first resonator, keeping the frequency of the second constant, for different values of the coupling k.





- "Coupling" only when the 2 resonators are close in frequency.
- For $f_1=f_2$, maximum spacing between the 2 frequencies ($=kf_0$)



case of 3 different coupling factors (0.1%, 5%, 10%)

For an elliptical coupling slot:

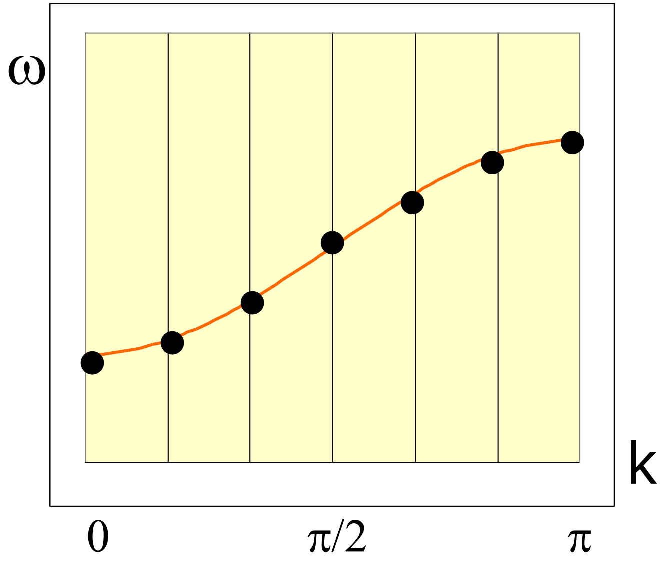
$$k \approx F\, l^3 \left(\frac{H_1}{\sqrt{U_1}}\right)\left(\frac{H_2}{\sqrt{U_2}}\right)$$

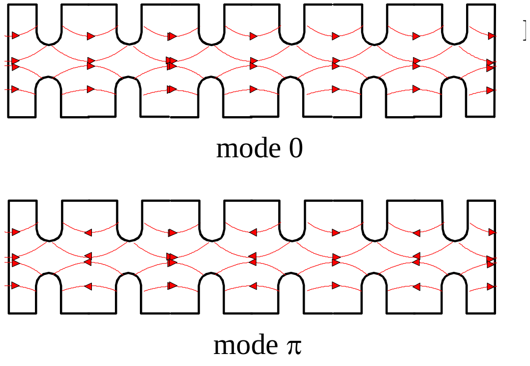
F = slot form factor
l = slot length (in the direction of H)
H = magnetic field at slot position
U = stored energy

The coupling k is:
- Proportional to the 3rd power of slot length.
- Inv. proportional to the stored energies.

Long chains of linac cells

- To reduce RF cost, linacs use high-power RF sources feeding a large number of coupled cells (DTL: 30-40 cells, other high-frequency structures can have >100 cells).
- But long linac structures (operating in 0 or π mode) become extremely sensitive to mechanical errors: small machining errors in the cells can induce large differences in the accelerating field between cells.





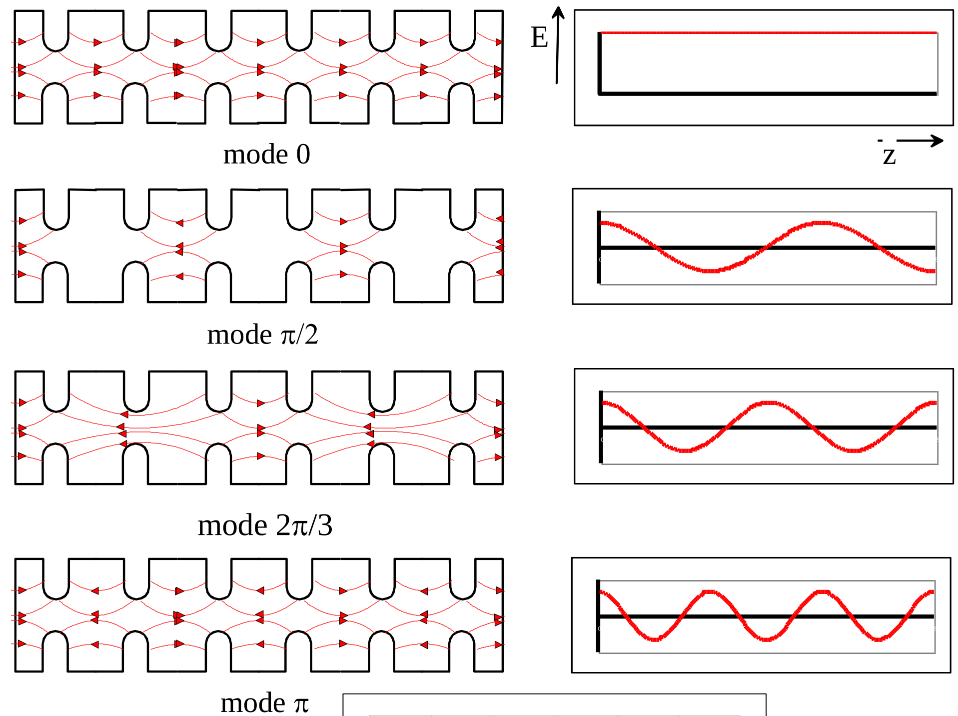
Stability of long chains of coupled resonators

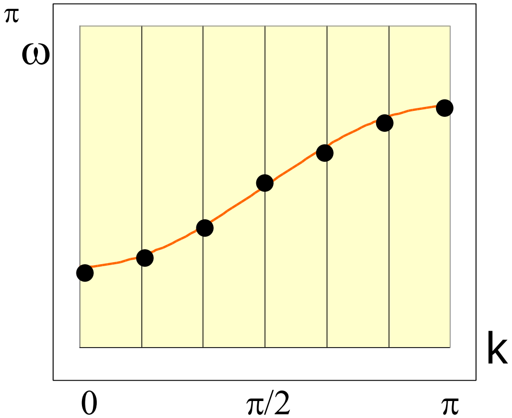
Mechanical errors → differences in frequency between cells →
to respect the new boundary conditions the electric field will be a linear combination of all modes, with weight

$$\frac{1}{f^2 - f_0^2}$$

(general case of small perturbation to an eigenmode system,
the new solution is a linear combination of all the individual modes)

The nearest modes have the highest effect, and when there are many modes on the dispersion curve (number of modes = number of cells, but the total bandwidth is fixed = k !) the difference in E-field between cells can be extremely high.

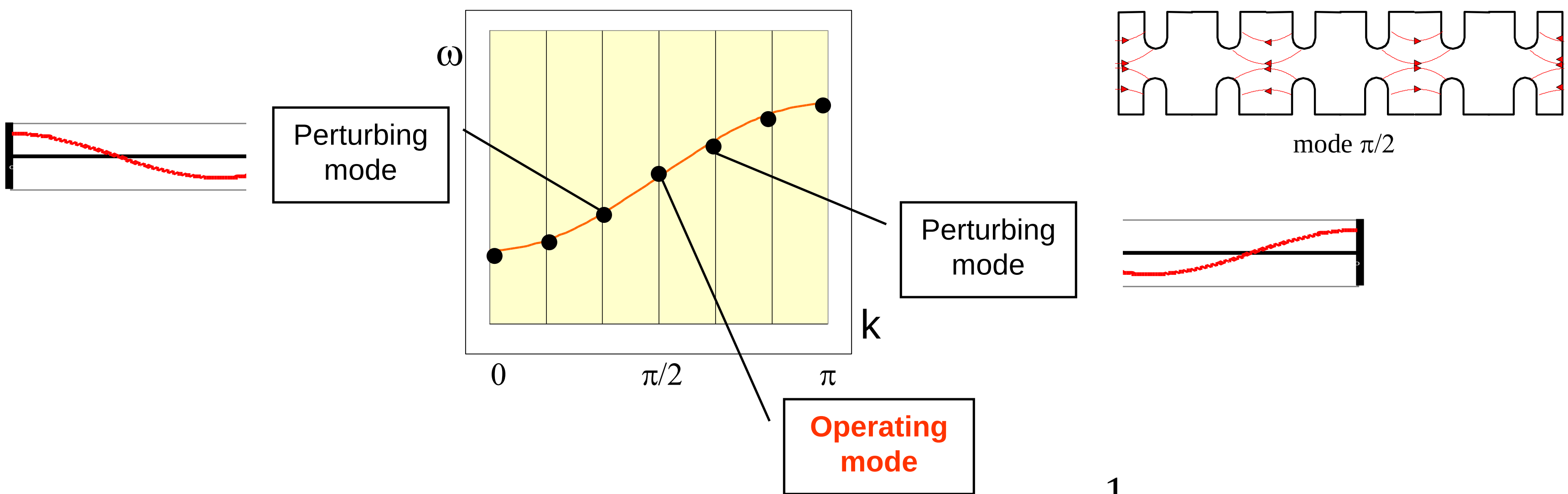




Stabilization of long chains: the $\pi/2$ mode

Long chains of linac cells can be operated in the $\pi/2$ mode, which is intrinsically insensitive to mechanical errors = differences in the cell frequencies.

In presence of errors, the E-field will have components from the adjacent modes, with amplitude proportional to the error and to the mode separation.



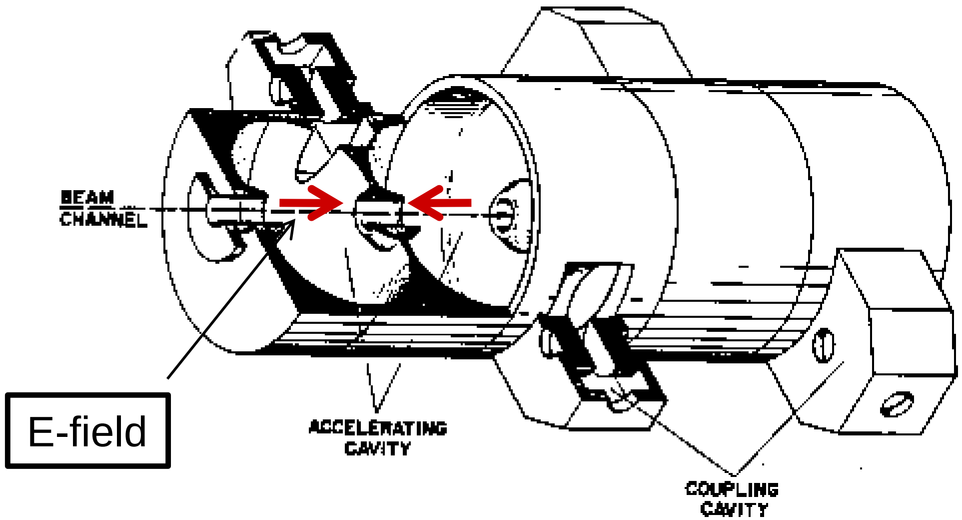
→ Contribution from adjacent modes proportional to $\frac{1}{f^2 - f_0^2}$ with the sign !!!

The perturbation will add a component $\Delta E/(f^2 - f_o^2)$ for each of the nearest modes.

Contributions from equally spaced modes in the dispersion curve will cancel each other !!

Pi/2 mode structures: the Side Coupled Linac

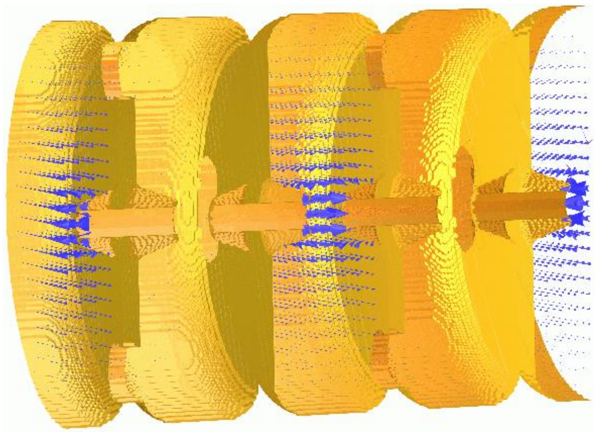
To operate efficiently in the $\pi/2$ mode, the cells that are not excited can be removed from the beam axis $\rightarrow$ they are called "coupling cells", as for the Side Coupled Structure.



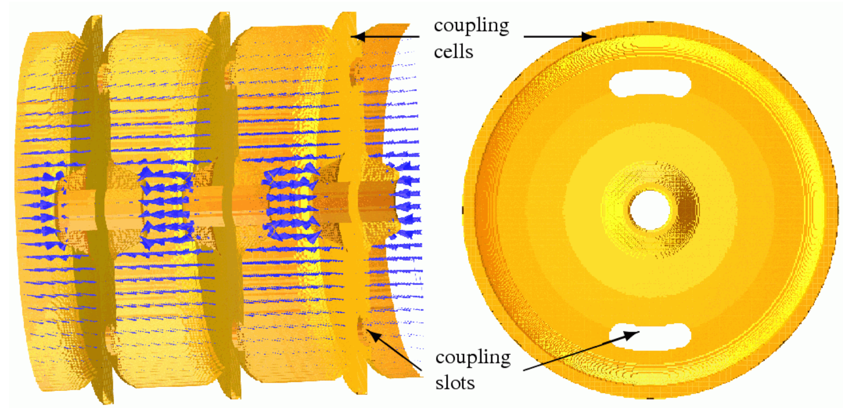
Example: the Cell-Coupled Linac at the SNS linac, 805 MHz, 100-200 MeV, >100 cells/module

Examples of $\pi/2$ structures

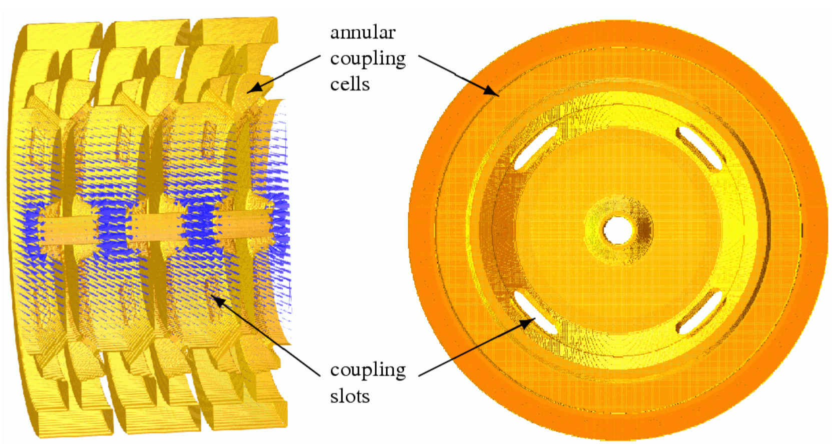
$\pi/2$-mode in a coupled-cell structure



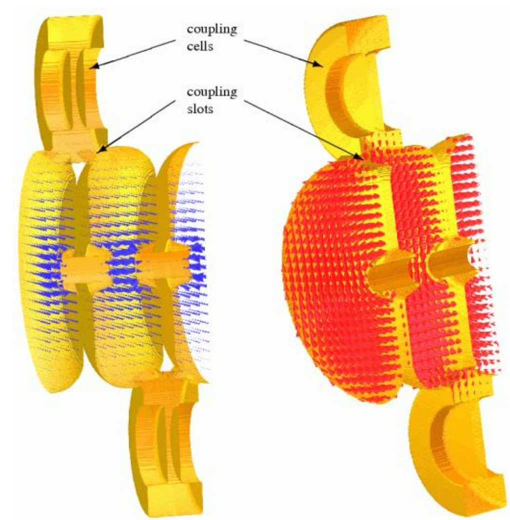
On axis Coupled Structure (OCS)



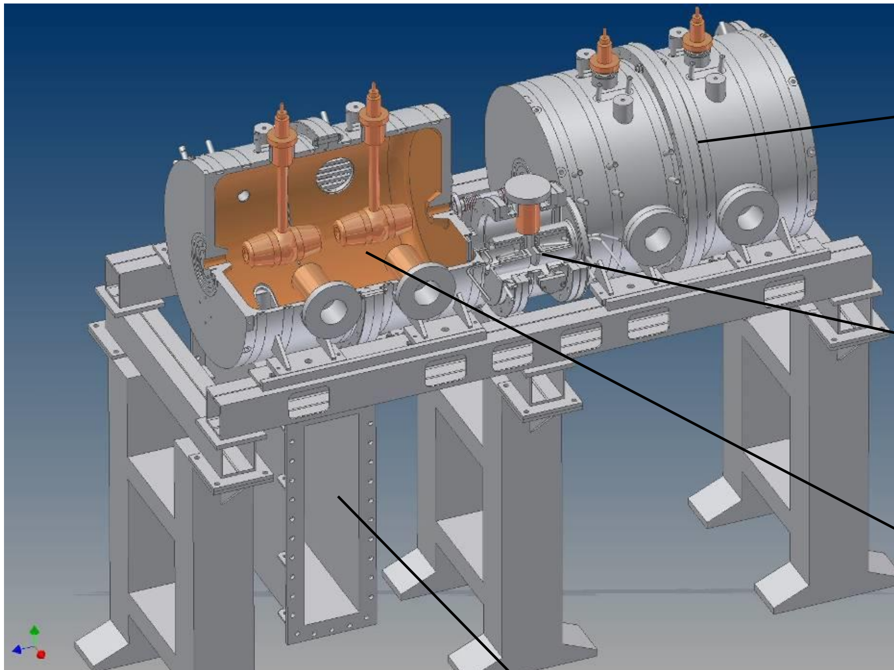
Annular ring Coupled Structure (ACS)



Side Coupled Structure (SCS)



A mixed case: the Cell-Coupled Drift Tube Linac

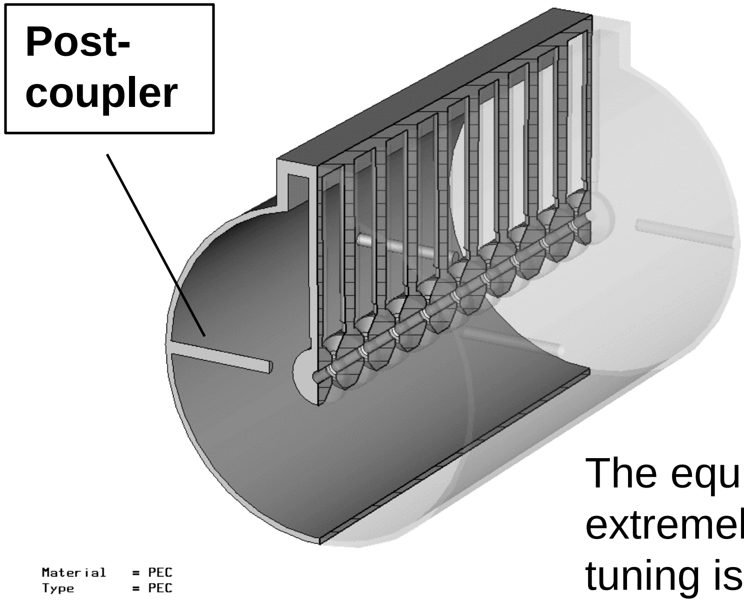


Series of DTL-like tanks with 3 cells (operating in 0-mode), coupled by coupling cells (operation in $\pi/2$ mode)

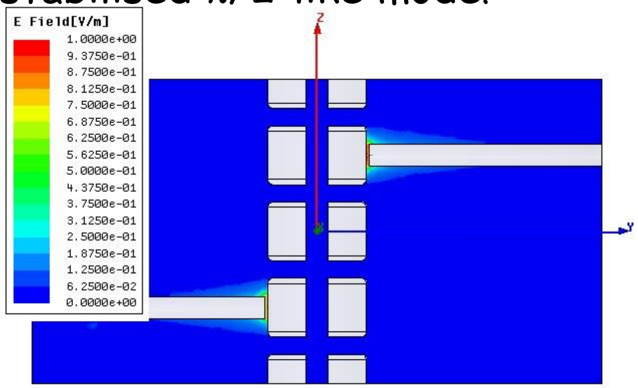
352 MHz, will be used for the CERN Linac4 in the range 40-100 MeV.

The coupling cells leave space for focusing quadrupoles between tanks.

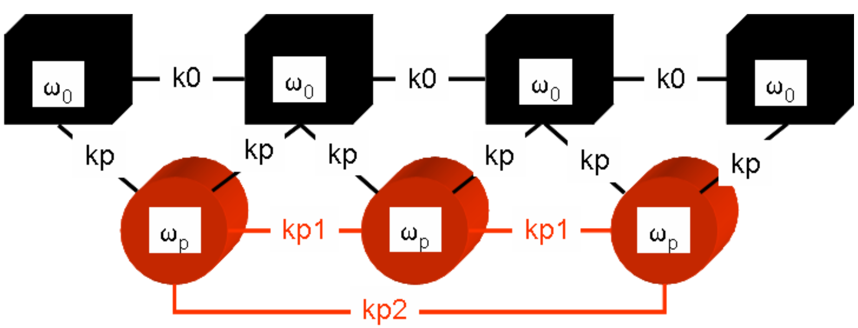
Pi/2 mode: the DTL with post-couplers

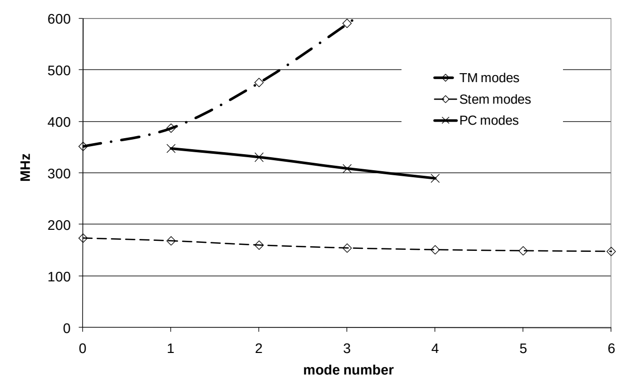


In a DTL, can be added "post-couplers" on a plane perpendicular to the stems.
Each post is a resonator that can be tuned to the same frequency as the main 0-mode and coupled to this mode to double the chain of resonators allowing operation in stabilised $\pi/2$-like mode!

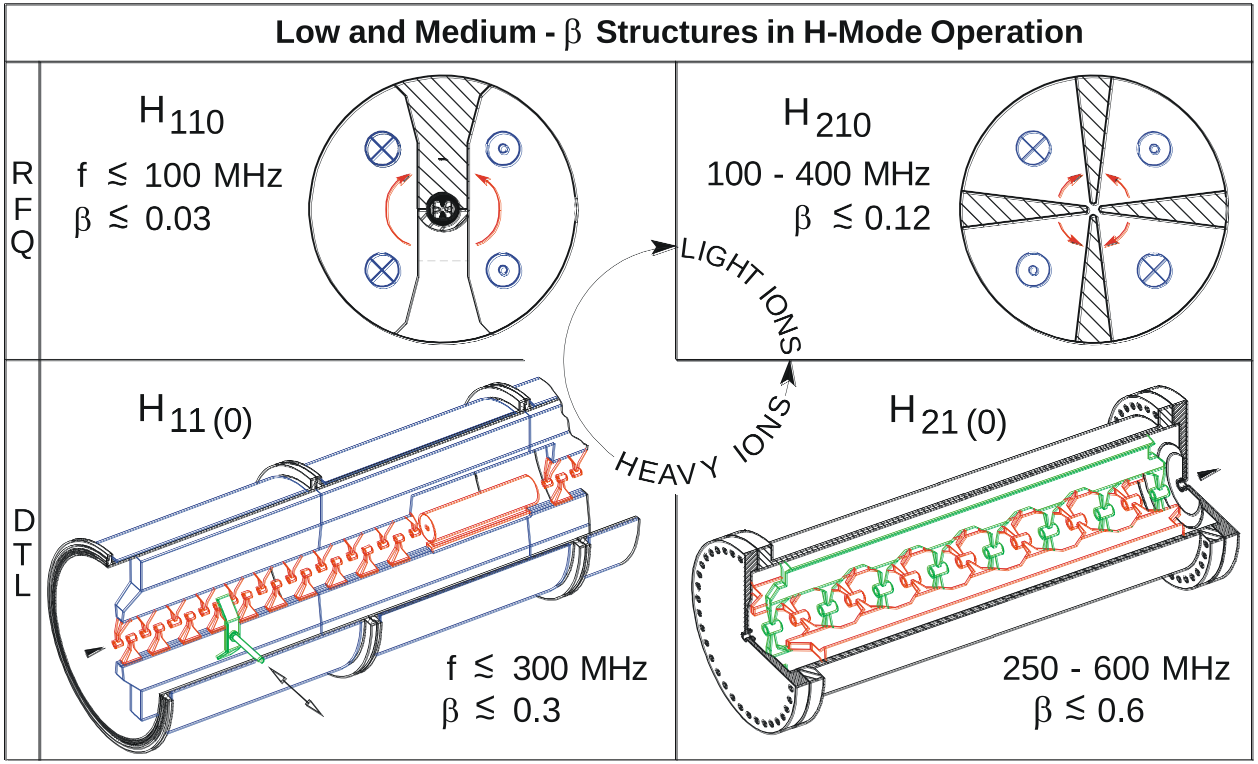


The equivalent circuit becomes extremely complicated and tuning is an issue, but $\pi/2$ stabilization is very effective and allows having long DTL tanks!





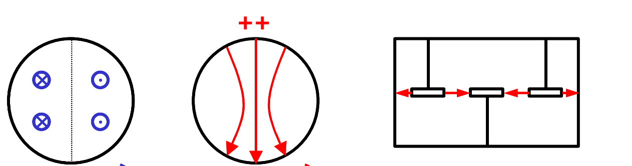
Alternative modes: H-mode structures



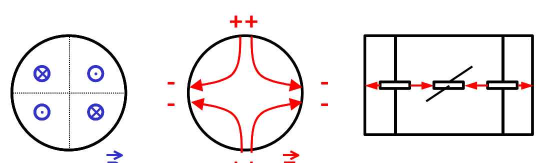
Interdigital-H Structure
Operates in TE110 mode
Transverse E-field
  "deflected" by adding
  drift tubes
Used for ions, β<0.3

CH Structure operates
  in TE210, used for
  protons at β<0.6

High ZT² but more
  difficult beam
  dynamics (no space for
  quads in drift tubes)

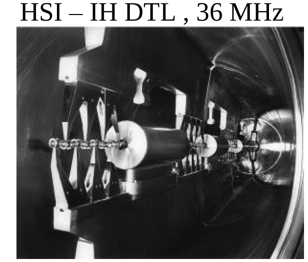


Interdigital H-Mode (IH)



Crossbar H-Mode (CH)

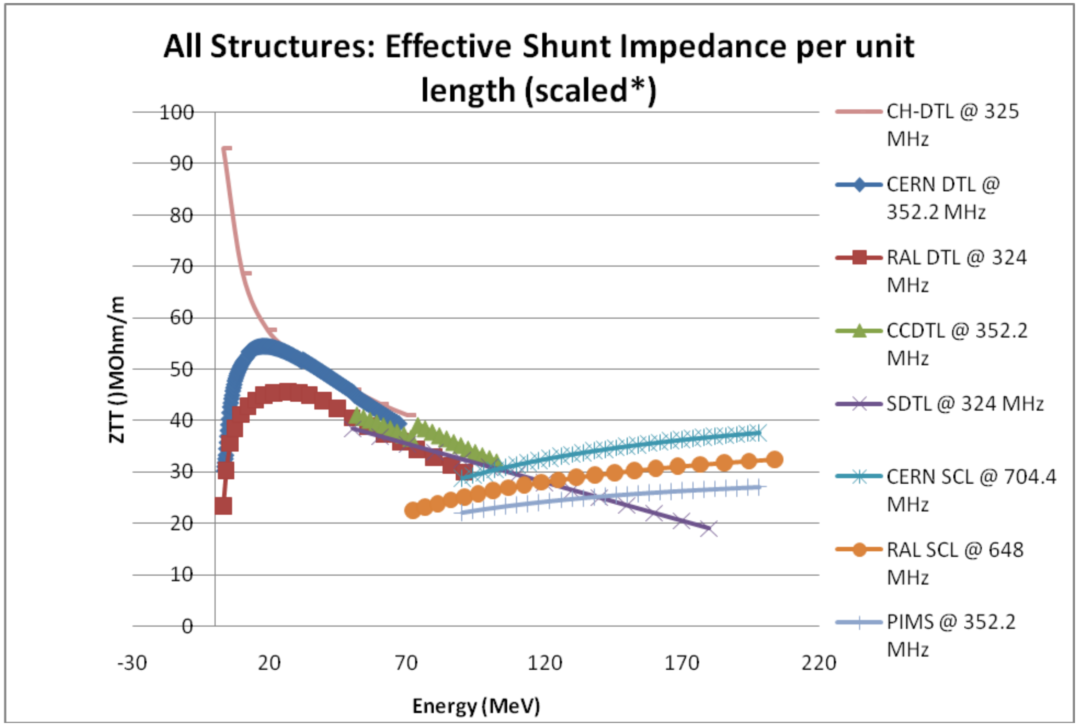
HSI – IH DTL , 36 MHz



Comparison of structures – Shunt impedance

Main figure of merit is the shunt impedance
Ratio between energy gain (square) and power dissipation, is a measure of the energy efficiency of a structure.
Depends on the beta, on the energy and on the mode of operation.

However, the choice of the best accelerating structure for a certain energy range depends also on beam dynamics and on construction cost.



Comparison of shunt-impedances for different low-beta structures done in 2005-08 by the "HIPPI" EU-funded Activity.

In general terms, a DTL-like structure is preferred at low-energy, and π-mode structures at high-energy.
CH is excellent at very low energies (ions).

Traveling wave accelerating structures (electrons)

What happens if we have an infinite chain of oscillators?

$$\omega_q^2 = \frac{\omega_0^2}{1 + k\cos\frac{\pi q}{N}}, \qquad q = 0,\ldots,N$$

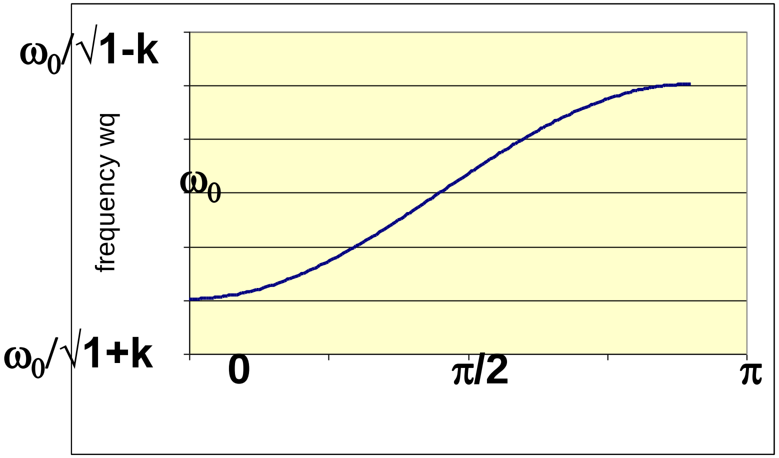
becomes (N→h)

$$\omega^2 = \frac{\omega_0^2}{1 + k\cos\varphi}$$

$$X_n^{(q)} = (const)\cos\frac{\pi q n}{N}e^{j\omega_q t} \qquad q = 0,\ldots,N$$

becomes (N→h)

$$X_i = (const)e^{j\omega_q t}$$



All modes in the dispersion curve are allowed, the original frequency degenerates into a continuous band. The field is the same in each cell, there are no more standing wave modes → only "traveling wave modes", if we excite the EM field at one end of the structure it will propagate towards the other end.

But: our dispersion curve remains valid, and defines the velocity of propagation of the travelling wave, $v_\phi = \omega d/\Phi$

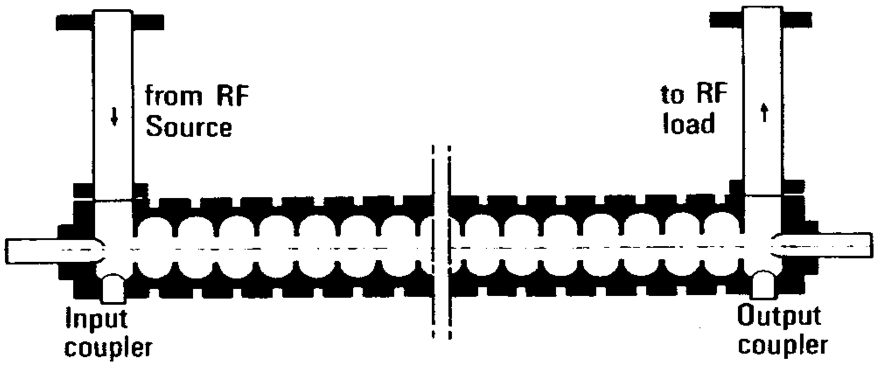
For acceleration, the wave must propagate at $v_\phi = c$

→ for each frequency ω and cell length d we can find a phase Φ where the apparent velocity of the wave v_ϕ is equal to c

Traveling wave accelerating structures

How to "simulate" an infinite chain of resonators? Instead of a singe input, exciting a standing wave mode, use an input + an output for the RF wave at both ends of the structure.

beam →



"Disc-loaded waveguide" or chain of electrically coupled cells characterized by a continuous band of frequencies. In the chain is excited a "traveling wave mode" that has a propagation velocity $v_{ph} = \omega/k$ given by the dispersion relation.

For a given frequency ω, $v_{ph} = c$ and the structure can be used for particles traveling at β=1
The "traveling wave" structure is the standard linac for electrons from β~1.
→ Can not be used for protons at v<c:
1. constant cell length does not allow synchronism
2. structures are long, without space for transverse focusing

Example: the 3 GHz electron linac

A 3 GHz LIL accelerating structure used for CTF3. It is 4.5 meters long and provides an energy gain of 45 MeV. One can see 3 quadrupoles around the RF structure.

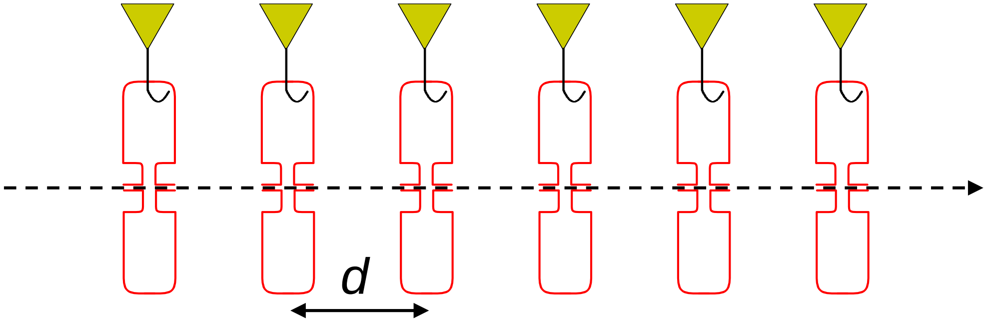
Superconducting low-beta structures

General architecture

For Superconducting structures:

1. Shunt impedance and power dissipation are not a concern.
2. Power amplifiers are small (and relatively inexpensive) solid-state units.

→ can be used the independent cavity architecture, which allows some flexibility in the range of beta and e/m of the particles to be accelerated.

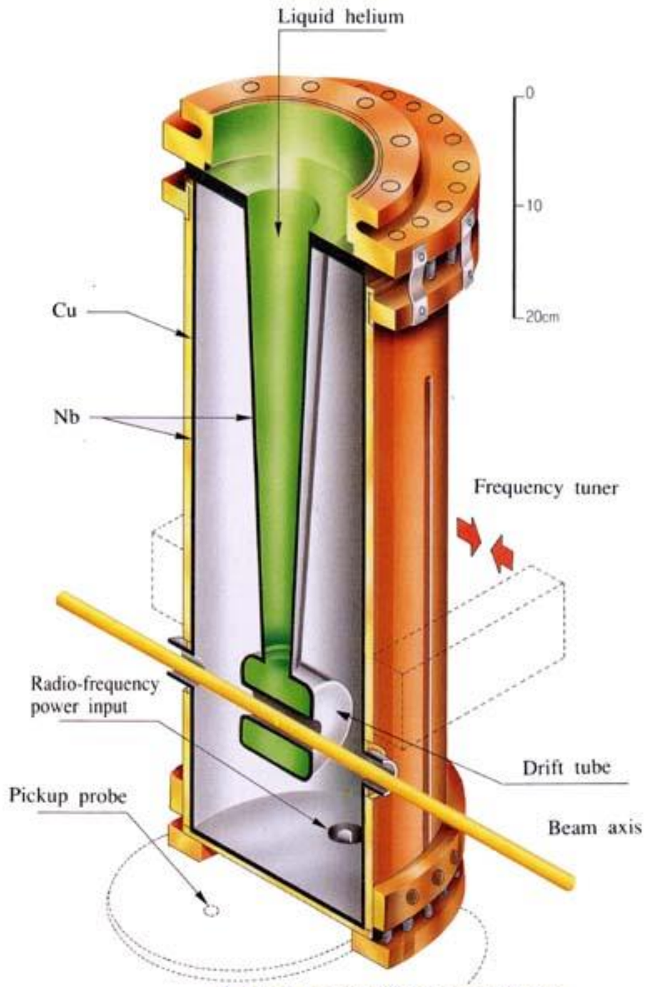


In particular, SC structures are convenient for machines operating at high duty cycle. At low duty, static cryogenic losses are predominant (many small cavities!)

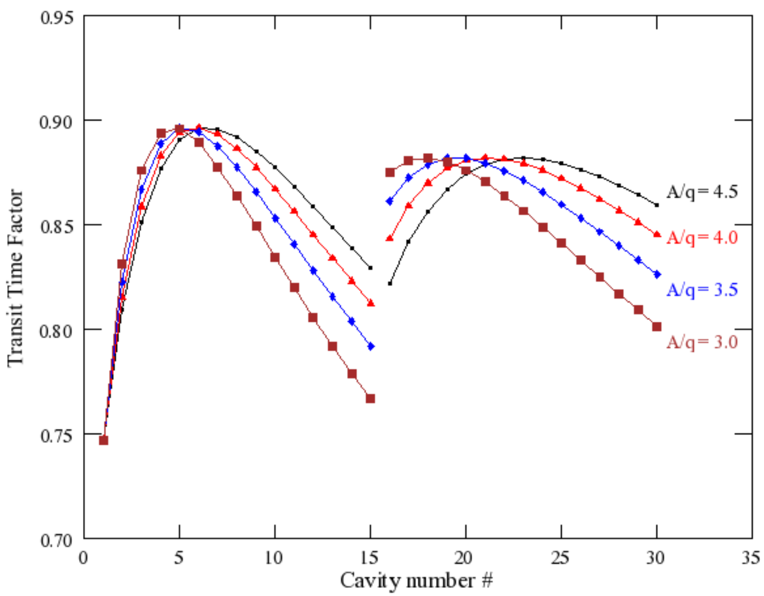
However, even for SC linacs single-gap cavities are expensive to produce (and lead to larger cryogenic dissipation) → double or triple-gap resonators are commonly used!

Quarter Wave Resonators

Simple 2-gap cavities commonly used in SC version (lead, niobium, sputtered niobium) for low beta protons or ion linacs, where ~CW operation is required.
Synchronicity (distance $\beta\lambda/2$ between the 2 gaps) is guaranteed only for one energy/velocity, while for easiness of construction a linac is composed by series of identical QWR's $\rightarrow$ reduction of energy gain for "off-energy" cavities, Transit Time Factor (= ratio between actual energy gained and maximum energy gain) curves as below:
"phase slippage"



▲ Cross section of the quarter wave superconducting resonator



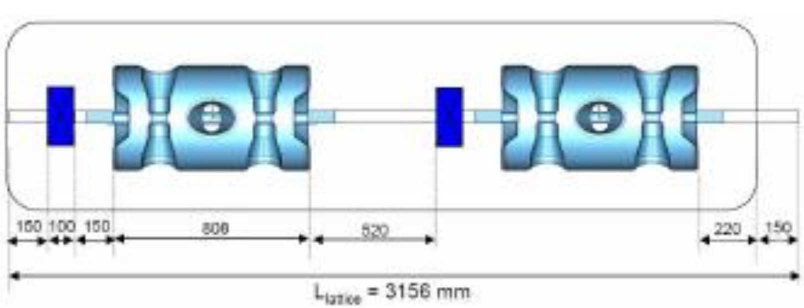
HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

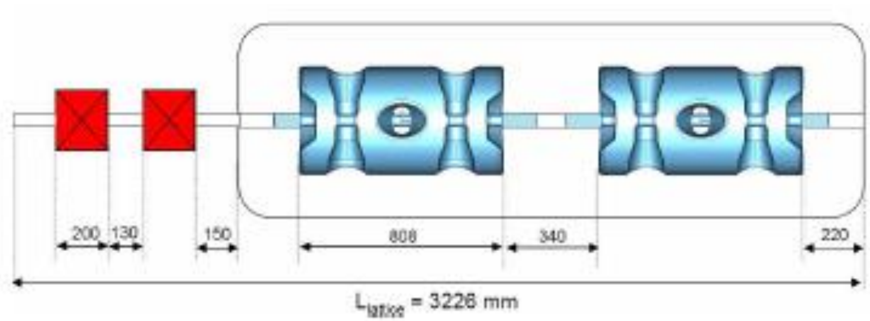
The Spoke cavity

CERN

HIPPI Triple-spoke cavity prototype built at FZ Jülich, now under test at IPNO

Another option:
Double or triple-spoke cavity, can be used at higher energy (100-200 MeV for protons and triple-spoke).

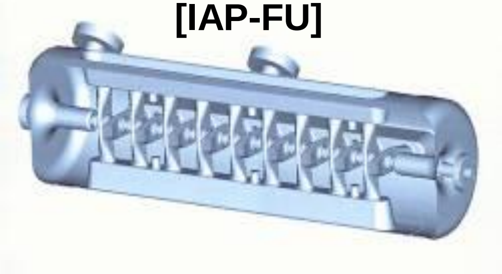




The superconducting zoo

Spoke (low beta)
[FZJ, Orsay]

CH (low/medium beta)
[IAP-FU]



QWR (low beta)
[LNL, etc.]

HWR (low beta)
[FZJ, LNL, Orsay]

Re-entrant
[LNL]

Superconducting linacs for low and medium beta ions are made of multi-gap (1 to 4) individual cavities, spaced by focusing elements. Advantages:
can be individually phased → linac can accept different ions
Allow more space for focusing → ideal for low β CW proton linacs

Questions on Module 3 ?

- Coupled resonator chains
- Stabilization
- Periodic structures
- Superconducting low-beta structures