HGS-HIRe for FAIR

Helmholtz Graduate School for Hadron and Ion Research

CERN

Module 5

A quick overview of beam dynamics in linear accelerators

Introduction

A linear accelerator requires an accurate beam optics design, in order to:

- Minimize emittance growth (remember Liouville: emittance can only increase!);
- Minimize beam loss, to: a) avoid activation of the accelerator (of the linac and of the following machine!) and b) reduce the requirements on the ion source.

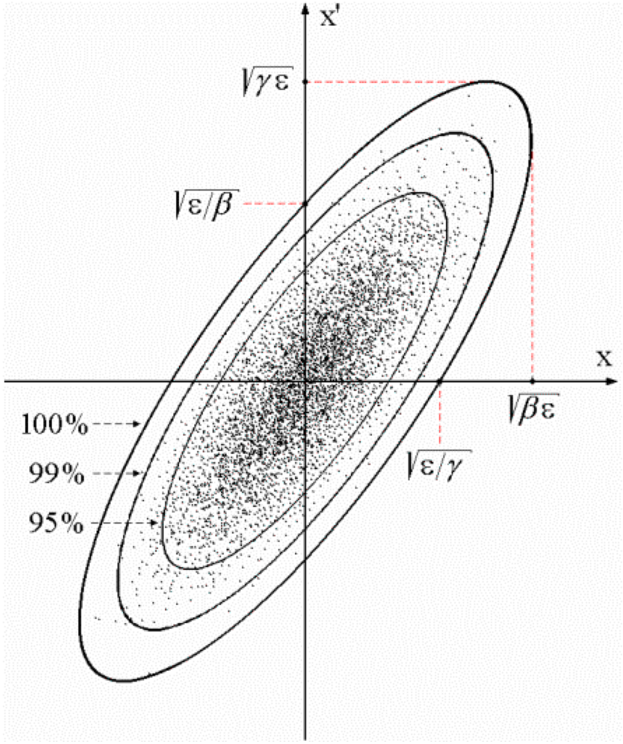
Note that the operating regimes in linacs can be very different, between

- μa peak currents for heavy ion linacs (~ 10^5 particles / bunch)
- mA peak currents for proton linacs (~ 10^8 particles / bunch)
- 100's mA peak currents for high-power proton linacs (~ 10^{11} particles / bunch)
- A's peak currents for electron linacs

Beam emittance

Recall a basic definition that we will need in the following:

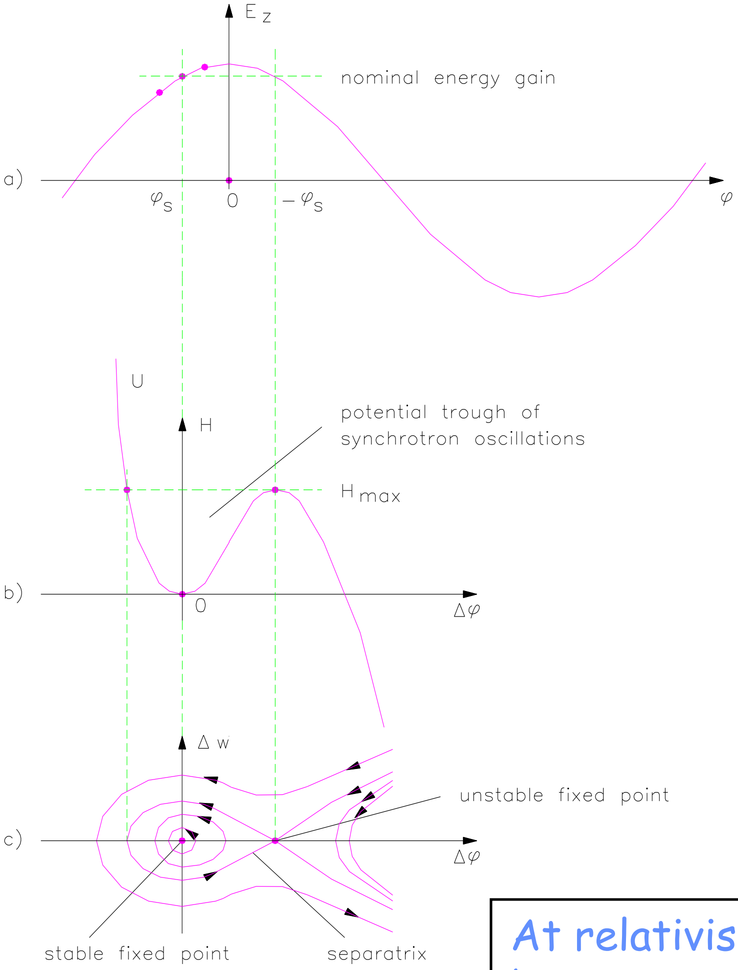
Beam emittance: is the volume in the phase space occupied by the particle beam. The space is 6-dimensional (x,x',y,y',W,ϕ) but are important the projections in the 3 dimensions: transverse emittance in x and y (ε_x, ε_y) and longitudinal emittance (ΔW, ϕ).



Liouville theorem: under the action of linear forces, the beam emittance is constant. (BUT: in presence of non-linear forces, it increases). Consequence: in an accelerator, where non-linear forces are often present, the emittance will progressively increase.

Beam brightness: beam current/transverse emittance.
Corresponds to the "density" in phase space. Preserving brightness means minimizing emittance growth.

Longitudinal dynamics



→ Ions are accelerated around a (negative = linac definition) synchronous phase.

→ Particles around the synchronous one perform oscillations in the longitudinal phase space.

→ Frequency of small oscillations:

$$\omega_l^2 = \omega_0^2 \frac{qE_0T\sin(-\varphi)\lambda}{2\pi mc^2\beta\gamma^3}$$

→ Tends to zero for relativistic particles $\gamma \gg 1$.

→ Note phase damping of oscillations:

$$\Delta\varphi = \frac{const}{(\beta\gamma)^{3/4}} \qquad \Delta W = const \times (\beta\gamma)^{3/4}$$

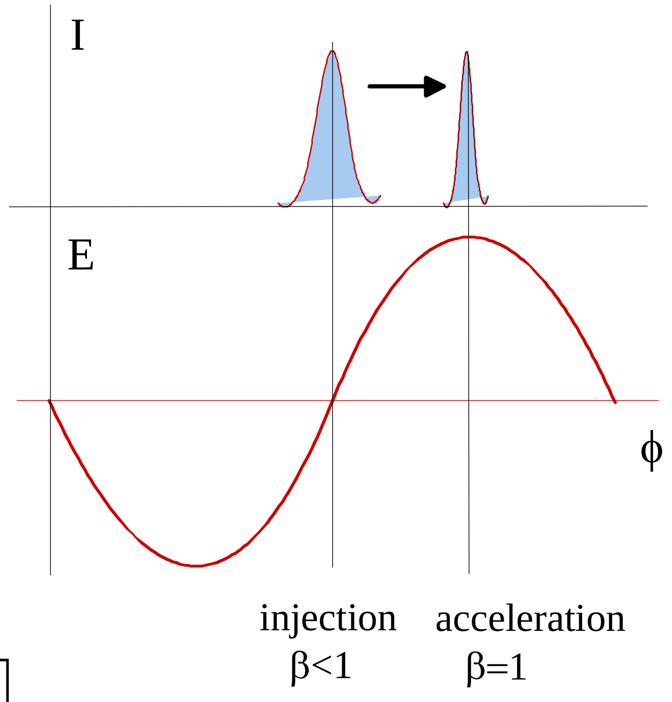
At relativistic velocities phase oscillations stop, and the beam is compressed in phase around the initial phase. The crest of the wave can be used for acceleration.

Longitudinal dynamics - electrons

- Electrons at v=c remain at the injection phase.
- Electrons at v<c injected into a TW structure designed for v=c will move from injection phase φ_0 to an asymptotic phase φ, which depends only on gradient and β_0 at injection.
- The beam can be injected with an offset in phase, to reach the crest of the wave at β=1
- **Capture condition**, relating E_0 and β_0:

$$\frac{2\pi}{\lambda_g}\frac{mc^2}{qE_0}\left[\sqrt{\frac{1-\beta_0}{1+\beta_0}}\right]=1$$

$$\sin\varphi = \sin\varphi_0 + \frac{2\pi}{\lambda_g}\frac{mc^2}{qE_0}\left[\sqrt{\frac{1-\beta_0}{1+\beta_0}} - \sqrt{\frac{1-\beta}{1+\beta}}\right]$$



Example: λ=10cm, W_{in}=150 keV and E_0=8 MV/m.

In high current linacs, a bunching and pre-acceleration sections up to 4-10 MeV prepares the injection in the TW structure (that occurs already on the crest)

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Transverse dynamics - Space charge

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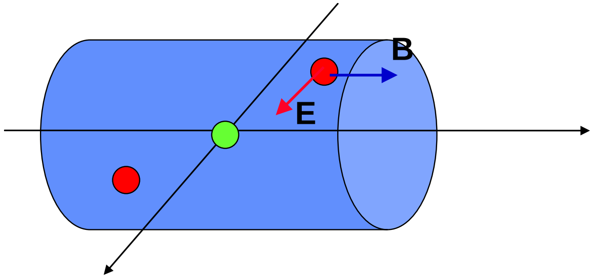
- → Large numbers of particles per bunch (~10^{10}).
- → Coulomb repulsion between particles (space charge) plays an important role and **is the main limitation to the maximum current in a linac**.
- → But space charge forces ~ $1/\gamma^2$ disappear at relativistic velocity

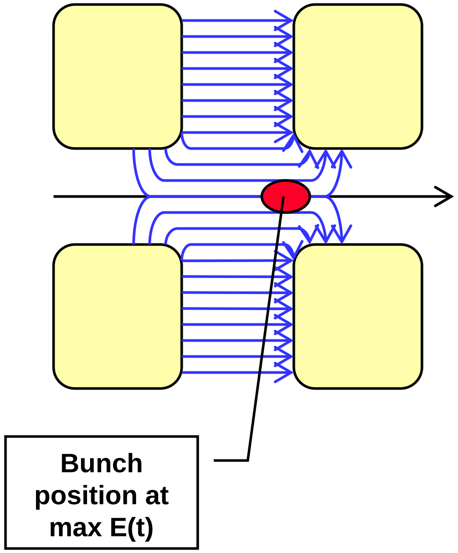


Force on a particle inside a long bunch with density n(r) traveling at velocity v:

$$E_r = \frac{e}{2\pi\varepsilon\, r}\int_0^r n(r)\, r\, dr \qquad B_\varphi = \frac{\mu}{2\pi}\frac{e\,v}{r}\int_0^r n(r)\, r\, dr$$

$$F = e(E_r - vB_\varphi) = eE_r\left(1-\frac{v^2}{c^2}\right) = eE_r(1-\beta^2) = \frac{eE_r}{\gamma^2}$$

Transverse dynamics - RF defocusing



- → RF defocusing experienced by particles crossing a gap on a longitudinally stable phase.
- → In the rest frame of the particle, only electrostatic forces → no stable points (maximum or minimum) → radial defocusing.
- → Lorentz transformation and calculation of radial momentum impulse per period (from electric and magnetic field contribution in the laboratory frame):

$$\Delta p_r = -\frac{\pi\, e\, E_0\, T\, L\, r \sin\varphi}{c\, \beta^2\, \gamma^2\, \lambda}$$

- → Transverse defocusing ~ $1/\gamma^2$ disappears at relativistic velocity (transverse magnetic force cancels the transverse RF electric force).

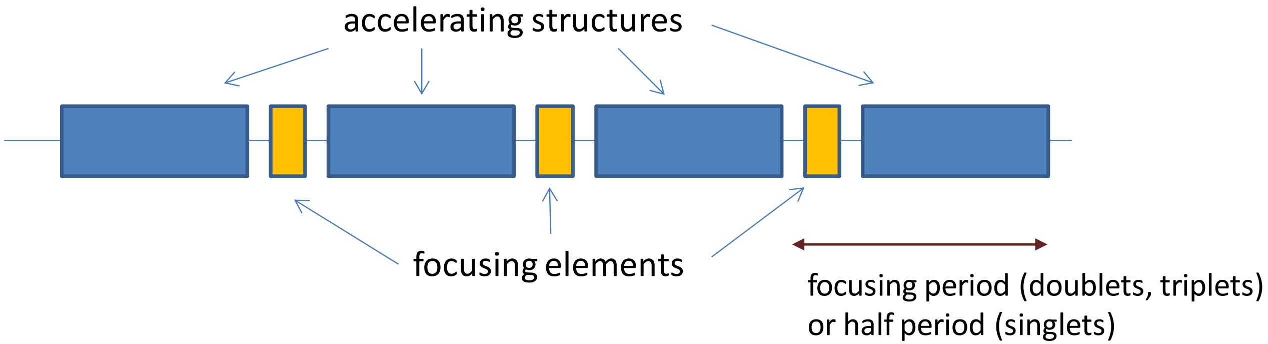
Focusing

Defocusing forces need to be compensated by focusing forces → alternating gradient focusing provided by quadrupoles along the beam line.

A linac alternates accelerating sections with focusing sections. Options are: one quadrupole (singlet focusing), two quadrupoles (doublet focusing) or three quadrupoles (triplet focusing).

Focusing period=length after which the structure is repeated (usually as $N\beta\lambda$).

The accelerating sections have to match the increasing beam velocity → the basic focusing period increases in length (but the beam travel time in a focusing period remains constant). The maximum allowed distance between focusing elements depends on beam energy and current and change in the different linac sections (from only one gap in the DTL to one or more multi-cell cavities at high energies).



Transverse beam equilibrium in linacs

The equilibrium between external focusing force and internal defocusing forces defines the frequency of beam oscillations.

Oscillations are characterized in terms of ***phase advance per focusing period*** σ_t **or *phase advance per unit length*** k_t.

Ph. advance = Ext. quad focusing - RF defocusing - space charge

$$k_t^2 = \left(\frac{\sigma_t}{N\beta\lambda}\right)^2 = \left(\frac{qGl}{2mc\,\beta\gamma}\right)^2 - \frac{\pi q E_0 T \sin(-\varphi)}{mc^2\lambda\,\beta^3\gamma^3} - \frac{3qI\lambda(1-f)}{8\pi\varepsilon_0\, r_0^{\,3} mc^3\beta^2\gamma^3}$$

q=charge
G=quad gradient
l=length foc. element
f=bunch form factor
r_0=bunch radius
λ=wavelength
…

Approximate expression valid for:
FODO lattice, smooth focusing approximation, space charge of a uniform 3D ellipsoidal bunch.

A “low-energy” linac is dominated by space charge and RF defocusing forces !!

Phase advance per period must stay in reasonable limits (30-80 deg), phase advance per unit length must be continuous (smooth variations) → at low β, we need a strong focusing term to compensate for the defocusing, but the limited space limits the achievable G and l → needs to use short focusing periods N βλ.

Note that the RF defocusing term $\propto$f sets a higher limit to the basic linac frequency (whereas for shunt impedance considerations we should aim to the highest possible frequency, Z $\propto\sqrt{}$f) .

Electron linacs

Ph. advance = Ext. quad focusing - RF defocusing - space charge – Instabilities

$$k_t^2 = \left(\frac{\sigma_t}{N\beta\lambda}\right)^2 = \left(\frac{q\,G\,l}{2\,mc\,\beta\gamma}\right)^2 - \frac{\pi\,q\,E_0 T\sin(-\varphi)}{mc^2\lambda\,\beta^3\gamma^3} - \frac{3q\,I\,\lambda(1-f)}{8\pi\varepsilon_0\,r_0^{\ 3}mc^3\beta^2\gamma^3} - \ldots$$

Electron Linac:

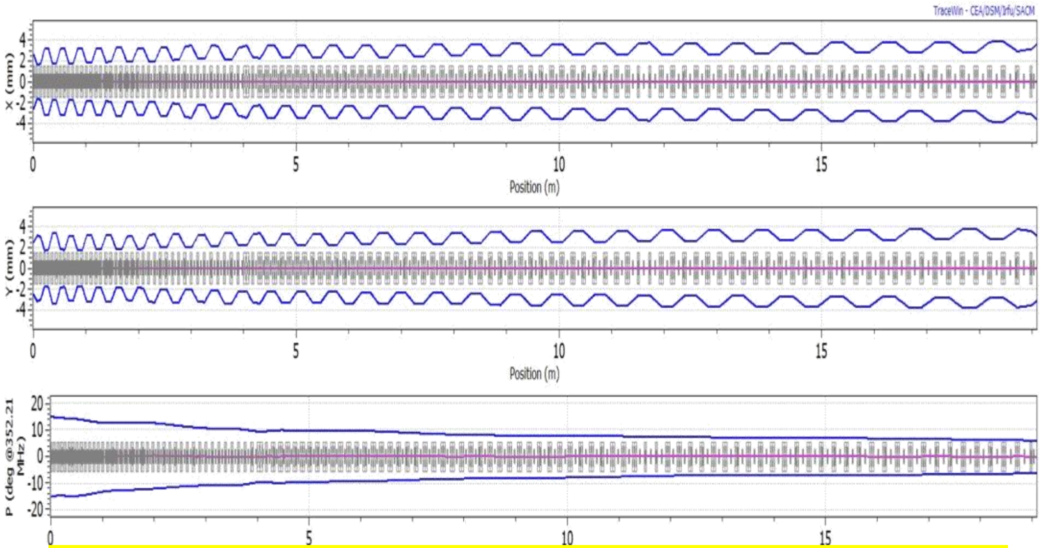
Ph. advance = Ext. focusing + RF ~~defocusing~~ + ~~space charge~~ + Instabilities

For $\gamma \gg 1$ (electron linac): RF defocusing and space charge disappear, *phase advance →0*.

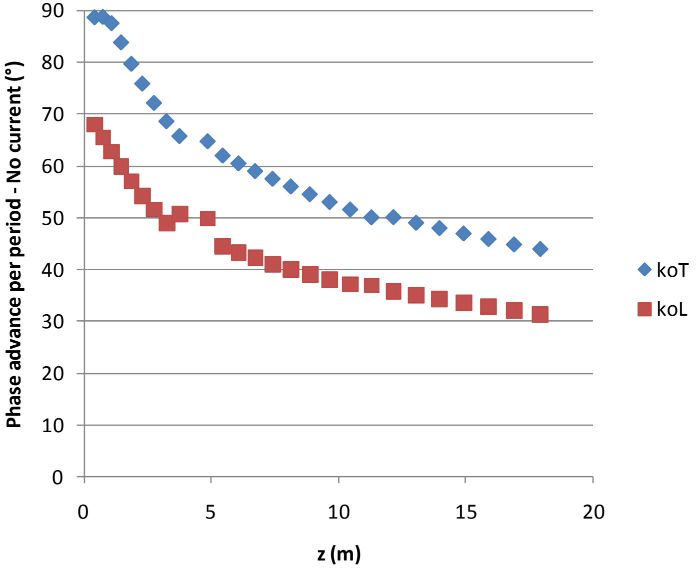
External focusing is required only to control the emittance and to stabilize the beam against instabilities (as wakefields and beam breakup).

Phase advance – an example

Beam optics of the Linac4 Drift Tube Linac (DTL): 3 to 50 MeV, 19 m, 108 focusing quadrupoles (permanent magnets).



Oscillations of the beam envelope (coordinates of the outermost particle) along the DTL (x, y, phase)



Corresponding phase advance per period

Design prescriptions:

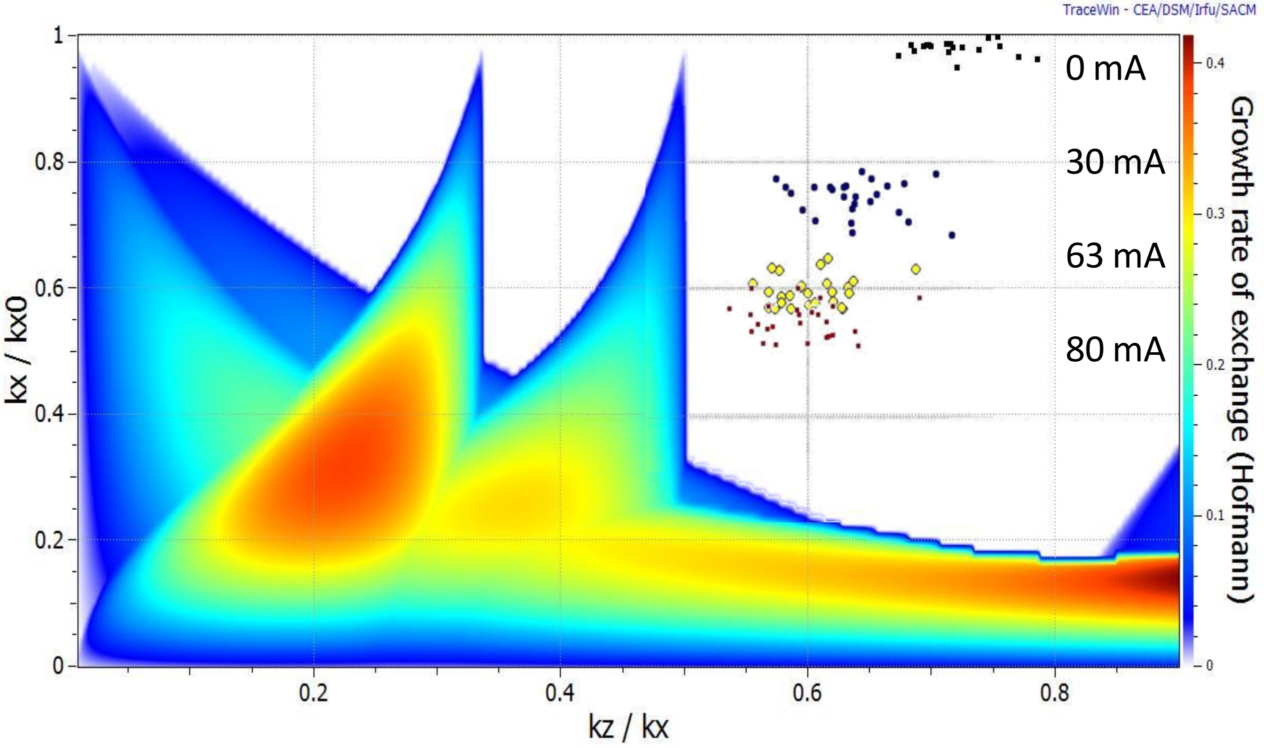
•Tranverse phase advance at zero current always less than 90°.

• Smooth variation of the phase advance.

• Avoid resonances (see next slide).

Instabilities in linacs – the Hoffman plot

Ratio between transverse phase advance and zero current phase advance (a measure of space charge)



The "tune diagram" of a linac

Ratio between longitudinal and transverse phase advance

Linac4 DTL: the operating point(s) for all possible current levels are far from the resonances between transverse and longitudinal oscillations which are enhanced by space charge.

Effect of the resonances: emittance exchange, transverse emittance growth, migration of particles into the beam halo → particularly dangerous for high intensity machines (beam loss).

Focusing periods

Focusing usually provided by quadrupoles.

Need to keep the phase advance in the good range, with an approximately constant phase advance per unit length → The length of the focusing periods has to change along the linac, going gradually from short periods in the initial part (to compensate for high space charge and RF defocusing) to longer periods at high energy.

For Protons (high beam current and high space charge), distance between two quadrupoles (=1/2 of a FODO focusing period):

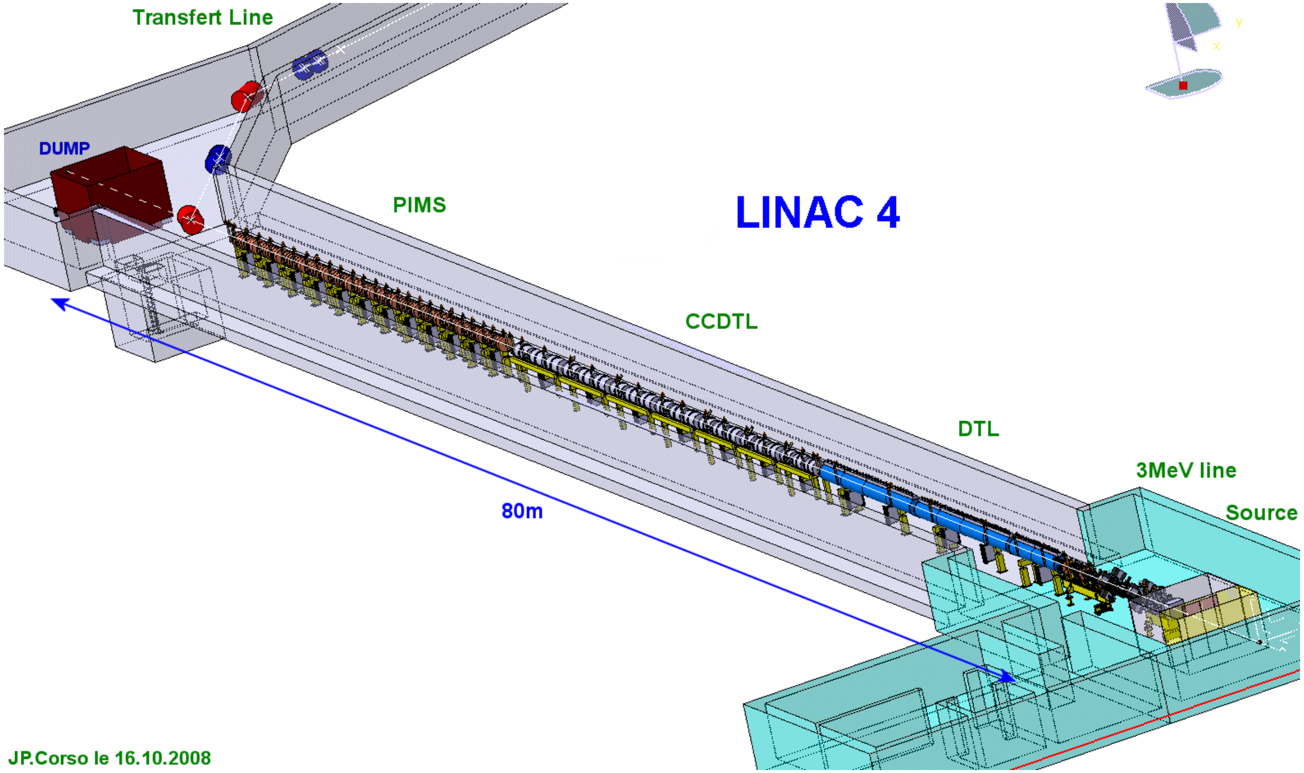
- $\beta\lambda$ in the DTL, from ~70mm (3 MeV, 352 MHz) to ~250mm (40 MeV),
- can be increased to 4-10$\beta\lambda$ at higher energy (>40 MeV).
- longer focusing periods require special dynamics (example: the IH linac).

For Electrons (no space charge, no RF defocusing):
focusing periods up to several meters, depending on the required beam conditions. Focusing is mainly required to control the emittance.

Architecture: cell length, focusing period

EXAMPLE: the Linac4 project at CERN. H-, 160 MeV energy, 352 MHz.
A 3 MeV injector + 22 multi-cell standing wave accelerating structures of 3 types

DTL, 3-50 MeV: every cell is different, focusing quadrupoles in each drift tube, 0-mode
CCDTL, 50-100 MeV: sequences of 2 identical cells, quadrupoles every 3 cells, 0 and $\pi/2$ mode
PIMS, 100-160 MeV: sequences of 7 identical cells, quadrupoles every 7 cells, $\pi/2$ mode



JP.Corso le 16.10.2008

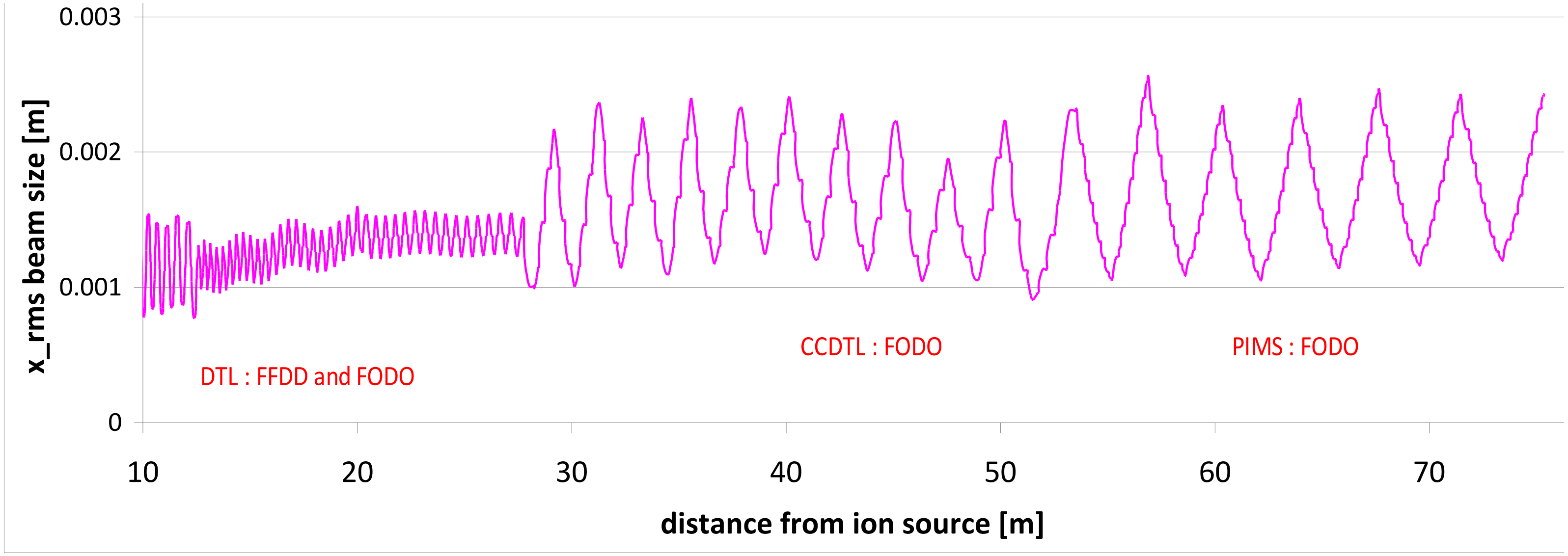
Two basic principles to remember:

1. As beta increases, phase error between cells of identical length becomes small → we can have short sequences of identical cells (lower construction costs).

2. As beta increases, the distance between focusing elements can increase.

High-intensity protons – the case of Linac4

Transverse (x) r.m.s. beam envelope along Linac4



Example: beam dynamics design for Linac4@CERN.

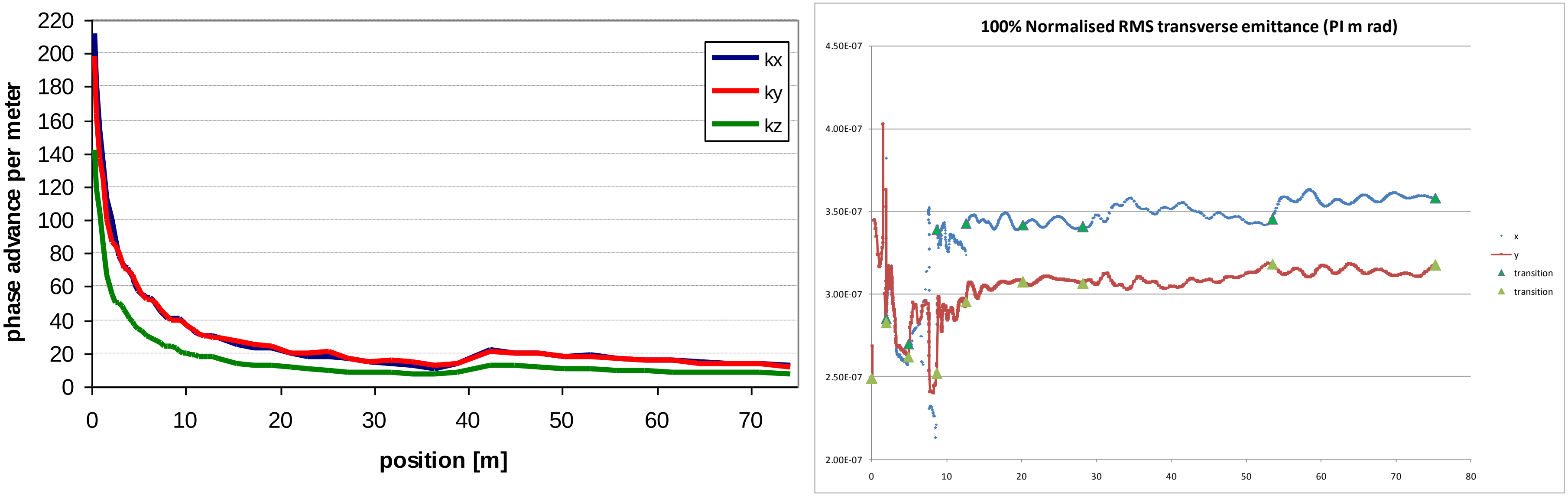
High intensity protons (60 mA bunch current, duty cycle could go up to 5%), 3 - 160 MeV

Beam dynamics design minimising emittance growth and halo development in order to:
1. avoid uncontrolled beam loss (activation of machine parts)
2. preserve small emittance (high luminosity in the following accelerators)

Beam Optics Design Guidelines

Prescriptions to minimise emittance growth and halo formation:

1. Keep zero current phase advance always below 90°, to avoid resonances
2. Keep longitudinal to transverse phase advance ratio 0.5-0.8, to avoid emittance exchange
3. Keep a smooth variation of transverse and longitudinal phase advance per meter.
4. Keep sufficient safety margin between beam radius and aperture



Transverse r.m.s. emittance and phase advance along Linac4 (RFQ-DTL-CCDTL-PIMS)

Halo and beam loss

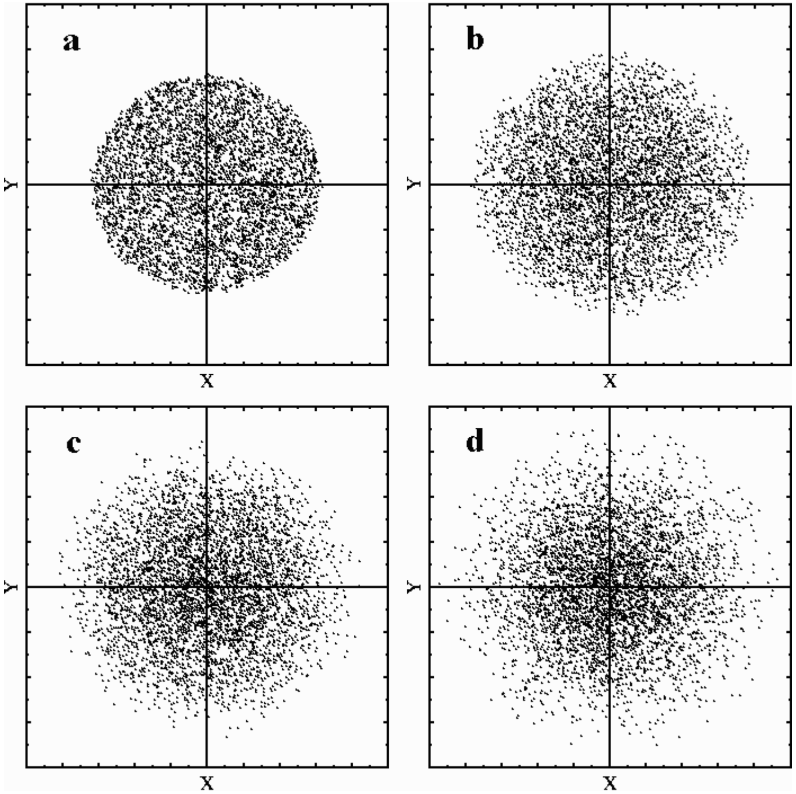
Additional challenge on beam dynamics:

Beam loss has to be avoided not because it reduces current, but because activation due to beam loss could come to levels preventing access to the machine.

Commonly accepted loss limit for hands-on maintenance is 1 W/m.

For example, in the case of the SPL design at CERN (5 GeV, 20 mA, 5% duty cycle) this corresponds to a maximum loss at 5 GeV of 0.2×10^{-6}/m, or 4 nA/m !!

In usual linacs, the beam distribution can be quite complicated, and presents a core surrounded by a “halo”. Halo formation has to be studied and controlled, but is at the limit of capability for modern computers

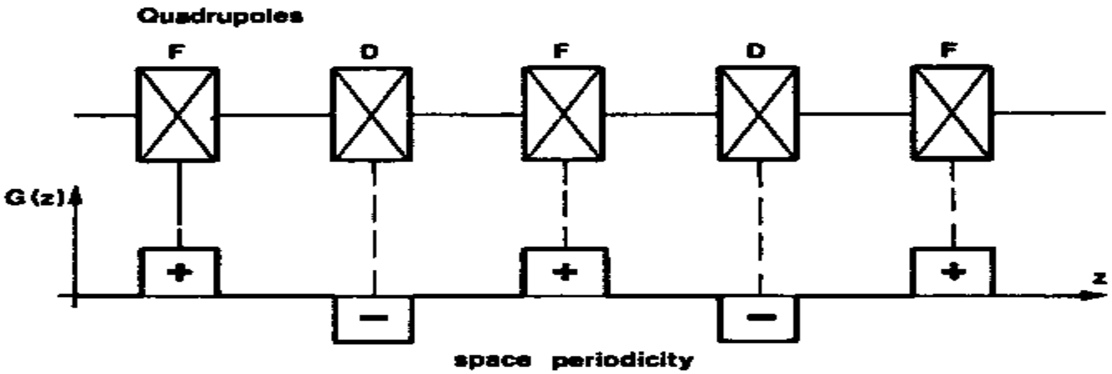


High brightness

Current and emittance are defined by the ion source: common ion sources have maximum currents ~ 80 mA for H-, ~ 200 mA for protons, in emittances of >0.02 π mm mrad.

In the linac, non-linearities in the focusing channel and in the space charge forces tend to increase the emittance and to decrease brightness.

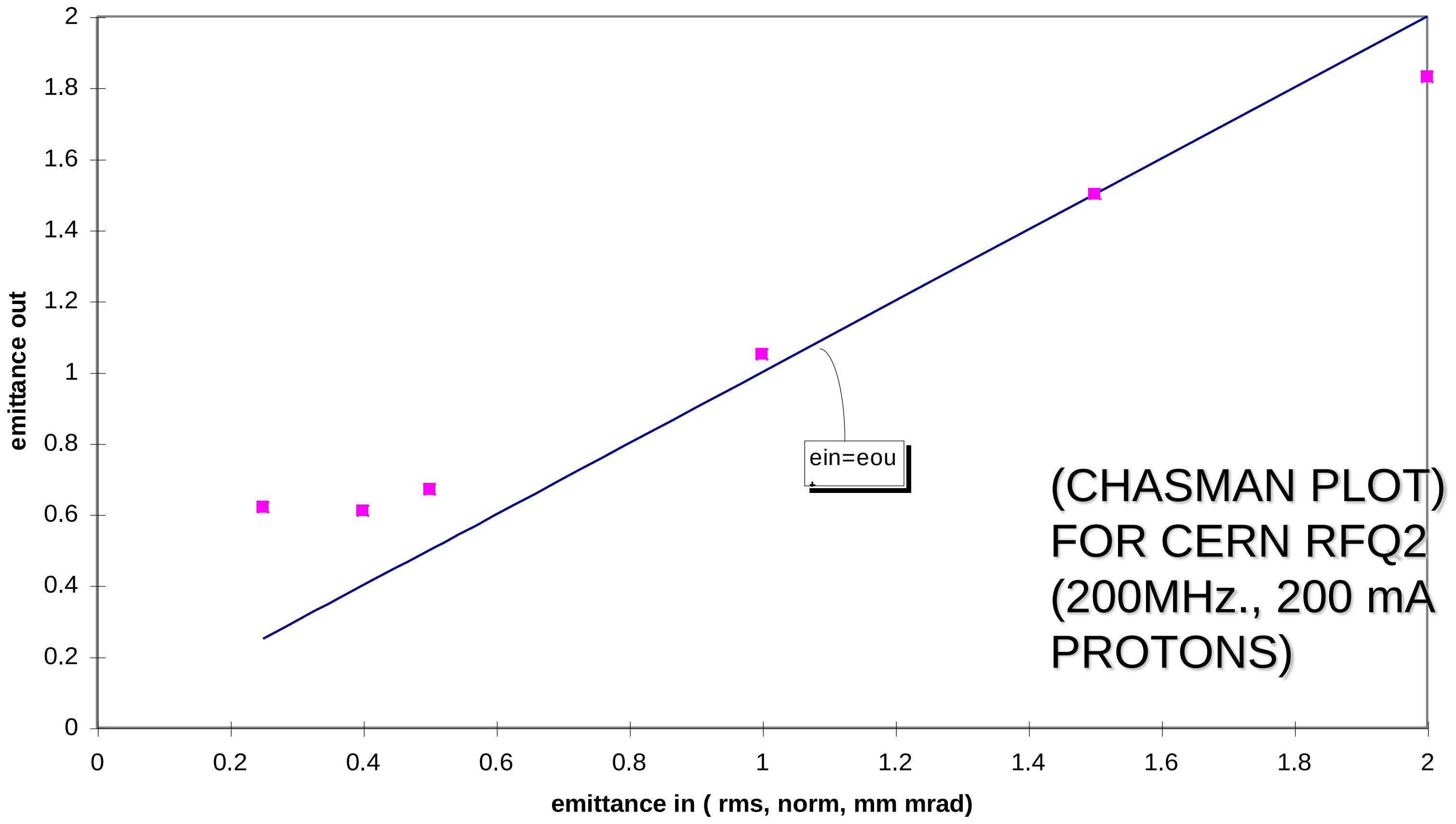
Beam brightness: beam current/transverse emittance.
Corresponds to the “density” in phase space. Preserving brightness means minimizing emittance growth.



space charge – non linear effects

- the more the beam is compressed in real space, the more the space charge effect is non linear
- non linear space charge effect generates emittance growth
- at low energy space charge is the limiting factor for the minimum emittance that can be produced out of an accelerator

Minimum emittance from space charge

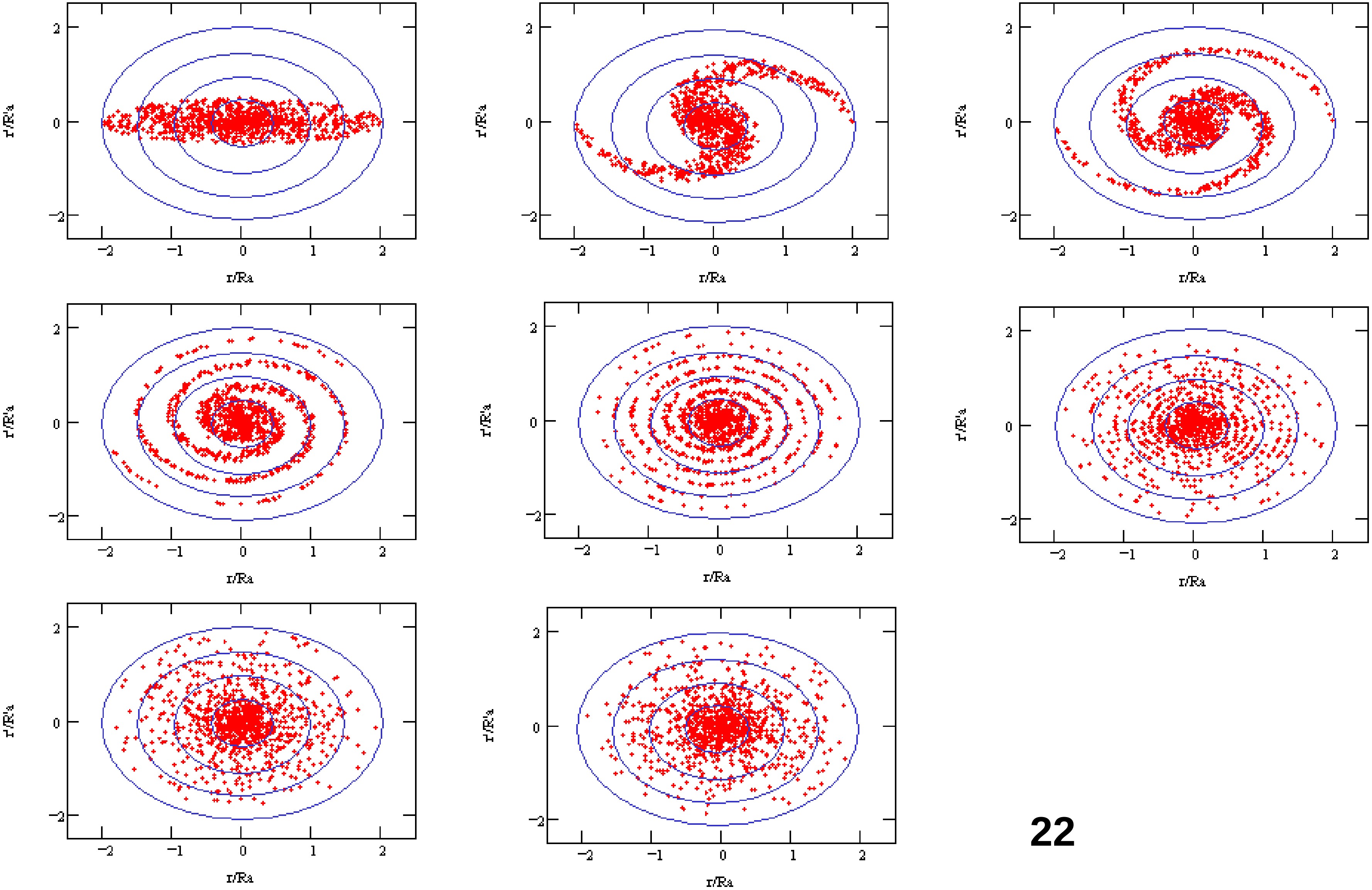


(CHASMAN PLOT)
FOR CERN RFQ2
(200MHz., 200 mA
PROTONS)

emittance growth due to filamentation

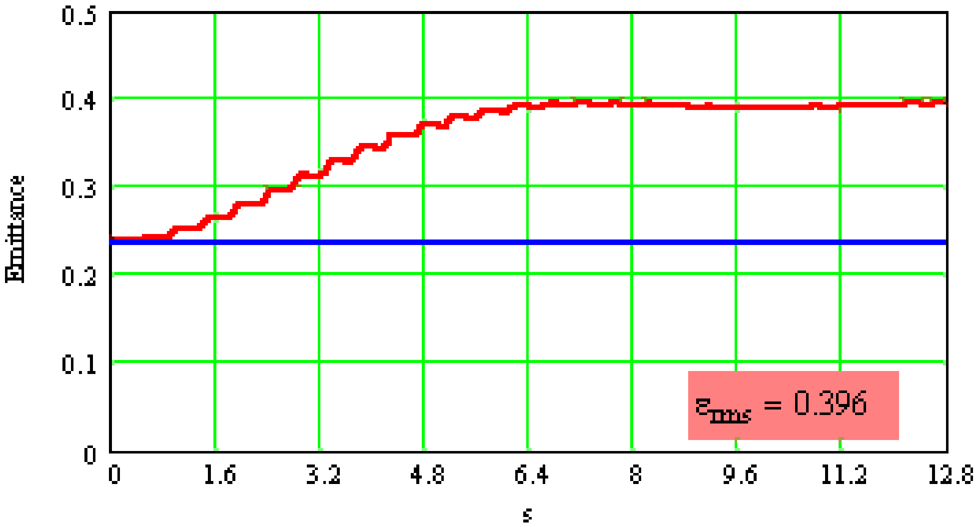
- velocity of rotation in the transverse phase space with no space charge doesn’t depend on the amplitude.
- with linear space charge it is lowered but it still doesn’t depend on the amplitude
- with non linear space charge it does depend on amplitude and therefore there are areas of the phase space move at different velocity. This generates emittance growth.

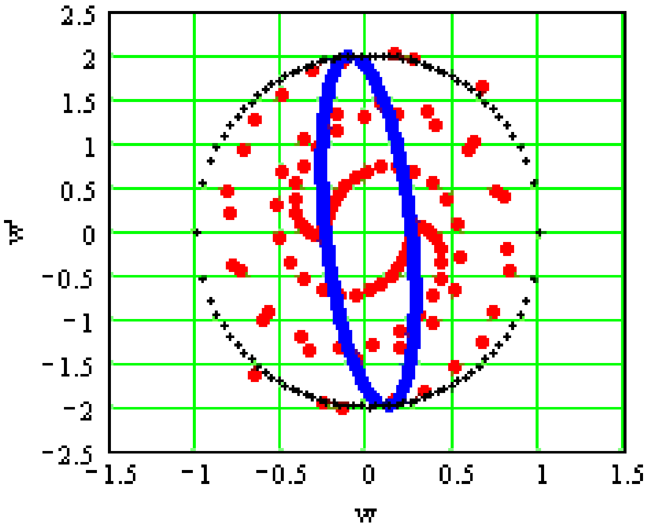
Non linear forces: filamentation



Filamentation: emittance increase

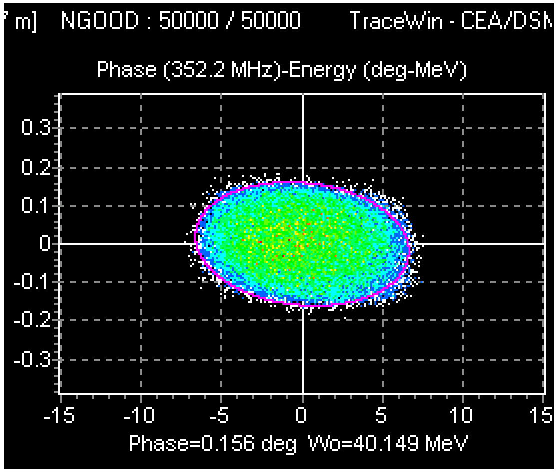
Evolution of the emittance along an accelerator under the influence of linear forces only (blue line) or non-linear forces (red line)



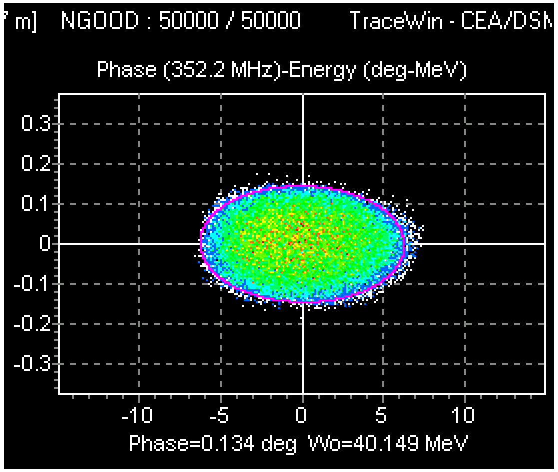


🟦 Linear force **🟥 Non linear force**

Space-charge filamentation in longitudinal plane



(1)



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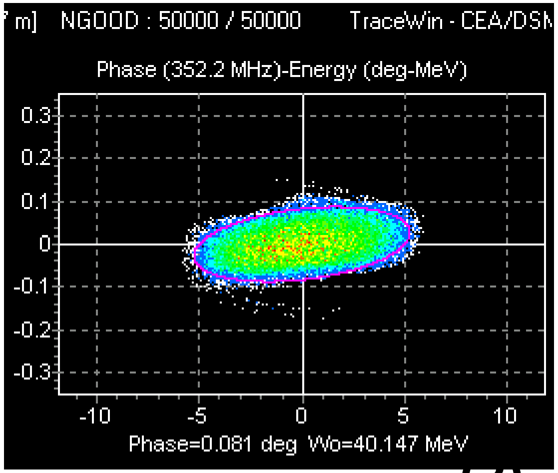

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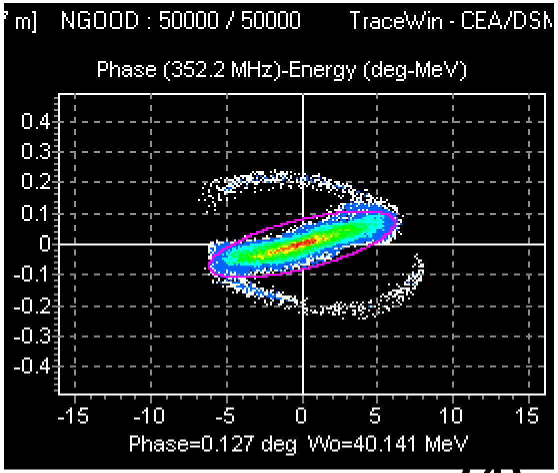

(4)

(1) el_in=0.21

(2) el_in=0.18

(3) el_in=0.12

(4) el_in=0.03

emittance r.m.s. in deg MeV

Questions on Module 5 ?

- Defocusing / focusing forces
- Space charge
- Focusing periods
- Filamentation