# Hydrodynamic modeling from pp to AA

Li Yan

Department of Physics, McGill University

Feb. 9, 2017, Chicago, Quark Matter 2017

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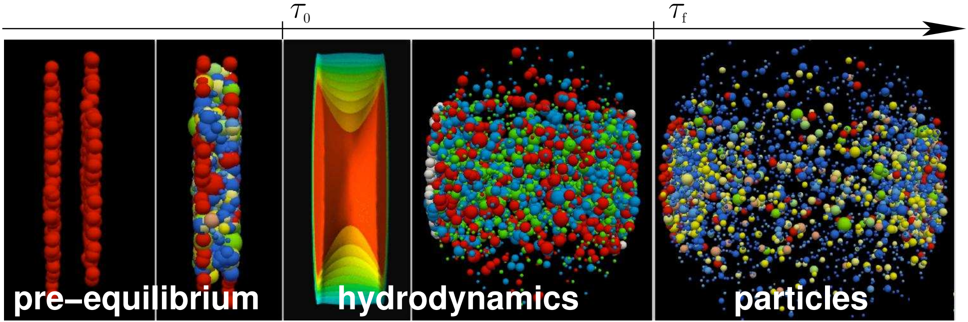
for hydro. in small systems, see B. Schenke's talk

# Brief overview of hydro modeling of heavy-ion collisions




Hydrodynamics applies when system evolve like a fluid: $e$, $\mathcal{P}$, $u^\mu$

- Hydrodynamic equation of motion:

$$\partial_\mu T^{\mu\nu} = 0, \qquad T^{\mu\nu} = e u^\mu u^\nu - (\mathcal{P} + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}, \quad \text{\& Lattice EoS}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu, \qquad \nabla^\mu = \Delta^{\mu\nu}\partial_\nu$$

$$\Pi = -\zeta \nabla \cdot u + \mathcal{O}(\nabla^2) \quad \text{and} \quad \pi^{\mu\nu} = \eta\sigma^{\mu\nu} + \mathcal{O}(\nabla^2) \quad \leftrightarrow \quad \text{gradient expansion}$$

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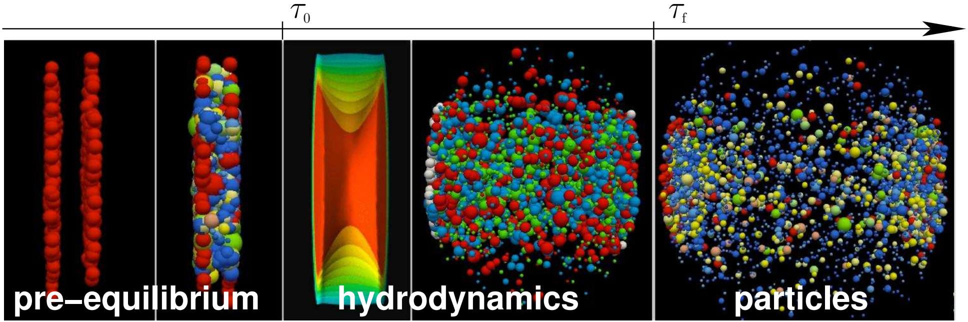


Some inputs of hydro simulations:

- Initial condition: geometrical property of QGP medium at $\tau_0$.

$$\text{eccentricity} \Rightarrow \mathcal{E}_n = \varepsilon_n e^{in\Phi_n} \equiv -\frac{\int dxdye(x, y, \eta_s, \tau_0)(x+iy)^n}{\int dxdye(x, y, \eta_s, \tau_0)|x+iy|^n}$$

* $\mathcal{E}_n$'s are model dependent! (IP-Glasma, MC-KLN, MC-Glauber, etc.)

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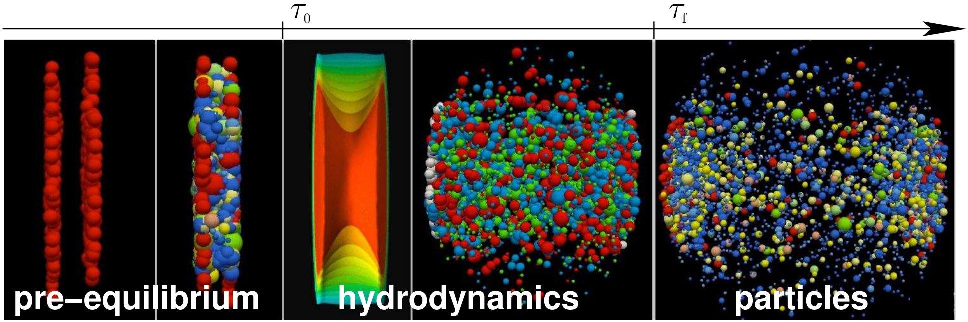


Some inputs of hydro simulations:

- QCD transport coefficients : dynamical property of QGP medium

  $\zeta \leftrightarrow$ bulk viscosity , $\eta \leftrightarrow$ shear viscosity

  * One of our interests is to extract $\eta$ and $\zeta$ from hydro modeling.

$$\frac{\eta}{s} \sim \mathcal{O}\left(\frac{\hbar}{4\pi k_B}\right)$$

P. Kovtun, D. Son and A. Sarinets, PRL94(2005) 111601
M.Luzum and P.Romatschke, PRC78(2008) 034915

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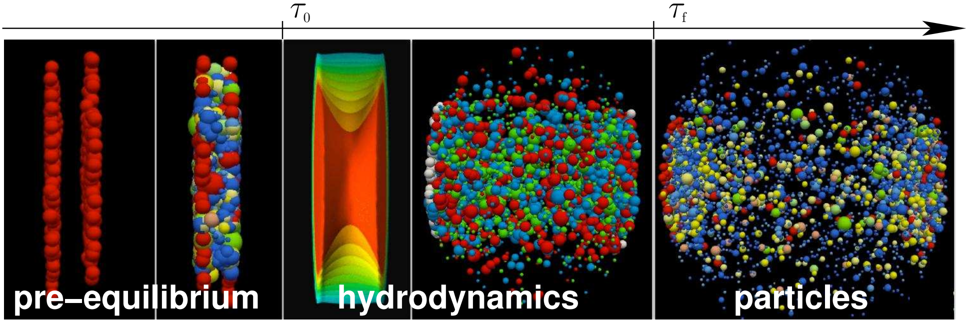



Some outputs of hydro simulations:

- Single-particle spectrum: $dN/dp_T$, mean $p_T$
- Long-range multi-particle correlations:

$$\text{Harmonic flow:} \quad V_n = v_n e^{in\Psi_n} \equiv \langle\langle e^{in\phi_p} \rangle\rangle$$

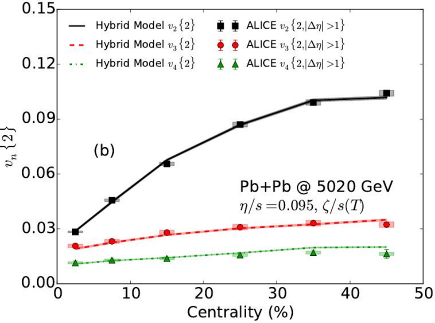
Hydro works successfully for the medium evolution in AA!

# Recent results from hydro simulations for A+A

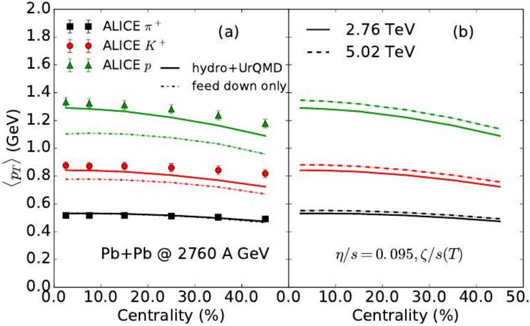
- $\sqrt{s_{NN}} = 5.02$ TeV PbPb: IP-Glasma+MUSIC+UrQMD

talk by S. McDonald

$v_n\{2\}$, $n = 2, 3, 4$




$\langle p_T \rangle$




* Constant $\eta/s = 0.095$, temperature dependent $\zeta/s$

see also J. Noronha-Hostler PRC93(2016)034912: $\eta/s$=0.08

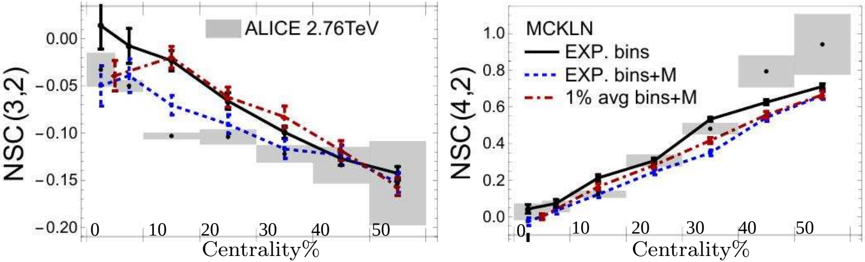
H. Niemi et al., PRC93(2016) 014912: $\eta/s(T)$ and $\eta/s = 0.2$

EKRT i.c. and $\eta/s(T)$, also see talk by K. Eskola

# Recent results from hydro. simulations for A+A

- Correlations of harmonic flow – Event-Plane (EP)
  talk by K. Eskola, S. McDonald
- Correlations of harmonic flow – Symmetric Cumulants (SC)

talk by F. Gardim




* (Normalized) Symmetric Cumulants: ALICE collaboration

$$\mathrm{NSC}(n,m) = \frac{\langle\langle v_n^2 v_m^2 \rangle\rangle - \langle\langle v_n^2 \rangle\rangle\langle\langle v_m^2 \rangle\rangle}{\langle\langle v_n^2 \rangle\rangle\langle\langle v_m^2 \rangle\rangle}$$

* NSC(n,m) gets stronger for larger value of $\eta/s$ X. Zhu et al, 1609.02628

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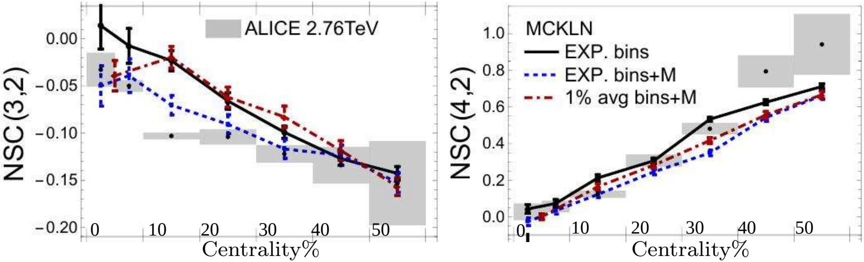


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G. Giacalone et. al. PRC94(2016) 014906

* NSC(n,m) gets stronger for larger value of $\eta/s$ X. Zhu et al, 1609.02628

# Formal developments in hydro modeling

Pushing the limits of hydro modeling further:

- Regarding an anisotropic background : anisotropic hydrodynamics
- Thermal fluctuations in a fluid system : hydrodynamic fluctuations
- Factorization of hydro. flow response : non-linear flow response

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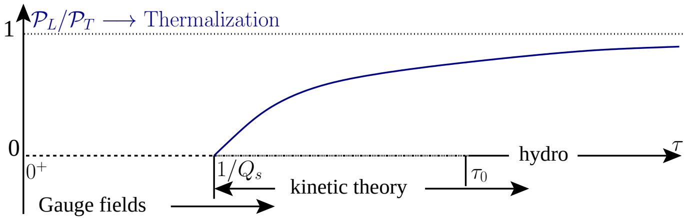
# Formal development – Anisotropic hydro




- Anisotropic hydro expands from an anisotropic background

$$\text{(Bjorken \& RTA):} \quad f(\tau,\vec{x},\vec{p}) = \underbrace{f_{\rm iso}\left(\sqrt{\vec{p}^{\,2}+\xi(\tau)p_z^2}/\Lambda(\tau)\right)}_{\text{anisotropic}} + \underbrace{\delta f'}_{\text{`viscous'}}$$

Martinez and Strickland NPA848 (2010); Florkowski and Ryblewski PRC83(2011)

* A part of pressure anisotropy absorbed in anisotropic background,

(reduce effects of $\delta f'$) $\Rightarrow$ improve hydro. applicability

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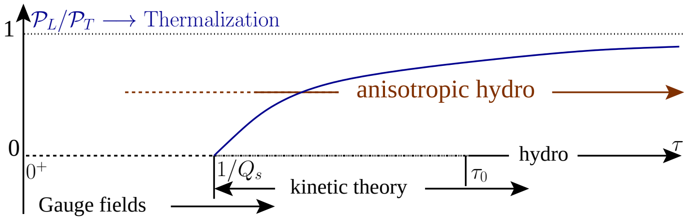


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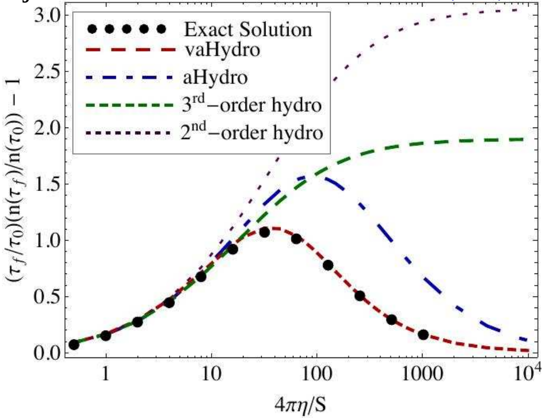
Bjorken + RTA D. Bazow et al. PRC90(2014)054910




- anisotropic hydro → viscous anisotropic hydro D. Bazow et. al. PRC(91) 064903; M. Nopoush et. al, PRD(91), 045007; D.Bazow, U. Heinz and M. StrickLand PRC(90), 054910(2014); M. Nopoush et. al, PRD91, 045007(2015); . . .
- 0+1D, 1+1D → realistic 3+1D poster by Nopoush #X03
- Closing ahydro with equation matching $\mathcal{P}_L$ talk by H. Niemi and M.Martinez

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- Regarding an anisotropic background : anisotropic hydrodynamics
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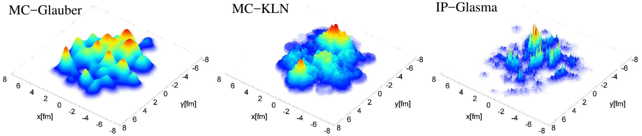
# Formal development – Hydrodynamic fluctuations

- Quantum fluctuations $\longrightarrow$ initial state geometrical configuration




**B. Schenke et. al., PRL108, 252301**

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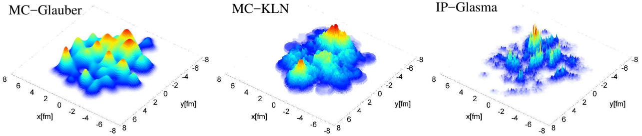
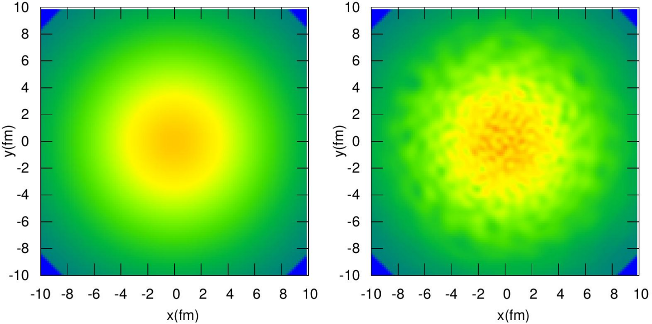



B. Schenke et. al., PRL108, 252301

- Thermal fluctuations $\longrightarrow$ throughout the whole system evolution

L.Y.



# Formal development – Hydrodynamic fluctuations

- Hydro fluctuations can be included by introducing a stochastic term.

$$T^{\mu\nu} = eu^\mu u^\nu - (\mathcal{P} + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} + S^{\mu\nu},$$

$$\langle S^{\mu\nu} \rangle = 0, \qquad \underbrace{\langle S^{\mu\nu}(x) S^{\alpha\beta}(x') \rangle \sim 2T\eta\delta(x - x') + \text{bulk visc.}}_{\text{fluctuation-dissipation}}$$

Landau and Lifshitz; Kapusta, Muller and Stephanov PRC85(2012)

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- Hydro fluctuations are conceptually significant !

$$G_R^{xy,xy}(\omega) \sim \underbrace{\mathcal{P}}_{\text{ideal+noise}} \underbrace{-i\omega\eta}_{\text{1st order+noise}} \underbrace{+\mathcal{O}(\omega^{3/2})}_{\text{noise}} \underbrace{+\eta\tau_{\pi}\omega^2}_{\text{2nd order}}$$ , talk by Y. Akamatsu

P.Kovtun et al. PRD84(2011), 025006

# Effects of hydrodynamic fluctuations

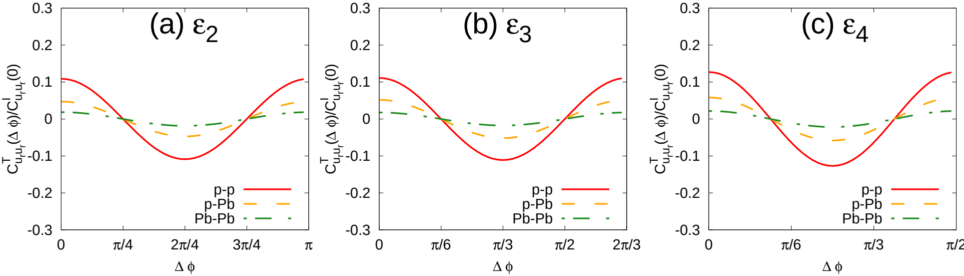
- Noisy hydro on top of an analytically solved 1+1D conformal medium
  LY and H. Grönqvist, JHEP 1603(2016)121

  * Effect of hydro fluctuations depends only on (1/multiplicity).

    ⇒ supports fluidity in high-multiplicity events, even in small systems

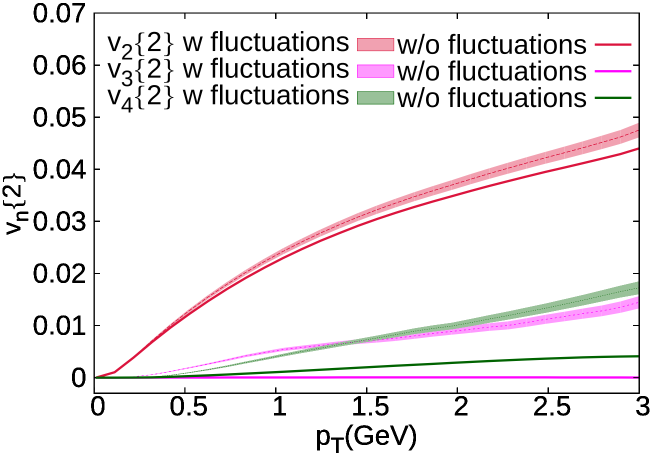
    ⇒ scaling of flow in high-multiplicity event of small systems
    G.Basar and D.Teaney, PRC90(2014)054903

  * Hydro. fluc. more significant for higher order flow and smaller systems.



# Effects of hydrodynamic fluctuations

- 3+1D MUSIC + hydrodynamic fluctuations




Poster by M. Singh (#M20)

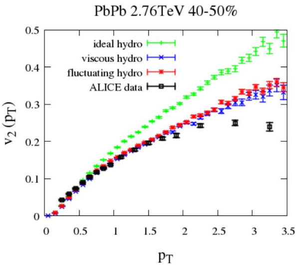
* Smooth Glauber i.c. $b = 3$ fm, $\langle\langle v_3 \rangle\rangle = 0$

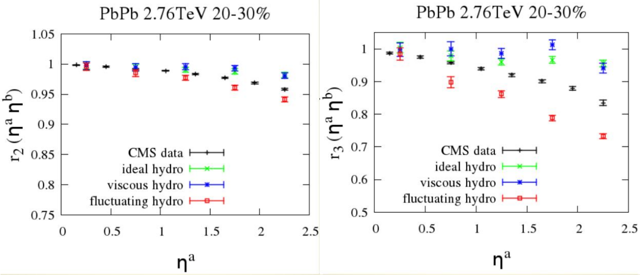
# Effects of hydrodynamic fluctuations

- 3+1D hydro simulations + hydrodynamic fluctuations

$v_2\{2\}$

$$r_n(\eta_a, \eta_b) = \frac{\mathrm{Re}\langle\langle V_n(-\eta_a) V_n^*(\eta_b) \rangle\rangle}{\mathrm{Re}\langle\langle V_n(\eta_a) V_n^*(\eta_b) \rangle\rangle},\ 3.0 < \eta_b < 4.0$$




talk by A. Sakai

* MC-Glauber + longitudinal profile

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Pushing the limits of hydro modeling further:

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# Non-linearities of medium flow response to $\mathcal{E}_n$

- Motivation: $\begin{cases} \text{disentangle } \mathcal{E}_n \text{ from medium property } (\eta,\ \zeta) \\ \text{extract model-independent properties of hydro. predictions} \end{cases}$

$$V_n(p_T, y) = V_n\Big( \underbrace{\mathcal{E}_n}_{\text{initial state}} \Big| \underbrace{\eta, \quad \zeta}_{\text{medium dynamics}} \Big), \qquad (V_n \text{ and } \mathcal{E}_n \text{ are complex!})$$

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$$= \kappa_n(\eta,\zeta)\mathcal{E}_n + \kappa'_n(\eta,\zeta)|\mathcal{E}_2|^2\mathcal{E}_n, \qquad \text{for } V_2,\ V_3\,,$$

Expand w.r.t. $\mathcal{E}_n$ ($|\mathcal{E}_n| < 1$, 2nd harmonic dominates)

J. Noronha-Hostler, LY, F. Gardim and J. Ollitrault(2016)

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J. Noronha-Hostler, LY, F. Gardim and J. Ollitrault(2016)

$$= V_n^L + \chi_{nab}(\eta, \zeta)V_aV_b\,, \quad n = a + b, \qquad \text{for } V_4,\ V_5\ \ldots$$

Projection of $V_n$ onto $V_aV_b$, such that $\langle\langle V_n(V_aV_b)^*\rangle\rangle = 0$

LY and J. Ollitrault(2015)

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$\Rightarrow$ e.g.: $\quad \chi_{422} \equiv \dfrac{\langle\langle V_4(V_2^*)^2\rangle\rangle}{\langle\langle |V_2|^4\rangle\rangle}$

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$$\Rightarrow \text{e.g.:} \quad \chi_{422} \equiv \frac{\langle\langle V_4(V_2^*)^2\rangle\rangle}{\langle\langle |V_2|^4\rangle\rangle} \leftrightarrow \frac{V_4,\ V_2 \text{ correlation}}{\text{fluctuation of } V_2} \Rightarrow \text{measurable in Exp.!}$$

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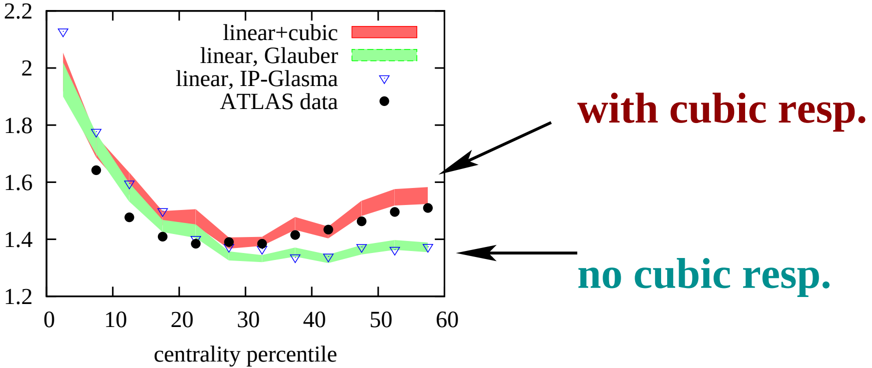
* Based on hydro simulations $\longrightarrow$ flow paradigm

$\kappa_n$, $\kappa'_n$ and $\chi$ depend only on the medium bulk property.

# Cubic response in fine structure of EbyE $v_n$ fluctuations

$$V_2 = \kappa_2 \mathcal{E}_2 + \kappa_2' |\mathcal{E}_2|^2 \mathcal{E}_2 \quad \longrightarrow \quad \mathcal{P}(v_2) \Leftrightarrow \mathcal{P}(V_n(\varepsilon_2))$$

- Ratio of moments from probability distribution $P(v_2)$ : $\langle\langle v_2^4 \rangle\rangle / \langle\langle v_2^2 \rangle\rangle^2$




J. Noronha-Hostler, LY, F. Gardim and J. Ollitrault(2016)

- From $\mathcal{P}(v_2)$ one can in principle solve $\kappa_2$ and $\kappa_2'$ independently of i.c.

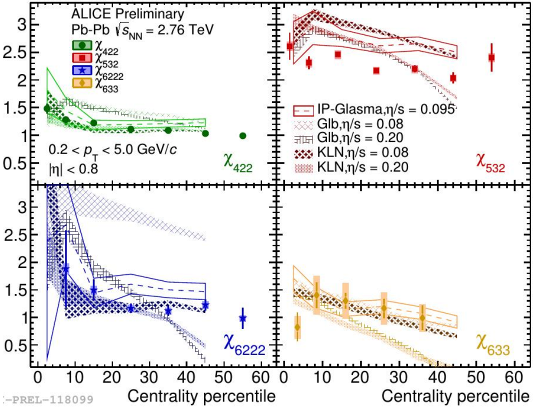
  G. Giacalone, LY, J. Noronha-Hostler J. Ollitrault, PRC95(2017), 014913;

  talk by J. Castle

# Non-linear response coefficients from hydro.

- $\chi$'s from hydro see also LY, J. Ollitrault and S.Pal

figure from talk by Y. Zhou (ALICE)




Combinatorial counting from freeze-out surface in ideal hydro

N. Borghini, J. Ollitrault(2006)

$$\frac{1}{2}\chi_{523} \sim \chi_{422} \sim \chi_{633} \ldots \quad \Rightarrow \quad \text{domintated at freeze-out surface}$$

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# Interesting topics not covered in this talk

- Bayesian analysis of flow and estimate of parameters.
  talk by G. Denicol and talk by S. Moreland
- Hydro modeling for BES and searching for critical point.
  talk by C. Shen, L. Jiang and A. Monnai
- Effects of vorticity in hydrodynamics.
  talk by L. Pang
- Rapidity-dependent fluctuations and flow from hydro. modeling.
  talk by M. Luzum
- Theoretical calculations of transport coefficients.
  talk by A. Czjka and talk by J. Ghiglieri
- Validity of hydro modeling in various aspects.
  talk by P. Romatschke and J. Noronha

# Summary and conclusions

- Remarkable success of hydro. modeling in AA!
- More details in the hydro framework:

  * Anisotropic hydro. : extend the applicability of hydro. modeling in AA

  * Hydrodynamic fluctuations

    → conceptually significant – effective "1.5th" order viscous correction

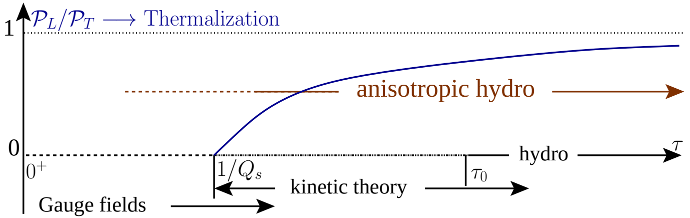
    → phenomenologically important to harmonic flow.

  * Non-linearities of hydro flow response

  → Extract i.c. model-independent properties or observables in hydro modeling.

# Back-up slides

# Anisotropic (viscous) hydro.




- Hydro. emerges from gradient exp. from an isotropic background

$$f(\tau,\vec{x},\vec{p}) = \underbrace{f_{eq}(\tau,\vec{x},\vec{p})}_{\text{isotropic}} + \delta f, \quad \begin{cases} f_{eq} \rightarrow & T^{\mu\nu}_{\text{ideal}} = e u^\mu u^\nu - \mathcal{P}\Delta^{\mu\nu} \\ \delta f \rightarrow & T^{\mu\nu}_{\text{visc.}} = \eta\sigma^{\mu\nu} + \zeta\Delta^{\mu\nu}\theta + \mathcal{O}(\nabla^2) \end{cases}$$

* Equation of State is identified from $T^{\mu\nu}_{\text{ideal}}$.
* Truncation of the exp. relies on scale seperation: $\frac{l_{\text{mfp}}}{L_{\text{macro.}}} = \text{Kn} < 1$
* Pressure anisotropy fully captured by viscous corrections.

large momentum anistropy → large Kn → hydro. breaks down

# Apply hydro. fluctuations to heavy-ion collisions

- Treat thermal fluctuations perturbatively (linearized hydro.)

1. $\underbrace{\partial_\mu \langle T^{\mu\nu} \rangle = 0}_{\text{background}}$ and $\underbrace{\partial_\mu \delta T^{\mu\nu} = 0}_{\text{Langevin type}} \Rightarrow (\delta e, \delta u^\mu)$

2. freeze-out procedure w.r.t. hydro fluctuations

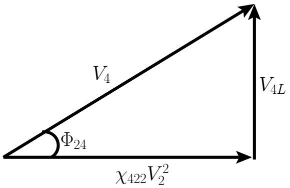
3. Two-point correlations from ensemble average:

$$\langle \delta e(x) \delta e(x') \rangle,\ \langle \delta u^\mu(x) \delta u^\nu(x') \rangle \ldots \Rightarrow v_n\{2\}$$

NB1: Difficult to realize in numerical simulations.

NB2: NOT applicable for medium system close to critical point !

# Higher order harmonics : $v_4$, $v_5$, $v_6$, ...




- E.g., project $V_4$ (complex) onto orientation of $(V_2)^2$

$$V_4 = \underbrace{V_{4L}}_{\text{“Linear } V_4\text{”} \sim \mathcal{E}_4 \text{ ?}} + \underbrace{\chi_{422}}_{\text{nonlinear flow resp.}} \times (V_2)^2$$

since $V_{4L}$ is uncorrelated to $(V_2)^2$, one may extract $\chi_{422}$

LY and J. Ollitrault, PLB744(2015) 82

$$\Rightarrow \chi_{422} \equiv \frac{\langle V_4 (V_2^*)^2 \rangle}{\langle |V_2|^4 \rangle}$$

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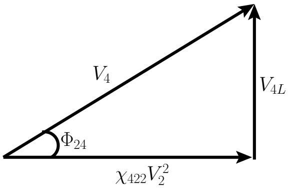


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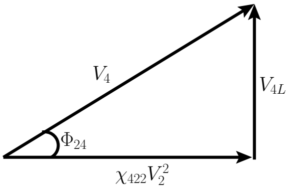


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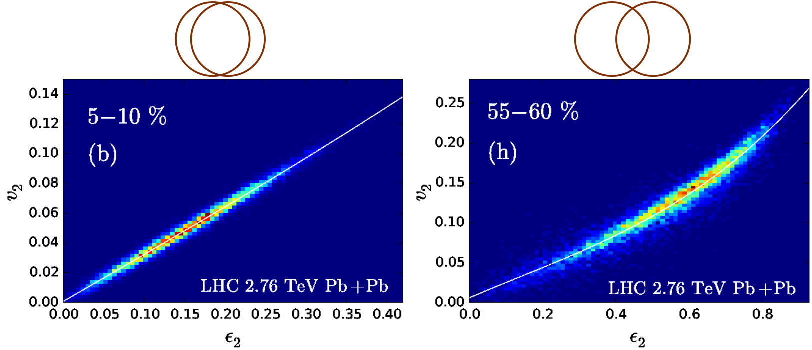
- Similarly, nonlinear mode mixing leads to $\chi_{523}$, $\chi_{633}$, ...

# $v_2$ and $v_3$: linear + cubic corrections

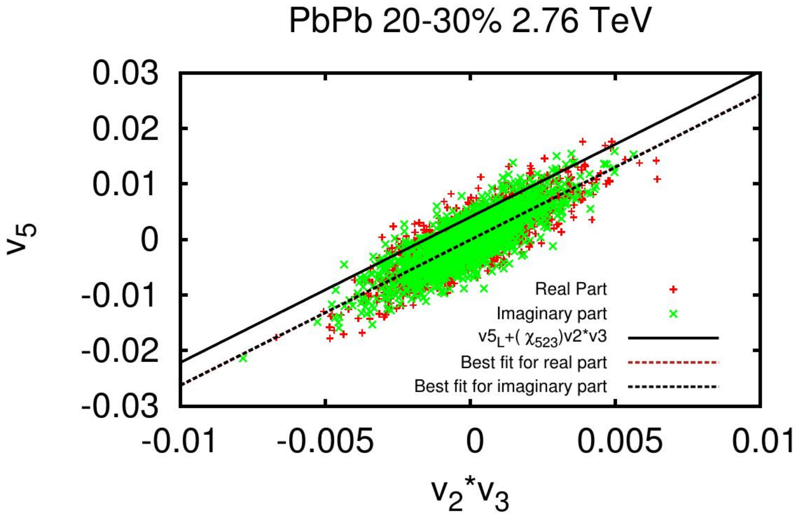
- Density plot of $v_2$ and $\varepsilon_2$ from event-by-event hydro simulations:




H. Niemi et. al., PRC93(2016)024907

$$V_n = \underbrace{\kappa_n \mathcal{E}_n + \kappa_n' |\mathcal{E}_2|^2 \mathcal{E}_n}_{\text{response}} + \underbrace{\delta_n}_{\text{fluctuations}} \quad , \quad \begin{cases} \kappa_n \rightarrow & \text{linear resp. coefficient} \\ \kappa_n' \rightarrow & \text{cubic resp. coefficient} \end{cases}$$

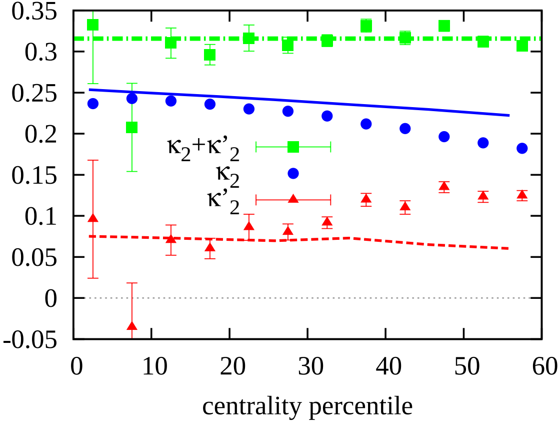
# Density plot of $V_5$ vs. $V_2V_3$




talk by S. McDonald

# $v_2$ and $v_3$: linear + cubic corrections

- $\kappa_2$ and $\kappa_2'$ from hydro.: $\eta/s = 0.08$, $\zeta = 0$, $\sqrt{s_{NN}} = 2.76$ TeV PbPb, MC-Glb




J. Noronha-Hostler, LY, F. Gardim and J. Ollitrault(2016)

$\kappa_2' > 0$, $\kappa_3'$ is compatible with zero in all centralities.

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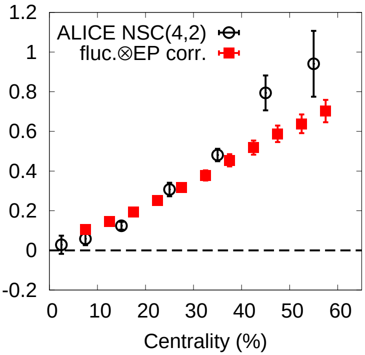
# Non-linear flow response and correlations of $V_n$

- Symmetric cumulants can decomposed with help of non-linear resp.,

$$\mathrm{NSC}(4,2) = \underbrace{\left(\frac{\langle\langle v_2^6\rangle\rangle}{\langle\langle v_2^4\rangle\rangle\langle\langle v_2^2\rangle\rangle} - 1\right)}_{\text{flow fluc.}} \times \underbrace{\langle\langle \cos 4(\Psi_4 - \Psi_2)\rangle\rangle^2}_{\text{event-plane correlations.}}$$




G. Giacalone, LY, J. Noronha-Hostler J. Ollitrault, PRC94(2016), 014906

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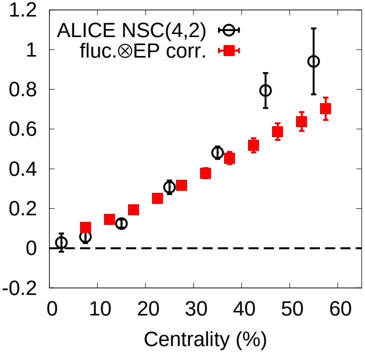
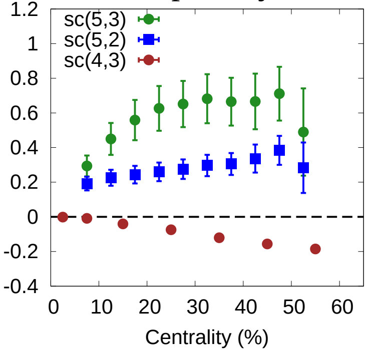



I.C. indep. + Hydro!

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