Lattice QCD results on soft and hard probes of strongly interacting matter

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I) Equation of State at $\mu_B > 0$ and cumulants of conserved charges
   → thermal conditions at freeze-out
   → radius of convergence and critical point
   → skewness and kurtosis of net-baryon number

II) Spectral and Transport properties in the QGP
   → thermal dilepton and photon rates
   → electrical conductivity and heavy-quark diffusion

Quark Matter 2017, Chicago, 09.02.2017
Continuum extrapolated results of pressure & energy density & entropy density

HISQ: [A. Bazavov et al. (hotQCD), PRD90 (2014) 094503] stout: [S. Borsanyi et al. (BMW), PLB730, 99 (2014)]

consistent results from hotQCD (HISQ) and Budapest-Wuppertal (stout)

Hadron resonance gas (HRG) model using all known hadronic resonances from PDG describes the EoS quite well up to cross-over region

QCD results systematically above HRG

QCD is quite different from HRG thermodynamics at T > 160MeV
Hadron resonance gas

HRG: thermal gas of uncorrelated hadrons

partial pressure of each hadron:

\[ \hat{P}_h \sim f(\hat{m}_h) \cosh [B_h \hat{\mu}_h + Q_h \hat{\mu}_Q + S_h \hat{\mu}_S] \]

total pressure given by the sum over all (known) hadrons

\[ \hat{P}_{\text{total}} = \sum_{\text{all hadrons}} \hat{P}_h \]

are we sensitive to this?

Quark Model predicts more strange baryons:

PDG-HRG uses states listed in the particle data tables

QM-HRG uses states calculated in the quark model

large enhancement of the partial baryonic pressure from additional strange baryons

PDG-HRG uses states listed in the particle data tables

QM-HRG uses states calculated in the quark model

use QCD thermodynamics instead of HRG to describe physics at freeze-out
Taylor expansion of pressure in terms of chemical potentials related to conserved charges

\[
\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k
\]

defines generalized susceptibilities:

\[
\chi_{ijk}^{BQS} = \frac{\partial^{(i+j+k)}[P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \bigg|_{\bar{\mu}=0}
\]

correlations of strangeness with baryon number fluctuations:

\[
\chi_{11}^{BS} = \frac{\partial^2 [P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B \partial \hat{\mu}_S} \bigg|_{\bar{\mu}=0}
\]

second cumulant of net strangeness fluctuations:

\[
\chi_2^S = \frac{\partial^2 [P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_S^2} \bigg|_{\bar{\mu}=0}
\]

suitable ratios like

\[
\frac{\chi_{11}^{BS}}{\chi_2^S}
\]

strange baryon density

(in a hadron gas)

dominated by strange mesons

are sensitive probes of the strangeness carrying degrees of freedom

\[
\frac{\mu_S}{\mu_B} = -\frac{\chi_{11}^{BS}}{\chi_2^S} + \mathcal{O}(\mu^2)
\]
Equation of state of (2+1)-flavor QCD - $\mu_B/T > 0$

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k$$

defines generalized susceptibilities:

$$\chi_{ijk}^{BQS} = \frac{\partial^{(i+j+k)}[P(T, \mu_B, \mu_Q, \mu_S)/T^4]}{\partial \mu_B^i \partial \mu_Q^j \partial \mu_S^k} \bigg|_{\mu=0}$$

for $\mu_Q = \mu_S = 0$ this simplifies to

$$\frac{\Delta P(T)}{T^4} = \frac{P(T, \mu_B) - P(T, 0)}{T^4} = \frac{\chi_2^B}{2} \left( \frac{\mu_B}{T} \right)^2 \left( 1 + \frac{1}{12} \frac{\chi_4^B}{\chi_2^B} \left( \frac{\mu_B}{T} \right)^2 \right) + \mathcal{O} \left( \mu_B^6 \right)$$

variance of net-baryon number distribution

\[ \kappa_B \sigma_B^2 \]

kurtosis*variance

[Bielefeld-BNL-CCNU, arXiv:1701.04325]
Taylor expansion of pressure in terms of chemical potentials related to conserved charges:

\[
P \frac{T^4}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k
\]

defines generalized susceptibilities:

\[
\chi_{ijk}^{BQS} = \left. \frac{\partial(i+j+k)[P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B \partial \hat{\mu}_Q \partial \hat{\mu}_S} \right|_{\bar{\mu} = 0}
\]

for \( \mu_Q = \mu_S = 0 \) this simplifies to

\[
\frac{\Delta P(T)}{T^4} = \frac{P(T, \mu_B) - P(T, 0)}{T^4} = \frac{\chi_2^B}{2} \left( \frac{\mu_B}{T} \right)^2 \left( 1 + \frac{1}{12} \frac{\chi_4^B}{\chi_2^B} \left( \frac{\mu_B}{T} \right)^2 \right) + O(\mu_B^6)
\]

Net-baryon number fluctuations are "Skellam" only for \( T < 155 \text{ MeV} \) (consistent with ALICE data - see talk by Anar Rustamov (Wed.))
Taylor expansion of pressure in terms of chemical potentials related to conserved charges

\[ \frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k \]

defines generalized susceptibilities:

\[ \chi_{ijk}^{BQS} = \frac{\partial^{i+j+k}[P(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)/T^4]}{\partial \hat{\mu}_B \partial \hat{\mu}_Q \partial \hat{\mu}_S} \bigg|_{\bar{\mu}=0} \]

for \( \mu_Q = \mu_S = 0 \) this simplifies to

\[ \Delta P(T) = \frac{P(T, \mu_B) - P(T, 0)}{T^4} = \frac{\chi_2^B}{2} \left( \frac{\mu_B}{T} \right)^2 \left( 1 + \frac{1}{12} \chi_4^B \left( \frac{\mu_B}{T} \right)^2 \right) + \mathcal{O}(\mu_B^6) \]

\( \chi_2 \) vs. \( T \) (MeV)

6th-order becomes negative in the cross-over region

(consistent with central STAR data - see talk by Roli Esha (Wed.))
Equation of state of (2+1)-flavor QCD - $\mu_B/T > 0$

Constraints in heavy-ion collisions:

\[ n_s = 0 \ , \ \frac{n_Q}{n_B} = r = 0.4 \]

strangeness neutrality and fixed electric charge to baryon-number ratio

Continuum estimated results of pressure & energy density up to 6th-order:

Parametrization of the EoS for $T \in [130\text{MeV}, 280\text{MeV}]$ & $\mu_B/T \geq 0$ in [Bielefeld-BNL-CCNU, arXiv:1701.04325]

Consistent results from two different actions, HISQ and stout, in the continuum

$\rightarrow$ Equation of state well controlled for $\mu_B/T \lesssim 2 \ \leftrightarrow \ \sqrt{s_{NN}} \geq 14.5 \text{ GeV}$
Constraints in heavy-ion collisions:

strangeness neutrality and fixed electric charge to baryon-number ratio

Continuum estimated results of pressure & energy density up to 6th-order:

\[ n_S = 0 \quad , \quad \frac{n_Q}{n_B} = r = 0.4 \]

\[ \mu_B / T > 0 \]

Consistent results from two different actions, HISQ and stout, in the continuum

\[ \rightarrow \text{Equation of state well controlled for } \mu_B / T \lesssim 2 \iff \sqrt{s_{NN}} \geq 14.5 \text{ GeV} \]

**HISQ:** [Bielefeld-BNL-CCNU, arXiv:1701.04325]

**stout:** [Wuppertal-Budapest, arXiv:1607.02493]
Thermal conditions at chemical freeze-out / hadronization characterized by lines of constant pressure, energy and entropy densities?

\[
T_f(\mu_B) = T_0 \left(1 - \kappa_2^f \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4^f \left(\frac{\mu_B}{T_0}\right)^4\right)
\]

\[
0.0064 \leq \kappa_2^P \leq 0.0101
\]
\[
0.0087 \leq \kappa_2^E \leq 0.012
\]
\[
0.0074 \leq \kappa_2^S \leq 0.011
\]

\[
T_c(\mu_B) = T_c(0) \left(1 - \kappa_2^C \left(\frac{\mu_B}{T_c(0)}\right)^2\right)
\]

\[
0.0066 \leq \kappa_2^C \leq 0.020
\]

Thermal conditions at chemical freeze-out / hadronization characterized by lines of constant pressure, energy and entropy densities? 

\[
\begin{align*}
T &= 165 \text{ MeV} \\
\epsilon &= 0.556(57) \text{ GeV/fm}^3 \\
\end{align*}
\]

\[
\begin{align*}
T &= 155 \text{ MeV} \\
\epsilon &= 0.346(41) \text{ GeV/fm}^3 \\
\end{align*}
\]

\[
\begin{align*}
T &= 145 \text{ MeV} \\
\epsilon &= 0.203(27) \text{ GeV/fm}^3 \\
\end{align*}
\]

compare well with estimates of the crossover line:

Radius of convergence and the critical point

estimated from Taylor expansion of pressure or baryon-number susceptibility

Fodor, Katz, 2004
Datta et al., 2016
D’Elia et al., 2016

[A critical point at \( \mu_B < 2T \) is disfavored for \( 135 \text{ MeV} \leq T \leq 155 \text{ MeV} \)
and seems to be ruled out at \( T > 155 \text{ MeV} \)]

Higher order cumulants required for the search of a critical point

[M.D’Elia, G.Gagliardi, F.Sanfilippo, arXiv:1611.08285]
[Bielefeld-BNL-CCNU, arXiv:1701.04325]
Cumulants of net-baryon number and freeze-out

\( R_{12}^{B}(T, \mu_B) = \frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \frac{M_B}{\sigma_B^2} \)

\( R_{31}^{B}(T, \mu_B) = \frac{\chi_3^B(T, \mu_B)}{\chi_1^B(T, \mu_B)} = \frac{S_B\sigma_B^3}{M_B} \)

\( R_{42}^{B}(T, \mu_B) = \frac{\chi_4^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \kappa_B\sigma_B^2 \)

leading order expansion coefficients:

\( r_{12}^{B} \)

\( \mu = \text{mean} \quad \sigma = \text{variance} \quad S = \text{skewness} \quad \kappa = \text{kurtosis} \)

(M_B = \text{mean} \quad \sigma_B = \text{variance} \quad S_B = \text{skewness} \quad \kappa_B = \text{kurtosis})

Free quark gas

HRG

HRG preliminary

\( n_s = 0, n_Q/n_B = 0.4 \)

\( m_s/m_l = 27 \) (filled)

20 (open)

free quark gas

\( T_c,0 = 154(9) \text{ MeV} \)

fit to prel. STAR data

STAR: 0.4 GeV < p_t < 2.0 GeV

SB/MP

PDG-HRG

QM-HRG

N_τ = 6

8

12

16

[continuum extrap.]

[MB = mean \quad \sigma_B = variance \quad S_B = skewness \quad \kappa_B = kurtosis]

(mean \quad \sigma_B = variance \quad S_B = skewness \quad \kappa_B = kurtosis)
Cumulants of net-baryon number and freeze-out

\( R_{12}^B(T, \mu_B) = \frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \frac{M_B}{\sigma_B^2} \)

\( R_{31}^B(T, \mu_B) = \frac{\chi_3^B(T, \mu_B)}{\chi_1^B(T, \mu_B)} = \frac{S_B \sigma_B^3}{M_B} \)

\( R_{42}^B(T, \mu_B) = \frac{\chi_4^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \kappa_B \sigma_B^2 \)

Leading order expansion coefficients:

\( r_{12}^B \)

\( n_s = 0, n_Q/n_B = 0.4 \)

\( m_s/m_l = 27 \) (filled) 20 (open)

free quark gas

\[ \text{for } \mu/T \rightarrow 0 \rightarrow T_{f,0} = 153(4) \text{ MeV} \]
Cumulants of net-baryon number and freeze-out

NLO expansion of ratios of cumulants on lines of constant physics:

\[
R_{12}^B(T, \mu_B) = r_{12}^{B,1} \hat{\mu}_B + r_{12, f}^{B,3} \hat{\mu}_B
\]

\[
R_{31}^B(T, \mu_B) = r_{31}^{B,0} + r_{31, f}^{B,2} \hat{\mu}_B
\]

\[
R_{42}^B(T, \mu_B) = r_{42}^{B,0} + r_{42, f}^{B,2} \hat{\mu}_B
\]

Ratio of slopes from STAR = 3.9(2.1)

compares well with QCD predictions

\[
\frac{r_{42, f}^{B,2}}{r_{31, f}^{B,2}} = 3 - 4.5
\]

on lines of constant physics

[Bielefeld-BNL-CCNU, in preparation]
- Equation of state well controlled for $\frac{\mu_B}{T} \lesssim 2$ using expansion up to 6th-order

- A critical point at $\mu_B < 2T$ is disfavored for $135 \text{ MeV} \leq T \leq 155 \text{ MeV}$ and seems to be ruled out at $T > 155 \text{ MeV}$

- Higher order cumulants required for the search of a critical point

- Decrease of skewness and kurtosis for $\sqrt{s_{NN}} > 19.6 \text{ GeV}$ in accordance with QCD

- Physics above 160MeV much different from a hadron gas
Part II: Spectral and transport properties in the QGP

Thermal dilepton rate
\[
\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2(e^{\omega/T} - 1)} \rho_V(\omega, T)
\]

Thermal photon rate
\[
\omega \frac{dN_\gamma}{d^4x d^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T} - 1} \rho_V(\omega = |\vec{k}|, T)
\]

Transport coefficients are encoded in the same spectral function
→ Kubo formulae

Diffusion coefficients:
\[
DT = \frac{T}{2\chi_q} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega)}{\omega}
\]

Need to determine vector-meson spectral functions

On the lattice only correlation functions can be calculated
→ spectral reconstruction required
Vector-meson spectral function – hard to separate different scales

\[ G(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \vec{p}, T) K(\tau, \omega, T) \]

\[ K(\tau, \omega, T) = \frac{\cosh \left( \omega (\tau - \frac{1}{2T}) \right)}{\sinh \left( \frac{\omega}{2T} \right)} \]

Different contributions and scales enter in the spectral function

- continuum at large frequencies
- possible bound states at intermediate frequencies
- transport contributions at small frequencies
- in addition cut-off effects on the lattice

Spectral functions in the QGP

notoriously difficult to extract from correlation functions

\[ G_{\mu\nu}(\tau, \vec{x}) = \langle J_\mu(\tau, \vec{x}) J^\dagger_\nu(0, \vec{0}) \rangle \]

\[ J_\mu(\tau, \vec{x}) = 2\kappa Z_V \bar{\psi}(\tau, \vec{x}) \Gamma_\mu \psi(\tau, \vec{x}) \]

→ large lattices and continuum extrapolation needed
→ still only possible in the quenched approximation
→ use perturbation theory to constrain the UV behavior

(narrow) transport peak at small \( \omega \):

\[ \rho(\omega \ll T) \approx 2\chi_{00} \frac{T}{M} \frac{\omega \eta}{\omega^2 + \eta^2}, \quad \eta = \frac{T}{MD} \]
Ansatz for the (non-perturbative) transport contribution:

\[ G(\tau, \bar{\rho}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \bar{\rho}, T) K(\tau, \omega, T) \]

\[ K(\tau, \omega, T) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})} \]

and perturbative constraints for the UV part of the spectral function

\[ \rho_{BW}(\omega) = \frac{2\chi_q C_{BW}}{\omega^2 + (\Gamma/2)^2} \]

(5-loop vacuum + LO thermal correction)

Fit to continuum extrapolated vector-meson correlation function \( G_{ii}(\tau, T) \)
Electrical conductivity of the QGP continuum estimate for the of the electrical conductivity lower and upper limits from analysis of different classes of spectral functions:

\[
\frac{\sigma_{el}}{C_{em} T} = \frac{1}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T')}{\omega T}
\]

[H-T.Ding, F.Meyer, OK, PRD94(2016)034504]

comparison of different lattice results (Plot courtesy of A.Francis)

Electrical conductivity of the QGP

Compared to calculations in partonic transport approaches

Progress in determining transport coefficients, although systematic uncertainties still need to be reduced in the future.

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Comparison of different lattice results (Plot courtesy of A. Francis)

- HRG, kin.th.: PRD 93, 096012
- BAMPS, PRD 90, 094014
- DQPM: I.J.M.Phys E 25, 1642003
- ChPT: PRD 73, 045025
- PHSD: PRC 89, 035203
- Mainz lattice
- Bielefeld quenched continuum lattice

M. Greif, C. Greiner, G. Denicol, PRD 93 (2016) 096012

G. Aarts et al., PRL 99 (2007) 022002,
H-T. Ding, F. Meyer, OK, PRD 94 (2016) 034504,
B. B. Brandt et al., JHEP 1303 (2013) 100,
Brandt et al., PRD 93 (2016) 054510,
A. Amato et al., PRL 111 (2013) 172001
Dilepton rate directly related to vector spectral function:

\[
\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2(e^{\omega/T} - 1)} \rho_V(\omega, T)
\]

Hard thermal loop (HTL) [E. Braaten, R.D. Pisarski (1990)]

[H-T. Ding, F. Meyer, OK, PRD94(2016)034504]
Photon rate directly related to vector spectral function (at finite momentum):

\[ \omega \frac{dN_\gamma}{d^4xd^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T} - 1} \rho_V(\omega = |\vec{k}|, T) \]

**pQCD spectral function used to constrain the UV**

interpolation between different (perturbative) regimes:

3T < \omega < 10T: [J.Ghiglieri, G.D.Moore, JHEP 1412 (2014) 029]

\[ \omega > 10T: \quad [I. Ghisoiu, M.Laine, JHEP 10 (2014) 84] \]

\[ \omega \gg 10T: \quad [M.Laine, JHEP 1311 (2013) 120] \]

to allow for non-perturbative effects

and to analyze how far pQCD can be trusted

we model the infrared behavior assuming smoothness at the light cone

and fit to continuum extrapolated lattice correlators
Using vector correlation functions on large and fine lattices up to $196^3 \times N_t$ with $N_t = 56, 64$ → continuum extrapolation at finite momentum $k$

Using best perturbative knowledge to constrain the spectral function at large $\omega$

→ fit a polynomial at small $\omega$ to extract the spectral function at the photon point $\omega = k$
The spectral function at the photon point $\omega = k$

$$D_{\text{eff}}(k) \equiv \begin{cases} \frac{\rho_\nu(k, k)}{2\chi_q k}, & k > 0 \\ \lim_{\omega \to 0^+} \frac{\rho^{ii}(\omega, 0)}{3\chi_q \omega}, & k = 0 \end{cases}.$$ 

can be used to calculate the photon rate

$$\frac{d\Gamma_\gamma(k)}{d^3 k} = \frac{2\alpha_{\text{em}}\chi_q}{3\pi^2} n_B(k) D_{\text{eff}}(k) + \mathcal{O}(\alpha_{\text{em}}^2)$$

becomes more perturbative at larger $k$, approaching the NLO prediction (valid for $k >> gT$)

but non-perturbative for $k/T < 3$

Electrical conductivity obtained in the limit $k \to 0$ between the results from

AdS/CFT: $DT = \frac{1}{2\pi}$

LO perturbation theory [Arnold, Moore Yaffe, JHEP 05 (2003)] using lattice value for $\chi_q/T^2$: $DT = 2.9 - 3.1$
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) “color-electric correlator”

\[ G_E(\tau) \equiv -\frac{1}{3} \sum_{i=1}^{3} \frac{\left\langle \text{Re} \ Tr \left[ \frac{1}{T} U(\tau,0) E_i(\tau,0) U(0,0) E_i(0,0) \right] \right\rangle}{\left\langle \text{Re} \ Tr[U(\frac{1}{T};0)] \right\rangle} \]

\[ \kappa = \lim_{\omega \to 0} \frac{2T \rho_E(\omega)}{\omega} \]

NLO perturbative calculation:

[Caron-Huot, G. Moore, JHEP 0802 (2008) 081]

\[ \rightarrow \text{large correction towards strong interactions} \]

\[ \rightarrow \text{non-perturbative lattice methods required} \]
Heavy Quark Momentum Diffusion Constant $\kappa$

**Heavy-quark momentum diffusion $\kappa$**

\[
G_E(\tau) \equiv \int_0^{\infty} \frac{d\omega}{\pi} \rho_E(\omega) \cosh \left( \frac{1}{2} - \tau T \right) \frac{\omega}{\sinh \frac{\omega}{2T}}
\]

\[
\kappa = \lim_{\omega \to 0} \frac{2T \rho_E(\omega)}{\omega}
\]

Combine continuum-extrapolated lattice results and perturbative input at large $\omega$ and model corrections in the non-perturbative low-$\omega$ region by power series in $\omega$.
Heavy Quark Momentum Diffusion Constant – systematic uncertainties

A.Francis, OK et al., PRD92(2015)116003

close to $T_c$, $\kappa$ appears to be almost as fast as that of light partons.

Detailed analysis of systematic uncertainties

continuum estimate of $\kappa$:

$$\frac{\kappa}{T^3} = \lim_{\omega \to 0} \frac{2T \rho_E(\omega)}{\omega} = 1.8...3.4$$

Related to diffusion coefficient $D$ and drag coefficient $\eta_D$ (in the non-relativistic limit)

$$2\pi TD = 4\pi \frac{T^3}{\kappa} = 3.7...7.0$$

$$\eta_D = \frac{\kappa}{2M_{kin}T} \left(1 + O \left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

time scale associated with the kinetic equilibration of heavy quarks:

$$\tau_{kin} = \frac{1}{\eta_D} = (1.8...3.4) \left(\frac{T_c}{T}\right)^2 \left(\frac{M}{1.5 \text{ GeV}}\right) \text{ fm/c}$$

close to $T_c$, $\tau_{kin} \approx 1 \text{ fm/c}$ and therefore charm quark kinetic equilibration appears to be almost as fast as that of light partons.

\[ \begin{align*}
T &= 0 \\
T &= 147\text{MeV} \\
T &= 163\text{MeV} \\
T &= 184\text{MeV} \\
T &= 218\text{MeV} \\
T &= 257\text{MeV}
\end{align*} \]

\[ \begin{align*}
\beta &= 7.28 \ \Upsilon(3S_1) \ T > 0, \ n = 8
\end{align*} \]

Lattice-NRQCD using two versions of Bayesian reconstruction: [A.Rothkopf, see Talk on Tuesday]

\[ \begin{align*}
\beta &= 7.825 \ \Upsilon(3S_1) \ T > 0, \ n = 4
\end{align*} \]

From full QCD with u,d,s quarks based on asqtad action
Charmonium spectral and transport properties

Using vector correlation functions on large and fine lattices up to $196^3 \times N_t$ with $N_t=96,48$ and Stochastic Analytical Interference method (SAI) based on Bayes’ theorem:

From a careful analysis of systematic uncertainties: $2\pi TD = 1.6 - 7.0$
Conclusions – part II

using **continuum extrapolated correlation functions** from Lattice QCD and using phenomenologically inspired and **perturbatively constrained Ansätze** allows to extracted **transport properties** and **spectral properties**

we obtained continuum estimates for

→ **Electrical conductivity / Diffusion coefficients**

→ **Thermal dilepton rates**

→ **Thermal photon rates**

**next goals**: continuum extrapolation for charm and bottom correlators

→ quark mass dependence of diffusion coefficient + sequential melting of quarkonia

The methodology developed in this studies within the quenched approximation shall be extended to full QCD calculations for a realistic QGP medium as close to $T_c$ dynamical fermion degrees of freedom will become important