Universität Bielefeld

Lattice QCD results on soft and hard probes of strongly interacting matter

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I) Equation of State at $\mu_B > 0$ and cumulants of conserved charges

→ thermal conditions at freeze-out

→ radius of convergence and critical point

→ skewness and kurtosis of net-baryon number

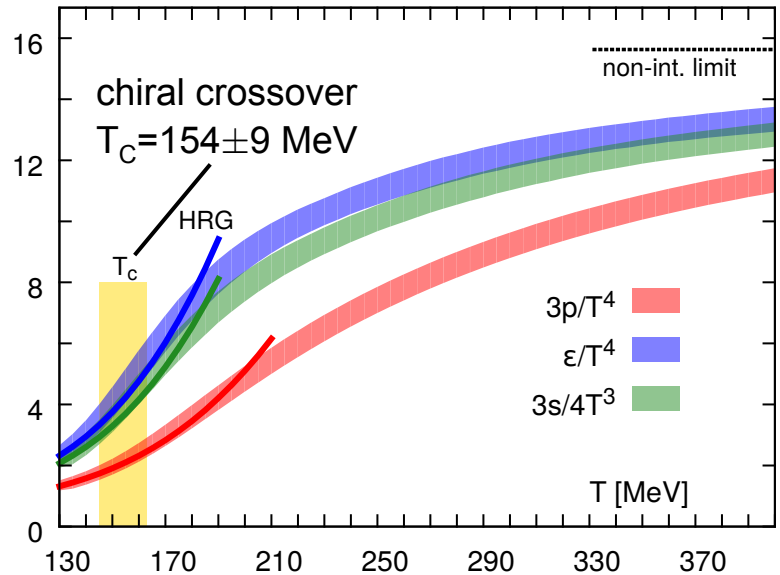
II) Spectral and Transport properties in the QGP

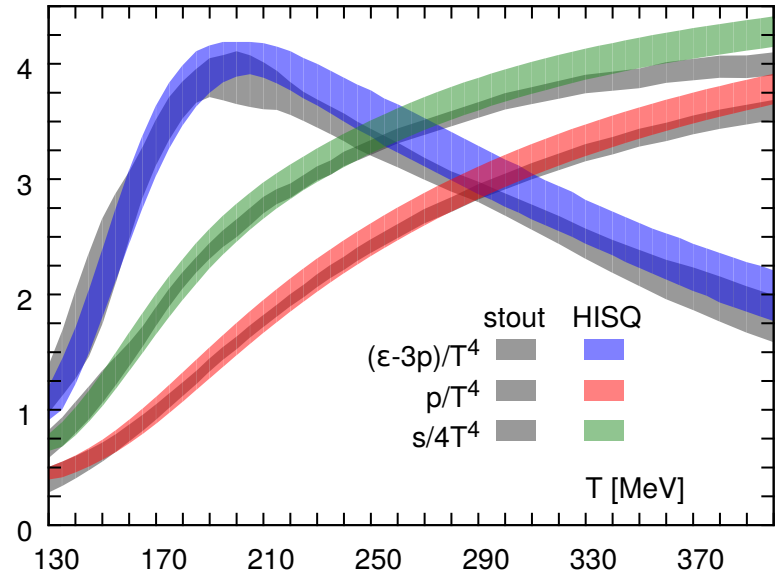
→ thermal dilepton and photon rates

→ electrical conductivity and heavy-quark diffusion

Quark Matter 2017, Chicago, 09.02.2017

Continuum extrapolated results of **pressure** & **energy density** & **entropy density**





HISQ: [A. Bazavov et al. (hotQCD), PRD90 (2014) 094503] **stout:** [S. Borsanyi et al. (BMW), PLB730, 99 (2014)]

consistent results from hotQCD (HISQ) and Budapest-Wuppertal (stout)

Hadron resonance gas (HRG) model using all known hadronic resonances from PDG

describes the EoS quite well up to cross-over region

QCD results systematically above HRG

QCD is quite different from HRG thermodynamics at T > 160MeV

HRG: thermal gas of uncorrelated hadrons

partial pressure of each hadron:

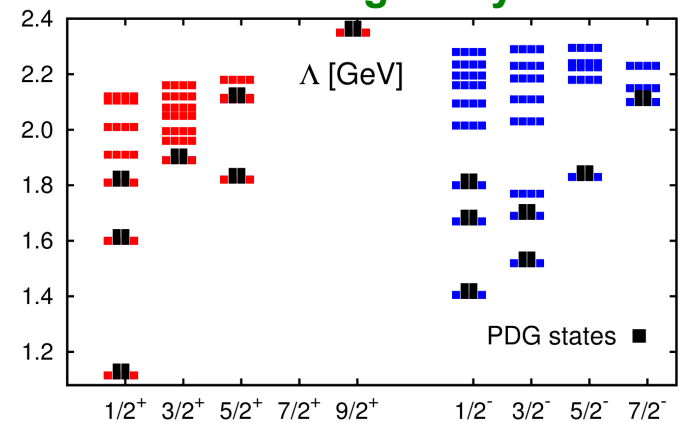
$$\hat{P}_h \sim f(\hat{m}_h)\cosh\left[B_h\hat{\mu}_h + Q_h\hat{\mu}_Q + S_h\hat{\mu}_S\right]$$

total pressure given by the sum over all (known) hadrons

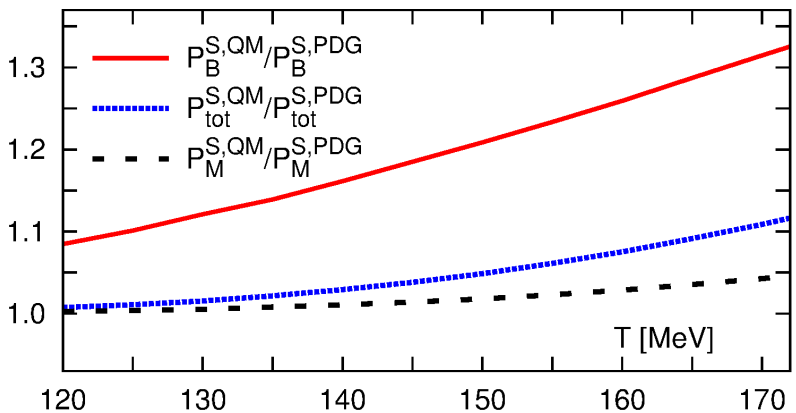
$$\hat{P}_{total} = \sum_{all\ hadrons} \hat{P}_h$$

are we sensitive to this?

Quark Model predicts more strange baryons:



[Capstick-Isgur, PRD34 (1986) 2809]



[Bielefeld-BNL-CCNU, PRL113(2014)072001]

$$P_{\rm tot}^{S,X} = P_M^{S,X} + P_B^{S,X}$$

X = { QM resonances / PDG resonances

$$P_{M/B}^{S,X}(T,\vec{\mu}) = \frac{T^4}{2\pi^2}\sum_{i\in X} g_i\left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \times \cosh\left(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S\right)$$

large enhancement of the partial baryonic pressure from additional strange baryons

PDG-HRG uses states listed in the particle data tables

QM-HRG uses states calculated in the quark model

[see also poster by Paolo Alba and P.Alba, R.Bellwied et al., arXiv:1702.01113]

use QCD thermodynamics instead of HRG to describe physics at freeze-out

Equation of state of (2+1)-flavor QCD – strange sector

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

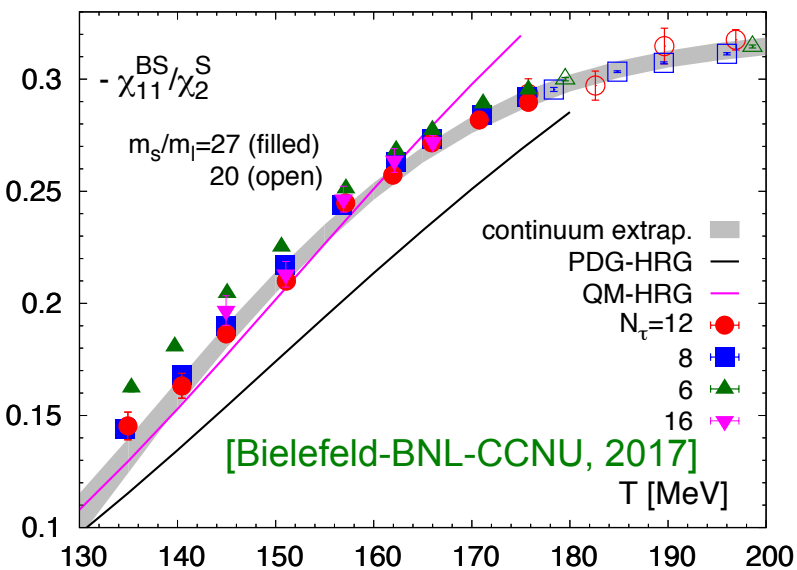
defines generalized susceptibilities: $\chi_{ijk}^{BQS} = \left.\frac{\partial^{(i+j+k)}[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\right|_{\vec{\mu}=0}$

correlations of strangeness with baryon number fluctuations:

$$\chi_{11}^{BS} = \left.\frac{\partial^2[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_B \partial\hat{\mu}_S}\right|_{\vec{\mu}=0}$$

second cumulant of net strangeness fluctuations:

$$\chi_2^{S} = \left.\frac{\partial^2[P(T,\hat{\mu}_B,\hat{\mu}_Q,\hat{\mu}_S)/T^4]}{\partial\hat{\mu}_S^2}\right|_{\vec{\mu}=0}$$



suitable ratios like

$$\frac{\chi_{11}^{BS}}{\chi_2^S}$$

χ_{11}^{BS}: strange baryon density (in a hadron gas)

χ_2^S: dominated by strange mesons

are sensitive probes of the strangeness carrying degrees of freedom

$$\frac{\mu_S}{\mu_B} = -\frac{\chi_{11}^{BS}}{\chi_2^S} + \mathcal{O}(\mu^2)$$

Equation of state of (2+1)-flavor QCD - μ_B/T > 0

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

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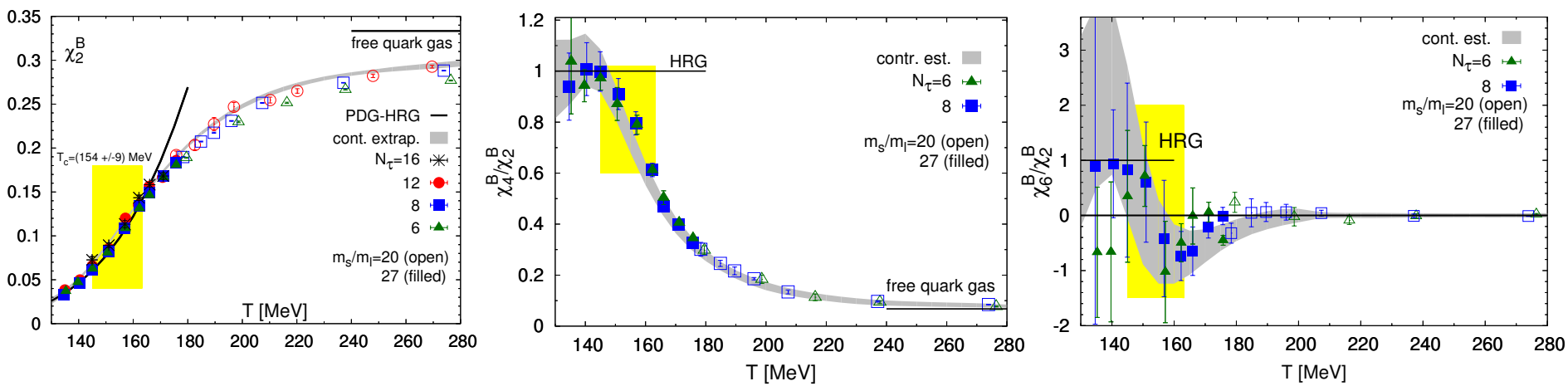
for $\mu_Q = \mu_S = 0$ this simplifies to

variance of net-baryon number distribution

kurtosis*variance $\kappa_B \sigma_B^2$

$$\frac{\Delta P(T)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right) + \mathcal{O}\left(\mu_B^6\right)$$



[Bielefeld-BNL-CCNU, arXiv:1701.04325]

good agreement with HRG up to crossover region

deviations from HRG in crossover region

Equation of state of (2+1)-flavor QCD - μ_B/T > 0

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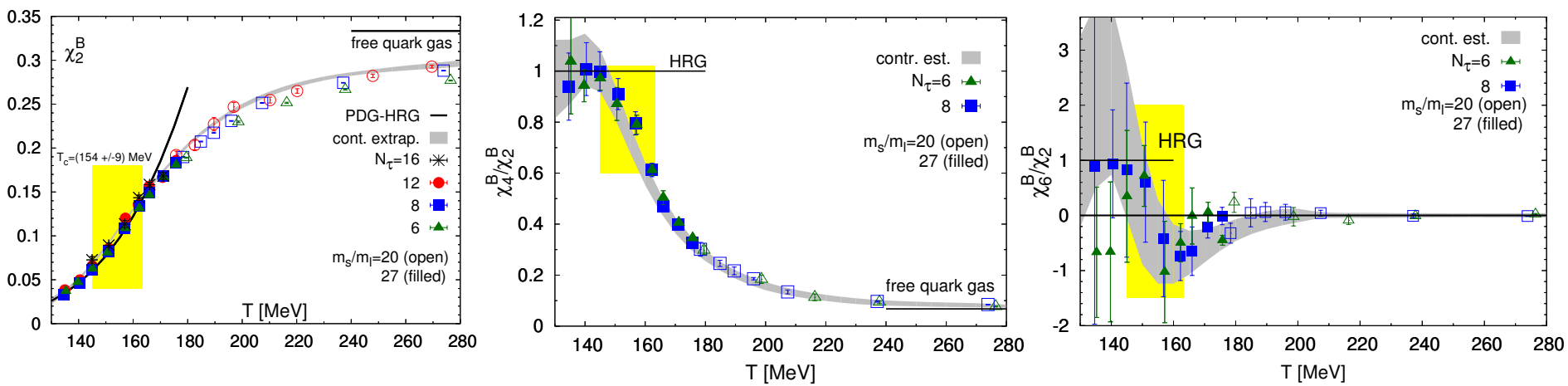

Net-baryon number fluctuations are "Skellam" only for T<155 MeV (consistent with ALICE data - see talk by Anar Rustamov (Wed.))

Equation of state of (2+1)-flavor QCD - μ_B/T > 0

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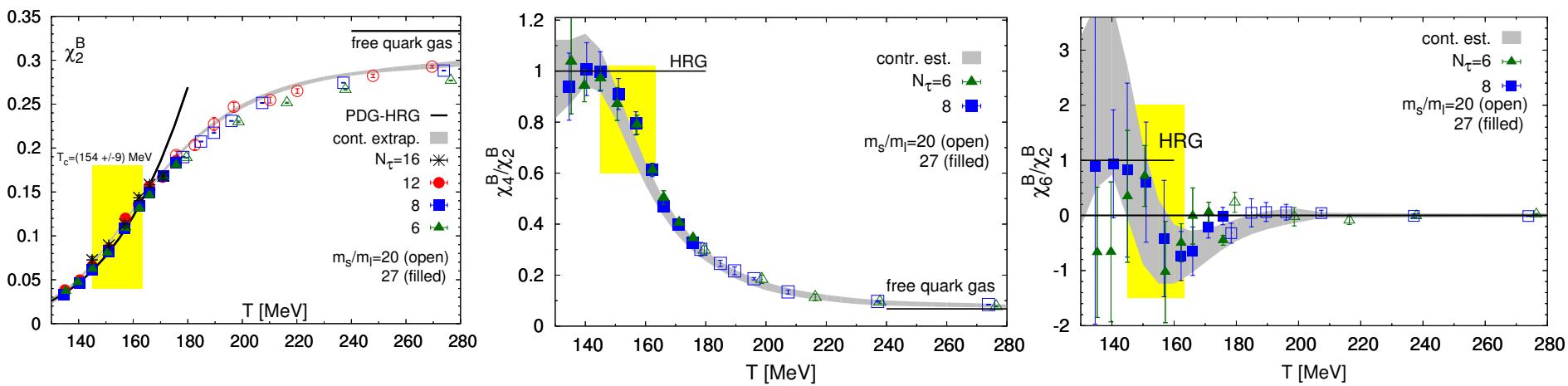


6th-order becomes negative in the cross-over region

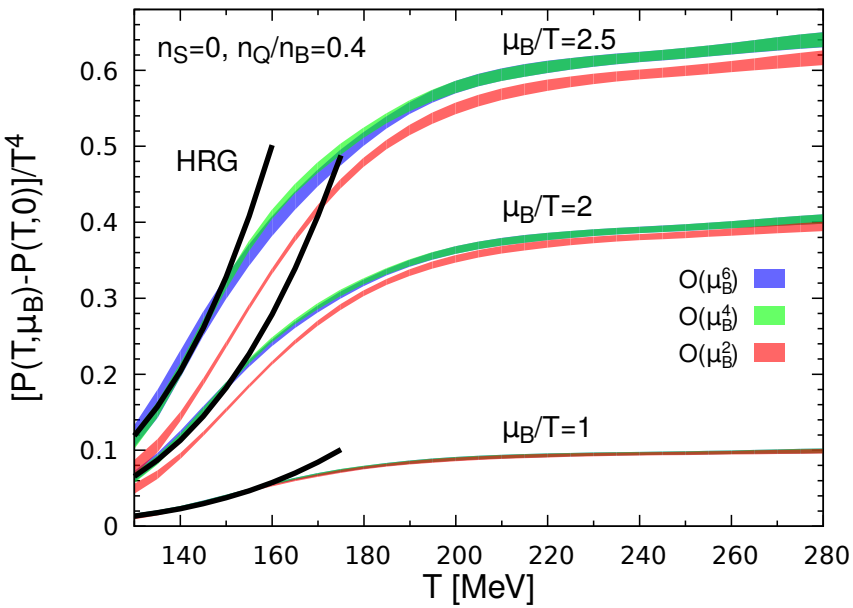
(consistent with central STAR data - see talk by Roli Esha (Wed.))

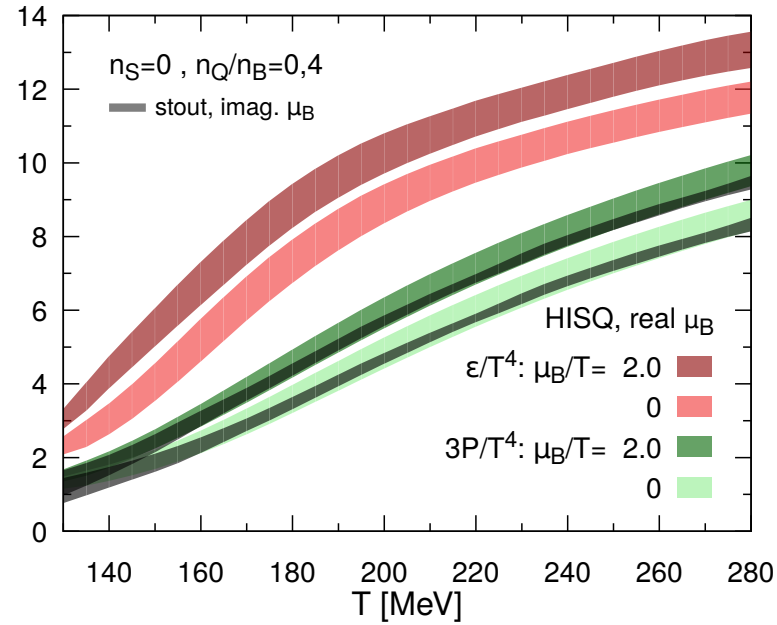
Equation of state of (2+1)-flavor QCD - μ_B/T > 0

Constraints in heavy-ion collisions: $n_S = 0 \quad , \quad \frac{n_Q}{n_B} = r = 0.4$

strangeness neutrality and fixed **electric charge to baryon-number ratio**

Continuum estimated results of **pressure** & **energy density** up to 6th-order:





Parametrization of the EoS for T∈ [130MeV,280MeV] & μ_B/T ≥ 0 in [Bielefeld-BNL-CCNU, arXiv:1701.04325]

HISQ: [Bielefeld-BNL-CCNU, arXiv:1701.04325]
stout: [Wuppertal-Budapest, arXiv:1607.02493]

Consistent results from two different actions, HISQ and stout, in the continuum

→ Equation of state well controlled for $\mu_B/T \lesssim 2$ ←→ $\sqrt{s_{NN}} \geq 14.5\ GeV$

Constraints in heavy-ion collisions: $n_S = 0 \quad , \quad \frac{n_Q}{n_B} = r = 0.4$

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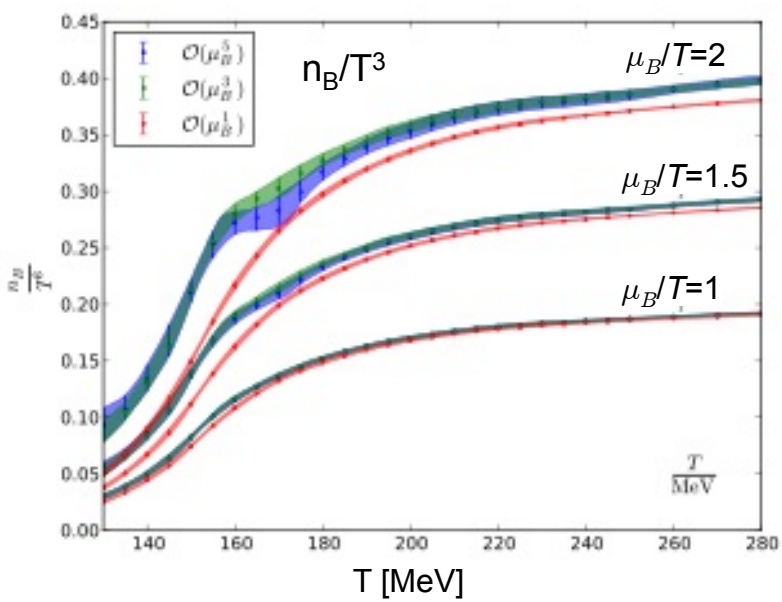

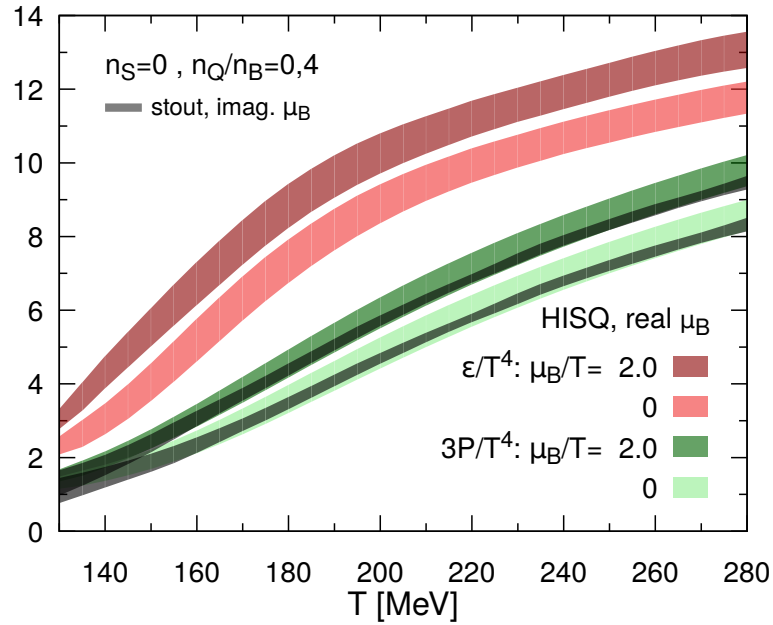

stout: [Wuppertal-Budapest, see Talk by Jana Günther]

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Lines of constant physics and chemical freeze-out / hadronization

Thermal conditions at chemical freeze-out / hadronization characterized by lines of constant pressure, energy and entropy densities?

$$T_f(\mu_B) = T_0 \left(1 - \kappa_2^f \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4^f \left(\frac{\mu_B}{T_0}\right)^4\right)$$

$0.0064 \leq \kappa_2^P \leq 0.0101$

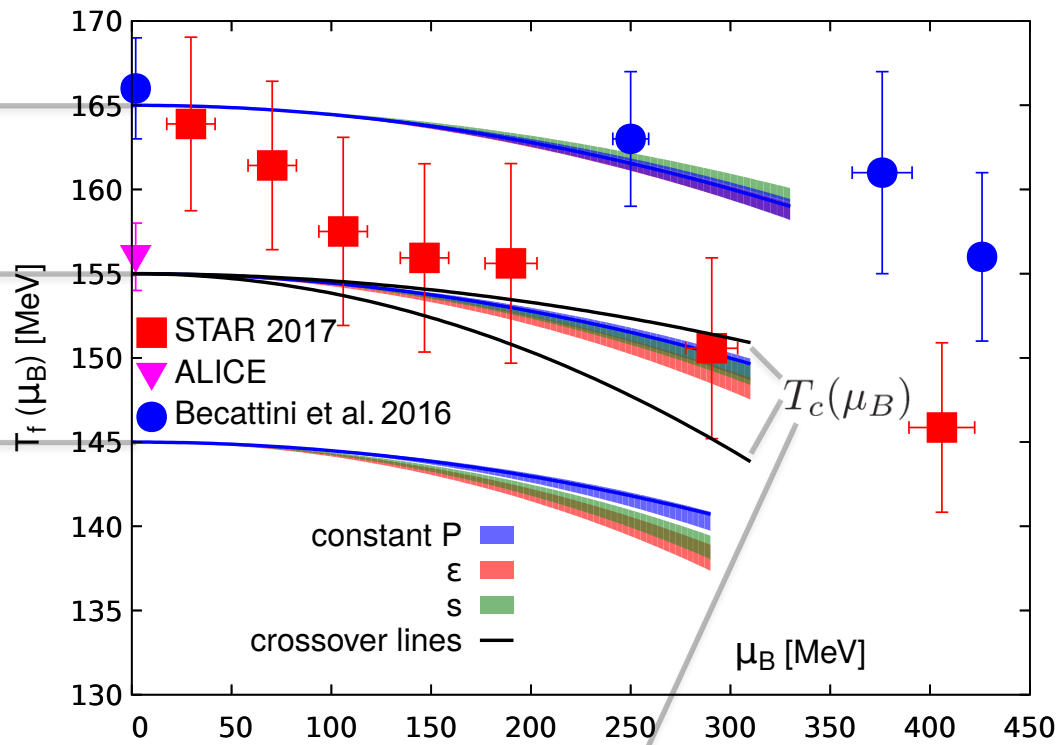
$0.0087 \leq \kappa_2^\epsilon \leq 0.012$

$0.0074 \leq \kappa_2^s \leq 0.011$

T=165 MeV
ϵ = 0.556(57) GeV/fm^3

T=155 MeV
ϵ = 0.346(41) GeV/fm^3

T=145 MeV
ϵ = 0.203(27) GeV/fm^3



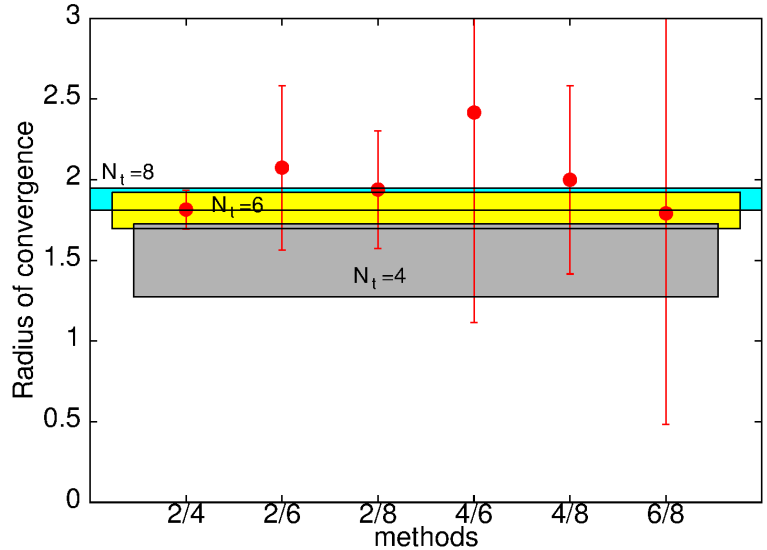
compare well with estimates of the crossover line: $T_c(\mu_B) = T_c(0)\left(1 - \kappa_2^c \left(\frac{\mu_B}{T_c(0)}\right)^2\right)$

$0.0066 \leq \kappa_2^c \leq 0.020$

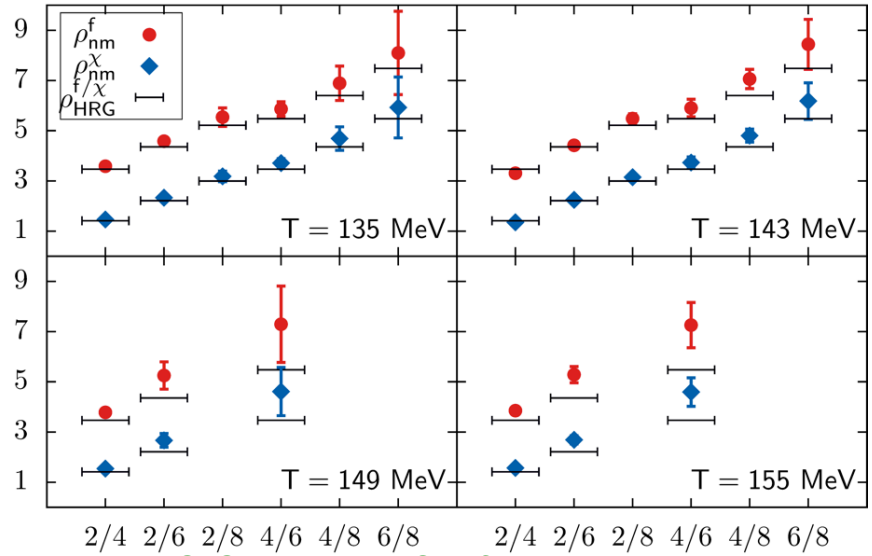
[Bielefeld-BNL-CCNU (2011), Endrodi et a. (2011), Bellwied et al. (2015), Cea et al. (2016), Bonati et al. (2015)]

Radius of convergence and the critical point

estimated from Taylor expansion of pressure or baryon-number susceptibilty

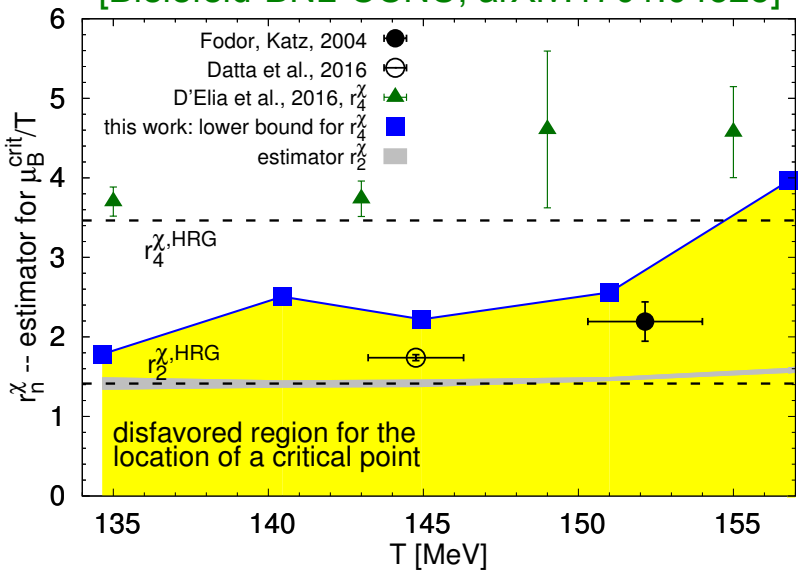


[S.Datta, R.V. Gavai, S. Gupta, arXiv:1612.06673]



[M.D'Elia, G.Gagliardi, F.Sanfilippo, arXiv:1611.08285]

[Bielefeld-BNL-CCNU, arXiv:1701.04325]



$$r_{2n}^{\chi} = \left| \frac{2n(2n-1)\chi_{2n}^{B}}{\chi_{2n+2}^{B}} \right|^{1/2}$$

A critical point at $\mu_B < 2T$ is disfavored for 135 MeV $\leq T \leq$ 155 MeV

and seems to be ruled out at T>155 MeV

Higher order cumulants required for the search of a critical point

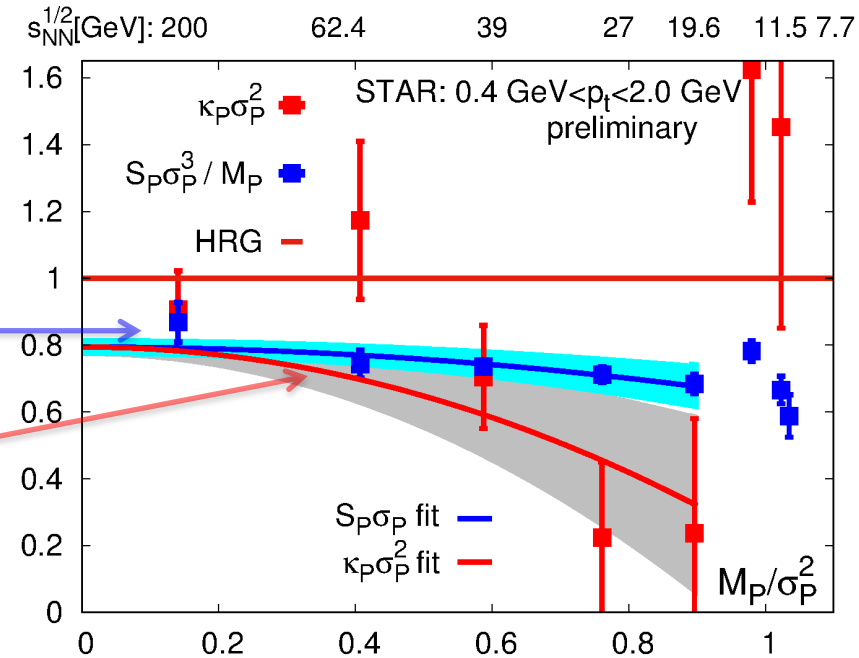
Cumulants of net-baryon number and freeze-out

(M_B=mean σ_B=variance S_B=skewness κ_B=kurtosis)

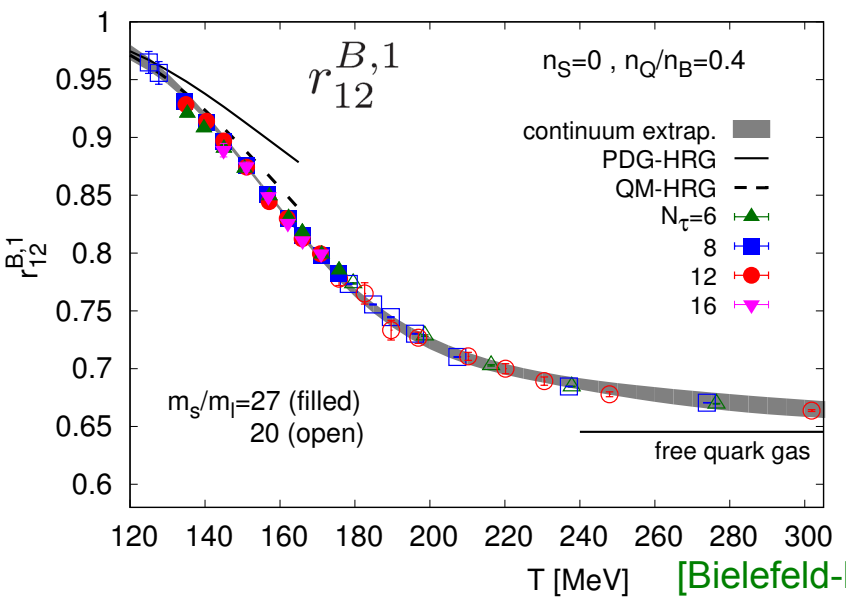
$$R^B_{12}(T,\mu_B) = \frac{\chi^B_1(T,\mu_B)}{\chi^B_2(T,\mu_B)} = \frac{M_B}{\sigma^2_B}$$

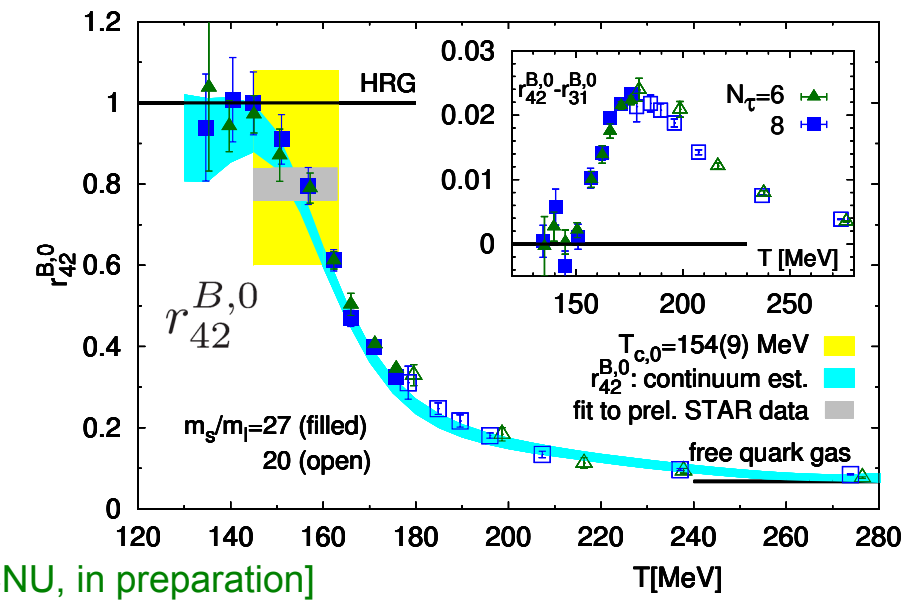
$$R^B_{31}(T,\mu_B) = \frac{\chi^B_3(T,\mu_B)}{\chi^B_1(T,\mu_B)} = \frac{S_B\sigma^3_B}{M_B}$$

$$R^B_{42}(T,\mu_B) = \frac{\chi^B_4(T,\mu_B)}{\chi^B_2(T,\mu_B)} = \kappa_B\sigma^2_B$$



Leading order expansion coefficients:





[Bielefeld-BNL-CCNU, in preparation]

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(M_B=mean σ_B=variance S_B=skewness κ_B=kurtosis)

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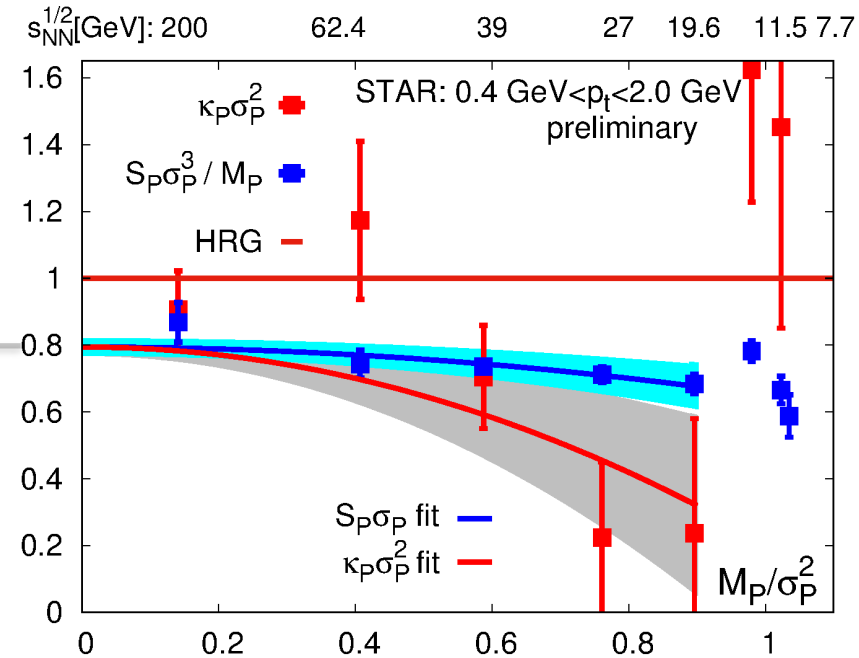
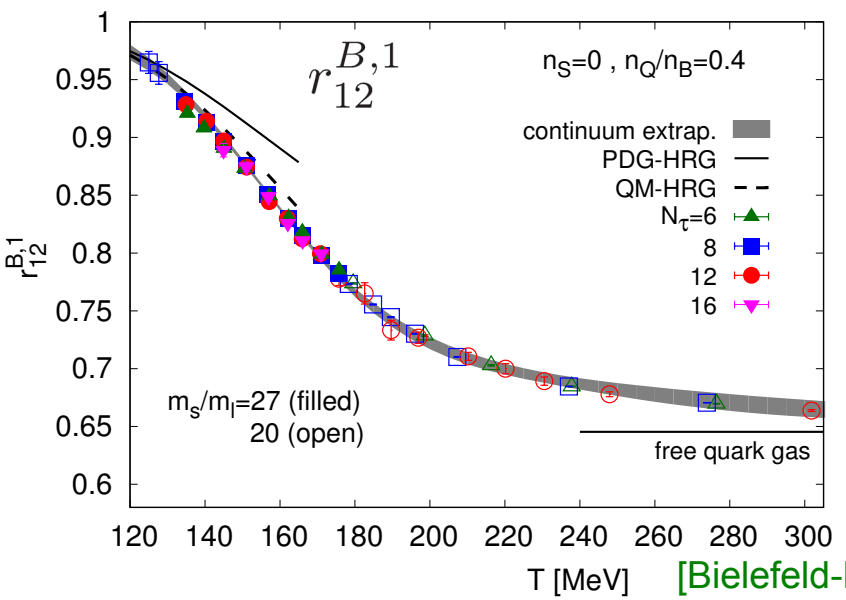
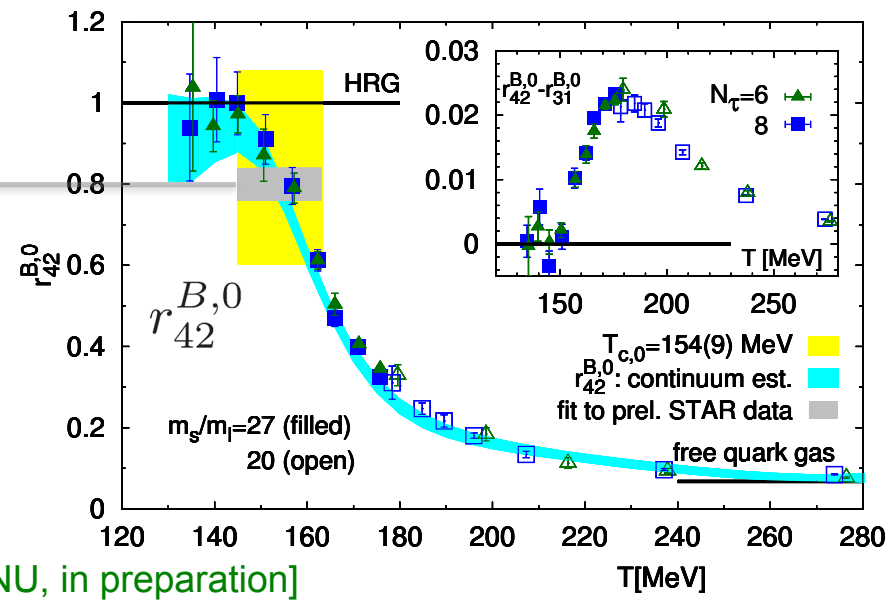


for $\mu/T \to 0$ → $T_{f,0}$ = 153(4) MeV

Leading order expansion coefficients:





[Bielefeld-BNL-CCNU, in preparation]

Cumulants of net-baryon number and freeze-out

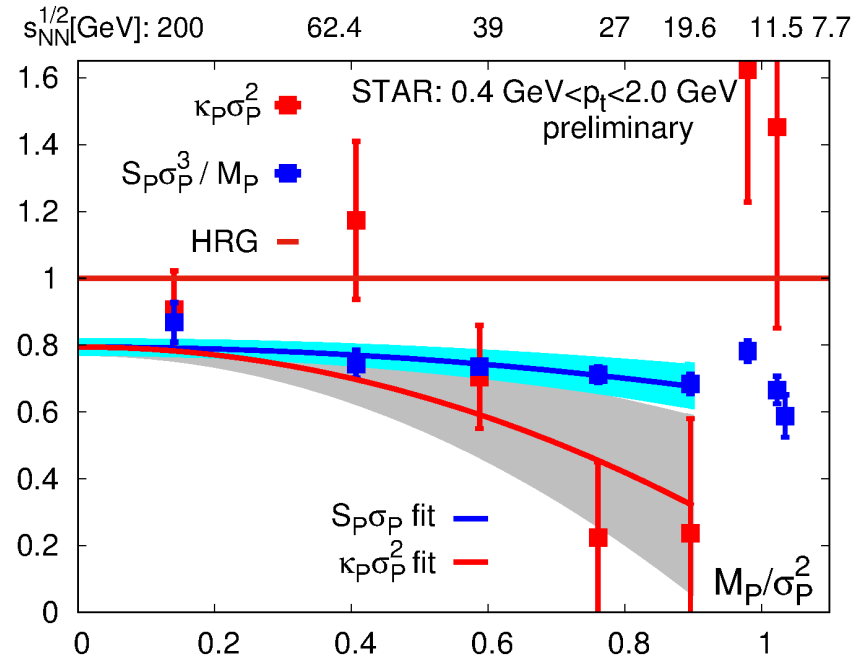
NLO expansion of ratios of cumulants on lines of constant physics:

$$R^B_{12}(T,\mu_B) = r^{B,1}_{12}\hat{\mu}_B + r^{B,3}_{12,f}\hat{\mu}^3_B$$

$$R^B_{31}(T,\mu_B) = r^{B,0}_{31} + r^{B,2}_{31,f}\hat{\mu}^2_B$$

$$R^B_{42}(T,\mu_B) = r^{B,0}_{42} + r^{B,2}_{42,f}\hat{\mu}^2_B$$

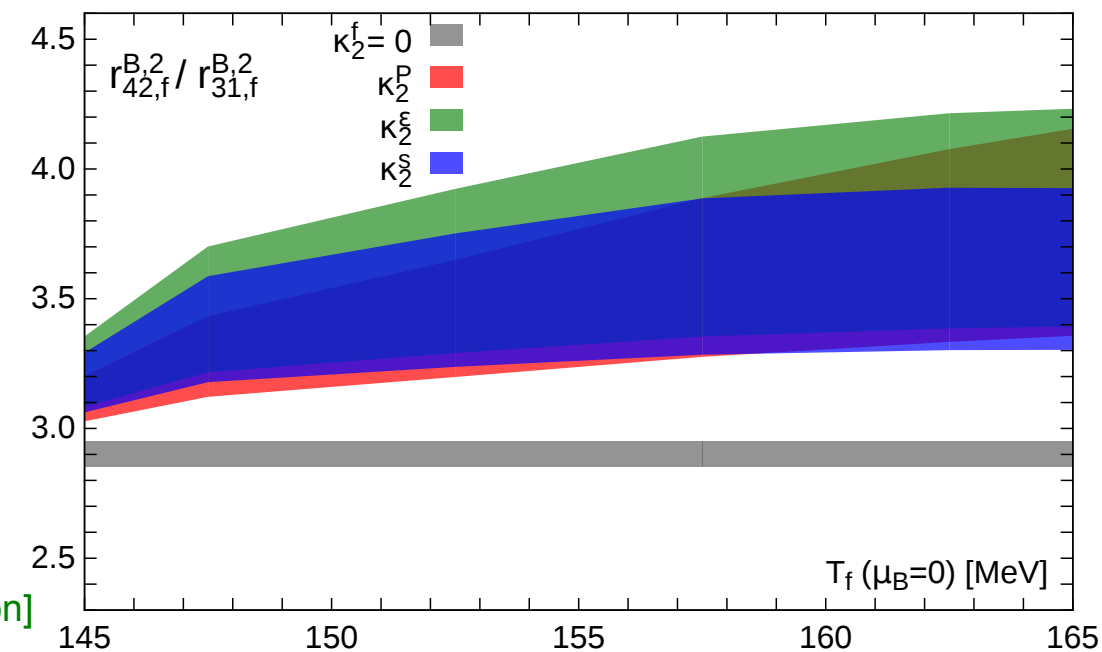
Ratio of slopes from STAR = 3.9(2.1)



compares well with QCD predictions

$$\frac{r^{B,2}_{42,f}}{r^{B,2}_{31,f}} = 3 \; - \; 4.5$$

on lines of constant physics



[Bielefeld-BNL-CCNU, in preparation]

Conclusions – part I

- Equation of state well controlled for $\mu_B/T \lesssim 2$ using expansion up to 6th-order

- A critical point at $\mu_B < 2T$ is disfavored for 135 MeV $\leq T \leq$ 155 MeV and seems to be ruled out at T>155 MeV

- Higher order cumulants required for the search of a critical point

- Decrease of skewness and kurtosis for $\sqrt{s_{NN}} > 19.6\ GeV$ in accordance with QCD

- Physics above 160MeV much different from a hadron gas

Part II: Spectral and transport properties in the QGP

Thermal dilepton rate

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\,\rho_{\mathbf{V}}(\omega,\mathbf{T})$$

Thermal photon rate

$$\omega\frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x\mathrm{d}^3q} = \frac{5\alpha}{6\pi^2}\frac{1}{e^{\omega/T}-1}\rho_V(\omega=|\vec{k}|,T)$$

Transport coefficients are encoded in the same spectral function

→ Kubo formulae

Diffusion coefficients:

$$DT = \frac{T}{2\chi_q}\lim_{\omega\to 0}\frac{\rho_{\mathrm{ii}}(\omega)}{\omega}$$

Need to determine vector-meson spectral functions

On the lattice only correlation functions can be calculated

→ spectral reconstruction required

Vector-meson spectral function – hard to separate different scales

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

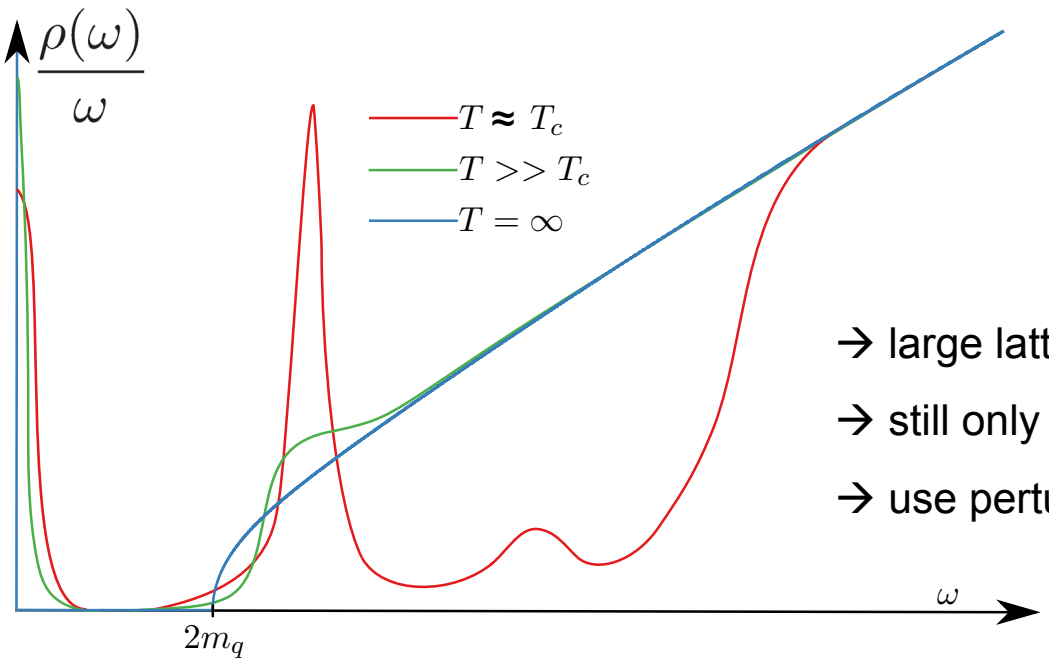
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Different contributions and scales enter in the spectral function

- **continuum at large frequencies**
- **possible bound states at intermediate frequencies**
- **transport contributions at small frequencies**
- **in addition cut-off effects on the lattice**

notoriously difficult to extract from correlation functions

Spectral functions in the QGP



$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$
$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V\bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

→ large lattices and continuum extrapolation needed

→ still only possible in the quenched approximation

→ use perturbation theory to constrain the UV behavior

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}, \qquad \eta = \frac{T}{MD}$

Spectral function and electrical conductivity – light quark sector

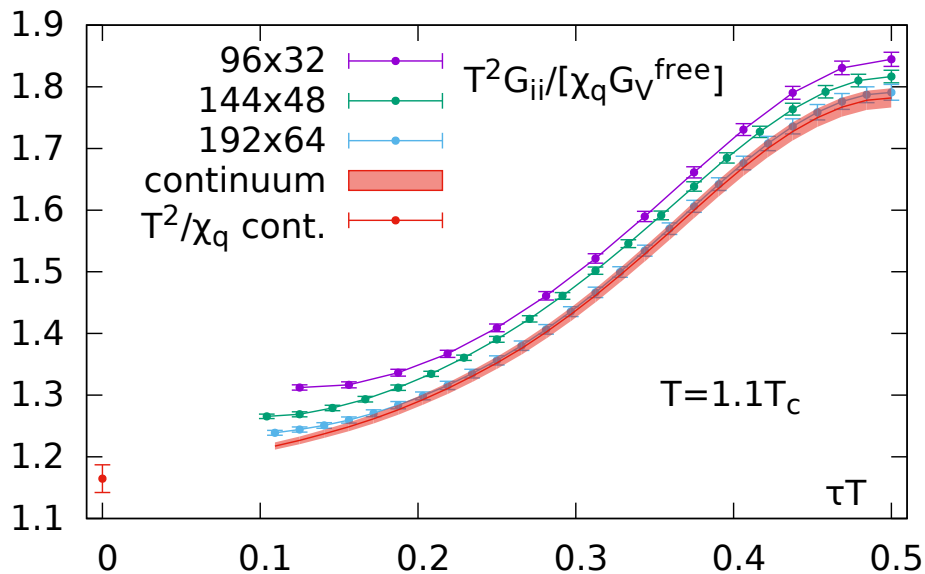
$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T) \qquad K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

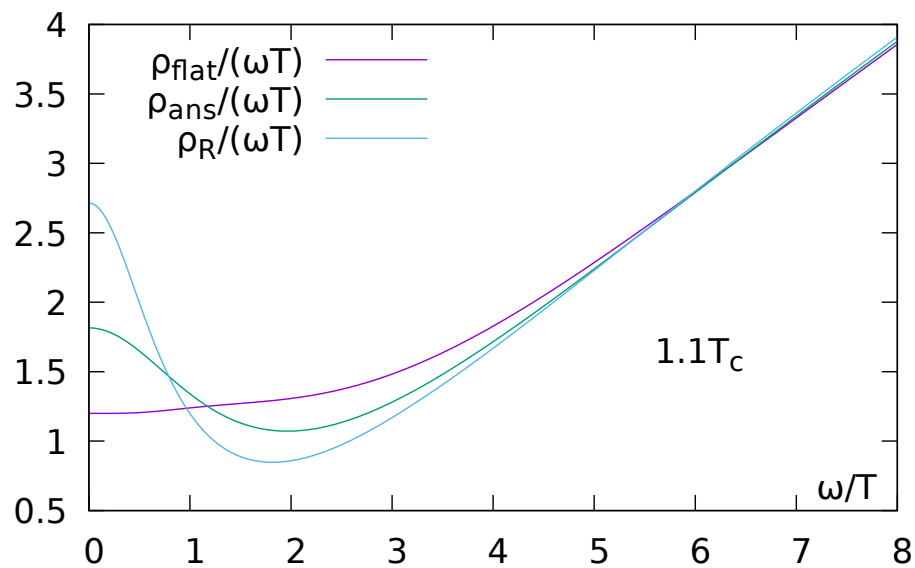
Ansatz for the (non-perturbative) transport contribution: $\rho_{BW}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2}$

and perturbative constraints for the UV part of the spectral function

$$\rho_R(\omega) = \rho_{BW}(\omega) + \frac{3\omega^2}{4\pi}[1-2n_F(\frac{\omega}{2})]R(\omega^2)$$ (5-loop vacuum + LO thermal correction)

Fit to continuum extrapolated vector-meson correlation function $G_{ii}(\tau,T)$

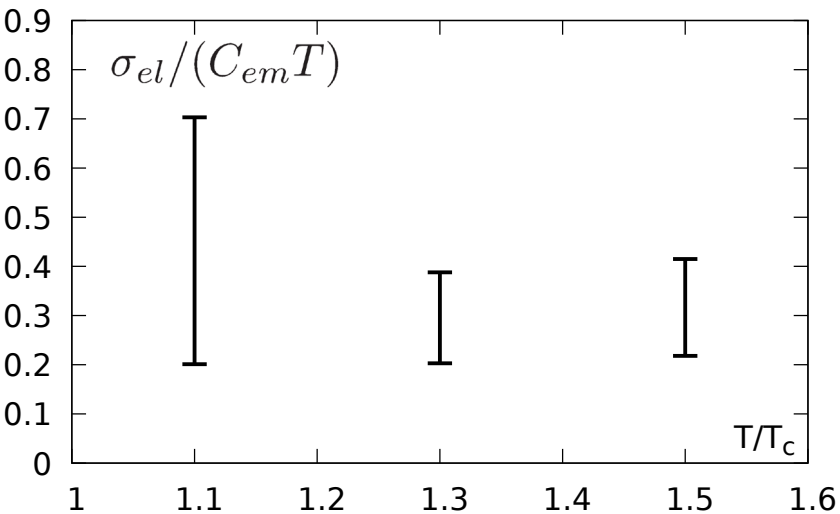




continuum estimate for the of the electrical conductivity

lower and upper limits from analysis of different classes of spectral functions:

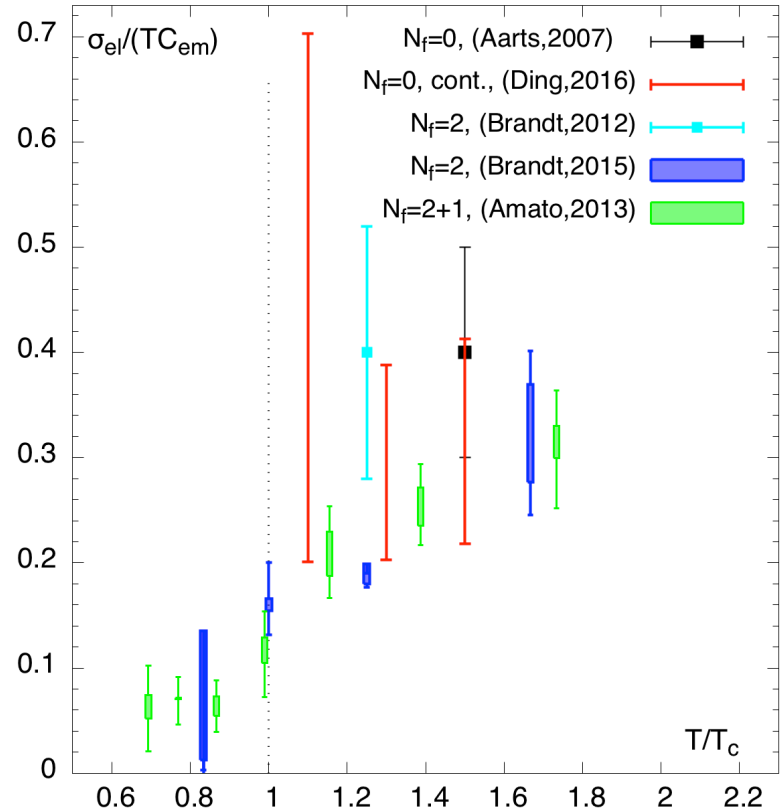
$$\frac{\sigma_{el}}{C_{em}T} = \frac{1}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$



[H-T.Ding, F.Meyer, OK, PRD94(2016)034504]

comparison of different lattice results

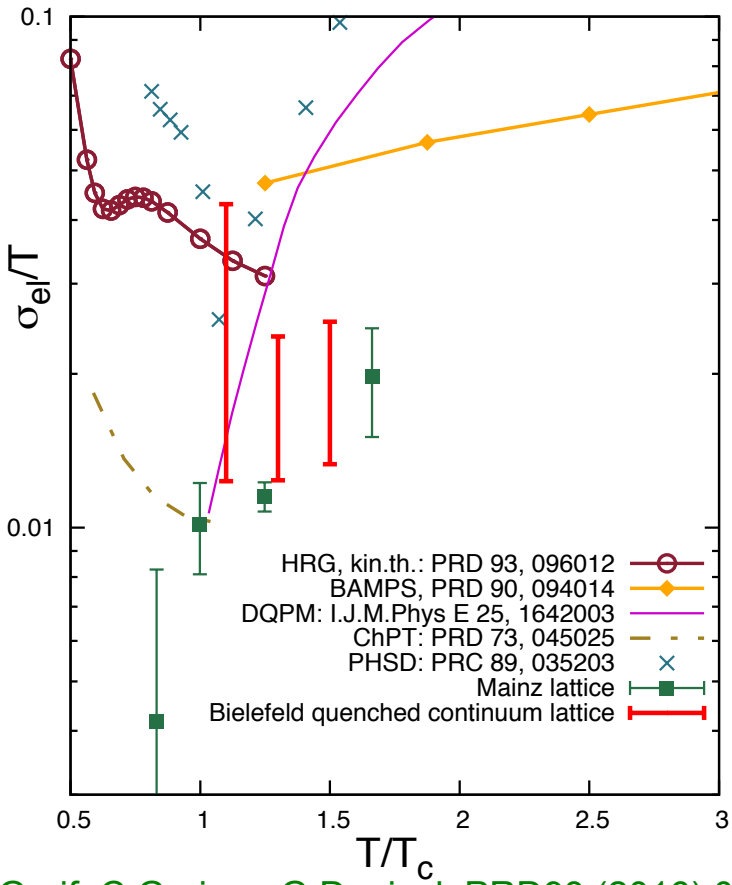
(Plot courtesy of A.Francis)



[G.Aarts et al., PRL 99 (2007) 022002,
H-T.Ding, F.Meyer, OK, PRD94(2016)034504,
B.B.Brandt et al., JHEP 1303 (2013) 100,
Brandt et al., PRD93 (2016) 054510,
A.Amato et al., PRL 111 (2013) 172001]

Electrical conductivity of the QGP

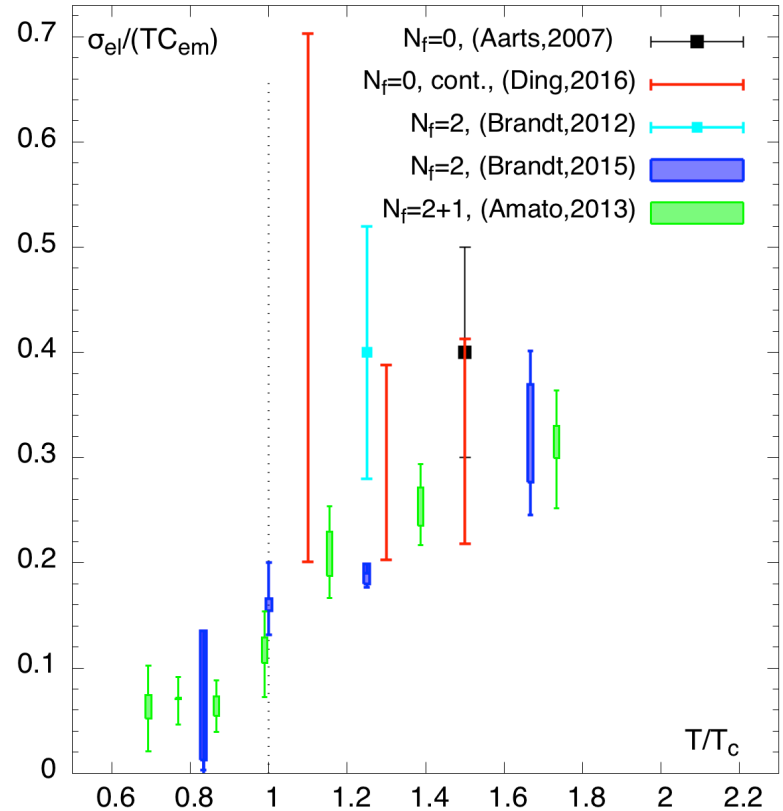
compared to calculations in partonic transport approaches



[M.Greif, C.Greiner, G.Denicol, PRD93 (2016) 096012]

comparison of different lattice results

(Plot courtesy of A.Francis)

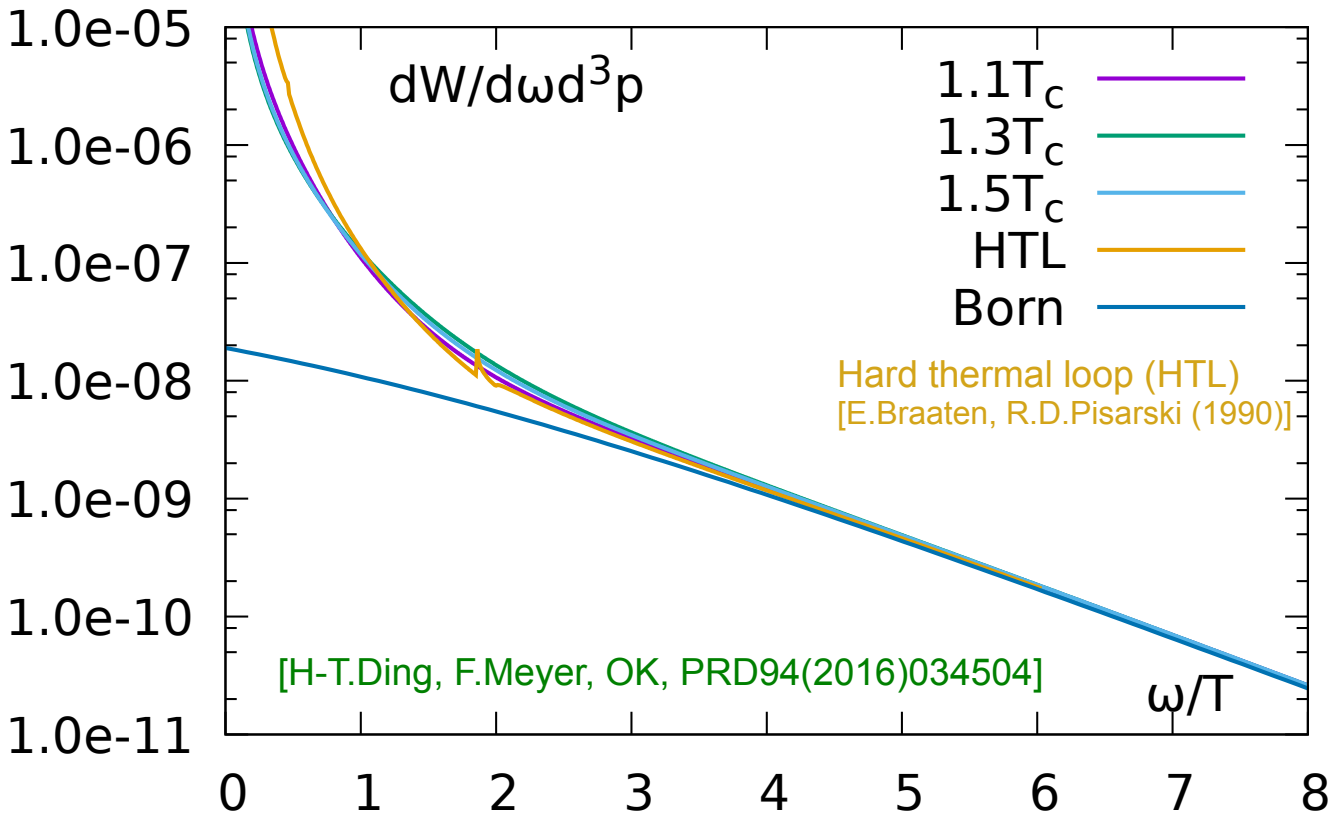


[G.Aarts et al., PRL 99 (2007) 022002,
H-T.Ding, F.Meyer, OK, PRD94(2016)034504,
B.B.Brandt et al., JHEP 1303 (2013) 100,
Brandt et al., PRD93 (2016) 054510,
A.Amato et al., PRL 111 (2013) 172001]

Progress in determining transport coefficients, although systematic uncertainties still need to be reduced in the future.

Dileptonrate directly related to vector spectral function:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\,\rho_{\mathbf{V}}(\omega,\mathbf{T})$$



Lattice constraints on thermal photon rates

Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x\mathrm{d}^3q} = \frac{5\alpha}{6\pi^2}\frac{1}{e^{\omega/T}-1}\rho_V(\omega = |\vec{k}|, T)$$

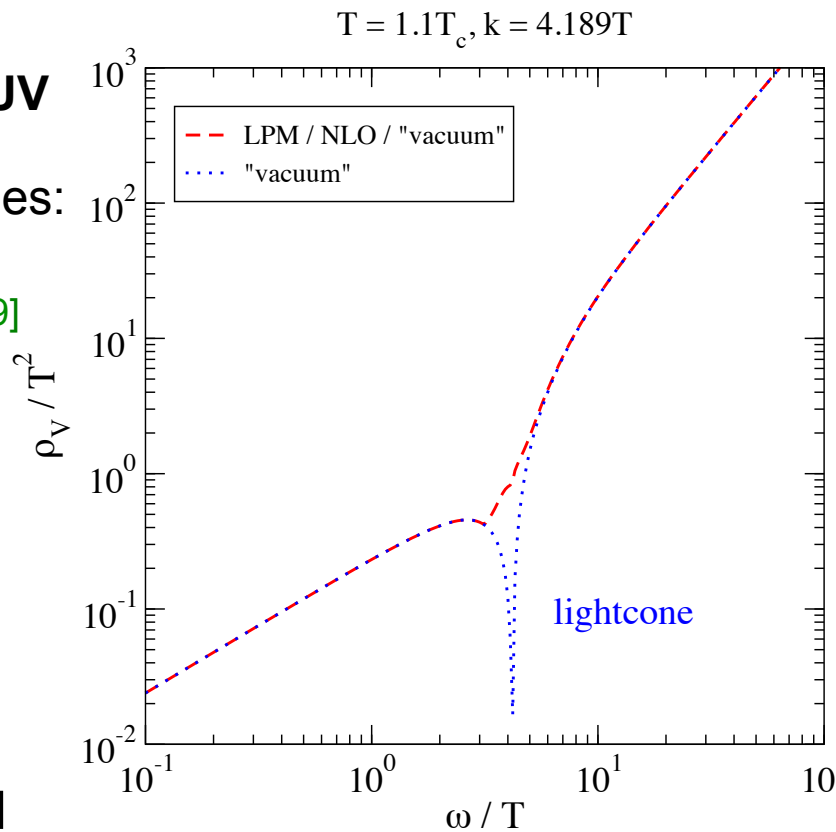
pQCD spectral function used to constrain the UV

interpolation between different (perturbative) regimes:

3T < ω < 10T: [J.Ghiglieri, G.D.Moore, JHEP 1412 (2014) 029]

ω > 10T: [I. Ghisoiu, M.Laine, JHEP 10 (2014) 84]

ω >> 10T: [M.Laine, JHEP 1311 (2013) 120]



to allow for non-perturbative effects

and to analyze how far pQCD can be trusted

we model the infrared behavior assuming smoothness at the light cone

and fit to continuum extrapolated lattice correlators

Lattice constraints on thermal photon rates

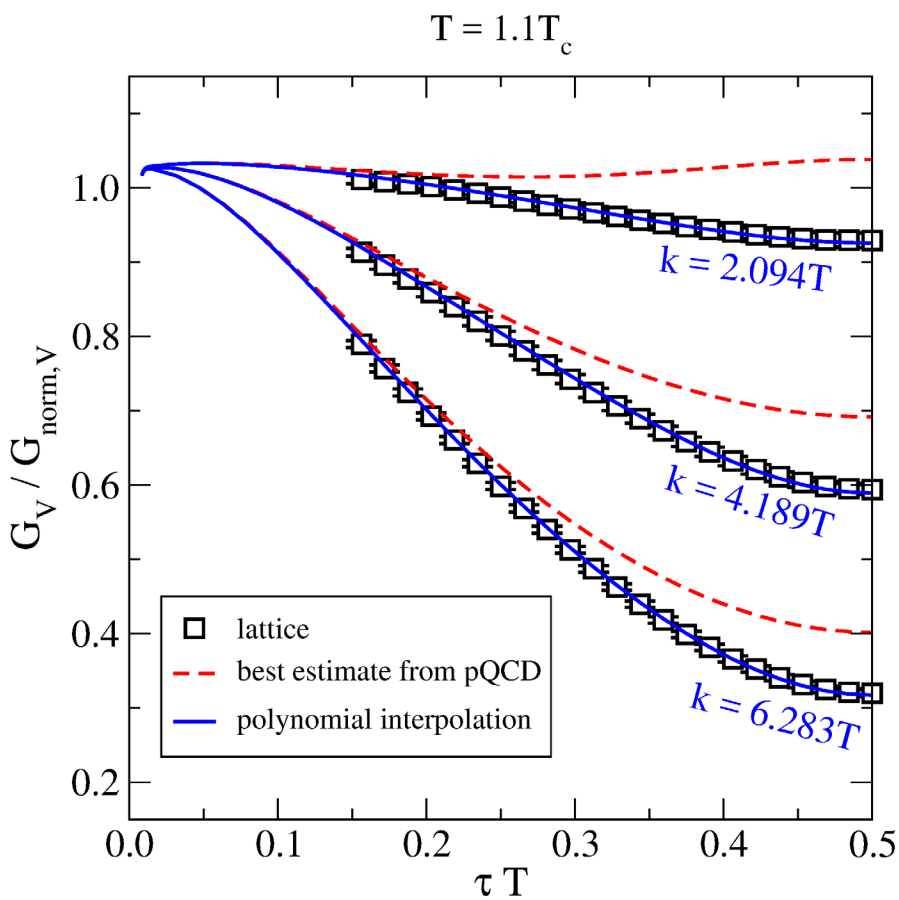
[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

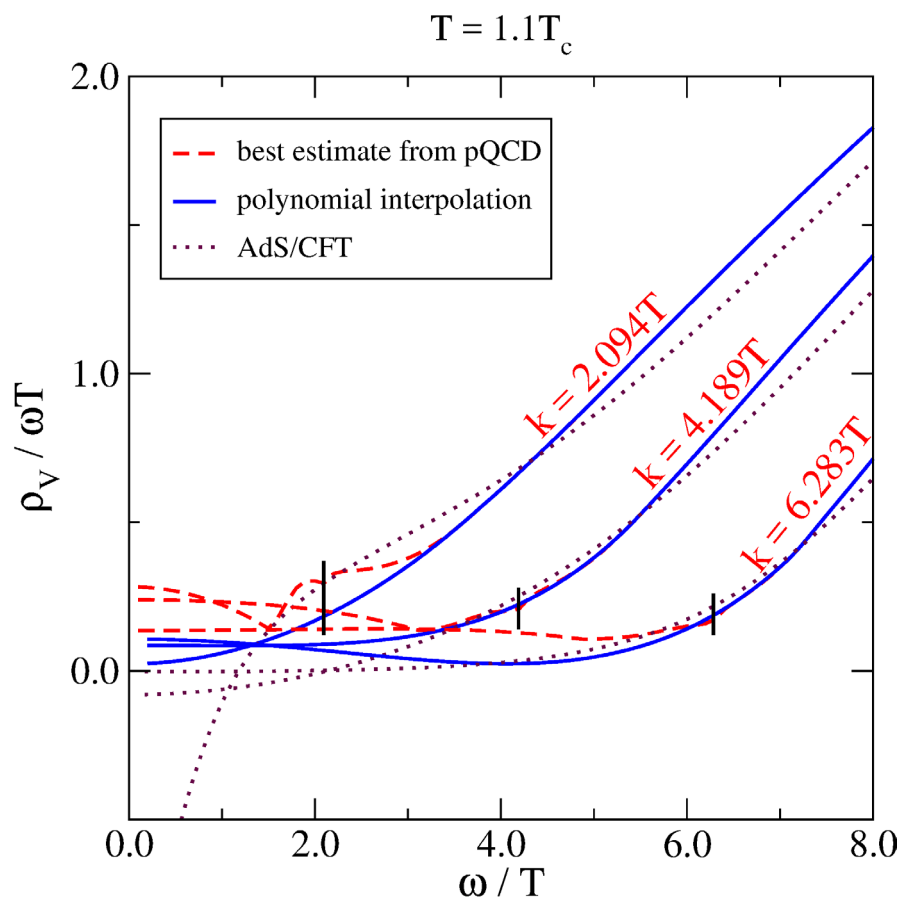
Using vector correlation functions on large and fine lattices up to $196^3 \times N_t$ with N_t=56,64

→ continuum extrapolation at finite momentum k

Using best perturbative knowledge to constrain the spectral function at large ω

→ fit a polynomial at small ω to extract the spectral function at the photon point ω = k



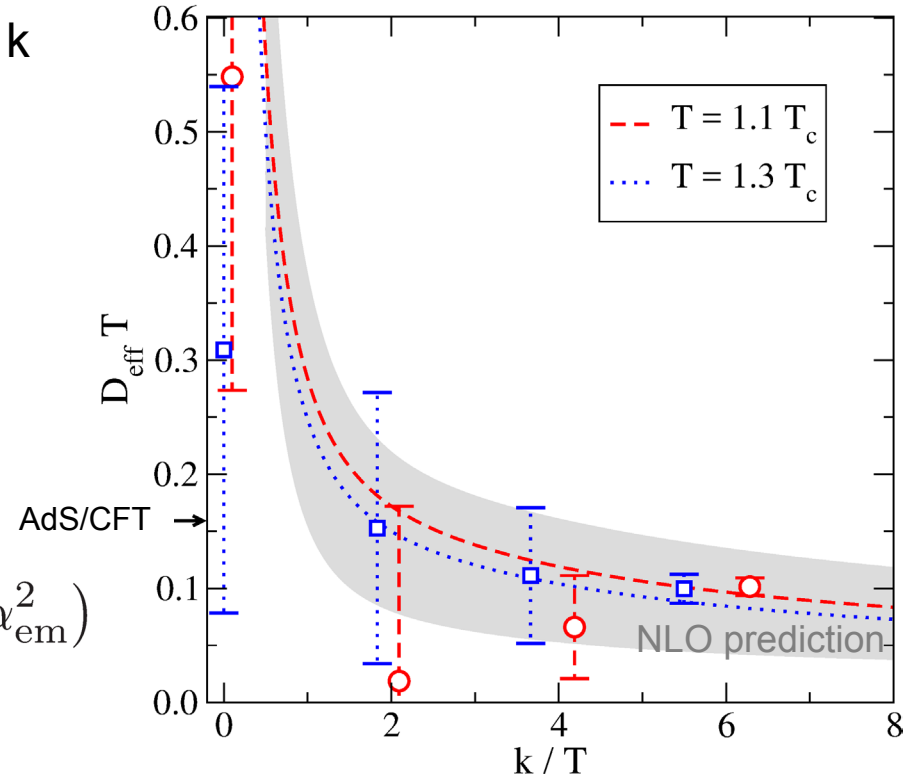


Lattice constraints on thermal photon rates

[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

The spectral function at the photon point ω = k

$$D_{\rm eff}(k) \equiv \begin{cases} \dfrac{\rho_{\rm V}(k,\mathbf{k})}{2\chi_{\rm q} k} & , \quad k>0 \\ \lim\limits_{\omega\to 0^+} \dfrac{\rho^{ii}(\omega,\mathbf{0})}{3\chi_{\rm q}\omega} & , \quad k=0 \end{cases} .$$



can be used to calculate the <u>photon rate</u>

$$\frac{{\rm d}\Gamma_\gamma(\mathbf{k})}{{\rm d}^3\mathbf{k}} = \frac{2\alpha_{\rm em}\chi_{\rm q}}{3\pi^2} n_{\rm B}(k) D_{\rm eff}(k) + \mathcal{O}\left(\alpha_{\rm em}^2\right)$$

becomes more perturbative at larger k, approaching the NLO prediction (valid for k>>gT)

[J. Ghiglieri, G.D. Moore, JHEP12 (2014) 029]

but non-perturbative for k/T < 3

<u>Electrical conductivity</u> obtained in the limit k→0 between the results from

AdS/CFT: $DT = \dfrac{1}{2\pi}$

[G.Policastro, D.T.Son,A.O.Starinets, JHEP09(2002)043]

LO perturbation theory [Arnold, Moore Yaffe, JHEP 05 (2003)]

using lattice value for χ_q/T^2 : $DT = 2.9 - 3.1$

Heavy Quark Momentum Diffusion Constant κ

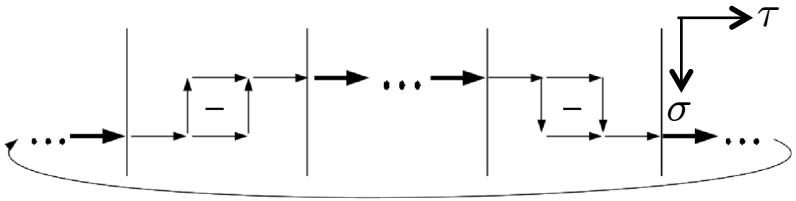
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

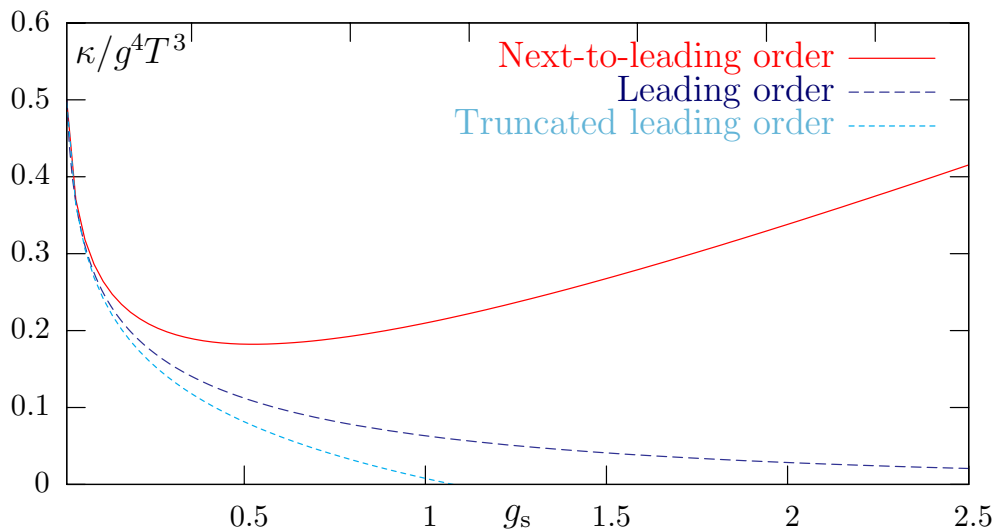
leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012,
S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]

$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\frac{1}{T};\tau)\, g\, E_i(\tau,\mathbf{0})\, U(\tau;0)\, g\, E_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\frac{1}{T};0)]\right\rangle}$$



$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$



NLO perturbative calculation:

[Caron-Huot, G. Moore, JHEP 0802 (2008) 081]

→ large correction towards strong interactions

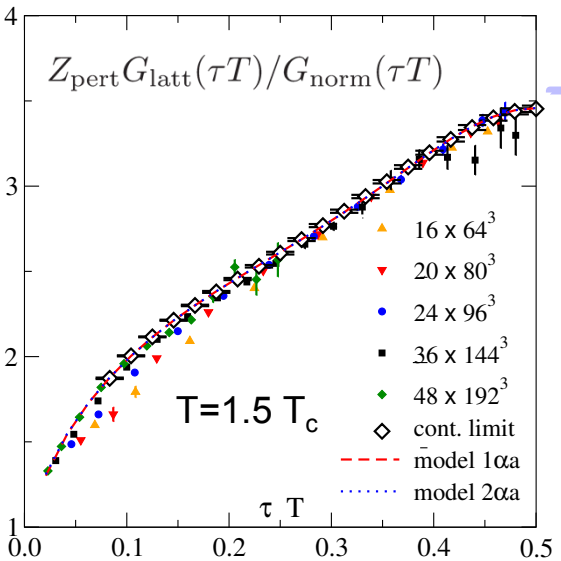
→ non-perturbative lattice methods required

Heavy-quark momentum diffusion κ

$$G_E(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi} \rho_E(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}} \qquad \kappa = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$

Combine continuum-extrapolated lattice results and perturbative input at large ω and model corrections in the non-perturbative low-ω region by power series in ω

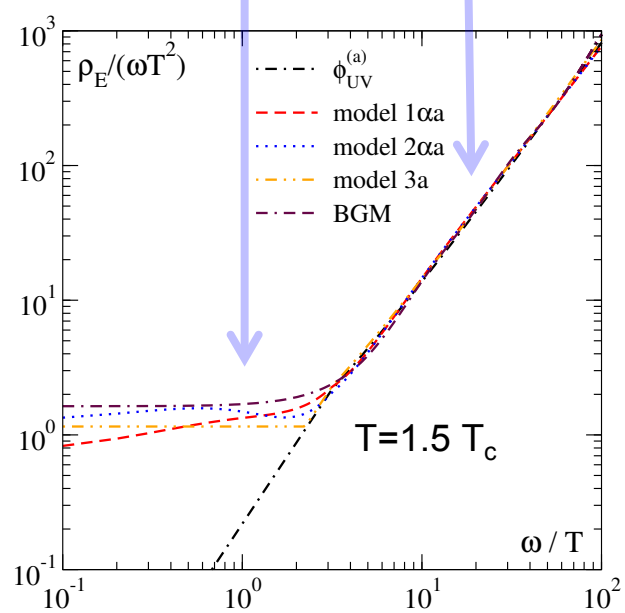
continuum extrapolated lattice correlator



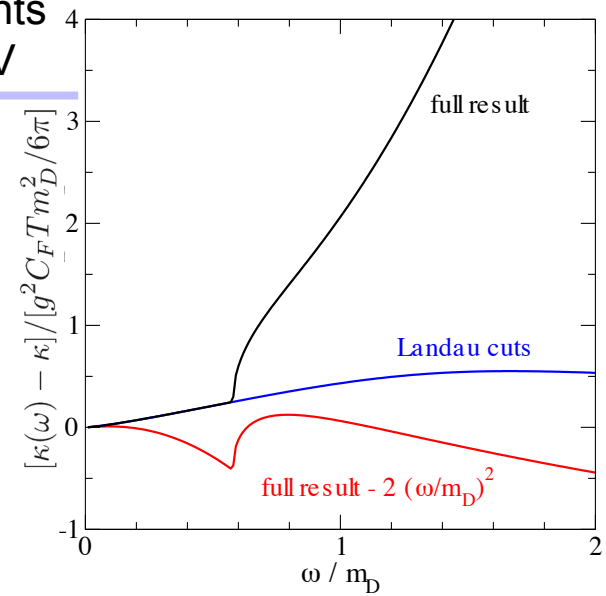
[A.Francis, OK et al., PRD92(2015)116003]

non-perturbative input from lattice QCD

perturbative constraints in the UV



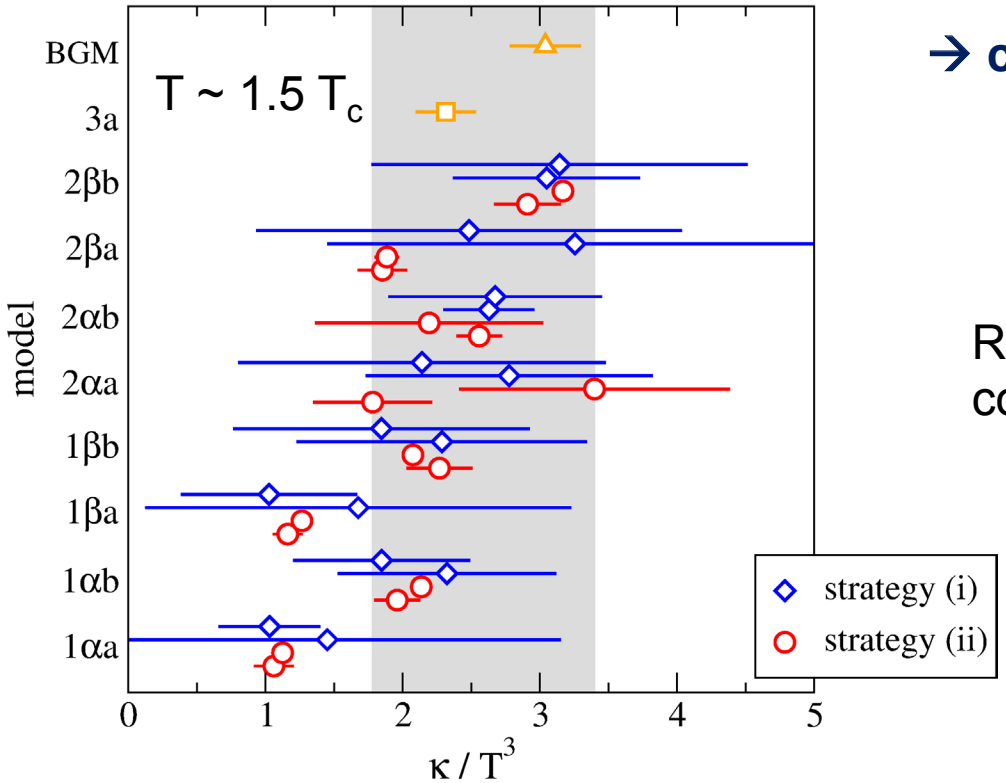
perturbative spectral function



[Caron-Huot, Laine, Moore, JHEP04 (2009) 053, Burnier, Laine et al, JHEP08 (2010) 094]

Heavy Quark Momentum Diffusion Constant – systematic uncertainties

[A.Francis, OK et al., PRD92(2015)116003]



Detailed analysis of systematic uncertainties

→ **continuum estimate of κ :**

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega} = 1.8...3.4$$

Related to diffusion coefficient D and drag coefficient η_D (in the non-relativistic limit)

$$2\pi T D = 4\pi \frac{T^3}{\kappa} = 3.7...7.0$$

$$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

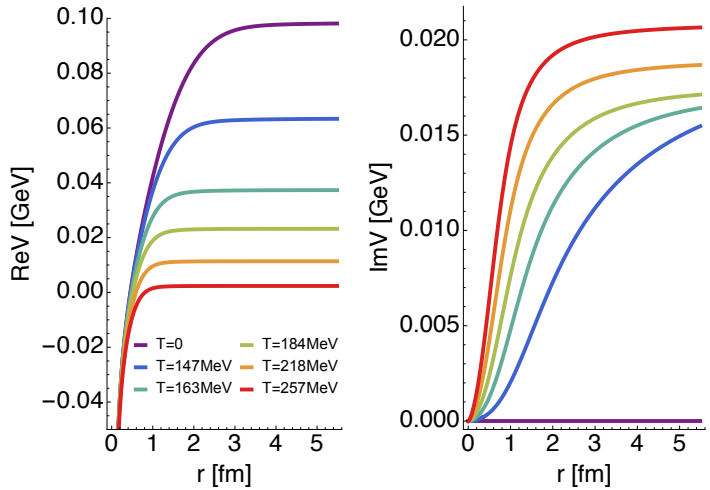
time scale associated with the kinetic equilibration of heavy quarks:

$$\tau_{\rm kin} = \frac{1}{\eta_D} = (1.8\ldots3.4)\left(\frac{T_{\rm c}}{T}\right)^2\left(\frac{M}{1.5\ {\rm GeV}}\right){\rm fm/c}$$

→ **close to T$_c$, $\tau_{kin} \simeq$ 1fm/c and therefore charm quark kinetic equilibration appears to be almost as fast as that of light partons.**

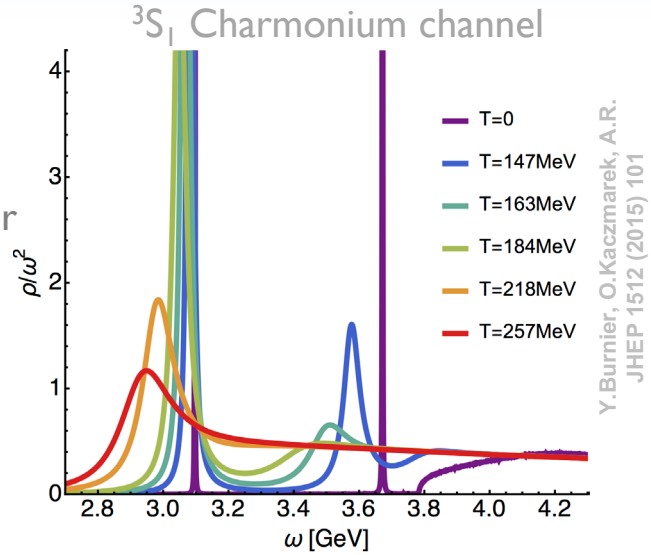
Charmonium spectral functions from pNRQCD and Lattice-NRQCD

Using complex heavy-quark potential in pNQCD: [Y. Burnier ,OK, A.Rothkopf, JHEP12(2015)101]



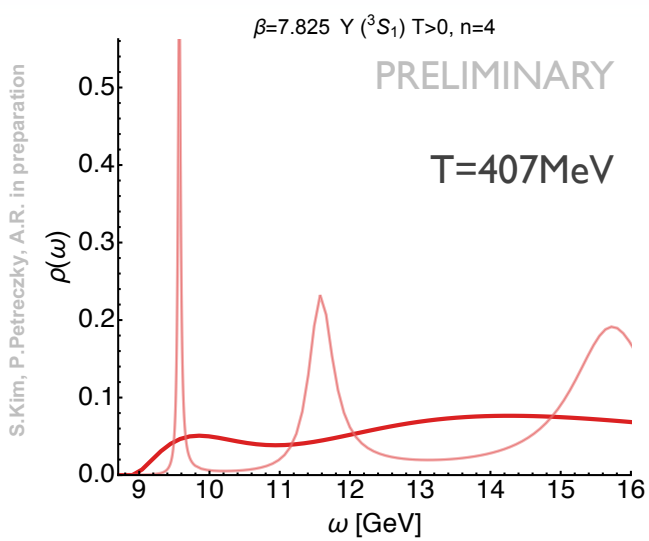
From full QCD with u,d,s quarks based on asqtad action

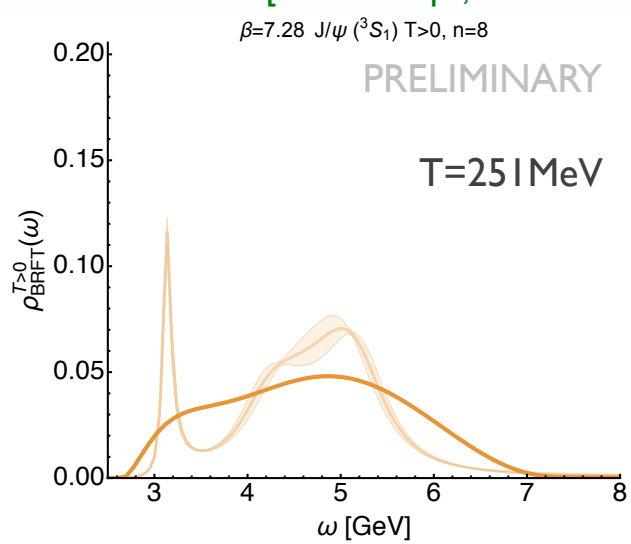
Schrödinger equation



Lattice-NRQCD using two versions of Bayesian reconstruction:

[A.Rothkopf, see Talk on Tuesday]

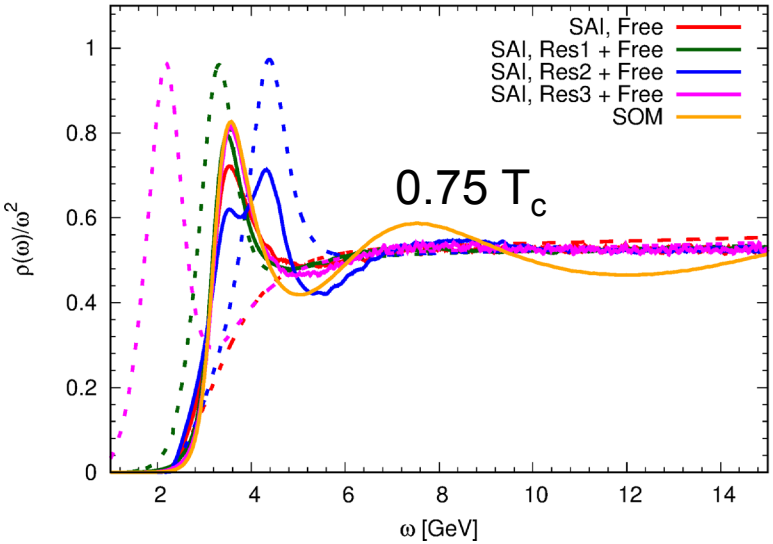


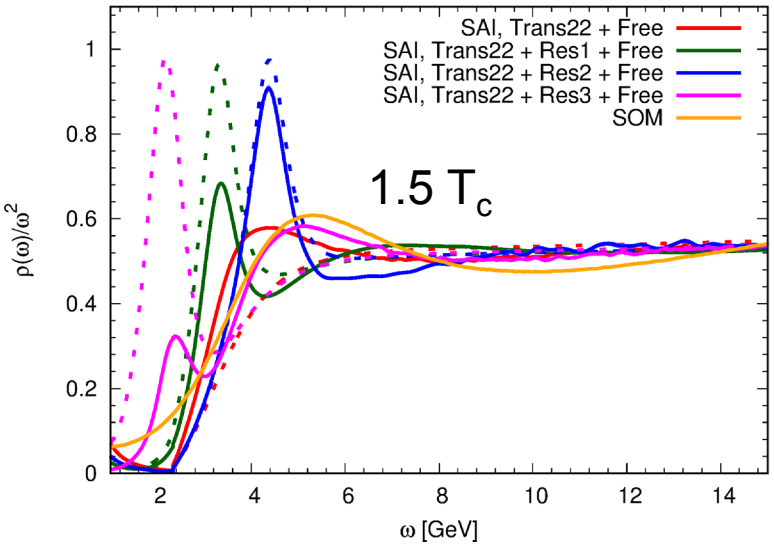


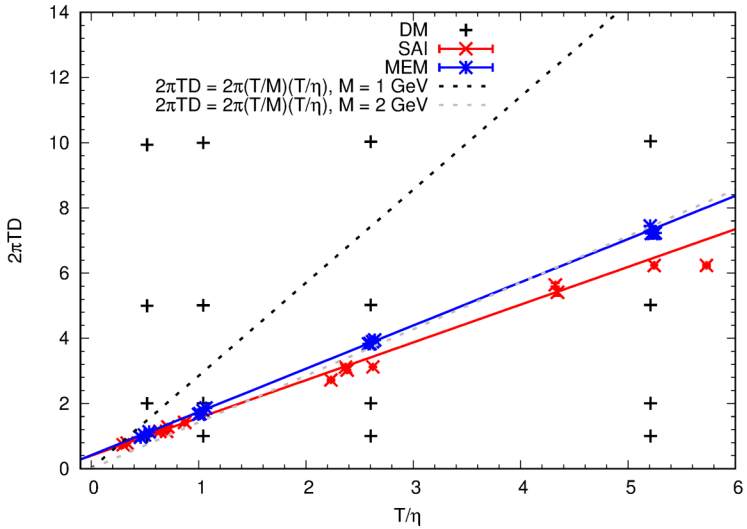
Charmonium spectral and transport properties

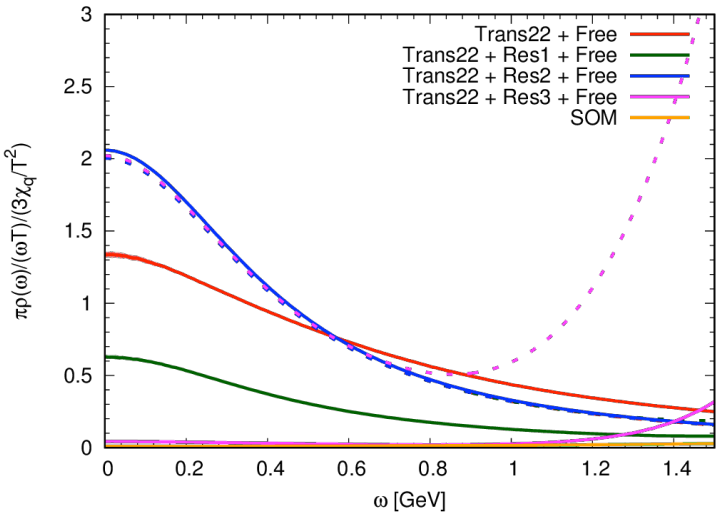
[H. Ohno, see Talk on Tuesday]

Using vector correlation functions on large and fine lattices up to $196^3 \times N_t$ with N_t=96,48 and Stochastic Analytical Interference method (SAI) based on Bayes' theorem:









From a careful analysis of systematic uncertainties: $2\pi TD = 1.6 - 7.0$

Conclusions – part II

using **continuum extrapolated correlation functions** from Lattice QCD and

using phenomenologically inspired and **perturbatively constrained Ansätze**

allows to extracted transport properties and spectral properties

we obtained continuum estimates for

→ Electrical conductivity / Diffusion coefficients

→ Thermal dilepton rates

→ Thermal photon rates

next goals: continuum extrapolation for charm and bottom correlators

→ quark mass dependence of diffusion coefficient + sequential melting of quarkonia

The methodology developed in this studies within the quenched approximation

shall be extended to full QCD calculations for a realistic QGP medium

as close to T_c dynamical fermion degrees of freedom will become important