Global and local spin polarization in heavy ion collisions

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Outline

• Introduction
• Theoretical models on particle polarization: [Spin-orbital coupling, Statistical-hydro, Kinetic]
• Experimental measurements of global polarization (recent STAR results)
• Prospect: correlation in $\Lambda$ polarization as probe to the most vortical fluid
• Summary
• Huge global orbital angular momenta are produced

\[ L \sim 10^5 \hbar \]

• Very strong magnetic fields are produced

\[ B \sim m^2_\pi \sim 10^{18} \text{ Gauss} \]

• Can and how does orbital angular momentum be transferred to the matter created?

• Any way to measure angular momentum?

Figure taken from Becattini et al, 1610.02506
Talks related to $\omega$, B and L

- Parellel talks, Feb.7:
  - Alexandru Florin Dobrin, Xu-Guang Huang, Prithwish Tribedy, Shuzhe Shi, Niklas Mueller, Liwen Wen, Isaac Upsal, Iurii Karpenko, Koichi Hattori, Long-gang Pang, Xingyu Guo
- Parellel talks, Feb.8:
  - Dmitri Kharzeev, Balthazar Peroutka
- Plenary talks, Feb.9-10:
  - Paul Sorensen, Qun Wang, Yuji Hirono

- 16 talks related to effects of vorticity, magnetic field and polarization, which can be classified into two themes
  1. Chiral Effects such as CME, CVE, CMW and other relevant topics [theory and experiments] (Sorensen’s talk)
  2. global polarization and Vortical structure of fluids in HIC (theory and experiments)
• **Barnett effect**: rotation to polarization
  uncharged object in rotation
  → spontaneous magnetization
  → polarization (spin-orbital coupling)
  [Barnett, Rev.Mod.Phys.7,129(1935)]

• **Einstein-de Haas Effect**: polarization to rotation
  magnetic field (impulse)
  → polarization of electrons
  → $\Delta L_{\text{electron}}$
  → $\Delta L_{\text{mechanical}} = -\Delta L_{\text{electron}}$
  [Einstein, de Haas, DPG Verhandlungen 17, 152(1915)]
With such correlation between rotation and polarization in materials, we expect the same phenomena in heavy ion collisions. Some early works along this line:

• Polarizations of $Λ$ hyperons and vector mesons through spin-orbital coupling in HIC from global OAM
  • -- Liang and Wang, PRL 94, 102301(2005), PRL 96, 039901(E) (2006) [nucl-th/0410079]
  • -- Liang and Wang, PLB 629, 20(2005) [nucl-th/0411101]

• Polarized secondary particles in un-polarized high energy hadron-hadron collisions
  • -- Voloshin, nucl-th/0410089

• Polarization as probe to vorticity in HIC
  • -- Betz, Gyulassy, Torrieri, PRC 76, 044901(2007) [0708.0035]

• Angular momentum conservation in HIC
  • -- Becattini, Piccinini, Rizzo, PRC 77, 024906 (2008) [0711.1253]
Spin-orbital coupling model
Global OAM in HIC

- Non-central collisions produce global orbital angular momentum

\[ L_y = -p_{in} \int x dx \left( \frac{dN_{part}^P}{dx} - \frac{dN_{part}^T}{dx} \right) \]

Liang & Wang, PRL 94, 102301(2005); PLB 629, 20(2005); Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008); Huang, Huovinen, Wang, PRC 84,054910(2011); Jiang, Lin, Liao, PRC 94,044910(2016); Deng,Huang, PRC 93,064907(2016); many others ……

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Global OAM in HIC

Number of participant nucleons per unit $x$ in projectile or target

Collective longitudinal momentum per produced parton

$$p_z(x, b) = \frac{\sqrt{s}}{2c(s)} \left( \frac{dN^P_{\text{part}}}{dx} - \frac{dN^T_{\text{part}}}{dx} \right) + \left( \frac{dN^P_{\text{part}}}{dx} \right)$$

Liang & Wang (2005); Gao, et al. (2008); Betz, Gyulassy, Torrieri (2007); Becattini, Piccinini, Rizzo (2008); Jiang, Lin, Liao (2016); Deng, Huang (2016); many others ……

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Quark scatterings in potential

- Quark scatterings at small angle in static potential with screening mass
- Unpolarized and polarized cross sections

\[ \frac{d\sigma}{d^2 \vec{x}_T} = \frac{d\sigma_+}{d^2 \vec{x}_T} + \frac{d\sigma_-}{d^2 \vec{x}_T} = 4C_T \alpha_s^2 K_0(\mu x_T) \]

\[ \frac{d\Delta\sigma}{d^2 \vec{x}_T} = \frac{d\sigma_+}{d^2 \vec{x}_T} - \frac{d\sigma_-}{d^2 \vec{x}_T} \propto \vec{n} \cdot (\vec{x}_T \times \vec{p}) \]

- Polarization vector
- OAM
- Spin-Orbital coupling

- Polarization for small angle scattering and \( m_q \gg p, \mu \)

\[ P_q \approx -\pi \frac{\mu p}{4m_q^2} \sim -\frac{\Delta E_{LS}}{E_0} \]

Quark-quark scattering

- Beyond small angle approximation with HTL gluon propagator

Quark polarization as functions of the square root of parton-parton scattering energy over $T \approx \text{local OAM or vorticity}$ which increases with $\alpha_s$

Statistical-hydro model
Rotation effect in non-inertial frame

- A particle of mass $m$ moves in a non-inertial rotating frame in potential $U(r)$

\[
L = \frac{1}{2}m(v_r + \omega \times r)^2 - U(r)
\]

\[
p = \frac{\partial L}{\partial v_r} = m(v_r + \omega \times r)
\]

\[
H = p \cdot v_r - L = H_0 - \omega \cdot J
\]

\[
\hat{\rho}_{GE} = \frac{1}{Z} \exp \left(-\beta \hat{H}_0 + \beta \omega \cdot \hat{J} + \beta \mu \hat{Q}\right)
\]

Assume no relative velocity between inertial and non-inertial rotating frame.

Canonical momentum

Angular momentum

Hamiltonian in inertial frame

Global equilibrium density operator

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Covariant form of quantum statistical physics (local equilibrium)

- To obtain covariant form in local equilibrium, we use principle of maximal entropy with conservation of total energy-momentum and particle number,

\[
\hat{\rho}_{LE} = \frac{1}{Z} \exp \left[ \int_{\Sigma} d\Sigma_{\mu} \left( -\hat{T}^{\mu\nu} \beta_{\nu} + \zeta \hat{j}^{\mu} \right) \right]
\]

- Given \(n^{\mu}\), one can determine \(\beta^{\mu}\) and \(\zeta\) by

\[
n_{\mu} \langle \hat{T}^{\mu\nu}(x) \rangle_{LE} (\beta^{\alpha}, \zeta) = n_{\mu} T^{\mu\nu}(x), \quad n_{\mu} \langle \hat{j}^{\mu}(x) \rangle_{LE} (\beta^{\alpha}, \zeta) = n_{\mu} j^{\mu}(x)
\]

Energy condition

Particle number condition

- where statistical average is defined by

\[
\langle \hat{O}(x) \rangle_{TE} = \text{Tr} \left[ \hat{\rho}_{TE} \hat{O}(x) \right]
\]
Stationary conditions

\[ \partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0, \quad \partial_\mu \zeta = 0 \]

Killing equation

\[ \beta^\mu = b^\mu + \omega^{\mu\nu} x_\nu \]

Killing vector solution

\[ b^\mu = \frac{1}{T} u^\mu \]

Density operator at global equilibrium

\[ \hat{\rho}_{\text{GE}} = \frac{1}{Z} \exp \left[ -\beta u_\nu \hat{P}^\nu + \frac{1}{2} \hat{J}_{\nu\rho} \omega_{\nu\rho} + \zeta \hat{Q} \right] \]

Total particle number

4-momentum vector operator

Total angular momentum tensor (OAM+spin)

Becattini (2012); Becattini, Bucciantini, Grossi, Tinti (2015; 2015)
Spin and polarization

- Spin (Pauli-Lubanski) pseudo-vector

\[
\hat{S}^\mu = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma} J_{\nu\rho}^S \hat{P}^\sigma \\
S^\mu = \text{Tr}(\hat{\rho}_{GE} \hat{S}^\mu) \\
\Pi^\mu = \frac{1}{S}S^\mu
\]

\[ [\hat{S}^\mu, \hat{P}^\nu] = 0, \quad \hat{S}^\mu \hat{P}_\mu = 0 \\
\hat{S}^\mu \hat{S}_\mu = -S(S+1) \]

properties of spin vector

phase space spin density for spin $\frac{1}{2}$-fermions

\[
S^\mu(x, p) = -\frac{1}{8m}[1 - n_F(x, p)]\epsilon^{\mu\rho\sigma\tau} p_\tau \omega_{\rho\sigma}
\]

particle number at freezeout

\[
N = \int \frac{d^3p}{E_p} \int d\Sigma p^\lambda n_F(x, p)
\]

spin at freezeout hypersurface

\[
S^\mu = \frac{1}{N} \int \frac{d^3p}{E_p} \int d\Sigma p^\lambda n_F(x, p) S^\mu(x, p)
\]

Becattini, et al., 1610.02506; Karpenko, Becattini, 1610.04717

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To describe polarization for massive spin $\frac{1}{2}$ fermions, we have to explicitly know their momentum $p$, therefore we need to know information in phase space $(t,x,p)$, that’s why we use kinetic approach.

- Classical kinetic approach: $f(t,x,p)$
- Quantum kinetic approach: $W(t,x,p)$
The Wigner function for spin 1/2 fermions in constant EM field satisfies EOM, which can be solved perturbatively in \((F_{\mu\nu})^i\) and \((\partial_x)^i\).

Wigner function can be decomposed in 16 generators of Clifford algebra

\[
W = \frac{1}{4} \left[ \mathcal{F} + i\gamma^5 \mathcal{P} + \gamma^\mu \mathcal{V}_\mu + \gamma^5 \gamma^\mu \mathcal{A}_\mu + \frac{1}{2} \sigma^{\mu\nu} \mathcal{I}_{\mu\nu} \right]
\]

\(4x4\) matrix  scalar  p-scalar  vector  axial-vector  tensor

\[j^\mu = \int d^4p \mathcal{V}^\mu, \quad j_5^\mu = \int d^4p \mathcal{A}_\mu, \quad T^{\mu\nu} = \int d^4p p^{\mu} \mathcal{V}^\nu\]

Heinz, Phys.Rev.Lett. 51, 351 (1983);
Vasak, Gyulassy and Elze, Annals Phys. 173, 462 (1987);
Spin tensor component

- Spin tensor component of Wigner function
  \[ M^{\alpha\beta}(x, p) \equiv \frac{1}{2} \text{Tr} \left[ \gamma_0 \sigma^{\alpha\beta} W(x, p) \right] \]
  \[ = \frac{1}{2} \left[ -\epsilon^{0\alpha\beta\rho} A_\rho + i g^{\alpha0} \gamma^\beta - i g^{\beta0} \gamma^\alpha \right] \]

- For \( \alpha\beta=ij \) (space indices)
  \[ M^{ij}(x, p) = \frac{1}{2} \epsilon^{ijk} A^k(x, p) \]

- We can regard **axial vector** as **spin vector** (up to \( \frac{1}{2} \))
  \[ \Pi^\mu(x) \sim \frac{1}{2} \int d^4p A^\mu(x, p) \]
  \[ \sim \frac{1}{2} \int d^4p \frac{|p_0|}{m} A^\mu(x, p) \]

Fang, Pang, QW, Wang, PRC 94,024904(2016); QW, et al, work in progress

Pauli matrices

Non-relativistic limit

To match Pauli-Lubanski pseudo-vector

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Axial vector component of Wigner function for massive fermions

- Axial vector component: zero \((i=0)\) and first \((i=1)\) order in \((F_{\mu\nu})^i\) and \((\partial_x)^i\):

  \[ A^\mu = \text{Tr}[\gamma^\mu \gamma^5 W] \]

  \[ A_{(0)}(x,p) = m [\theta(p_0)n^\mu(p,n) - \theta(-p_0)n^\mu(-p,-n)] \delta(p^2 - m^2) \]

  \[ A^{\alpha}(x,p) = -\frac{1}{2} \beta \Omega^{\alpha\sigma} p_\sigma \frac{dV}{d(p_0)} \delta(p^2 - m^2) - Q \hbar \tilde{F}\alpha\lambda p_\lambda V \delta(p^2 - m^2) \]

- Spin (pseudo-)vector in Lab frame

  \[ n^\mu(p,n) = \Lambda_\nu ^\mu (-v_p)n^\nu(0,n) = \left( \frac{n \cdot p}{m}, n + \left( \frac{n \cdot p}{m(m + E_p)} \right) \right) \]

  \[ \tilde{F}\alpha\lambda = \frac{1}{2} \epsilon^{\alpha\lambda\rho\sigma} F_{\rho\sigma} \]

  \[ \tilde{\Omega}\alpha\lambda = \frac{1}{2} \epsilon^{\alpha\lambda\rho\sigma} \Omega_{\rho\sigma} \]

  \[ \Omega_{\rho\sigma} = \frac{1}{2} (\partial_\rho u_\sigma - \partial_\sigma u_\rho) \]

where \(A\) and \(V\) are related to distribution functions

Spin in Lab frame

Spin in cms frame

Lorentz boost from cms to Lab frame

Fang, Pang, QW, Wang, PRC 94,024904(2016);
Fang, Pang, QW, Wang, PRD 95, 014032(2017)
Polarization at zeroth order is vanishing if we assume that the chemical potential for spin-up and spin-down fermions are equal.

Polarization vector at the first order

\[
\Pi_\alpha^{(1)} \approx \frac{1}{2m} \hbar \beta \int \frac{d^3 p}{(2\pi)^3} \left\{ [E_p \omega^\alpha + Q B^\alpha] \frac{e^{\beta (E_p - \mu)}}{[e^{\beta (E_p - \mu)} + 1]^2} + [E_p \omega^\alpha - Q B^\alpha] \frac{e^{\beta (E_p + \mu)}}{[e^{\beta (E_p + \mu)} + 1]^2} \right\}
\]

Polarization at freezeout

\[
E_p \frac{d\Pi_\alpha^\alpha(p)}{d^3 p} \approx \frac{\hbar}{2m} \beta \int d\Sigma \chi p^\lambda \left( \tilde{\Omega}^{\alpha \sigma} p_\sigma \pm Q \tilde{F}^{\alpha \sigma} u_\sigma \right) f_{FD}^\pm(x, p) \left[ 1 - f_{FD}^\pm(x, p) \right]
\]

Fang, Pang, QW, Wang, PRC(2016);
Aristova, Frenklakh, Gorsky, Kharzeev, JHEP(2016);
QW, et al, work in progress

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Experimental measurements of global polarization by Λ’s weak decay

- Related talks:
  - Isaac Upsal’s talk on Feb.7, Global polarization of Lambda hyperons in Au+Au Collisions at RHIC BES
  - Iurii Karpenko’s talk on Feb.7, Vorticity in the QGP liquid and Lambda polarization at the RHIC Beam Energy Scan
Polarization of Λ hyperon

• Λ is ‘self-analyzing’ in weak decay $\Lambda \rightarrow p + \pi^-$ which breaks parity (proton emission preferentially along Λ spin in Λ’s rest frame)

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{4\pi} \left( 1 + \alpha_H \bar{\Pi}_\Lambda \cdot \hat{p}_p^* \right)$$

$$= \frac{1}{4\pi} \left( 1 + \alpha_H \Pi_\Lambda \cos \Theta^* \right)$$

Decay constant Polarization vector

• Λ polarization can be determined by event average of proton momentum direction in Λ’s rest frame

$$\Pi_\Lambda = \Pi_\Lambda \hat{S}_\Lambda^*, \quad (\Pi_\Lambda \in [0, 1])$$

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Measurement of $\Lambda$ polarization

- If $\vec{\Pi}_\Lambda \parallel \vec{L}$, then $\Theta^* = \theta^* = \hat{p}_p^* \cdot \hat{L}$, $\Lambda$ polarization can be measured in a simpler way by

$$\frac{dN}{d\phi_p^*} = \int_0^\pi d\theta_p^* \sin \theta_p^* \frac{dN}{d\Omega^*} (\cos \theta^*)$$

$$= \frac{1}{2\pi} + \frac{1}{8} \alpha_H \Pi_\Lambda \sin(\psi_{RP} - \phi_p^*)$$

$$\cos \theta^* = \sin \theta_p^* \sin(\psi_{RP} - \phi_p^*)$$

$\Pi_\Lambda = \frac{8}{\pi \alpha_H} \langle \sin(\psi_{RP} - \phi_p^*) \rangle_{ev}$

STAR, PRC 76,024915 (2007) (Erratum for wrong sign)

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Corrections for event plane

Reaction plane can be estimated by event plane → needs corrections by reaction plane resolution

Azimuthal angle of event plane determined by direct flow

Decay constant

$\alpha_\Lambda = -\alpha_{\bar{\Lambda}} = 0.642 \pm 0.013$

Reactions plane resolution from direct flow

$R_{EP}^{(1)} = \langle \cos(\psi_{RP} - \psi_{EP}) \rangle$

STAR, PRC 76,024915 (2007); 1701.06657

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STAR data for Λ polarization

- At each energy, a positive polarization for Λ and $\bar{\Lambda}$ at 1.1-3.6σ level. The polarizations decrease with energies. On average over all data,

$$\mathcal{P}_\Lambda = (1.08 \pm 0.15)\%$$

$$\mathcal{P}_{\bar{\Lambda}} = (1.38 \pm 0.30)\%$$

- Systematic uncertainties are smaller than statistical ones and are mainly from estimated combinatoric background of proton-pion pairs.
- Other small systematic uncertainties in the overall scale: a) Λ decay parameter $\alpha_H$ (2%); b) the reaction-plane resolution (2%); c) detector efficiency corrections (3.5%)
- The data contain both primary and those feed-down contributions from heavier particles. The effect of feed-down is about 20% difference between the polarization of primary and all hyperons.

STAR collab., 1701.06657
Global polarization $\Lambda$ hyperons

- STAR data at low energies
  $P \approx 1$-$8\%$ from 7.7 to 62.4 GeV
  $\Delta P (\overline{\Lambda} - \Lambda) \approx 0.03\% - 0.2\%$
- $P_\Lambda$ is anti-parallel to $B$ due to negative magnetic moment
- Magnetic field that leads to
  \[
  \Delta P \approx \frac{1}{2} \beta \frac{B^\alpha}{m_\Lambda} \sum_{e=\pm} \frac{\int d^3p f^e_{FD} (1 - f^e_{FD})}{\int d^3p f^e_{FD}} \approx O(1)
  \]
  \[
  \approx \beta \frac{B^\alpha}{m_\Lambda} \Rightarrow B \sim T m_\Lambda \Delta P \sim (0.1 \sim 0.01) m_\pi^2
  \]
  too large for low energy HIC in freezeout scenario.

- From vorticity, there is more Pauli blocking effect for fermions than anti-fermions in lower energy HIC

Coutesy of Isaac Upsal for STAR Collab.

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Largest vorticity ever observed

- The fluid vorticity may be estimated from the data using the hydrodynamic relation with a systematic uncertainty of a factor of 2, mostly due to uncertainties in the temperature

\[ \omega \sim \frac{k_B T (\mathcal{P}_\Lambda + \mathcal{P}_{\Lambda})}{\hbar} \approx (9 \pm 1) \times 10^{21} \text{ s}^{-1} \]

- This far surpasses the vorticity of all other known fluids

<table>
<thead>
<tr>
<th>Solar subsurface flow</th>
<th>(10^{-7} \text{ s}^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large scale terrestrial atmospheric patterns</td>
<td>(10^{-7} - 10^{-5} \text{ s}^{-1})</td>
</tr>
<tr>
<td>Great Red Spot of Jupiter</td>
<td>(10^{-4} \text{ s}^{-1})</td>
</tr>
<tr>
<td>Supercell tornado cores</td>
<td>(10^{-1} \text{ s}^{-1})</td>
</tr>
<tr>
<td>Rotating, heated soap bubbles</td>
<td>(100 \text{ s}^{-1})</td>
</tr>
<tr>
<td>Turbulent flow in bulk superfluid He-II</td>
<td>(150 \text{ s}^{-1})</td>
</tr>
<tr>
<td>Superfluid nanodroplets</td>
<td>(10^7 \text{ s}^{-1})</td>
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STAR Collab., 1701.06657; Becattini et al., 1610.02506; Pang et al., PRC 94, 024904(2016); Aristova, Frenklakh, Gorsky, Kharzeev, JHEP(2016);
Prospect: Turbulence and vortices in high energy HIC

Spin-spin correlation of $\Lambda$ can probe the vortical structure of sQGP

Pang, Petersen, QW, Wang, PRL 117, 192301 (2016)

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• (a) The offset of transverse spin correlation indicates that global polarization are stronger at lower beam energies and peripheral collisions.

• (b) $\cos(\Delta \phi)$ azimuthal distribution in transverse spin correlation is due to circular structure of $\omega$ along beam direction.

• (c) Longitudinal spin correlation (pair structure) is due to transverse energetic particles. The beam energy dependence for longitudinal spin is weak.

Pang, Petersen, QW, Wang, PRL 117, 192301 (2016)
• Λ polarization provides a measurement of global angular momentum in HIC

• STAR data in Beam Energy Scan program show a clear non-vanishing global polarization for Λ

• There are a few theoretical models for hadron polarization: microscopic spin-orbital coupling model, statistical-hydro models, kinetic approach etc.

• “Discovery of global Λ polarization opens new directions in the study of the hottest, least viscous – and now, most vortical – fluid ever produced in the laboratory.” --- from STAR Collab., 1701.06657

It is just the beginning, stay tuned!