Gravitational waves and the nuclear equation of state

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Quark Matter 2017 @ Hyatt Regency, Feb. 5-11, 2017
Preface: GW astronomy era comes!

- **GW150914**: The first direct detection of GWs from BH-BH has opened the era of **GW astronomy**

Abbott et al. (2017)
Preface: GW astronomy era comes!

- **GW150914**: The first direct detection of GWs from BH-BH have opened the era of **GW astronomy**
- The observed GWs (BH-BH: GW150914 + GW151226) are consistent with GR
- GR seems to be the (sufficiently) correct theory of gravity at this time

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**Abbott et al. (2017)**

- **Inspiral** (theory + NR)
- **Before merger** (NR modelling)
- **Merger ringdown** (NR modelling)
EoS of NS matter is the only model

- **EoS of NS will be the only ‘model’ for GWs from NS/BH-NS merger**
  - We could constrain the physics of NS matter by GWs (topic of this talk)

 Hybrid star

**Hyperon star**

Neutron star

**Pion cond.**

Quark star

**Kaon cond.**

Weber (2005)

Yagi & Yunes (2016)
Event rate of NS-NS merger

- Event rate is quite uncertain
  - Galactic binary pulsar
  - SGRB = NS-NS merger
  - Origin r-process = NS-NS
  - Kilonova = NS-NS merger
  - Population synthesis

- Merger rate: \( \sim 10^{-1000}/\text{Gpc}^3/\text{yr} \)
- Detection rate @ design sensitivity: \(0.1 - 10/\text{yr}\)
  - \(D = 200 \text{ Mpc}\)
  - Duty cycle 60%

- **aLIGO observation schedule**
  - O1: 2015-2016 (done)
  - O2: 2016-2017+ (80-120 Mpc)
  - O3: 2017-2018+ (120-170 Mpc)
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GWs from NS-NS may be detected in next few year and might provide us unique information on NS EoS:

- M and R information of NS
- Maximum mass constraints
- Composition of NS interiors

We should prepare for it

Abbott et al. (2016)
Gravitational waves from NS-NS merger

1.2-1.4 M\textsubscript{sun} NS-NS APR EoS density contour in orbital plane

Gravitational Waveform
Gravitational waves from NS-NS merger

- Inspiral Chirp signal
- Tidal deformation
- NS oscillation (BH formation)

- Point particle approximation
- Information of orbits, NS mass

- Finite size effects appear
- tidal deformability
  ⇒ radius of cold NS

- BH or NS ⇒ Maximum mass
- quasi-periodic GW frequency
  ⇒ radius of hot, massive NS

1.2-1.4 M_{\odot} NS-NS APR EoS
density contour in orbital plane

Gravitational Waveform

Animation: Hotokezaka
GWs from NS-NS merger would provide us

- Simultaneous determination of mass and radius of NS
  - **mass from inspiral chirp signal**
  - **radius of cold NS with canonical mass** by deviation of GWs from chirp signal due to tidal deformation
    - Canonical mass: 1.3-1.4 Msun
    - $T \sim 0$ MeV, $\rho_0 \sim (3-4)\rho_S$, $Ye = 0.1$
  - **radius of hot merger remnant NS** by quasi-periodic GWs frequency
    - $T \sim 30-60$ MeV (larger for softer EoS)
    - $\rho_0 \sim 10^{15}$ g/cm$^3$ (larger for softer EoS)
    - Ye $\sim 0.1$
GWs from NS-NS and QCD phase diagram
Gravitational waves from NS-NS merger

- Point particle approx.
- Information of orbits, NS mass
- Finite size effects appear
- tidal deformability
- radius

**BH or NS** $\Rightarrow$ **maximum mass**
**GWs from massive NS**
$\Rightarrow$ **NS radius of massive NS**
Tidal deformability of NS

- **Tidal deformability:** $\lambda$

- Response of quadrupole moment $(Q_{ij})$ to external tidal field $(E_{ij})$

  $$Q_{ij} = -\lambda E_{ij}$$

- stiffer EOS $\Rightarrow$ less compact NS $\Rightarrow$ larger $\lambda$

- GWs: described by the quadrupole formula
  - Orbit and GWs deviate from those in the point particle approximation due to tidal effects

- Information of radius can be extracted

  $$\lambda = \frac{C^5}{G} \Lambda R^5$$  
  
  $$(\Lambda : \text{nondim.})$$

  $$C = \frac{GM}{c^2 R}$$

Lackey & Wade (2015)
- **Soft Eos:**
  - APR EoS ($R_{1.35} = 11.1\text{km}$)
  - Tidal effects are less important
  - Small deviation of orbit and GWs
- **Stiff Eos:**
  - MS1 EoS ($R_{1.35} = 14.5\text{km}$)
  - Tidal effects are more important
  - Remarkable deviation of orbit and GWs

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Animation: Hotokezaka
Tidal effect in gravitational wave spectra

\[
\Delta h_{12} \sim \int df \frac{|h_{\text{EOS1}} - h_{\text{EOS2}}|^2}{S_n(f)}
\]

- distinguishability
  - \( \Delta h_{12} = 1 \) corresponds to 1σ
  - \( D = 200 \text{Mpc} \) (horizon distance of aLIGO)

<table>
<thead>
<tr>
<th></th>
<th>APR</th>
<th>SFHo</th>
<th>DD2</th>
<th>TMA</th>
<th>TM1</th>
</tr>
</thead>
<tbody>
<tr>
<td>APR</td>
<td>( \Delta R )</td>
<td>0.7</td>
<td>2.3</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>SFHo</td>
<td>0.8 km</td>
<td>—</td>
<td>1.9</td>
<td>2.7</td>
<td>3.3</td>
</tr>
<tr>
<td>DD2</td>
<td>2.1 km</td>
<td>1.3 km</td>
<td>—</td>
<td>1.3</td>
<td>2.5</td>
</tr>
<tr>
<td>TMA</td>
<td>2.8 km</td>
<td>2.0 km</td>
<td>0.7 km</td>
<td>—</td>
<td>1.7</td>
</tr>
<tr>
<td>TM1</td>
<td>3.4 km</td>
<td>2.6 km</td>
<td>1.3 km</td>
<td>0.6 km</td>
<td>( \Delta h_{12} )</td>
</tr>
</tbody>
</table>

- See also

Read et al. (2013); Del Pozzo et al. (2013); Agathos et al. (2015); Lackey & Wade (2015)

Hotokezaka et al. (2016)
• distinguishability
  - $\Delta h_{12} = 1$ corresponds to $1\sigma$

If the true EoS is stiff and typical radius of NS is $>13$ km, our analysis suggests that the radius will be constrained within $\approx 1$ km at $>2\sigma$ level for an event at 200 Mpc.

- APR: $R_{1.35} = 11.1$ km, $\Lambda_{1.35} = 320$
- SFHo: $R_{1.35} = 11.9$ km, $\Lambda_{1.35} = 420$
- TMA: $R_{1.35} = 13.9$ km, $\Lambda_{1.35} = 1200$
- APR: $\Delta R = 2.3$ km
- SFHo: $\Delta R = 1.9$ km
- DD2: $\Delta R = 2.1$ km
- TMA: $\Delta R = 2.8$ km
- TM1: $\Delta R = 3.4$ km

See also
Read et al. (2013); Del Pozzo et al. (2013); Agathos et al. (2015); Lackey & Wade (2015)
Gravitational waves from NS-NS merger

First suggested by Shibata (2005)

- Point particle approx.
- Information of orbits, **NS mass**
- Finite size effects appear
- **tidal deformability**
- GWs from massive NS
  - **NS radius of massive NS**
  - BH or NS ⇒ **maximum mass**
Listening GWs: APR EoS ($R_{1.35}=11.1\text{km}$)
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Listening GWs: H4 EoS (R_{1.35}=13.6\,\text{km})

\begin{figure}
\centering
\includegraphics[width=\textwidth]{H4-135135.png}
\caption{H4-135135}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{2.7\,kHz.png}
\caption{2.7\,kHz}
\end{figure}
Listening GWs: H4 EoS (R_{1.35}=13.6\text{km})

Graph showing the waveform and frequency over time.
GW spectra and characteristic peak $f_{GW}$

First suggested by Shibata (2005); Bauswein+ (2012); Hotokezaka+ (2013)

**GW spectra and characteristic peak**

<table>
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<tr>
<th>Model</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>APR</td>
<td>11.1</td>
</tr>
<tr>
<td>DD2</td>
<td>13.2</td>
</tr>
<tr>
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</table>

Clark et al. (2014)
GW spectra and characteristic peak $f_{GW}$

- The quasi periodic GWs makes characteristic peak in GW spectra
- Peak frequency depends on EoS
  - stiffer EOS $\Rightarrow$ larger NS radii, smaller mean density $\Rightarrow$ lower $f_{GW}$
  - softer EOS $\Rightarrow$ smaller NS radii, larger mean density $\Rightarrow$ higher $f_{GW}$
Empirical relation between $f_{GW}$ and $R_{NS}$

- The peak frequency $f_{GW}$ shows good correlation with a radius
  - **Radius of 1.6M{}_{\odot} NS**
    - Bauswein et al. (2012)
    - approx. GR study
  - **Radius of 1.8M{}_{\odot} NS**
    - Hotokezaka et al. (2013)
    - full GR study
- Correlation is fairly tight:
  - $\Delta R \sim 0.5$ km (Bauswein et al. 2012)
  - $\Delta R \sim 1$ km (Hotokezaka et al. 2013)
- Further developments and theoretical background => towards the empirical relation
  - Takami et al. (2015); Rezzolla et al. (2016)
  - Bauswein+ (2014, 2015, 2016)
More realistic modeling (MHD viscosity) is necessary

- Rotation and oscillation of non-axisymmetric NS formed after merger emits quasi periodic GWs and makes the peak in the GW spectra
More realistic modeling (MHD viscosity) is necessary

- Rotation and oscillation of non-axisymmetric NS formed after merger emits quasi periodic GWs and makes the peak in the GW spectra
- The NS may become axisymmetric by viscous angular momentum transport
- GW amplitude and the peak may decrease => further study for MHD viscosity
Detectability and event rate (full aLIGO)

- **Tidal deformability (Hotokezaka et al. 2016)**
  - If true $R_{1.35} > \sim 13$ km then $\Delta R < 1$ km for events within 200Mpc $\Rightarrow$ **0.1-10 events / yr**
  - If true $R_{1.35} < 12-13$ km, it would be difficult to constrain EoS models for event at 200 Mpc
    - Nearby events will be necessary
    $\Rightarrow$ less than 0.1 event /yr (depends on EoS)

- **$f_{GW}$ in quasi-periodic GW (Clark et al. 2014, 2016)**
  - $f_{GW}$ are located in higher frequency (lower sensitivity) $\Rightarrow$
    only nearby events within 30 Mpc (depends on EoS) could be detected
  - $\Delta f = \Delta f_{obs} + \Delta f_{model} \approx \Delta f_{model} = 140$ Hz
    for which $\Delta R = 500$ m (if detected)
  - **Expected rate : $< 0.04$ / yr** (aLIGO design sensitivity)
  - Future planned detectors will be necessary (or very lucky event)
Future planned update for LIGO

LIGO white paper: LIGO-T1600119
Appearance of hyperon encoded in GWs?

- **Nucleonic**: NS shrinks by angular momentum loss in a long GW timescale
- **Hyperonic**: GW emission $\Rightarrow$ NS shrinks $\Rightarrow$ More Hyperons appear $\Rightarrow$ EOS becomes softer $\Rightarrow$ NS shrinks more $\Rightarrow$ emits more GWs ...

- $\Rightarrow f_{GW}$ for hyperonic EOS might increase with time if EoS softening is significant
  - *Could* provide potential way to tell existence of hyperons (exotic phases)

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EoS not compatible with 2Msun NS
Appearance of hyperon encoded in GWs?

- Simulation with a hyperon EoS compatible with 2Msun NS

- Softening due to hyperon appearance is reduced

- Increase of $f_{GW}$ may not be prominent as previously expected

- Instead hyperon (exotic phase) appearance is encoded in GW luminosity => more study

Radice et al. (2017)
A scenario with a very nearby event

- In 20XX, GWs from NS-NS merger are detected
- Inspiral chirp signal suggest mass of 1.35-1.35 and D = 20Mpc
- Extracted tidal deformability parameter suggests $R_{1.35} = 12-12.5km$
- Many hyperon models which require stiff nucleonic EoS are discarded

Ohnishi @ JPS JPARK-HI Symp.
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- $f_{GW}$ is marginally detected suggesting $dM/dR > 0$ and stiffening of EoS
- Quark crossover model is preferred

Masuda et al. (2013); Kojo et al. (2015); Fukushima & Kojo (2016)
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- **Tension between directed flow data by STAR collaboration which indicates softening of EoS.**
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- QCD phase boundary should depend (both on $T$ and) isospin chemical potential

Ohnishi @ JPS JPARK-HI Symp.
Summary

• GW150914: The first direct detection of GWs from BH-BH
  • marks the dawn of GW astronomy era
  • GW data consistent with GR prediction

• GWs from binary NS mergers and EOS
  • Tidal deformation: provides radius of NS before merger
    • can constrain EoS with $R_{1.35} > 12.5\text{-}13 \text{ km}$
    • Need more accurate NR simulations
  • Characteristic frequency: provides radius of more massive NS
    • can constrain EoS only for nearby event $< 30\text{Mpc}$ with aLIGO
    • Time dependent analysis: constraint on exotic phase might obtained
Massive NS is important to explore high density region

- **core bounce in supernovae**
  - mass: 0.5~0.7Msun
  - $\rho_c$: a few $\rho_s$
- **canonical neutron stars**
  - mass: 1.35-1.4Msun
  - $\rho_c$: several $\rho_s$
- **massive NS (> 1.6 Msun)**
  - $\rho_c$: > 4$\rho_s$
- **massive NSs are necessary to explore higher densities**
  - Such a massive NS is very rare
  - **NS-NS merger:** NS with $M > 2$ Msun after the merger
GW : Simultaneous mass and radius measurement
- Inspiral waveform naturally provides the mass of each NS
- Degeneracy of M and R in EM observations: additional information (assumption) required
GW : contains multiple information
- Tidal deformation (radius): lower (~ρs) density
- Oscillation of NS after the merger: higher density
- Maximum mass: highest density

Simple in a complementary sense (GW obs. rare)
- GW: quadrupole formula, no interaction with matter
  - EOS (what we want to know) is only uncertain (provided GR is correct and GWs are detected) ⇒ could be smoking-gun
- EM: a number of parameters, models
  - Atmosphere, distance, column density, B-field, fc, ...
    (recent debate: Ozel et al., Steiner& Lattimer, Guillot et al.)

Radius is sensitive to relatively low density parts

Maximum mass depends on most dense parts
Further possibility?

- Exploring quark-hadron phase transition by GWs
  - 2\textsuperscript{nd} order (like hyperons) \(\Rightarrow\) frequency shift in time
  - 1\textsuperscript{st} order \(\Rightarrow\) frequency may jump NS to quark star
    \(\Rightarrow\) double peak in GW spectra?

- We need a ‘good’ quark-hadron EOS to explore it
In the shear layer formed at the time of merger, KH instability develops amplifying the B-field => seed of turbulent MHD viscosity
Kiuchi et al. 2014, 2015; see also Rasio & Shapiro 1999