DYNAMICS IN PHASES OF QCD

FOCUS

Discern or Validate Bulk Properties of QCD Matter through Heavy-Ion Collisions

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Supported by Dept. of Energy, Office of Science
DE-FG02-03ER41259

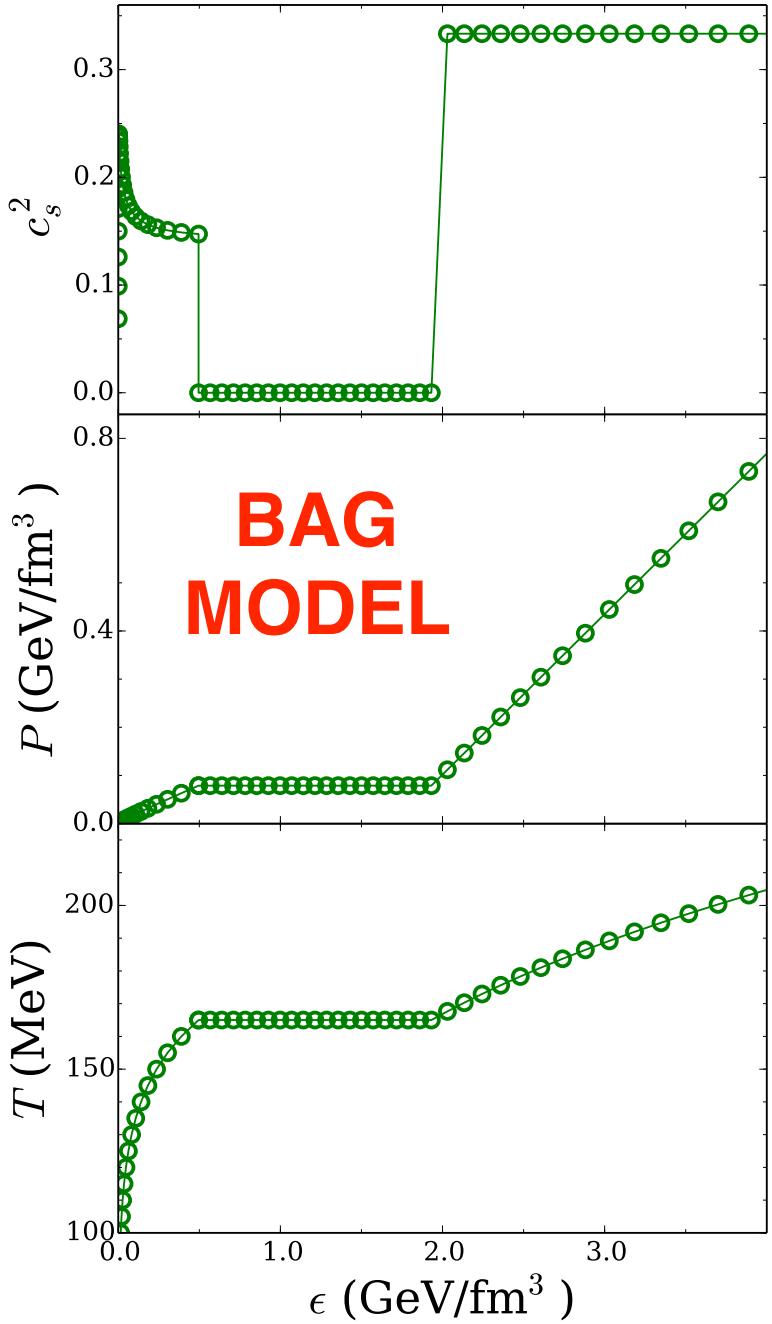
Fundamental Properties of QCD Matter

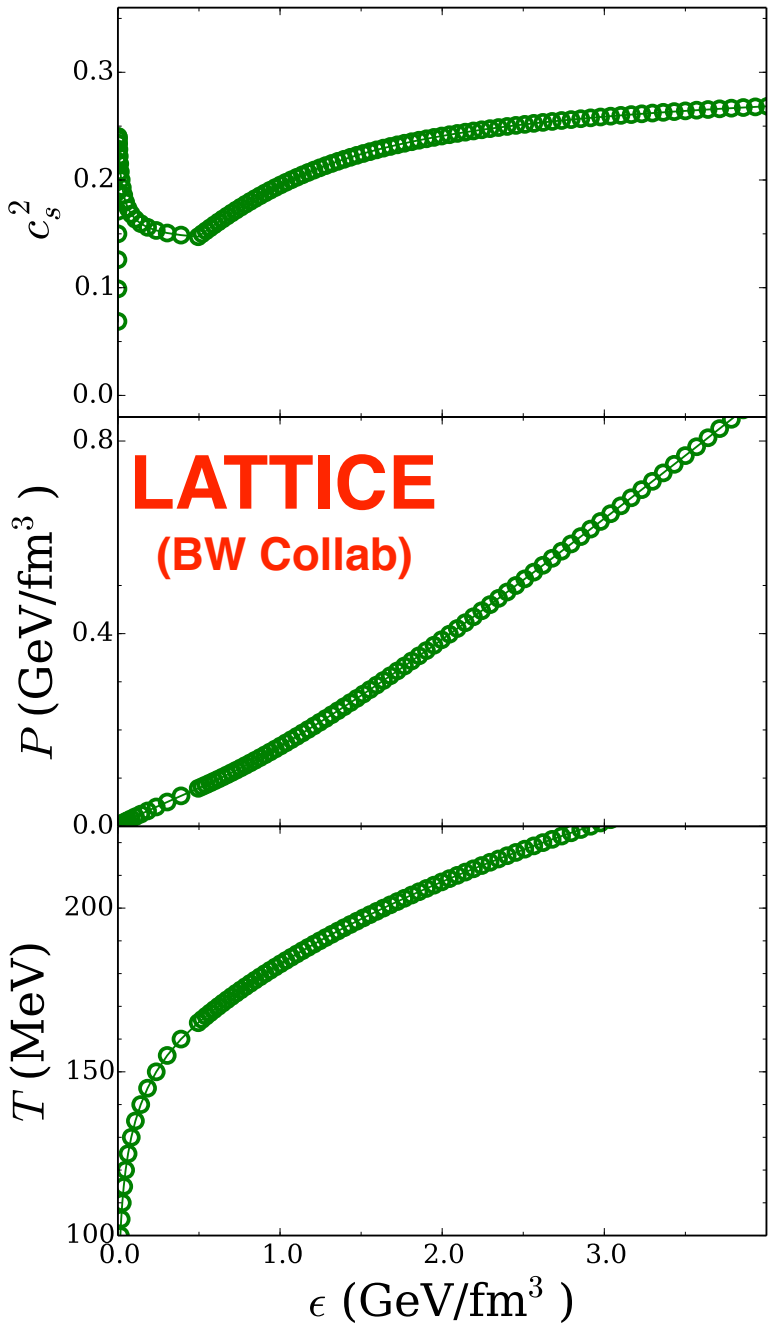
1. *Eq. of State
   + baryon number dependence
2. *Chemistry
   + Color Screening
3. *Chiral Symmetry Restoration
4. Viscosity
5. Diffusion Constant
6. Jet Damping

*Reliably extracted from lattice

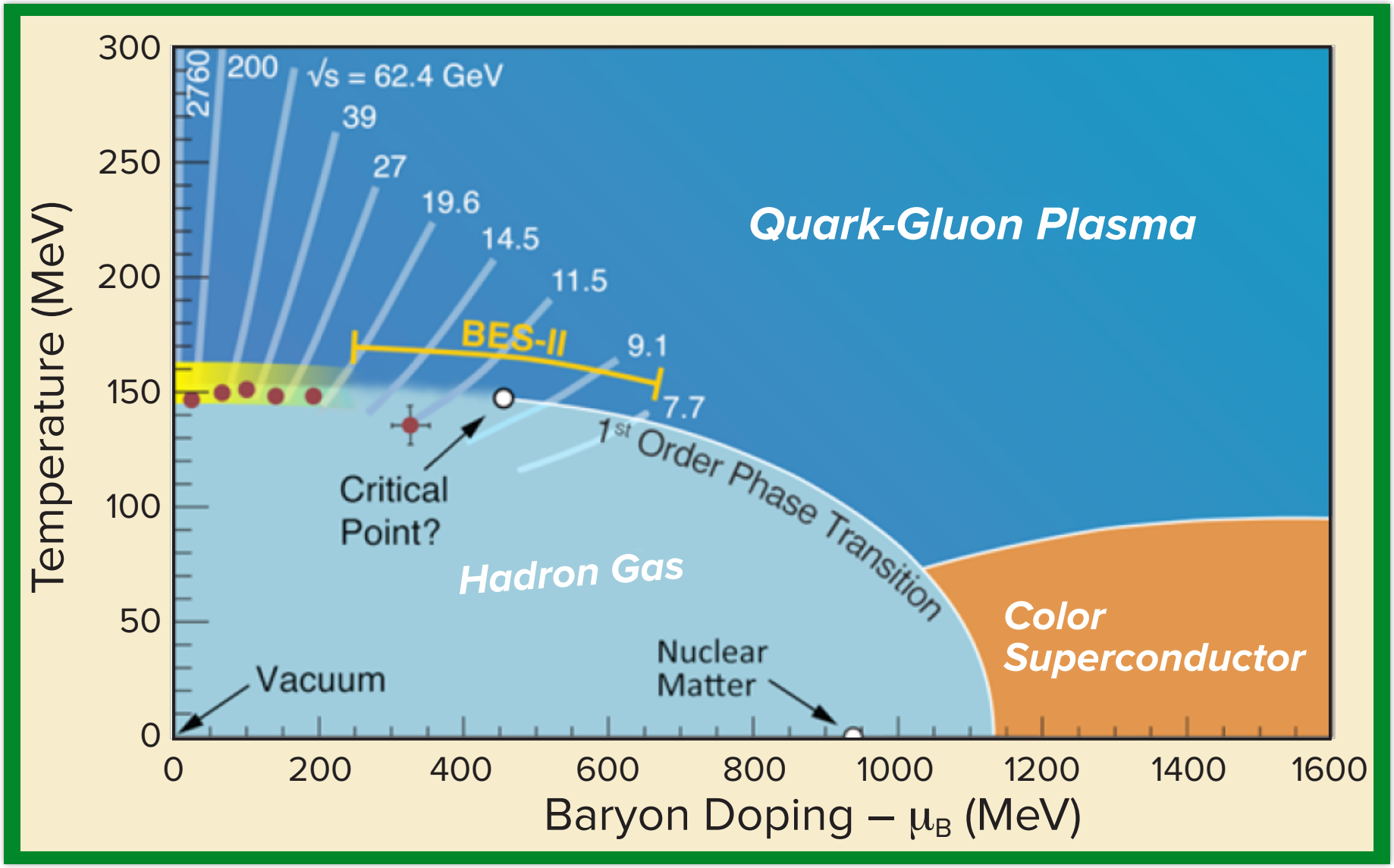
1. Eq. of State

First order?

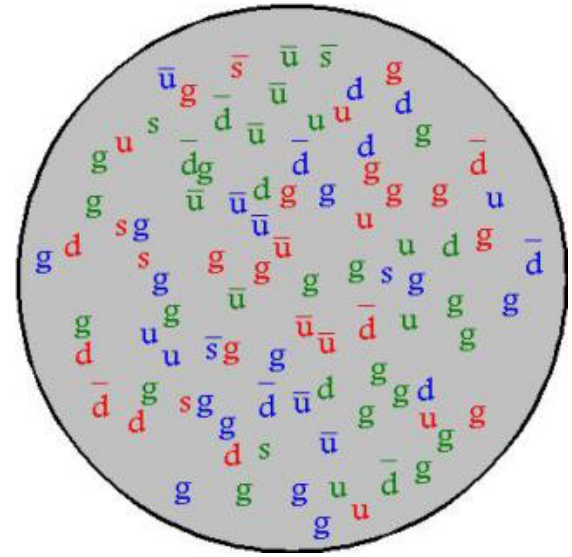




1. Eq. of State + Baryons



2. Chemistry



52 light degrees of freedom

- **36 quarks**
  (3 colors, 2 spins, part/antipart, uds)
- **16 gluons**
  (8 colors, 2 spins)
- **ignore leptons, photons or heavy quarks**

$$n \sim T^3$$

$$\epsilon \sim T^4$$

$$P = \epsilon / 3 \sim T^4$$

$$s = \frac{P + \epsilon}{T} \sim T^3$$

Inside 1 λ_{th}^3,

- **Bose condensed ^{4}He: one particle**
- **Photon gas: 2 particles**
- **Parton gas: 52 particles**

2. Chemistry

Parton number undefined in interacting system and $\langle \rho_{u,d,s} \rangle = 0$
so, considers fluctuations:

$$\chi_{ab} \equiv \frac{\langle Q_a Q_b \rangle}{V}$$

For parton gas (non-interacting)

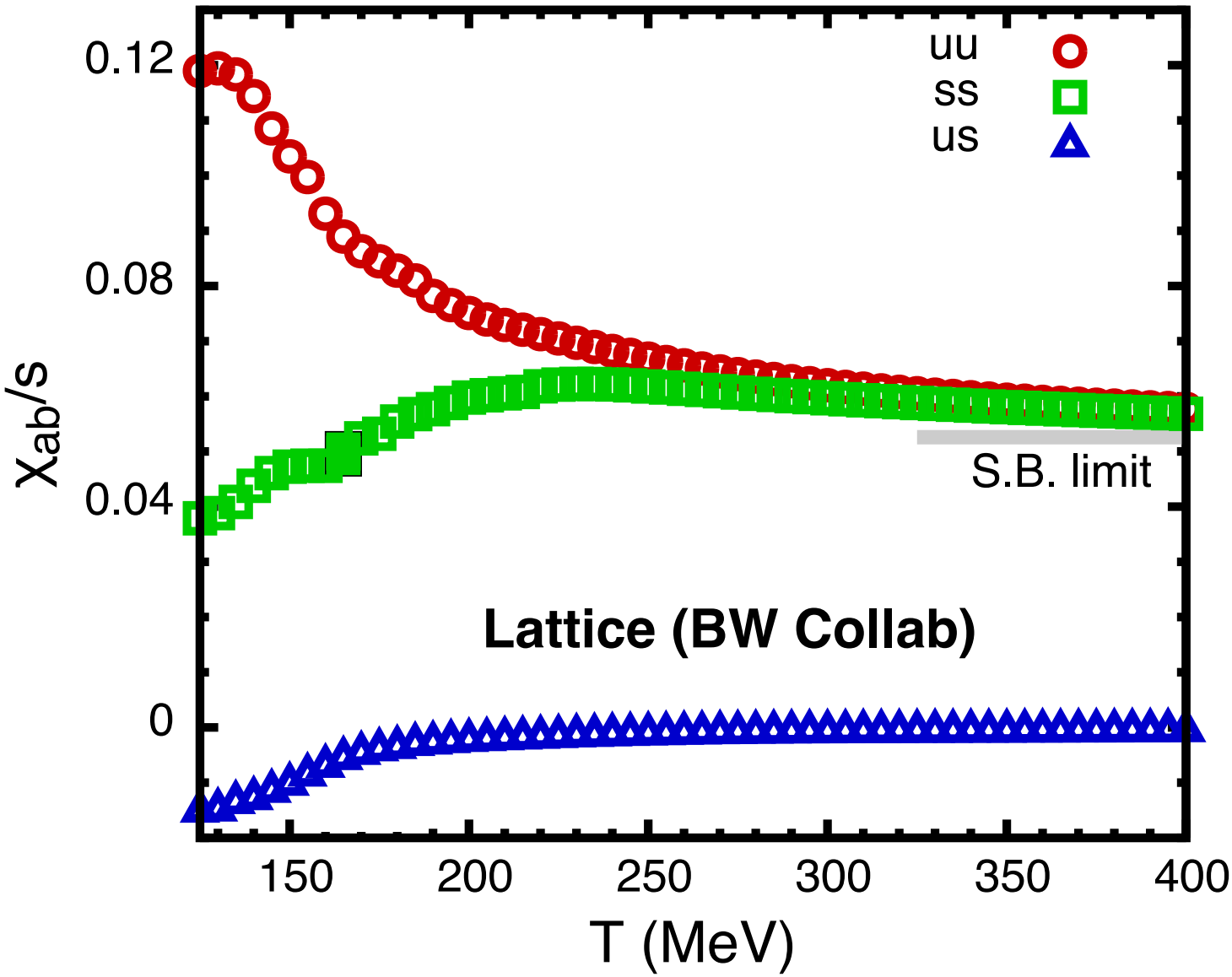
$$\chi_{ab} = (n_a + n_{\bar{a}})\delta_{ab} \ , \ \chi / s = \text{constant for } m = 0$$

For hadron gas (non-interacting)

$$\chi_{ab} = \sum_{\alpha} n_\alpha q_{\alpha a} q_{\alpha b}$$

2. Chemistry

behavior approaches parton gas at high *T*



2. Chemistry: Color Screening

Debye Screening: Charge $+Q_0$ in plasma, will attract negative charges

$$\Delta n_e(r) = n_e(e^{-V(r)/T} - 1),$$

$$\approx -n_0 V(r)/T$$

$$V(r) = \frac{-eQ_0}{4\pi\epsilon_0 r} e^{-r/\lambda},$$

Includes contribution from screening charges

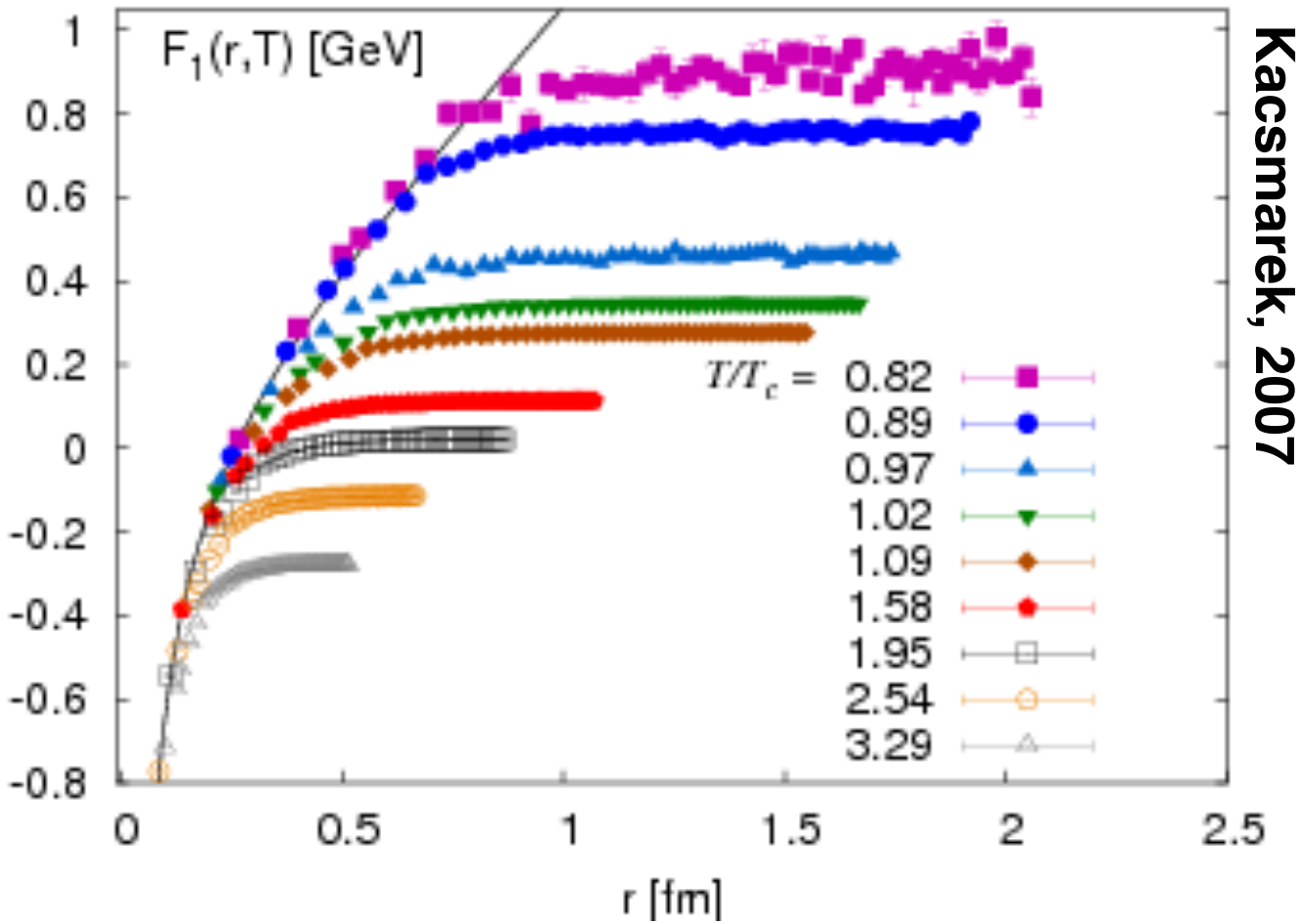
$$\lambda_{\text{Debye}} = \sqrt{\frac{\epsilon_0 T}{n_0 e^2}}$$

HW: Show this form is consistent with Gauss's Law.

Screens confining potential ➜ "free color charges"

2. Chemistry: Color Screening

Free energy vs. separation



Kacsmarek, 2007

For T> 200 MeV, charges can separate

3. Chiral Symmetry

$$\mathcal{L} = \bar{\Psi}_a (i\partial_\mu - eA_\mu)\gamma^\mu \Psi_a + \cdots$$

$$\Psi \to e^{i\gamma_5 \phi}\Psi$$

$$\Psi \to e^{i\gamma_5 \vec{\tau}\cdot\vec{\phi}}\Psi$$

Invariant to axial and iso-axial rotations (if *m*=0)

Noether's theorem leads to conserved currents

$$\cancel{j_5^\mu = \bar{\Psi}\gamma^\mu\gamma_5\Psi}$$

$$j_{5a}^\mu = \bar{\Psi}\gamma^\mu\gamma_5\tau_a\Psi$$

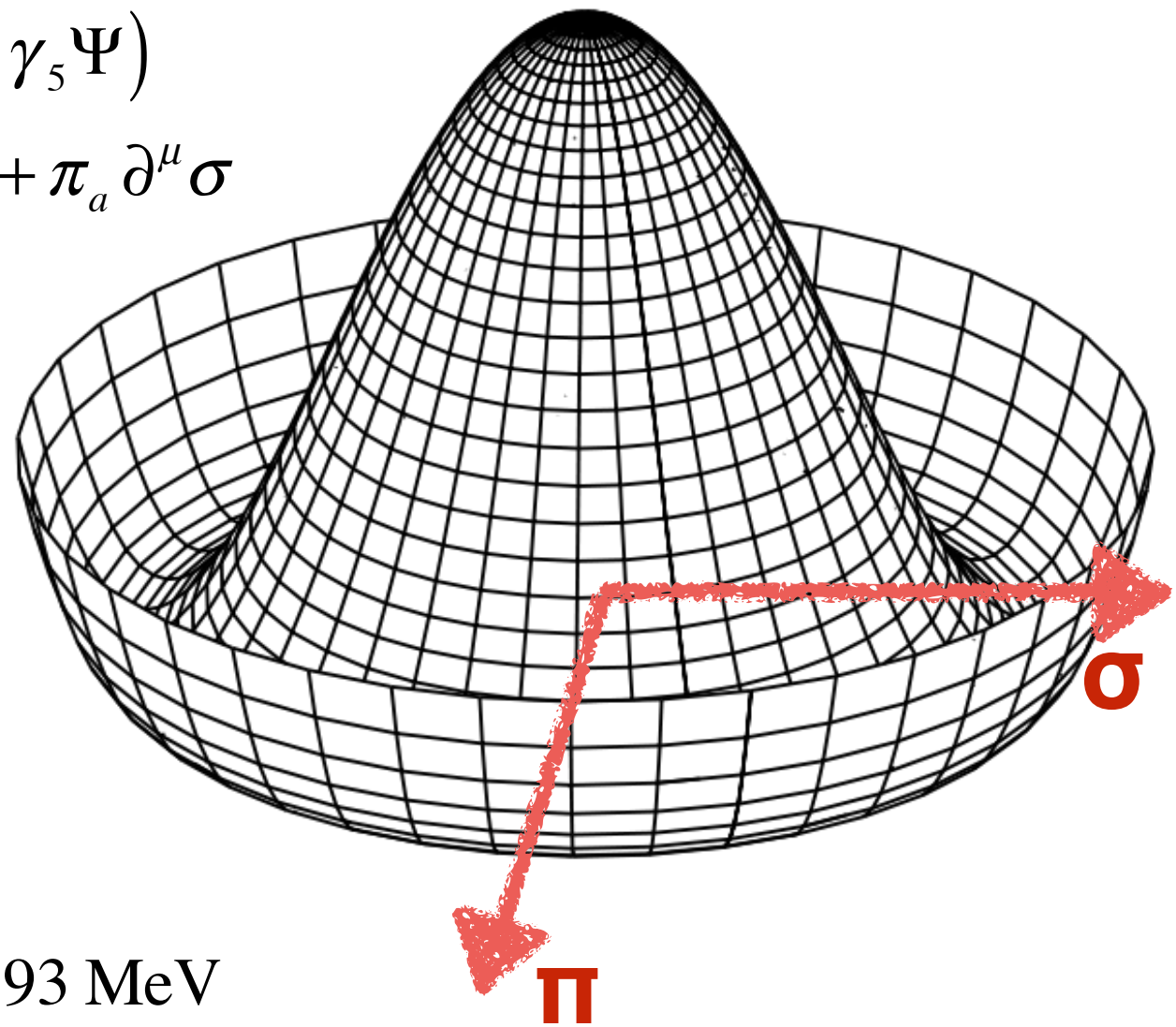
ruined by chiral anomaly

3. Chiral Symmetry (hadronic perspective)

$$\mathcal{L} = \frac{-1}{2}\left\{\sigma\,\partial^2\sigma + \vec{\pi}\,\partial^2\cdot\vec{\pi}\right\} + \frac{1}{2}M_0^2\left\{\sigma^2 + |\vec{\pi}|^2\right\} - \frac{\lambda}{4}\left\{\sigma^2 + |\vec{\pi}|^2\right\}^2$$

$$+\, g_{\pi N}\left(\sigma\,\bar{\Psi}\Psi + i\vec{\pi}\cdot\bar{\Psi}\vec{\tau}\,\gamma_5\Psi\right)$$

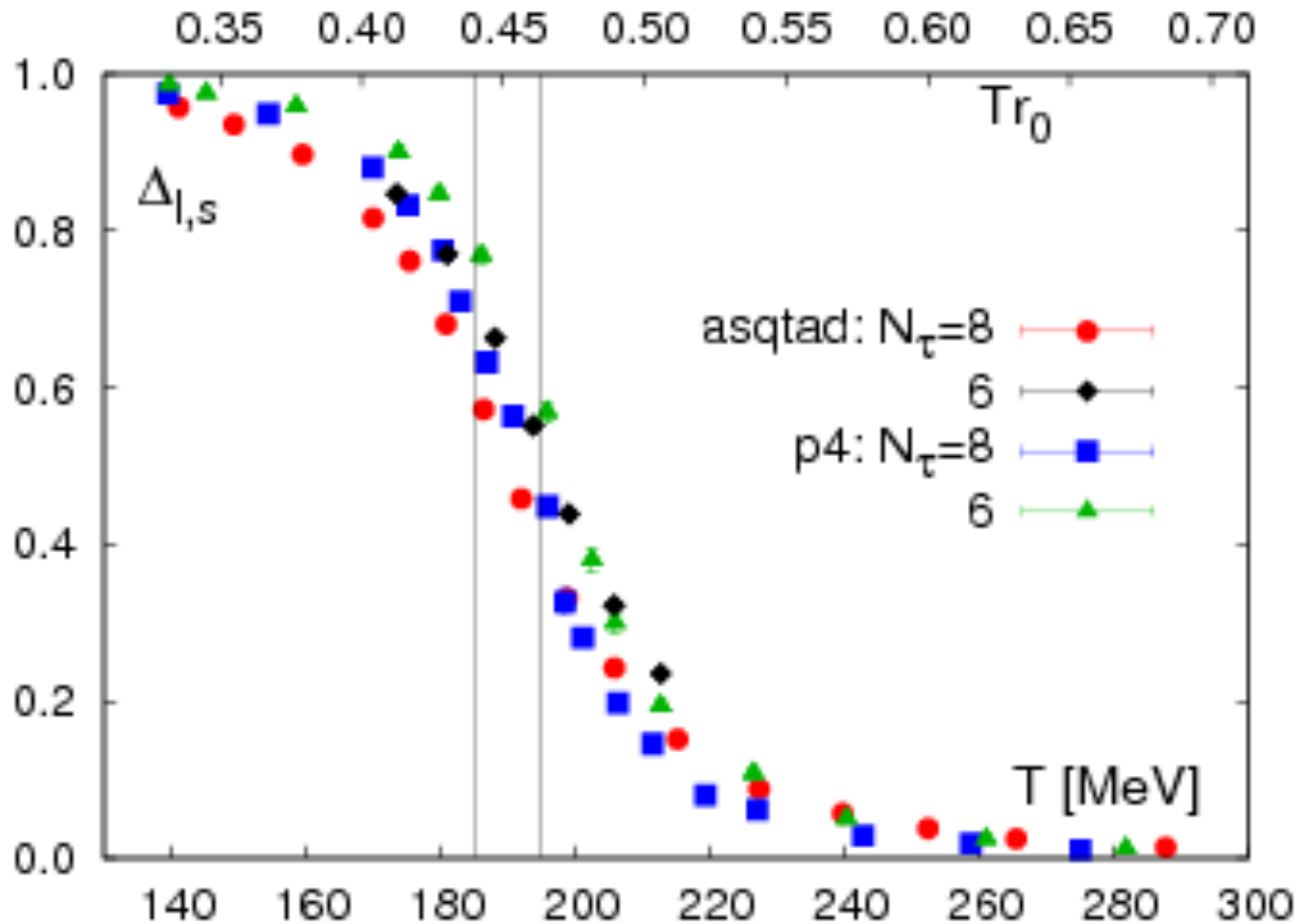
$$j_a^\mu = \bar{\Psi}\gamma_5\gamma^\mu\tau_a\Psi + \sigma\,\partial^\mu\pi_a + \pi_a\,\partial^\mu\sigma$$



$$M_N \approx g_{\pi N}\langle\sigma\rangle, \langle\sigma\rangle = f_\pi = 93\ \text{MeV}$$

3. Chiral Symmetry (lattice)

3. q-qbar condensate, is related to sigma condensate



Condensate couples to quarks and gives them mass

4. Viscosity

$$\partial_t T_{00} = -\partial_x T_{0x} - \partial_y T_{0y} + \partial_z T_{0z}$$
$$\partial_t T_{0x} = \partial_x T_{xx} + \partial_y T_{yx} + \partial_z T_{zx}$$

Local conservation of *E* and *P*

$$T_{i \neq j} = 0$$

Ideal hydro

$$T_{ij} = P\delta_{ij} - \eta(\partial_i v_j + \partial_j v_i) - \zeta \nabla \cdot \vec{v}$$

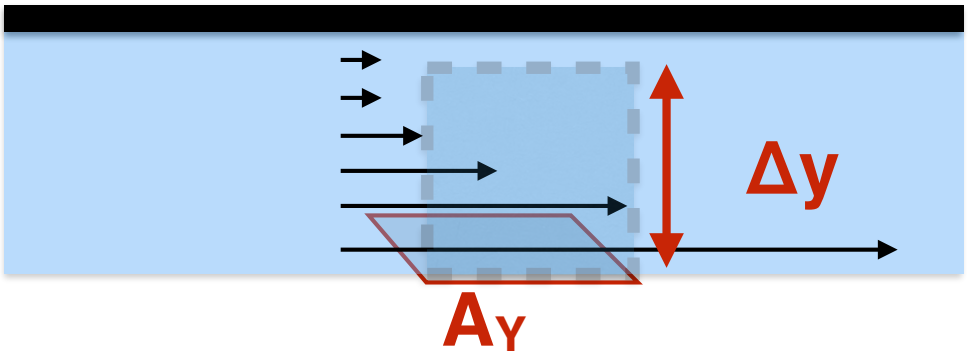
Navier-Stokes

η = shear viscosity

ζ = bulk viscosity

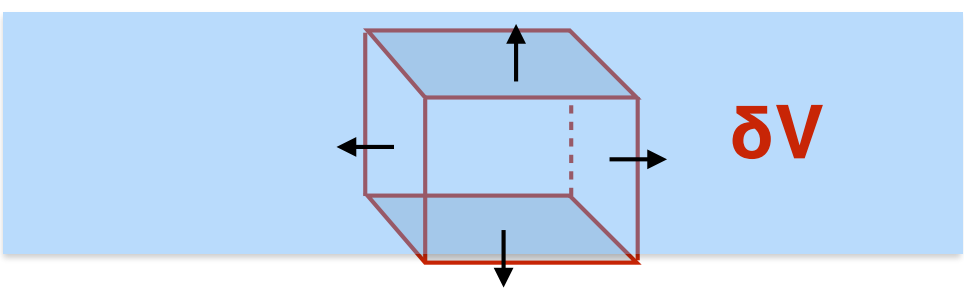
4. Viscosity

shear represents friction between layers of fluid



$$\frac{d}{dt}P_x = A_y \eta \partial_y v_x$$

bulk describes dissipation of diverging flow



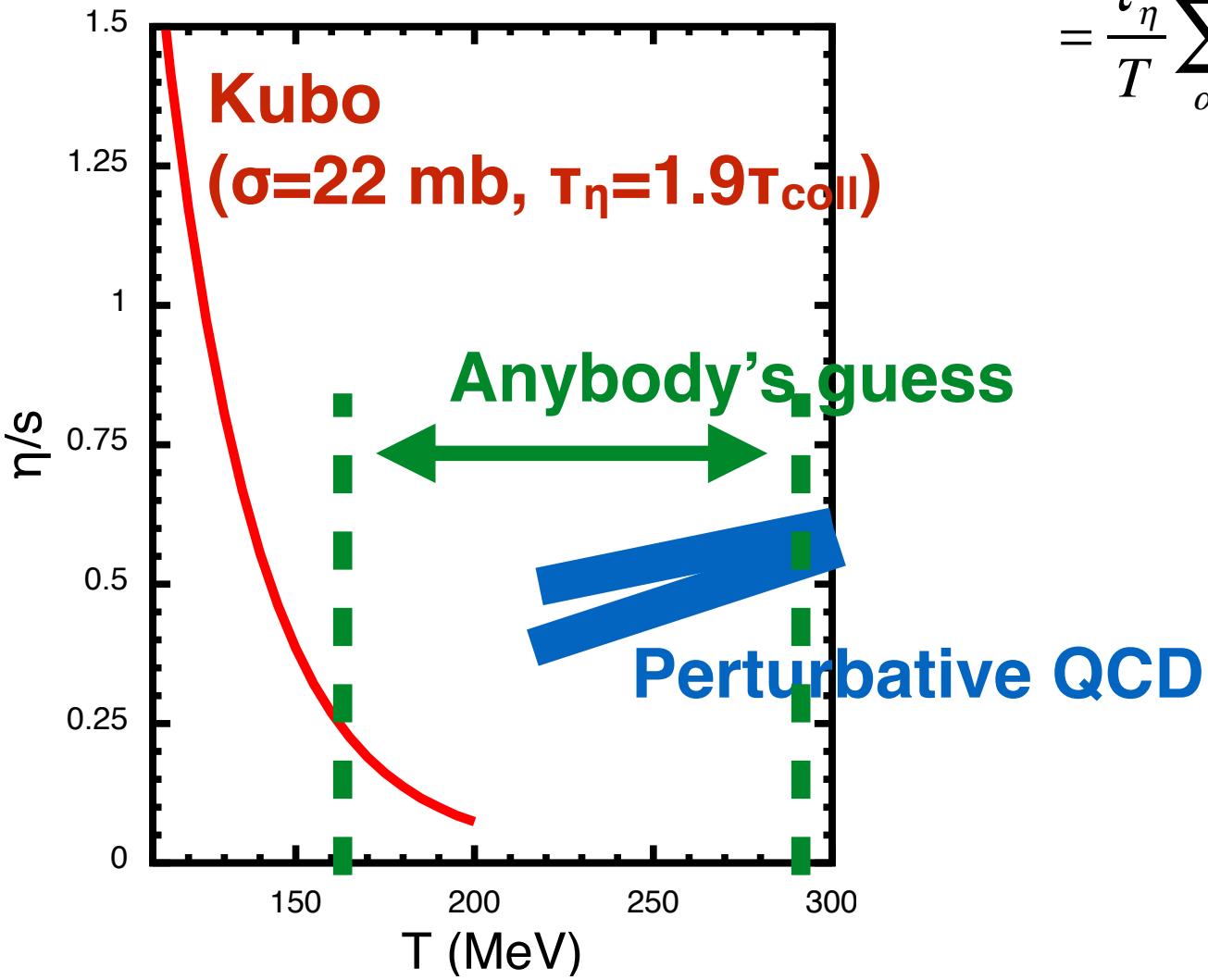
$$\delta E = -P\delta V + \zeta \nabla \cdot \vec{v} \delta V$$

4. Viscosity (Kubo relations)

For gas, correlation of particles with themselves multiplied by relaxation time:

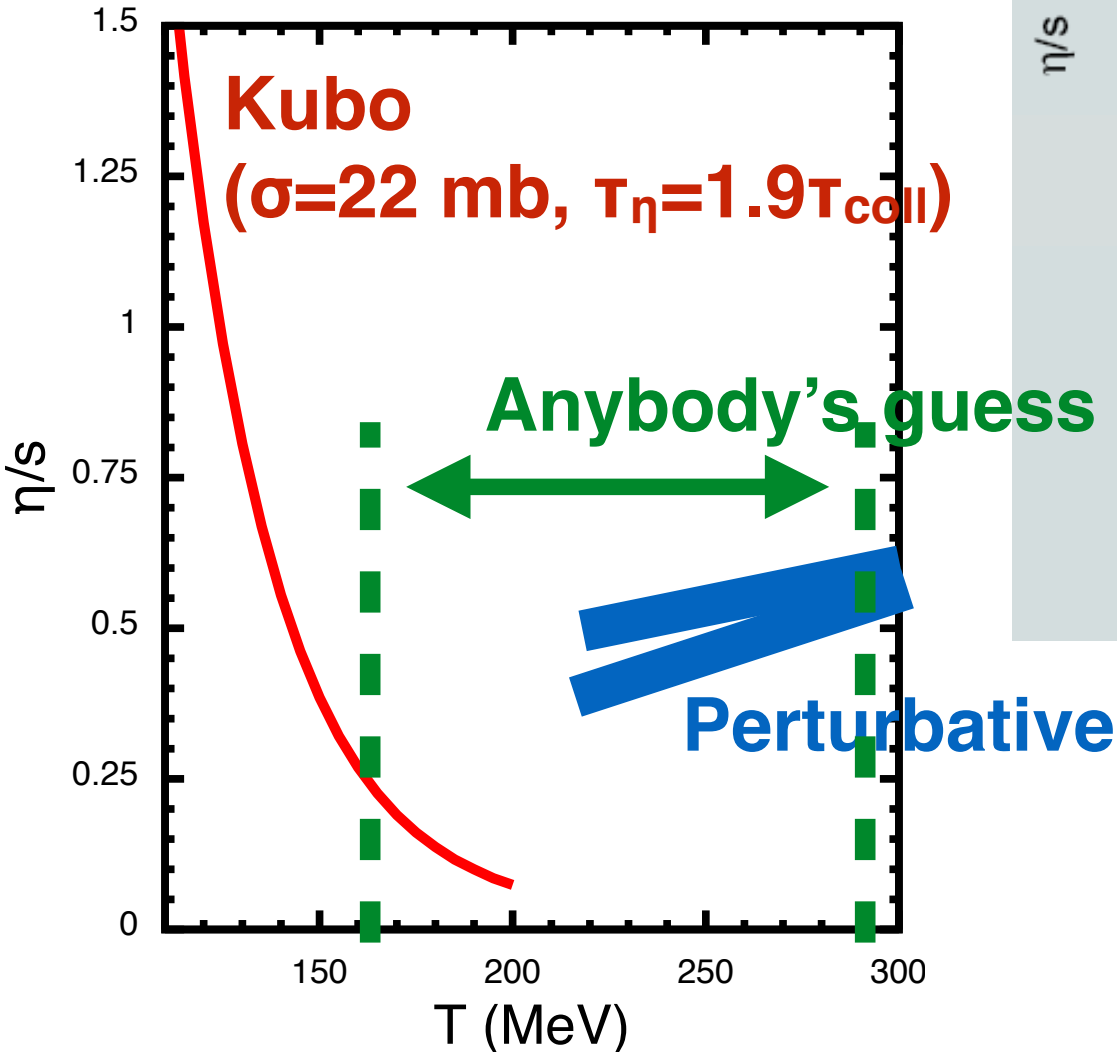
$$\eta = \frac{\tau_\eta}{T}\int d^3r \langle T_{xy}(0,0)T_{xy}(\vec{r},t=0)\rangle$$

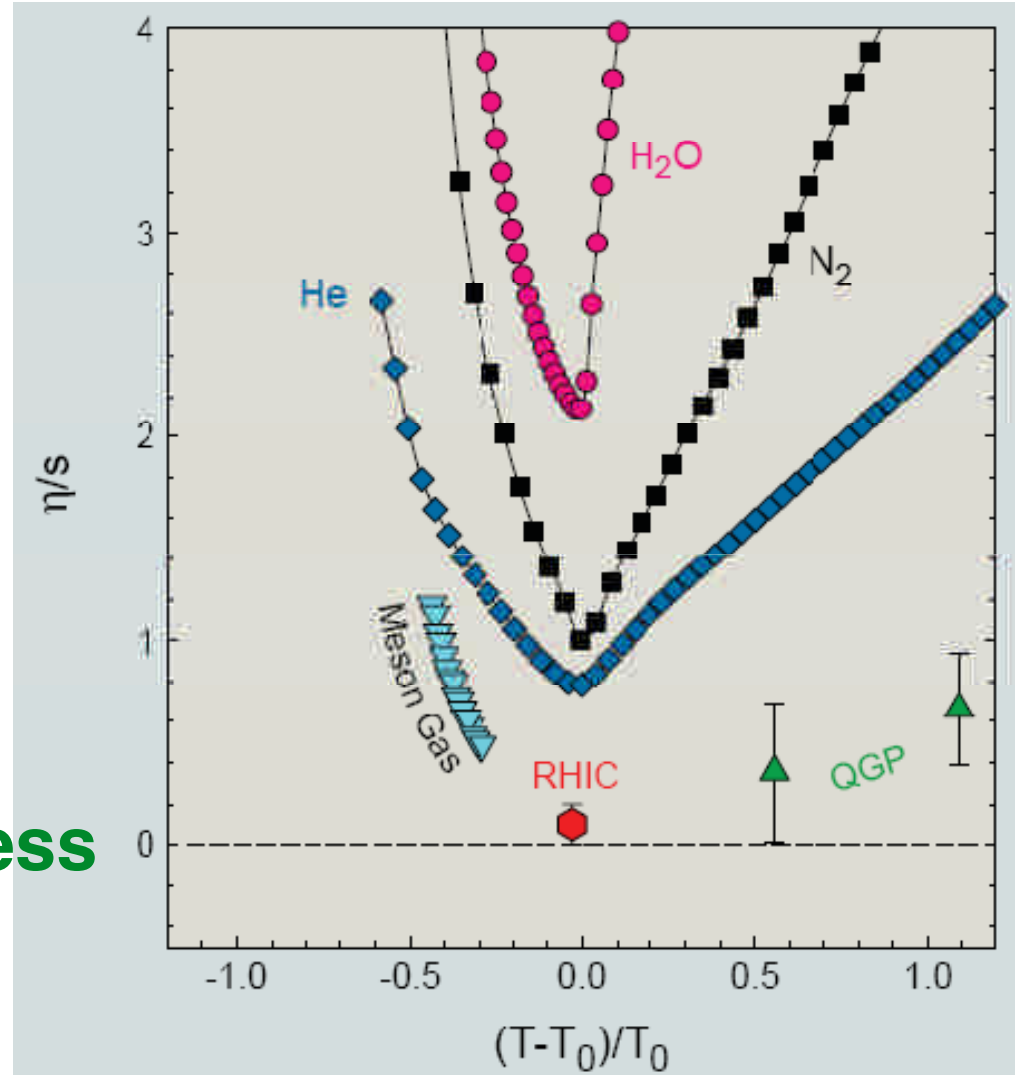
$$= \frac{\tau_\eta}{T}\sum_\alpha (2S_\alpha+1)\int \frac{d^3p}{(2\pi)^3} e^{-E/T}\frac{p_x^2 p_y^2}{E^2}$$



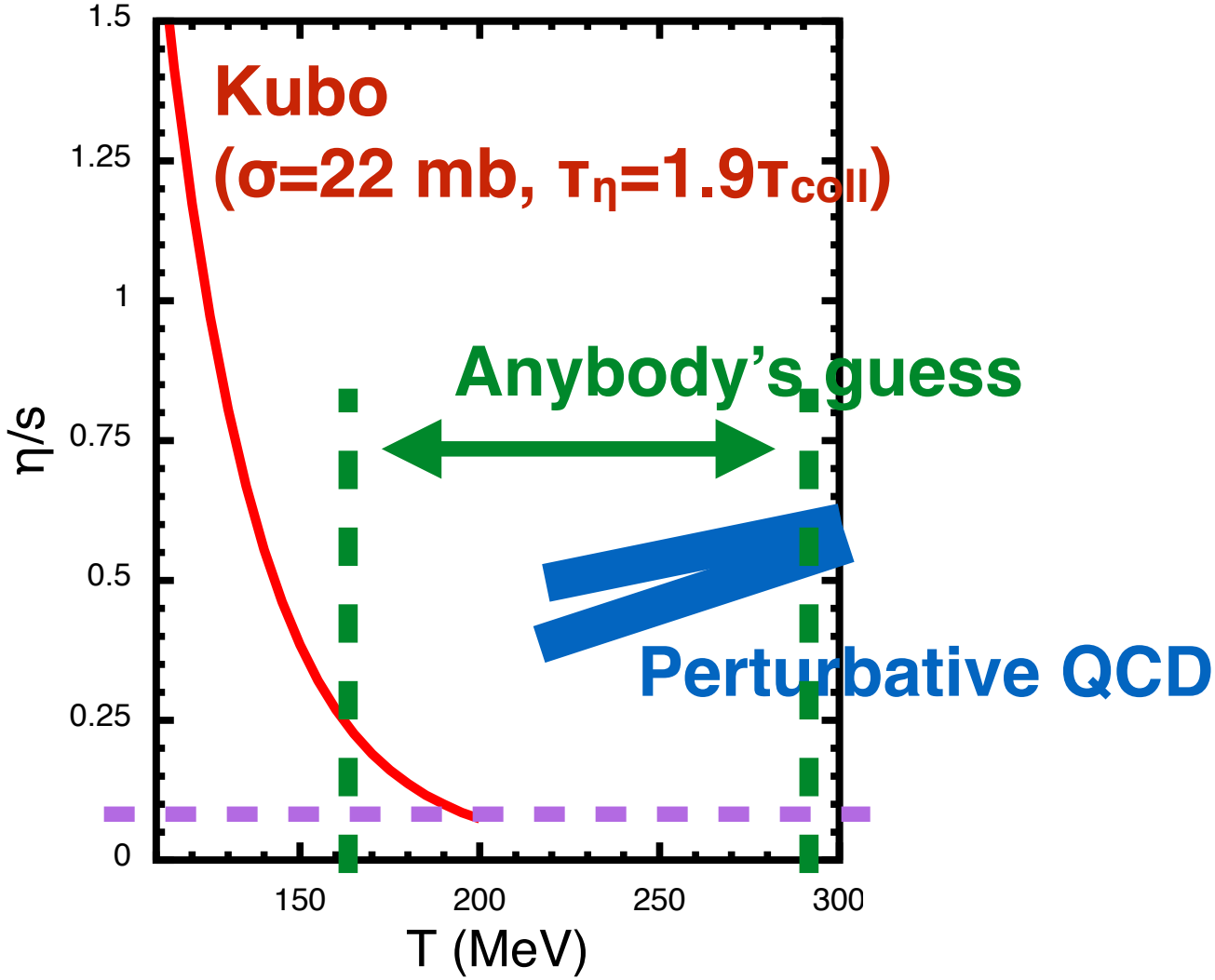
4. Viscosity

similar behavior to other fluids near T_c





4. Viscosity



Some values:

0.08 : λ_{therm} λ_{mfp} (Danielewicz and Gyulassy)

1/4π : AdS/CFT (Kovton, Starinets, Son)

in principal can go to zero

MODELS

Pre-Equilibrium ($0<\tau<1$ fm/c)
mixture of gluonic fields and partons
not decisively understood

Hydrodynamic (T>160 MeV)
relativistic, viscous
most strongly affects results

Hadronic cascade (T<160)
microscopic evolution of *f(p,r,t)* for each species
hydro can't handle 100 species flowing differently

Jets, rare probes, bulk correlations & EM probes
calculations overlaid on hydro evolution

Femtoscopic correlations
calculated from final *f(p,r,t)*

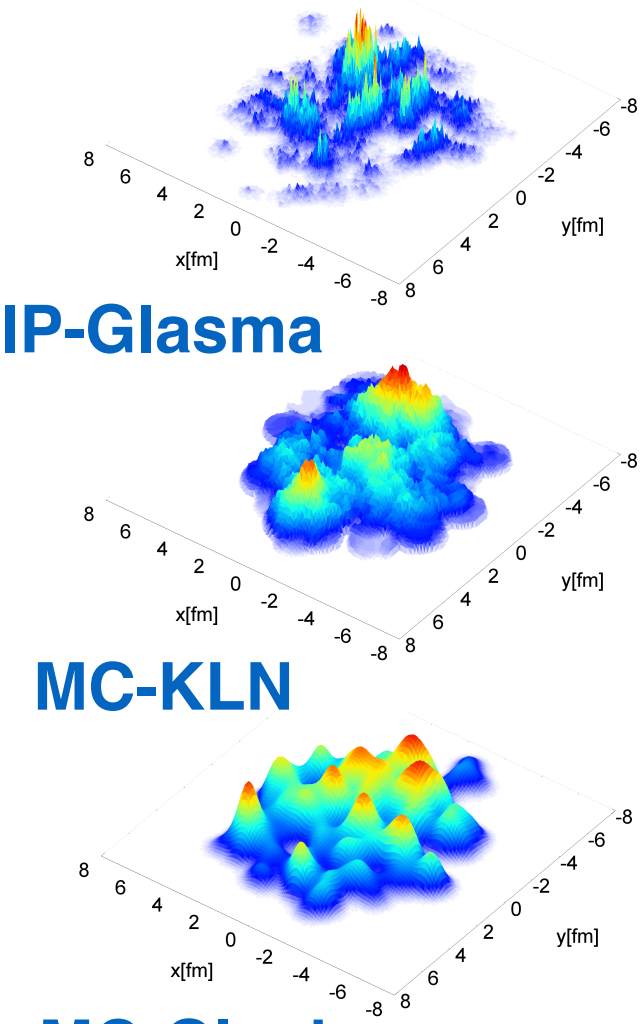
Modeling Status: Initial State

- **Low pt — coherent fields (CGC, flux tubes)**
  High pt — partonic
  Where is the boundary?

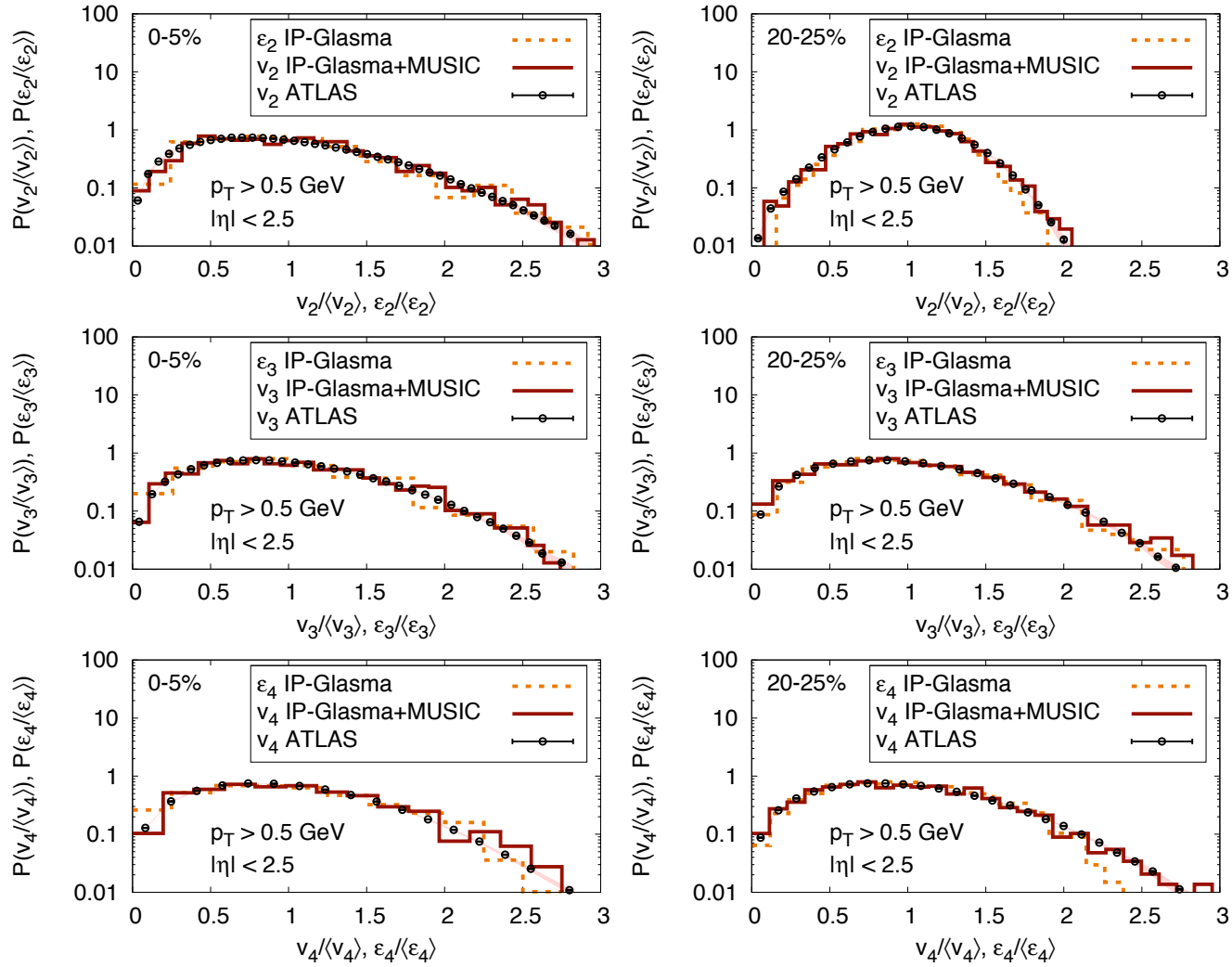
- **How and when do fields thermalize?**
  Quarks appear quickly — but how?
  Initial Flow?

- **Lower energy (BES) collisions**
  Not even good parameterizations

Modeling Status: Initial State Lumpy Hydro



IP-Glasma



Gale, Jeon, Schenke, Tribidy & Venugopalan, 2014

Something's right! — But what?

Modeling Status: Hydro

- **Non-ideal Hydro**

$$\partial_\mu T^{\mu\nu} = 0, \quad T_{ij} = P\delta_{ij} + \pi_{ij} + \Pi\delta_{ij}$$

- **Navier-Stokes**

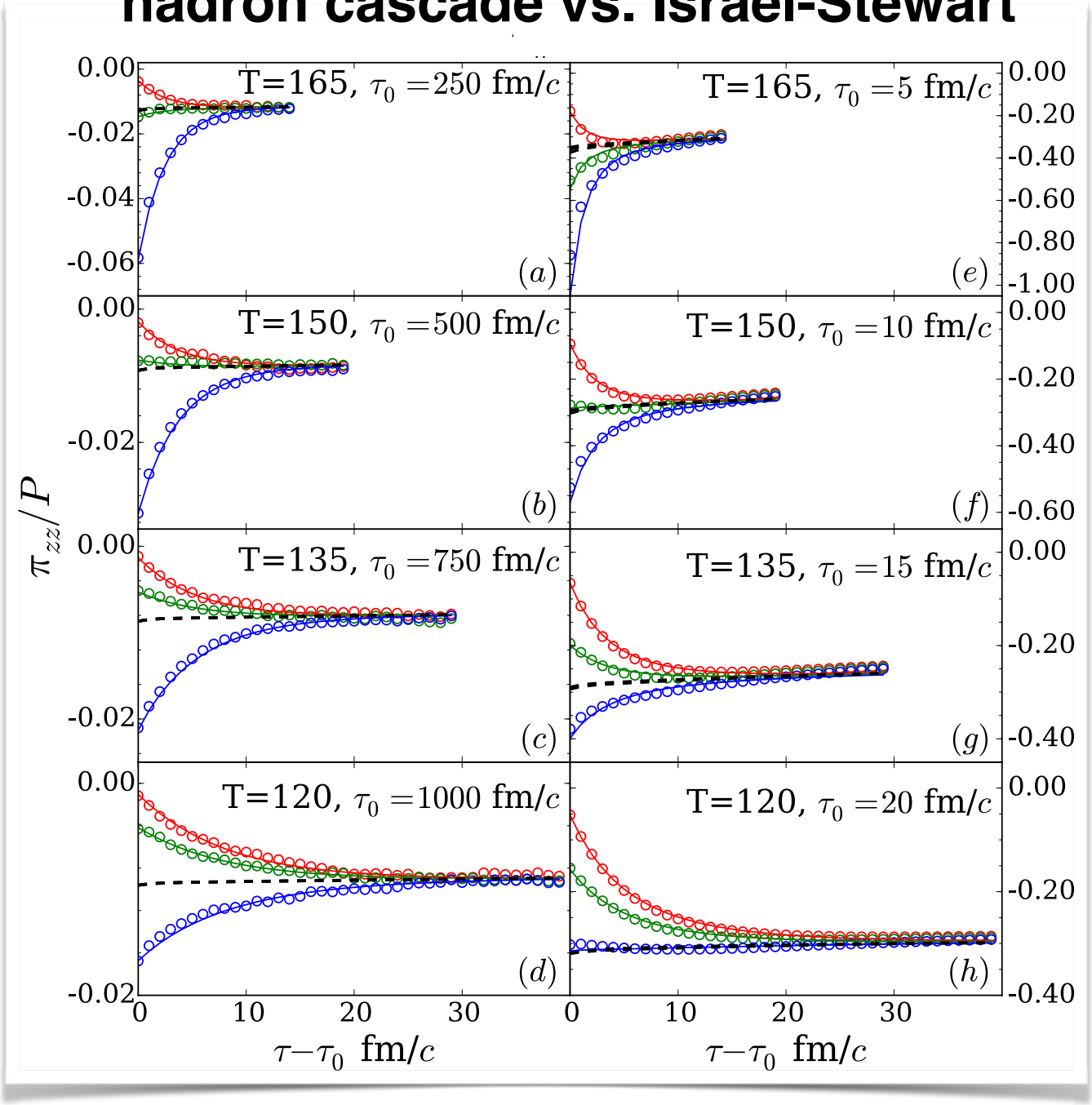
$$\pi_{ij} = \eta\omega_{ij}, \quad \omega_{ij} \equiv \partial_i v_j + \partial_j v_i - (2/3)\delta_{ij}\nabla\cdot v$$

- **Israel-Stewart (+ others)**

$$\partial_t \pi_{ij} = -\frac{1}{\tau_\pi}\left(\pi_{ij} - \eta\omega_{ij}\right) - \gamma_\pi \omega_{ik}\pi_{kj} - \kappa_\pi \nabla\cdot v \pi_{ij}$$

Modeling Status: Hydro

hadron cascade vs. Israel-Stewart



Israel-Stewart very successful for $\pi_{ij} < P/2$ (Minnesota, <u>Frankfurt</u>, McGill, Ohio State, Kent State,…)

S.P., A.Baez and J. Kim PRC 2017

Modeling Status: Hydro Correlations of Conserved Quantities

E_t, p_x, p_y, Q,B,S

Susceptibilities come from lattice (novel behavior at critical point)

$$C(\vec{r}_1,\vec{r}_2) = \langle(\rho(\vec{r}_1)-\bar{\rho}(\vec{r}_1))(\rho(\vec{r}_2)-\bar{\rho}(\vec{r}_2))\rangle$$
$$= \chi(\vec{r})\delta(\vec{r}_1-\vec{r}_2) + C'(\vec{r}_1,\vec{r}_2),$$
$$\partial_t C'(\vec{r}_1,\vec{r}_2) = \langle\nabla\cdot\vec{j}(\vec{r}_1)(\rho(\vec{r}_2)-\bar{\rho}(\vec{r}_2))\rangle + \langle(\rho(\vec{r}_1)-\bar{\rho}(\vec{r}_1))\nabla\cdot\vec{j}(\vec{r}_2)\rangle$$
$$- \partial_t\chi(\vec{r})\delta(\vec{r}_1-\vec{r}_2).$$

"source function"

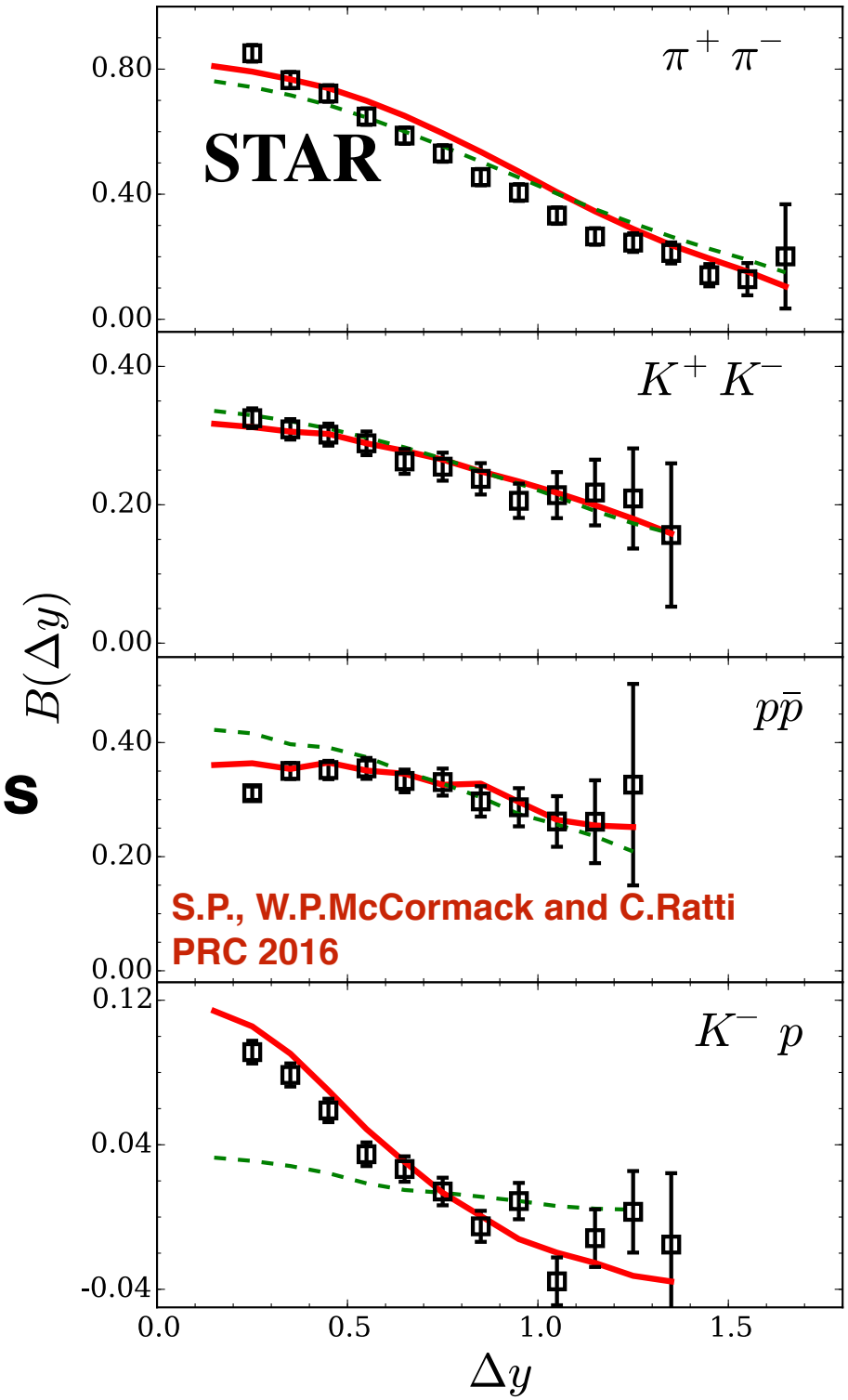
Given *j* one can predict measurable correlations in final state from lattice input (χ).

Modeling Status: Hydro Correlations of Conserved Quantities

Only simple models thus far, Need to:

1. **overlay onto 3D hydro**
2. **consider all conserved quantities**
3. **describe n-body correlations**

Charge Balance Functions



Modeling Status: Hydro
Correlations of Conserved Quantities

One promising approach: Noisy hydrodynamics

Mueller, Stephanov, Kapusta, Young, …

$$\vec{j}(\vec{r},t) = \langle \vec{j}(\vec{r},t) \rangle + \vec{j}^{(\mathrm{n})}(\vec{r},t),$$

$$\langle \vec{j}^{(\mathrm{n})}(\vec{r},t) \vec{j}^{(\mathrm{n})}(\vec{r}',t') \rangle = 2\sigma T \delta(\vec{r}-\vec{r}')\delta(t-t')$$

Allows 1-body description to reproduce 2-body correlations to linear level

But what non-linearity, or 3-body, 4-body…?

Modeling Status: Hadron Cascade

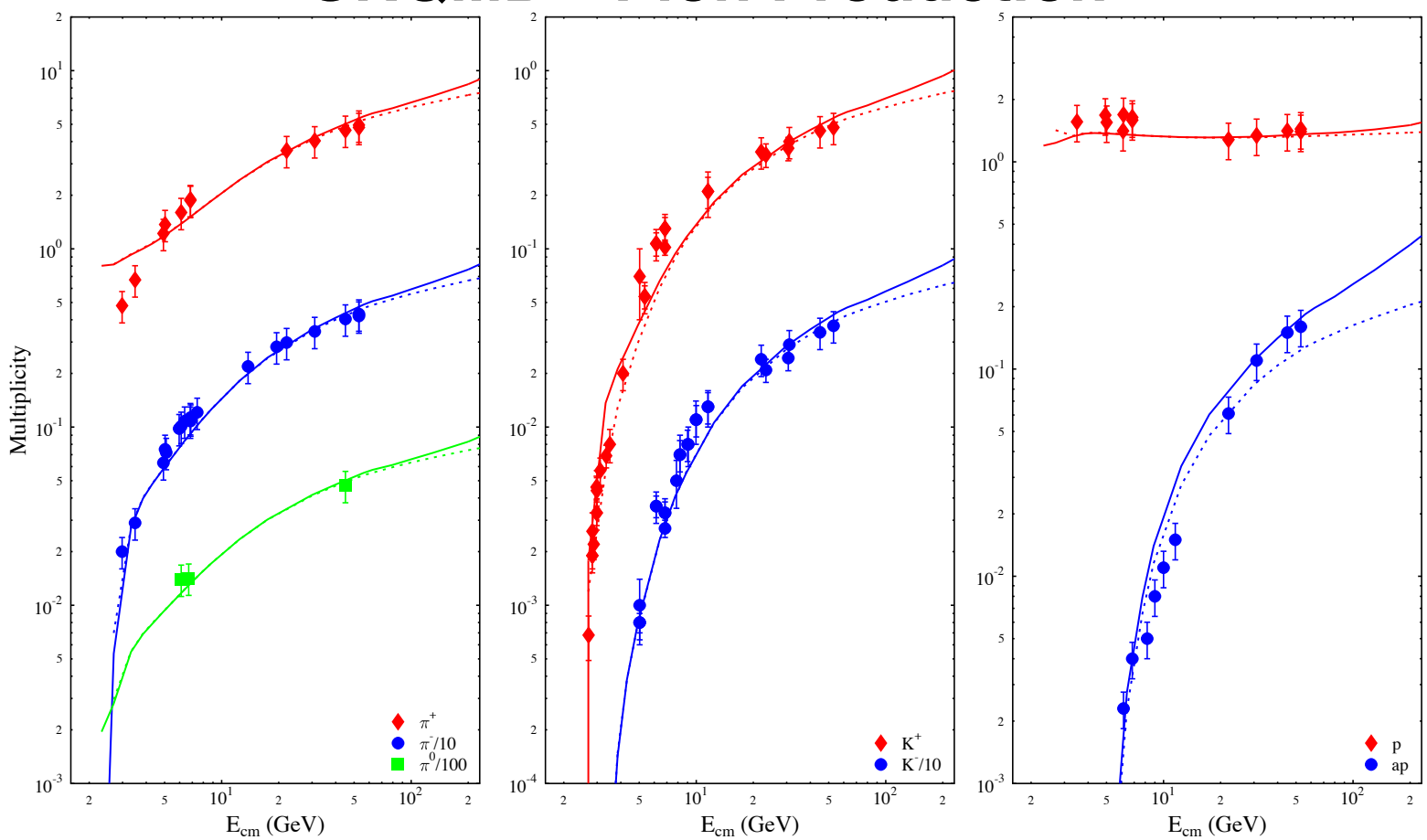
State of the Art: URQMD and similar…

straight-line trajectories + collisions

For Low energy

need mean fields (AMPT, PHSD….)

URQMD—Pion Production



Modeling Status: Hadron Cascade

Needed:

- More development of mean fields, Bose-enhancement "Open-source" or model comparison,
- Make scattering time delays and mean field consistent with EoS.

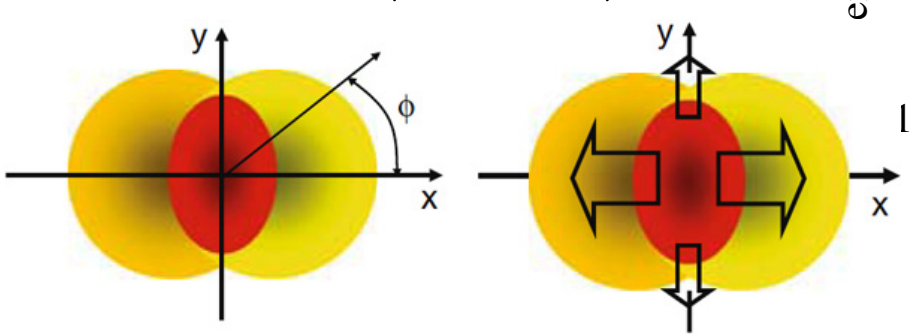
Observables and Critical Links

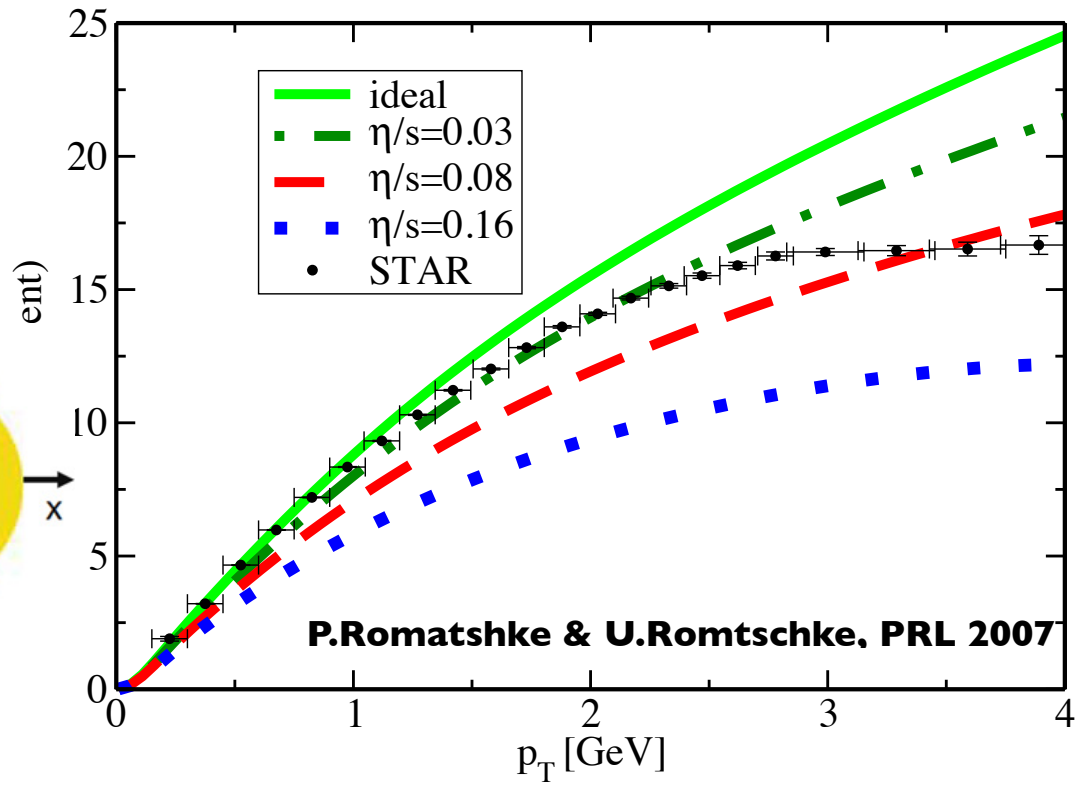
- **Spectra**
  sensitive to eq. of state, initial ε
- **Elliptic flow**
  strongly affected by viscosity
- **V_3,V_4,V_5…**
  initial state lumpiness
- **Femtoscopy**
  sensitive to eq. of state
- **Rare probes**
  color screening
- **EM probes**
  dissolution of rho, temperature
  charge composition, collision rate…
- **Longer-Range Correlations**
  sensitive to chemistry, phase structure
- **Jet Quenching**
  q-hat

Critical Links

Viscosity and Elliptic Flow

$$v_2 \equiv \langle \cos 2\phi \rangle$$





Critical Links

v2 depends on ....

- **viscosity**
- **saturation model**
- **pre-thermal flow**
- **Eq. of State**
- **T-dependence of η/s**
- **initial T_{xx}/T_{zz}**
- . . . .

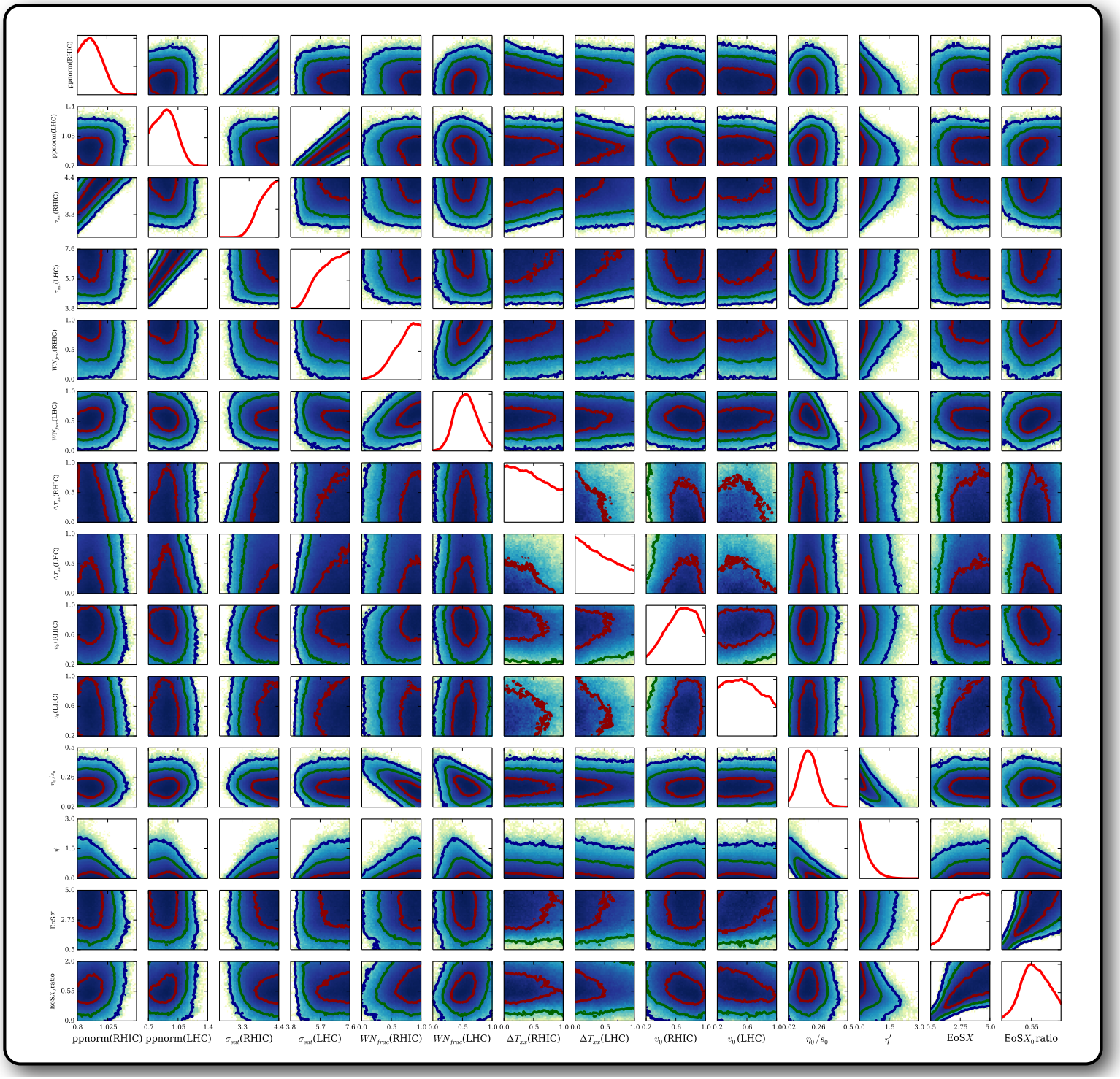
e.g. Drescher, Dumitru, Gombeaud and Ollitrault
PRC 2007

Correct Treatment:
***<u>GLOBAL</u>* MCMC analysis**

Strategy:
Model Emulators

Global Statistical Analysis

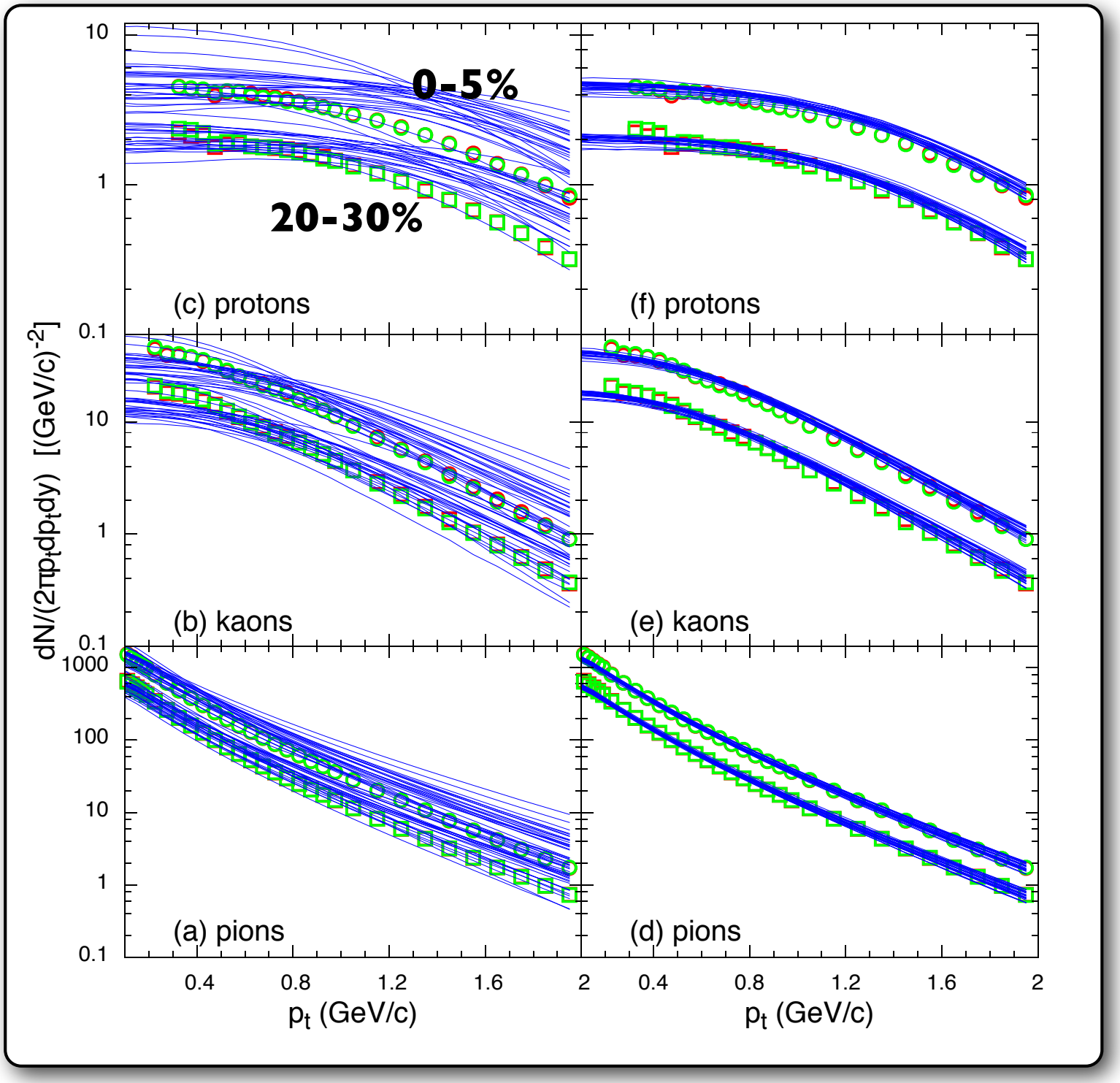
S.P., E. Sangaline, H. Wang & P.Sorensen
PRL 2015



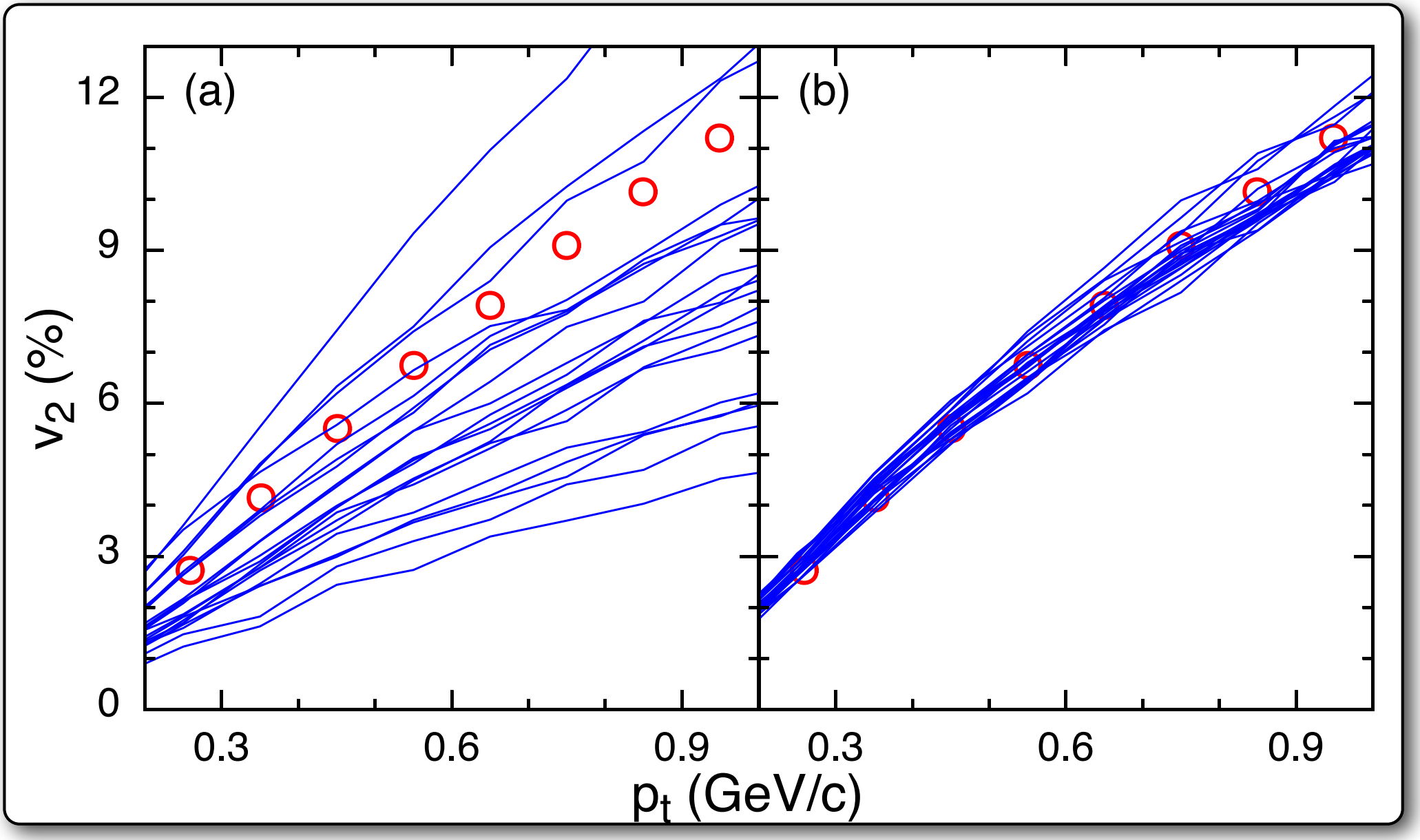
Goal: *N*-dimensional likelihood distribution

See Talk by Steffen Bass

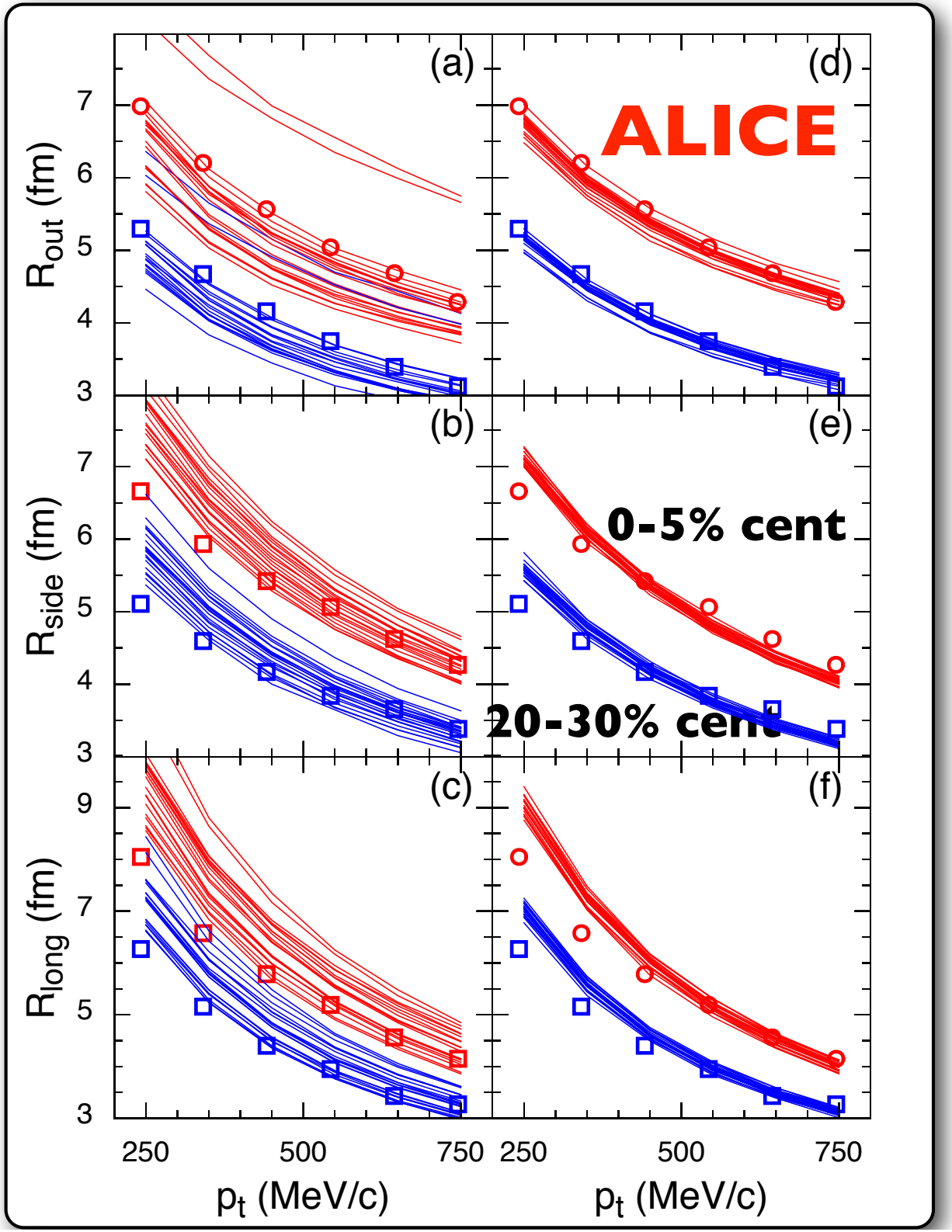
Spectra

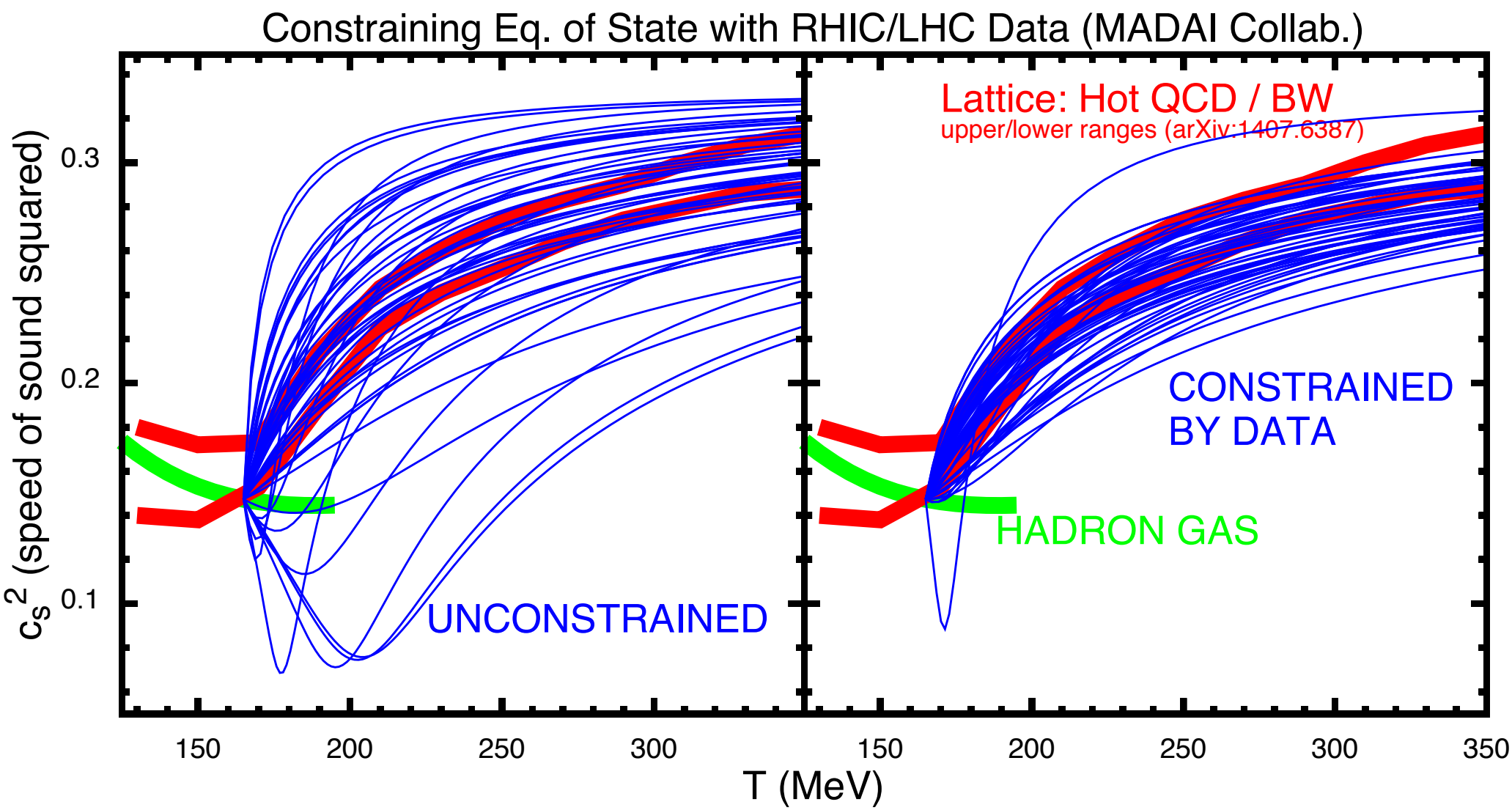

0-5%
20-30%
(c) protons
(f) protons
(b) kaons
(e) kaons
(a) pions
(d) pions
dN/(2πp_t dp_t dy) [(GeV/c)^-2]
p_t (GeV/c)
p_t (GeV/c)
10
1
0.1
1000
100
0.4
0.8
1.2
1.6
2

V_2 (elliptic flow)



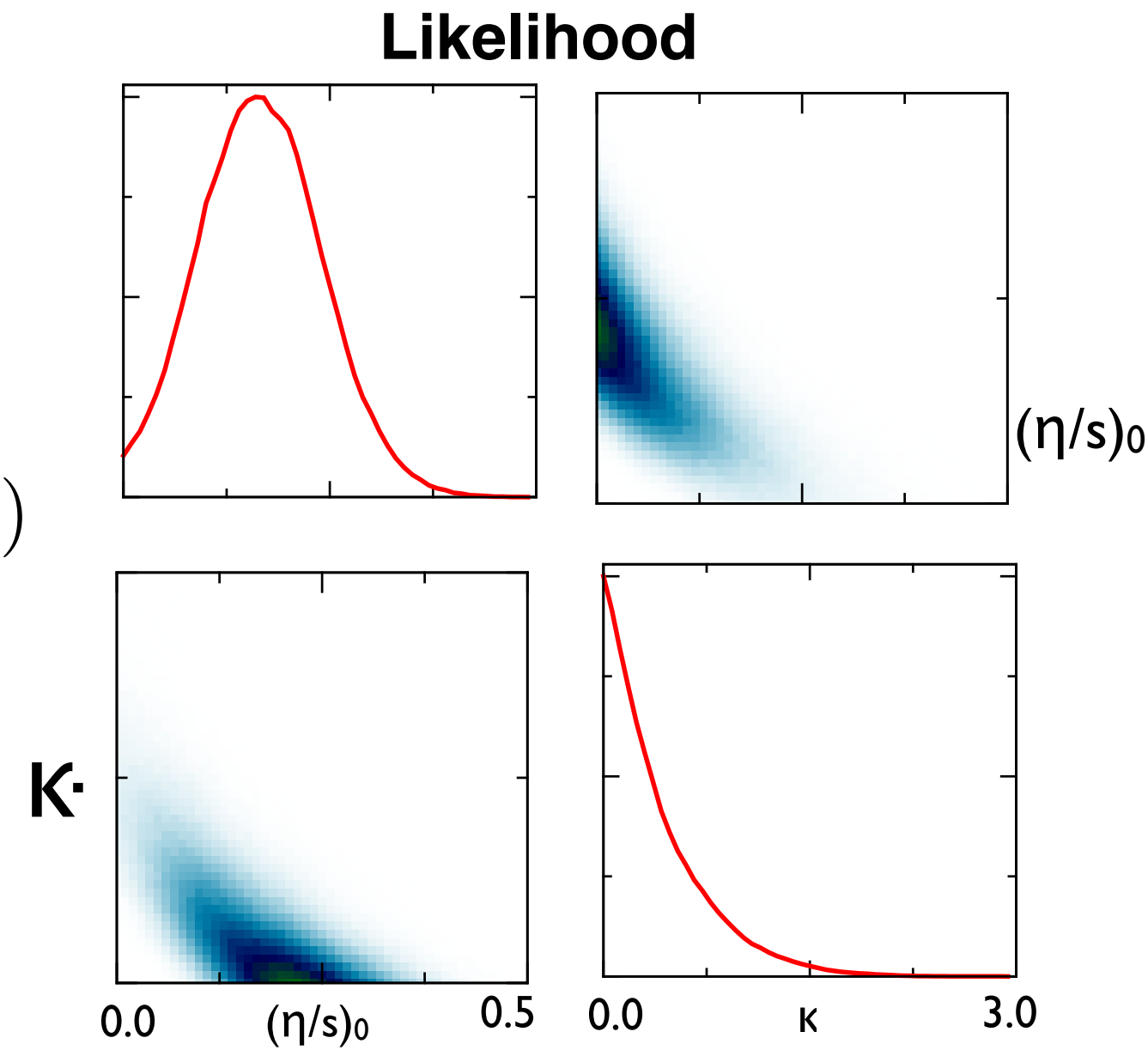
Femtoscopic Radii



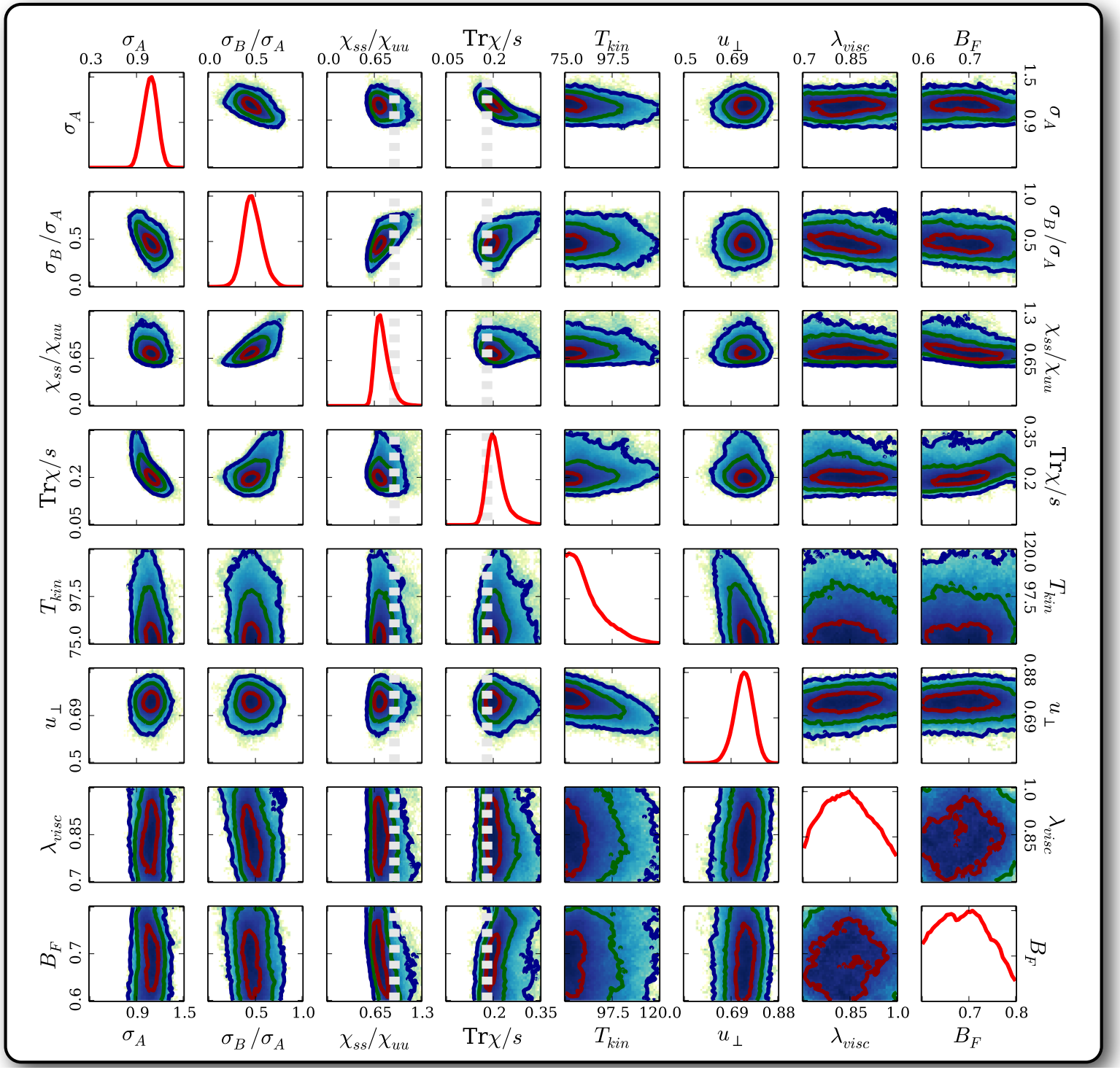

Constraining Eq. of State with RHIC/LHC Data (MADAI Collab.)
Lattice: Hot QCD / BW
upper/lower ranges (arXiv:1407.6387)
CONSTRAINED
BY DATA
HADRON GAS
UNCONSTRAINED
c_s^2 (speed of sound squared)
0.1
0.2
0.3
150
200
250
300
350
T (MeV)

Viscosity from spectra, femtoscopy, elliptic flow at RHIC & LHC

$$\eta/s = (\eta/s)_0 + \kappa \ln(T/165)$$

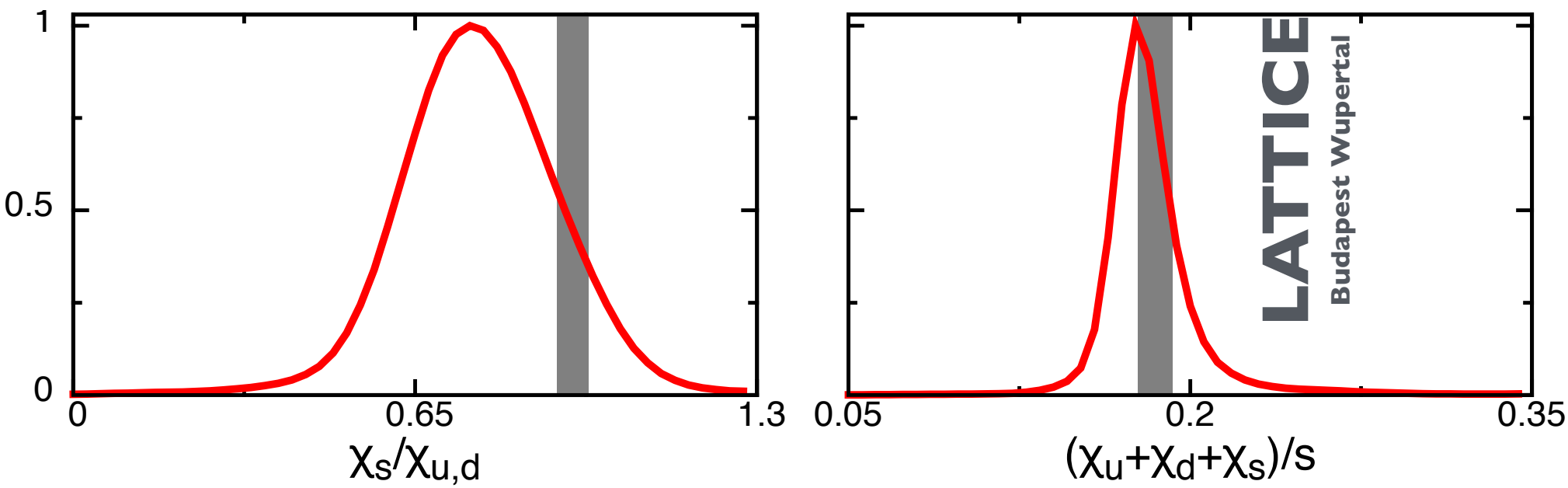


Analysis of STAR charge balance functions



Charge fluctuations from charge correlation measurements

Likelihood from Data Comparison



What we know (so far)

Strong Evidence for:

- SE tensor(**pressure**) near **equilibration**
- **chemically equilibrated** QGP
- hadrons melt (ρ and J/Ψ)
- extremely good **liquid** with low viscosity
- strong jet damping → **strongly interacting** liquid

What we don’t know (so far)

- **Baryon dependence of EoS (Critical point?)**
  Can significant fluctuations develop in heavy-ion environment?

- **Chiral symmetry:**
  Is there a “hadronic” phase with symmetry restored? What would that it look like? How would we study it experimentally?

- **q-hat**

- **Diffusion Constant**

What we need:

- **Ideas for signals of chiral symmetry restoration**
- **EM Probes Theory**
  Difficult for 150 < T < 225
- **BES Modeling**
  characterize initial state
  fluctuating hydro
  hadrons with mean field
  large-scale statistical analysis

Suggested Reading

◆C. Gale, S. Jeon, B.Schenke, P. Tribedy and R. Venugopalan,
"Event-by-event anisotropic flow in heavy-ion collisions from combined Yang-Mills and viscous fluid dynamics,"
PRL 110 no. 1, 012302 (2013)

◆J. I. Kapusta, B. Muller and M. Stephanov,
"Relativistic Theory of Hydrodynamic Fluctuations with Applications to Heavy Ion Collisions,"
PRC 85, 054906 (2012)

◆C. Young, J. I. Kapusta, C. Gale, S. Jeon and B. Schenke,
"Thermally Fluctuating Second-Order Viscous Hydrodynamics and Heavy-Ion Collisions,"
PRC 91, no. 4, 044901 (2015)

◆L.~P.~Csernai and J.~I.~Kapusta,
"Nucleation of relativistic first order phase transitions," PRD 46, 1379 (1992).

◆D. Bazow, U. W. Heinz and M. Strickland,
"Second-order (2+1)-dimensional anisotropic hydrodynamics," PRC 90, no. 5, 054910 (2014)

◆G. S. Denicol, H. Niemi, E. Molnar and D. H. Rischke,
"Derivation of transient relativistic fluid dynamics from the Boltzmann equation,"
Phys. Rev. D 85, 114047 (2012)

◆P. Danielewicz and M. Gyulassy,
"Dissipative Phenomena in Quark Gluon Plasmas," PRD 31, 53 (1985).

◆L. P. Csernai, J. I. Kapusta and L. D. McLerran,
"On the Strongly-Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions," PRL 97, 152303 (2006)

◆P. Romatschke and U. Romatschke,
"Viscosity Information from Relativistic Nuclear Collisions: How Perfect is the Fluid Observed at RHIC?,"
PRL 99, 172301 (2007)

◆B. Ling, T. Springer and M. Stephanov,
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◆S. Pratt, W. P. McCormack and C. Ratti,
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