DYNAMICS IN PHASES OF QCD

FOCUS
Discern or Validate Bulk Properties of QCD Matter through Heavy-Ion Collisions

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Supported by Dept. of Energy, Office of Science
DE-FG02-03ER41259
Fundamental Properties of QCD Matter

1. *Eq. of State  
   + baryon number dependence
2. *Chemistry  
   + Color Screening
3. *Chiral Symmetry Restoration
4. Viscosity
5. Diffusion Constant
6. Jet Damping

*Reliably extracted from lattice
1. Eq. of State

First order?

**BAG MODEL**

**LATTICE (BW Collab)**
1. Eq. of State + Baryons
2. Chemistry

52 light degrees of freedom
● 36 quarks
  (3 colors, 2 spins, part/antipart, uds)
● 16 gluons
  (8 colors, 2 spins)
● ignore leptons, photons or heavy quarks

Inside $1 \lambda_{th}^3$,
● Bose condensed $^4$He: one particle
● Photon gas: 2 particles
● Parton gas: 52 particles

\[
\begin{align*}
  n &\sim T^3 \\
  \epsilon &\sim T^4 \\
  P &= \epsilon / 3 \sim T^4 \\
  s &= \frac{P + \epsilon}{T} \sim T^3
\end{align*}
\]
2. Chemistry

Parton number undefined in interacting system and \( \langle \rho_{u,d,s} \rangle = 0 \)
so, considers fluctuations:

\[
\chi_{ab} \equiv \frac{\langle Q_a Q_b \rangle}{V}
\]

For parton gas (non-interacting)

\[
\chi_{ab} = (n_a + n_{\bar{a}}) \delta_{ab} , \ \chi / s = \text{constant for } m = 0
\]

For hadron gas (non-interacting)

\[
\chi_{ab} = \sum_\alpha n_\alpha q_{\alpha a} q_{\alpha b}
\]
2. Chemistry

behavior approaches parton gas at high $T$
2. Chemistry: Color Screening

Debye Screening: Charge $+Q_0$ in plasma, will attract negative charges

$$\Delta n_e(r) = n_e(e^{-V(r)/T} - 1),$$

$$\approx -n_0 V(r) / T$$

$$V(r) = \frac{-eQ_0}{4\pi\varepsilon_0 r} e^{-r/\lambda},$$

$$\lambda_{\text{Debye}} = \sqrt{\frac{\varepsilon_0 T}{n_0 e^2}}$$

Includes contribution from screening charges

HW: Show this form is consistent with Gauss’s Law.

Screens confining potential $\rightarrow$ “free color charges”
2. Chemistry: Color Screening

Free energy vs. separation

For $T > 200$ MeV, charges can separate
3. Chiral Symmetry

\[ \mathcal{L} = \overline{\Psi}_a (i \partial_\mu - eA_\mu) \gamma^\mu \Psi_a + \cdots \]

\[ \Psi \rightarrow e^{i \gamma_5 \phi} \Psi \]

\[ \Psi \rightarrow e^{i \gamma_5 \tau \cdot \phi} \Psi \]

Invariant to axial and iso-axial rotations (if \( m=0 \))

Noether’s theorem leads to conserved currents

\[ j_5^\mu = \overline{\Psi} \gamma^\mu \gamma_5 \Psi \]

ruined by chiral anomaly

\[ j_{5a}^\mu = \overline{\Psi} \gamma^\mu \gamma_5 \tau_a \Psi \]
3. Chiral Symmetry (hadronic perspective)

\[ \mathcal{L} = \frac{-1}{2} \left\{ \sigma \partial^2 \sigma + \bar{\pi} \partial^2 \cdot \bar{\pi} \right\} + \frac{1}{2} M_0^2 \left\{ \sigma^2 + |\bar{\pi}|^2 \right\} - \frac{\lambda}{4} \left\{ \sigma^2 + |\bar{\pi}|^2 \right\}^2 \\
+ g_{\pi N} \left( \sigma \bar{\Psi} \Psi + i \bar{\pi} \cdot \bar{\Psi} \bar{\tau} \gamma_5 \Psi \right) \\
j_a^\mu = \bar{\Psi} \gamma_5 \gamma^\mu \tau_a \Psi + \sigma \partial^\mu \pi_a + \pi_a \partial^\mu \sigma \]

\[ M_N \approx g_{\pi N} \langle \sigma \rangle, \langle \sigma \rangle = f_\pi = 93 \text{ MeV} \]
3. q-qbar condensate, is related to sigma condensate

Condensate couples to quarks and gives them mass
4. Viscosity

\[ \partial_t T_{00} = -\partial_x T_{0x} - \partial_y T_{0y} + \partial_z T_{0z} \]
\[ \partial_t T_{0x} = \partial_x T_{xx} + \partial_y T_{yx} + \partial_z T_{zx} \]

Local conservation of \( E \) and \( P \)

\[ T_{i\neq j} = 0 \]
\[ T_{ij} = P\delta_{ij} - \eta(\partial_i v_j + \partial_j v_i) - \zeta \nabla \cdot \vec{v} \]

Ideal hydro Navier-Stokes
\[ \eta = \text{shear viscosity} \]
\[ \zeta = \text{bulk viscosity} \]
4. Viscosity

Shear represents friction between layers of fluid:

\[
\frac{d}{dt} P_x = A_y \eta \partial_y v_x
\]

Bulk describes dissipation of diverging flow:

\[
\delta E = -P \delta V + \zeta \nabla \cdot \bar{v} \delta V
\]
4. Viscosity (Kubo relations)

For gas, correlation of particles with themselves multiplied by relaxation time:

\[ \eta = \frac{\tau_\eta}{T} \int d^3 r \left\langle T_{xy}(0,0)T_{xy}(\vec{r}, t = 0) \right\rangle \]

\[ = \frac{\tau_\eta}{T} \sum_\alpha (2S_\alpha + 1) \int \frac{d^3 p}{(2\pi)^3} e^{-E/T} \frac{p_x^2 p_y^2}{E^2} \]

Kubo

(\(\sigma=22\) mb, \(\tau_\eta=1.9\tau_{\text{coll}}\))

Anybody's guess

Perturbative QCD
4. Viscosity

similar behavior to other fluids near $T_c$

Kubo
($\sigma=22\,\text{mb, } \tau_\eta=1.9\tau_{\text{coll}}$)

Anybody's guess

Perturbative QCD

\begin{align*}
\eta/s & = 1.5 \\
\eta/s & = 1.25 \\
\eta/s & = 1.0 \\
\eta/s & = 0.75 \\
\eta/s & = 0.5 \\
\eta/s & = 0.25 \\
\eta/s & = 0 \\
\end{align*}

\begin{align*}
\text{T (MeV)} & = 150 \\
\text{T (MeV)} & = 200 \\
\text{T (MeV)} & = 250 \\
\text{T (MeV)} & = 300 \\
\end{align*}

\begin{align*}
\frac{(T-T_0)/T_0} & = -1.0 \\
\frac{(T-T_0)/T_0} & = -0.5 \\
\frac{(T-T_0)/T_0} & = 0.0 \\
\frac{(T-T_0)/T_0} & = 0.5 \\
\frac{(T-T_0)/T_0} & = 1.0 \\
\end{align*}
4. Viscosity

Kubo
(σ=22 mb, τ_η=1.9τ_{coll})

Anybody’s guess

Perturbative QCD

Some values:
0.08 : λ_{therm}  λ_{mfp} (Danielewicz and Gyulassy)
1/4π : AdS/CFT (Kovton, Starinets, Son)
in principal can go to zero
MODELS

Pre-Equilibrium (0<τ<1 fm/c)
mixture of gluonic fields and partons
not decisively understood

Hydrodynamic (T>160 MeV)
relativistic, viscous
most strongly affects results

Hadronic cascade (T<160)
microscopic evolution of \( f(p,r,t) \) for each species
hydro can’t handle 100 species flowing differently

Jets, rare probes, bulk correlations & EM probes
calculations overlaid on hydro evolution

Femtoscopic correlations
calculated from final \( f(p,r,t) \)
Modeling Status: Initial State

- **Low pt — coherent fields (CGC, flux tubes)**
  - High pt — partonic
  - Where is the boundary?

- **How and when do fields thermalize?**
  - Quarks appear quickly — but how?
  - Initial Flow?

- **Lower energy (BES) collisions**
  - Not even good parameterizations
Modeling Status: Initial State
Lumpy Hydro

MC-KLN

IP-Glasma

MC-Glauber

Gale, Jeon, Schenke, Tribidy & Venugopalan, 2014

Something’s right! — But what?
Modeling Status: Hydro

- Non-ideal Hydro

\[ \partial_\mu T^{\mu\nu} = 0, \quad T_{ij} = P \delta_{ij} + \pi_{ij} + \Pi \delta_{ij} \]

- Navier-Stokes

\[ \pi_{ij} = \eta \omega_{ij}, \quad \omega_{ij} \equiv \partial_i v_j + \partial_j v_i - (2/3) \delta_{ij} \nabla \cdot v \]

- Israel-Stewart (+ others)

\[ \partial_t \pi_{ij} = -\frac{1}{\tau_\pi} (\pi_{ij} - \eta \omega_{ij}) - \gamma_\pi \omega_{ik} \pi_{kj} - \kappa_\pi \nabla \cdot v \pi_{ij} \]
Modeling Status: Hydro

hadron cascade vs. Israel-Stewart

Israel-Stewart very successful for $\pi_{ij} < P/2$
(Minnesota, Frankfurt, McGill, Ohio State, Kent State,...)

S.P., A.Baez and J. Kim
PRC 2017
Correlations of Conserved Quantities

\[ E_t, p_x, p_y, Q, B, S \]

Susceptibilities come from lattice
(novel behavior at critical point)

\[ C(\vec{r}_1, \vec{r}_2) = \langle (\rho(\vec{r}_1) - \bar{\rho}(\vec{r}_1))(\rho(\vec{r}_2) - \bar{\rho}(\vec{r}_2)) \rangle \]
\[ = \chi(\vec{r}) \delta(\vec{r}_1 - \vec{r}_2) + C'(\vec{r}_1, \vec{r}_2), \]

\[ \partial_t C'(\vec{r}_1, \vec{r}_2) = \langle \nabla \cdot \vec{j}(\vec{r}_1)(\rho(\vec{r}_2) - \bar{\rho}(\vec{r}_2)) \rangle + \langle (\rho(\vec{r}_1) - \bar{\rho}(\vec{r}_1)) \nabla \cdot \vec{j}(\vec{r}_2) \rangle \]
\[ - \partial_t \chi(\vec{r}) \delta(\vec{r}_1 - \vec{r}_2). \]

“source function”

Given \( j \) one can predict measurable correlations in final state from lattice input (\( \chi \)).
Modeling Status: Hydro
Correlations of Conserved Quantities

Only simple models thus far, Need to:
1. overlay onto 3D hydro
2. consider all conserved quantities
3. describe n-body correlations
One promising approach: Noisy hydrodynamics
Mueller, Stephanov, Kapusta, Young, ...

\[ \vec{j}(\vec{r}, t) = \langle \vec{j}(\vec{r}, t) \rangle + \vec{j}^{(n)}(\vec{r}, t), \]
\[ \langle \vec{j}^{(n)}(\vec{r}, t) \vec{j}^{(n)}(\vec{r}', t') \rangle = 2\sigma T \delta(\vec{r} - \vec{r}') \delta(t - t') \]

Allows 1-body description to reproduce 2-body correlations to linear level
But what non-linearity, or 3-body, 4-body…?
Modeling Status: Hadron Cascade

State of the Art: URQMD and similar…
straight-line trajectories + collisions
For Low energy
need mean fields (AMPT, PHSD….)

**URQMD—Pion Production**
Modeling Status: Hadron Cascade

Needed:

‣ More development of mean fields, Bose-enhancement “Open-source” or model comparison,
‣ Make scattering time delays and mean field consistent with EoS.
Observables and Critical Links

- **Spectra**
  sensitive to eq. of state, initial $\varepsilon$

- **Elliptic flow**
  strongly affected by viscosity

- $V_3, V_4, V_5...$
  initial state lumpiness

- **Femtoscopy**
  sensitive to eq. of state

- **Rare probes**
  color screening

- **EM probes**
  dissolution of rho, temperature
  charge composition, collision rate...

- **Longer-Range Correlations**
  sensitive to chemistry, phase structure

- **Jet Quenching**
  q-hat
Critical Links

Viscosity and Elliptic Flow

\[ v_2 \equiv \langle \cos 2\phi \rangle \]

The above fit parameters to experimental data on the integrated and minimum bias elliptic flow in line with the trend found in \[19\].

Starting from ideal hydrodynamics with results for protons cannot be expected to match experimental data. Starting from ideal hydrodynamics with VH for finite baryon chemical potential, prohibiting us to distinguish particles from anti-particles. As a consequence, our model does not include a Bose enhancement effect from the vorticity term.

To further suppress effects the results shown. Note that this effect of rescaling, which we have ignored \[19\].

The results hardly change when assuming instead the only non-trivial vorticity is \( \eta \sim 0.02 \), which can be important\[27\]. From Fig. 3 it can be seen that the effect of rescaling, which we have ignored \[19\].

To make contact with experiment, the hydrodynamic model for various viscosity ratios \[25\] data on minimum bias collisions at \( s = 200 \text{ GeV} \), compared to our hydrodynamic model for various viscosity ratios \[25\] data on minimum bias collisions at \( s = 200 \text{ GeV} \).

Note that for simplicity our model does not include a Bose enhancement effect from the vorticity term. Since modeling any space-time dependence would necessarily introduce more unknown parameters. Therefore, results on total multiplicity and mean transverse momentum as mean values over the entire system evolution as a function of total number of participants .

Error bars for PHOBOS \[24\] data on integrated and minimum bias elliptic flow are chosen such that the experimental data on integrated and minimum bias elliptic flow is strong, in line with estimates from Ref. \[17\]. Data on integrated and minimum bias elliptic flow is shown in Fig. 1. We thus have some confidence that our numerical algorithm solves Eq. (2) correctly. Also, our code passes the fluctuation test from Ref. \[16\], see also \[17\]. For simplicity, we use a single Cooper-Frye freeze-out mechanism \[20\] (adapted to VH conditions). Particles produced by a viscosity of \( \eta/s = 0.16 \) for STAR only statistical errors are shown.

For the above initial conditions, we have noted that effects of rescaling, which we have ignored \[19\]. This is the relativistic generalization of the vorticity equation, well known in atmospheric sciences \[26\]. Shown are results for ideal hydrodynamics and VH for particles produced by a viscosity of \( \eta/s = 0.16 \) for STAR only statistical errors are shown.

Critical Links

Viscosity and Elliptic Flow

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Critical Links

$v^2$ depends on ....
- viscosity
- saturation model
- pre-thermal flow
- Eq. of State
- T-dependence of $\eta/s$
- initial $T_{xx}/T_{zz}$
- . . . .

Correct Treatment:

*GLOBAL* MCMC analysis

Strategy:
Model Emulators

e.g. Drescher, Dumitru, Gombeaud and Ollitrault
PRC 2007
Global Statistical Analysis

S.P., E. Sangaline, H. Wang & P. Sorensen
PRL 2015

Goal:
$N$-dimensional likelihood distribution

See Talk by Steffen Bass
Spectra

- 0-5%
- 20-30%

- (c) protons
- (f) protons
- (b) kaons
- (e) kaons
- (a) pions
- (d) pions

$\frac{dN}{2\pi dpd\Omega} [\text{GeV/c}^2]$ vs $p_t \text{ (GeV/c)}$
$V_2$ (elliptic flow)
Femtoscopic Radii
Viscosity from spectra, femtoscopy, elliptic flow at RHIC & LHC

\[ \frac{\eta}{s} = \left( \frac{\eta}{s} \right)_0 + \kappa \ln(T/165) \]
Analysis of STAR charge balance functions
Charge fluctuations from charge correlation measurements

Likelihood from Data Comparison

\[ \frac{\chi_s}{\chi_{u,d}} \]

\[ \frac{(\chi_u + \chi_d + \chi_s)}{s} \]
What we know (so far)

Strong Evidence for:
- SE tensor(\textit{pressure}) near \textit{equilibration}
- \textit{chemically equilibrated} QGP
- hadrons melt (\(\rho\) and \(J/\Psi\))
- extremely good \textit{liquid} with low viscosity
- strong jet damping \(\rightarrow\) \textit{strongly interacting} liquid
What we don’t know (so far)

‣ **Baryon dependence of EoS (Critical point?)**
  Can significant fluctuations develop in heavy-ion environment?

‣ **Chiral symmetry:**
  Is there a “hadronic” phase with symmetry restored? What would that look like? How would we study it experimentally?

‣ **q-hat**

‣ **Diffusion Constant**
What we need:

‣ Ideas for signals of chiral symmetry restoration
‣ **EM Probes Theory**
  Difficult for $150 < T < 225$
‣ **BES Modeling**
  characterize initial state fluctuating hydro
  hadrons with mean field
  large-scale statistical analysis
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