Quark self-energy in an ellipsoidally anisotropic quark-gluon plasma

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Introduction

- Anisotropic medium effects of the QGP contribute to the self-energy of the quarks.
- The real and imaginary parts of quark self-energy are related to the effective mass and the generalized decay rates of the quarks.
- The quark self-energy for the case of a QGP with spheroidal momentum anisotropy [1] has been calculated within HTL approximation before [2].
- Here, we calculate the quark self-energy for the more general case of the ellipsoidal momentum anisotropy [3].

Anisotropic Quark Self-Energy

The general expression for the gauge-independent retarded quark self-energy in a momentum-anisotropic system in the hard-loop (HL) approximation was first obtained in Ref. [4]

$$\Sigma(K) = \frac{C_F}{4} g^2 \int_{\mathbf{p}} \frac{f(\mathbf{p})}{|\mathbf{p}|} \frac{P \cdot \gamma}{P \cdot K},$$

where

- g is the QCD coupling, and the distribution function $f(\mathbf{p})$ is the sum of the momentum distributions for quarks and gluons $f(\mathbf{p}) \equiv 2\,(n(\mathbf{p}) + \bar{n}(\mathbf{p})) + 4n_g(\mathbf{p})$.
- $P = (\omega_p, \mathbf{p})$ and $K = (\omega, \mathbf{k})$ are the Minkowski-space partonic momentum four-vectors,
- $C_F \equiv (N_c^2 - 1)/2N_c$, and $\int_{\mathbf{p}} \equiv \int d^3p/(2\pi)^3$.

Ellipsoidal momentum-space anisotropy

For the case of ellipsoidal anisotropy, the local rest frame distribution function $f(\mathbf{p})$ is parametrized by

$$f(\mathbf{p}) = f_{\boldsymbol{\xi}}(\mathbf{p}) = f_{\rm iso}\left(\frac{1}{\lambda}\sqrt{\mathbf{p}^2 + \xi_x(\mathbf{p}\cdot\hat{\mathbf{x}})^2 + \xi_y(\mathbf{p}\cdot\hat{\mathbf{y}})^2 + \xi_z(\mathbf{p}\cdot\hat{\mathbf{z}})^2}\right),$$

where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are Cartesian unit vectors in the local rest frame of the matter, $\boldsymbol{\xi} \equiv (\xi_x, \xi_y, \xi_z)$ are anisotropy parameters corresponding to three spatial directions, and λ is a temperature-like scale.
In this parametrization, $f_{\rm iso}$ is a general isotropic distribution function which reduces to the appropriate equilibrium distribution function in the isotropic equilibrium limit ($\boldsymbol{\xi} = 0$).

Integrations

In the ellipsoidally-anisotropic case, for a conformal system there are only two physical anisotropy directions (transverse and longitudinal). This amounts to rearranging the parameters and setting $\frac{\xi_z - \xi_y}{1+\xi_y} \to \xi_1$, $\frac{\xi_x - \xi_y}{1+\xi_y} \to \xi_2$, $\frac{\lambda}{\sqrt{1+\xi_y}} \to \lambda$.
Then, we obtain

$$\Sigma(K) = \frac{m_q^2}{4\pi}\int d\Omega \left(1 + \xi_2(\hat{p}\cdot\hat{\mathbf{x}})^2 + \xi_1(\hat{p}\cdot\hat{\mathbf{z}})^2\right)^{-1} \frac{P\cdot\gamma}{P\cdot K}.$$

where

$$m_{\rm q}^2 = \frac{g^2 C_F}{8\pi^2}\int_0^\infty dp\, p\, f_{\rm iso}\left(\frac{p}{\lambda}\right).$$

As a result, all dependence on the form of the underlying isotropic distribution function is subsumed into m_q. Then, defining $\Sigma(K) = \gamma^0\Sigma_0 + \boldsymbol{\gamma}\cdot\boldsymbol{\Sigma}$, and $x = \cos\theta$,

$$\Sigma^i(K) = \frac{m_q^2}{4\pi}\int_0^{2\pi} d\phi \int_{-1}^{1} dx \, \frac{1}{A(1 + s\cos^2\phi)} \, \frac{v^i}{a - b\cos\phi - c\sin\phi},$$

where

$$a = \frac{\omega}{k} - x\cos\theta_k, \qquad A = 1 + \xi_1 x^2,$$
$$b = \sin\theta_k\cos\phi_k\sqrt{1-x^2}, \qquad s = \frac{\xi_2(1-x^2)}{A},$$
$$c = \sin\theta_k\sin\phi_k\sqrt{1-x^2}, \qquad v = (1,\ \sqrt{1-x^2}\cos\phi,\ \sqrt{1-x^2}\sin\phi,\ x).$$

Splitting the integrand using a partial-fraction decomposition, the self-energy components can be written as

$$\Sigma^i = \frac{m_q^2}{4\pi k}\int_{-1}^{1} dx \sum_{j=1}^{8} \lambda_j^i \, D_j \qquad (i = 0, 1, 2, 3),$$

where D_j's are single fractions which are easier to integrate over ϕ, and λ_j^i's depend only on the x variable.

Final Results

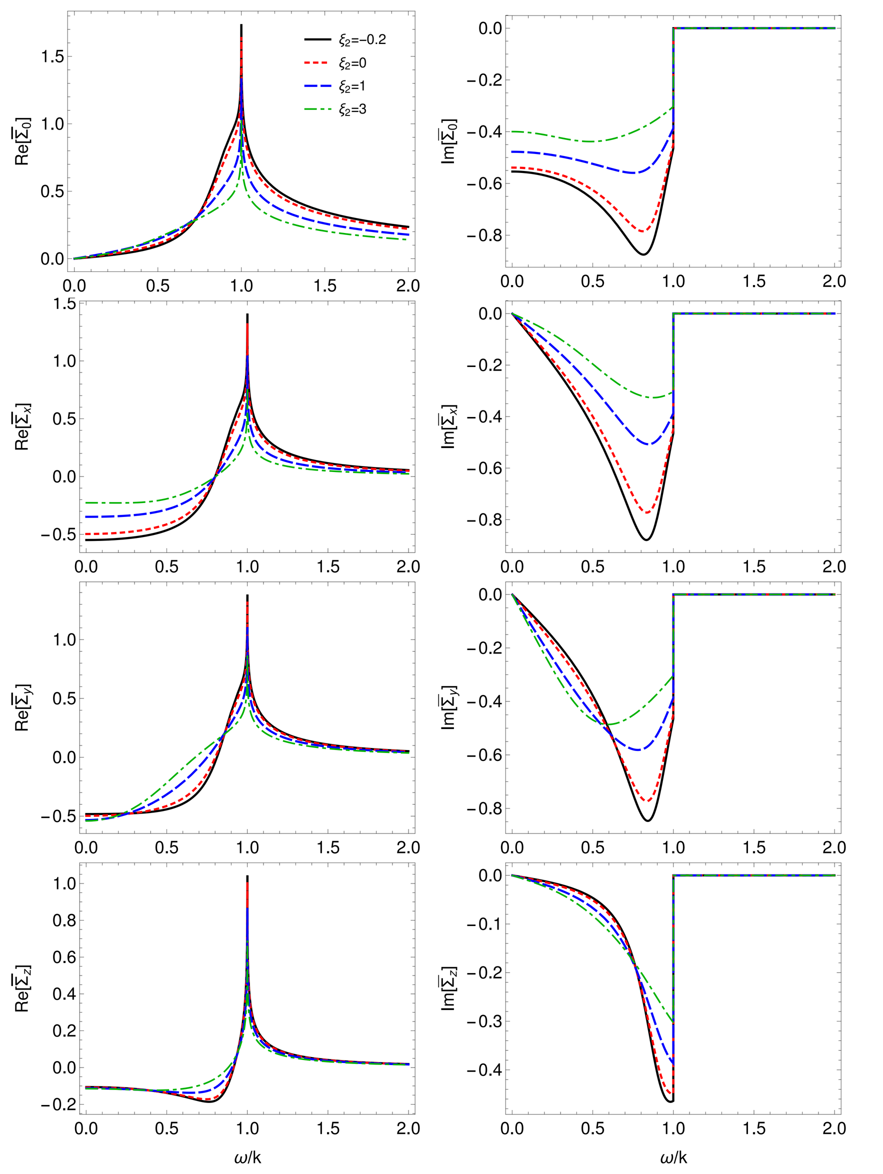
After integration over ϕ, one can express the self-energy components as one-dimensional integrals over the variable $x \equiv \cos\theta$, we get e.g.

$$\Sigma_0 = \frac{m_q^2}{2k}\int_{-1}^{1} dx\, \frac{1}{A\rho}\left[\alpha_0\sqrt{\frac{(a+b)^2}{a^2-(b^2+c^2)}} + s\beta_0\sqrt{\frac{1}{1+s}}\right],$$
$$\Sigma_x = \frac{m_q^2}{2k}\int_{-1}^{1} dx\, \frac{1}{A\rho}\left[\alpha_x\sqrt{\frac{(a+b)^2}{a^2-(b^2+c^2)}} + s\beta_x\sqrt{\frac{1}{1+s}}\right],$$
$$\Sigma_y = \frac{m_q^2}{2k}\int_{-1}^{1} dx\, \frac{1}{A\rho}\left[\alpha_y\sqrt{\frac{(a+b)^2}{a^2-(b^2+c^2)}} + s\beta_y\sqrt{\frac{1}{1+s}}\right],$$
$$\Sigma_z = \frac{m_q^2}{2k}\int_{-1}^{1} dx\, \frac{1}{A\rho}\left[\alpha_z\sqrt{\frac{(a+b)^2}{a^2-(b^2+c^2)}} + s\beta_z\sqrt{\frac{1}{1+s}}\right],$$

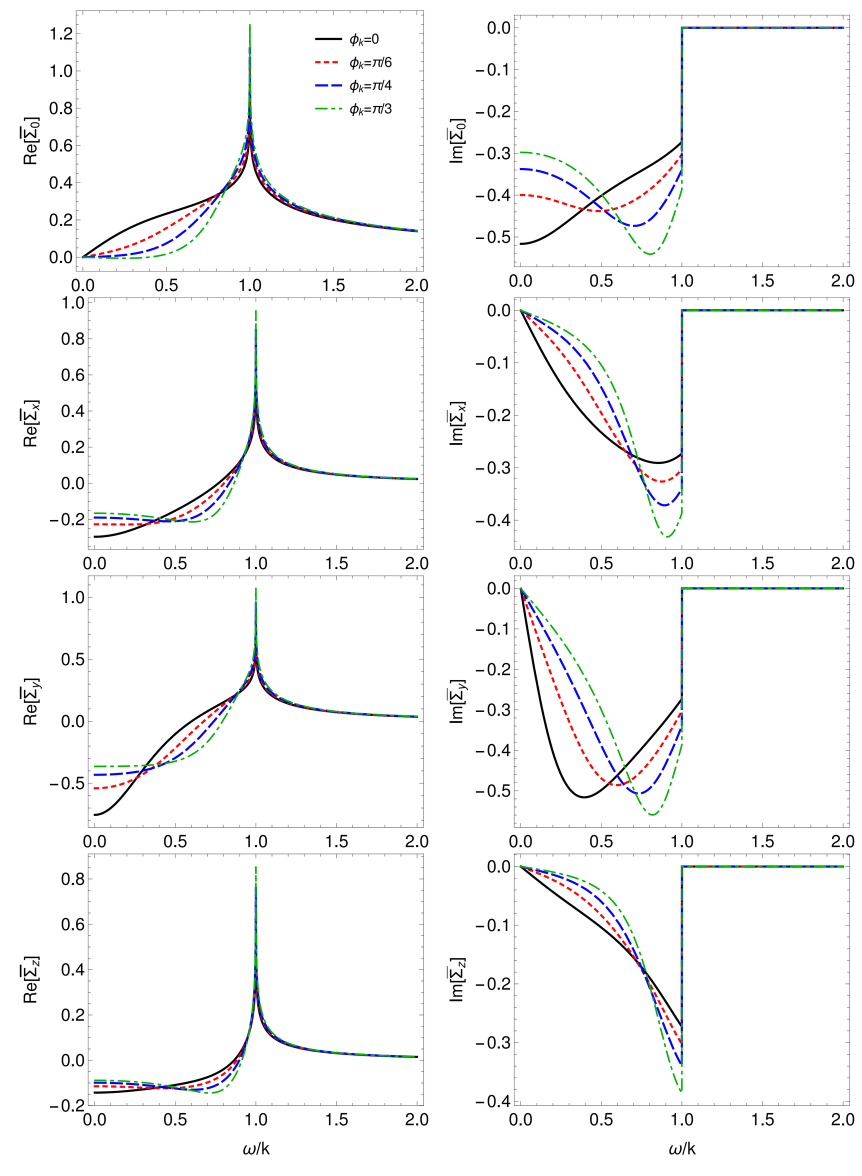
where the factors $\alpha_0, \alpha_x, \alpha_y, \alpha_z, \beta_0, \beta_x, \beta_y, \beta_z$, and ρ are also x-dependent.

Integrating numerically over x, we consider the following scaled components of quark self-energy

$$\bar{\Sigma}_0 \equiv \frac{k\Sigma_0}{m_{\rm q}^2}, \quad \bar{\Sigma}_x \equiv \frac{1}{\sin\theta_k\cos\phi_k}\frac{k\Sigma_x}{m_{\rm q}^2}, \quad \bar{\Sigma}_y \equiv \frac{1}{\sin\theta_k\sin\phi_k}\frac{k\Sigma_y}{m_{\rm q}^2}, \quad \bar{\Sigma}_z \equiv \frac{1}{\cos\theta_k}\frac{k\Sigma_z}{m_{\rm q}^2}.$$



- The real and imaginary parts of $\bar{\Sigma}_0$, $\bar{\Sigma}_x$, $\bar{\Sigma}_y$, and $\bar{\Sigma}_z$ as a function of ω/k for $\xi_1 = 10$, $\theta_k = \pi/3$, $\phi_k = \pi/6$, and $\xi_2 = \{-0.2, 0, 1, 3\}$.



- The real and imaginary parts of $\bar{\Sigma}_0$, $\bar{\Sigma}_x$, $\bar{\Sigma}_y$, and $\bar{\Sigma}_z$ as a function of ω/k for $\xi_1 = 10$, $\xi_2 = 3$, $\theta_k = \pi/3$, and $\phi_k = \{0, \pi/6, \pi/4, \pi/3\}$.

Conclusions

- We determined the self-energy of quarks in an ellipsoidally-anisotropic QGP by using the method of partial-fraction decomposition together with numerical evaluation of the resulting one-dimensional integrals.
- We find the dependence of self-energy components on both the polar and azimuthal angles.
- The self-energy modifications due to the transverse anisotropies induce additional angular dependence of the self-energy in transverse-momentum plane.
- These results set the stage for the calculation of the effects of ellipsoidal anisotropy on the QGP collective flow and photon spectra.

Acknowledgements/References

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