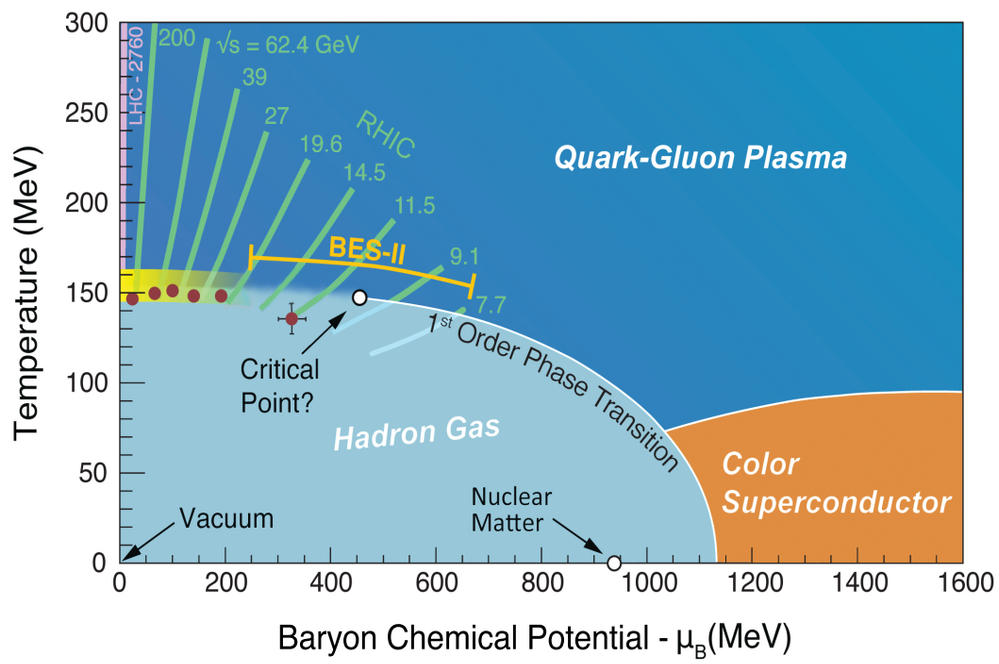
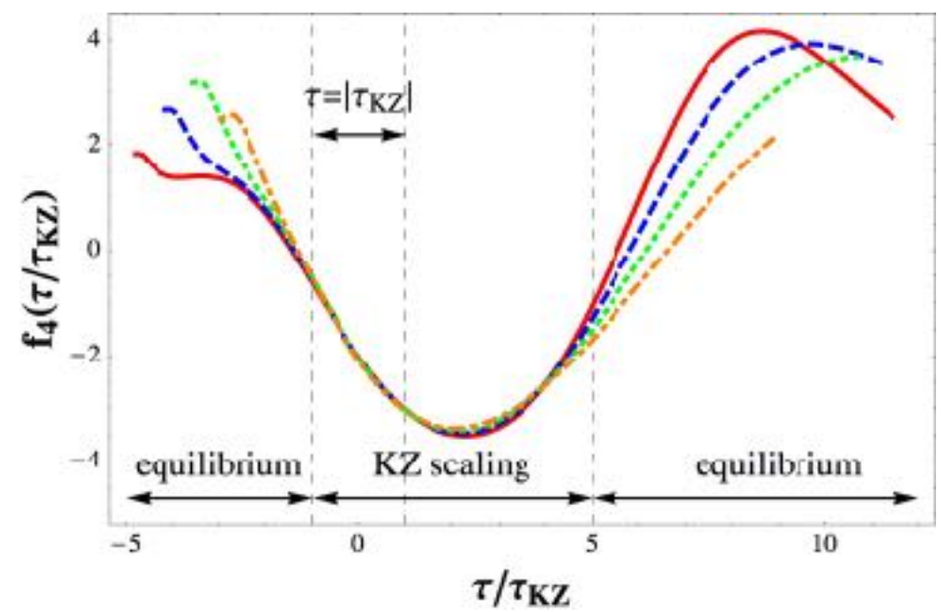
From complexity to universality: *Off-equilibrium scaling and search for QCD critical point*

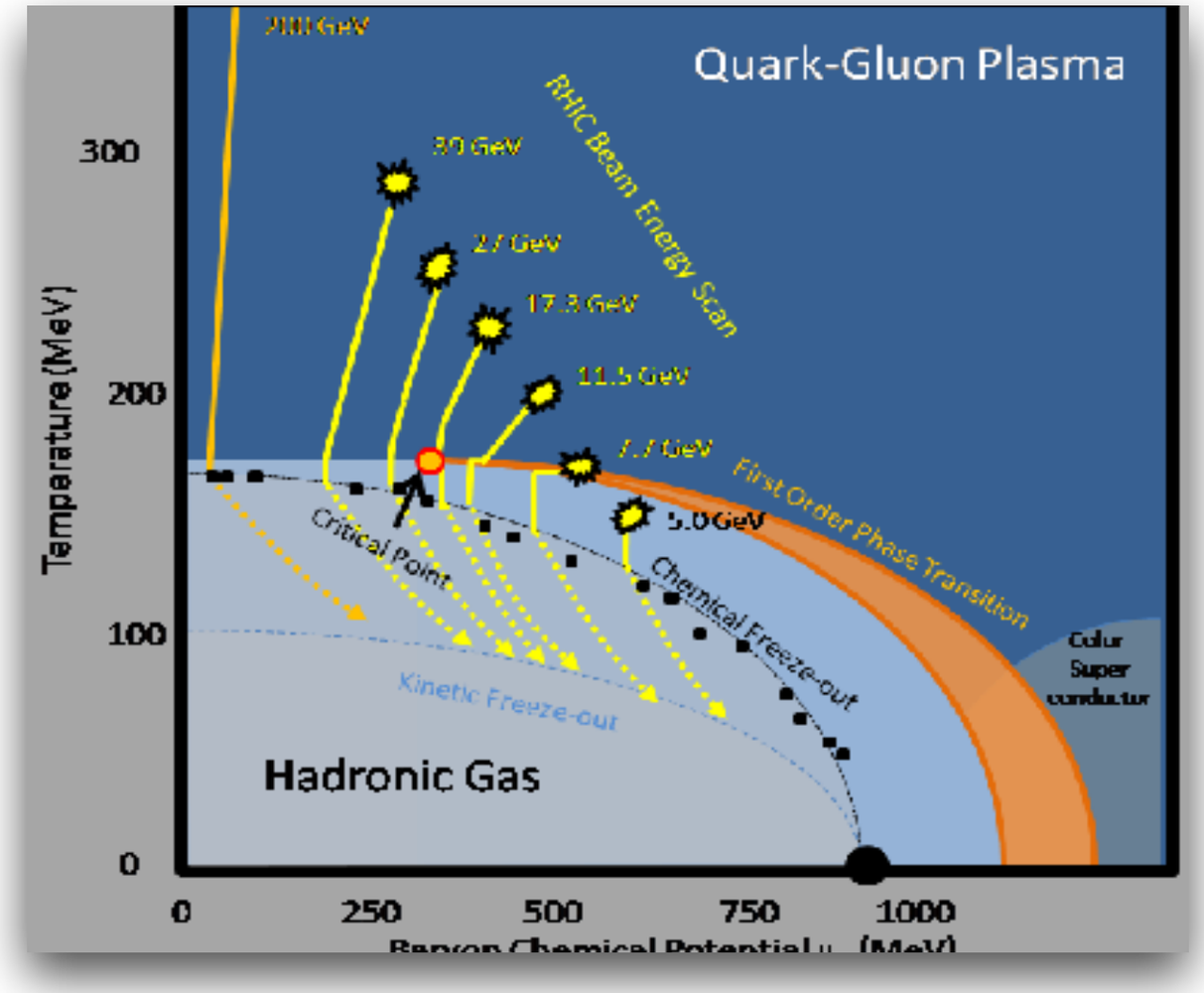




(PRL, Editors' suggestion, 16')

BEST COLLABORATION

Yi Yin



- Beam energy scan program: explore QCD phase diagram.
- **Key question**: what do we expect to see experimentally?

Thermodynamics (equilibrium)

- Correlation length ξ of the critical mode σ grows.
- Fluctuations: sensitive to the growth of ξ. $\kappa_n \sim \langle(\delta\sigma)^n\rangle$

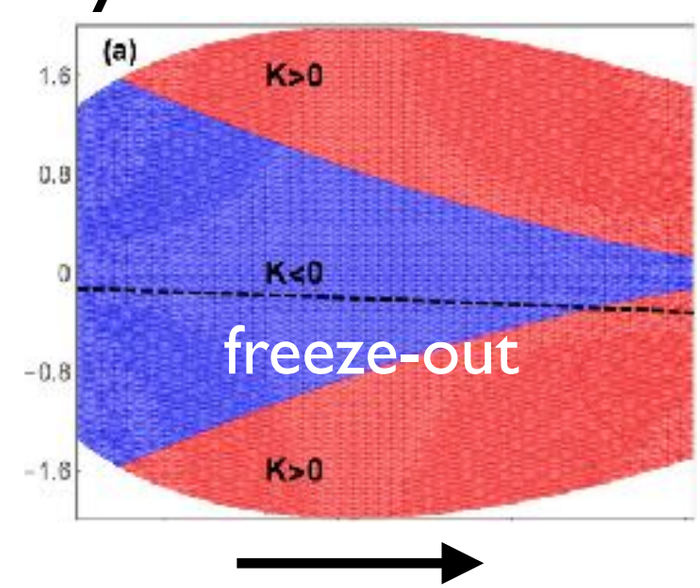
$$\kappa_2^{\rm eq} \sim \xi_{\rm eq}^2 \qquad \kappa_3^{\rm eq} \sim \xi_{\rm eq}^{9/2} \qquad \kappa_4^{\rm eq} \sim \xi_{\rm eq}^7$$

3d Ising model ⟷ QCD critical point (Berges, Rajagopal, 98)

Magnetization (critical mode) σ ⟷ A mixture of baryon density n_B energy density ε and chiral condensate.

Observables: $\langle (\delta N_B)^n \rangle \sim \kappa_n \sim \langle (\delta\sigma)^n \rangle$

- Non-Gaussian fluctuations: universal pattern in sign.

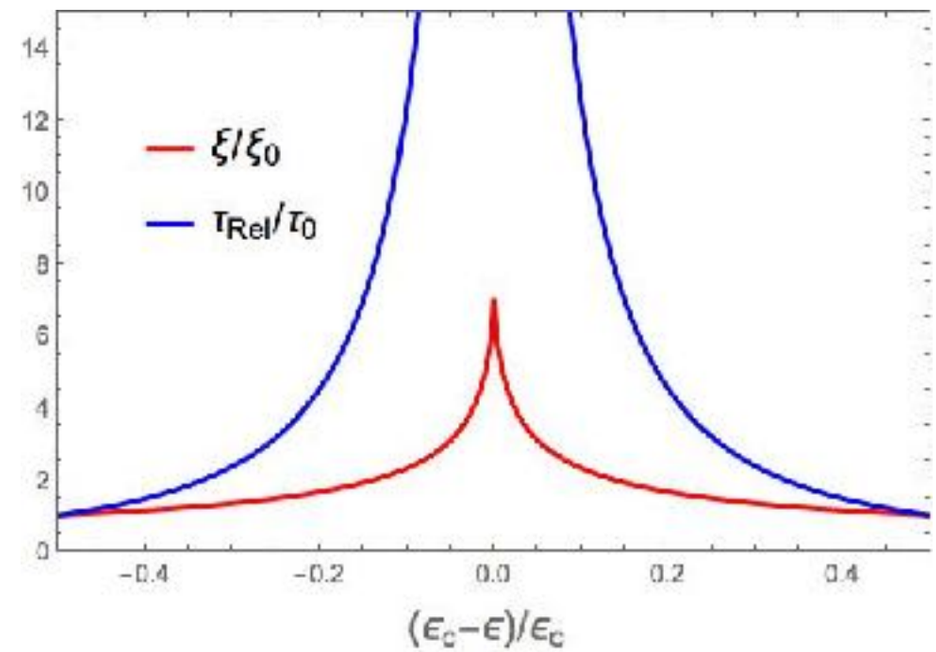


Decreasing

- Critical mode is off-equilibrium !

$$\tau_\sigma \sim \xi_{eq}^z, \qquad z \approx 3$$

(Son, Stephanov, 2008)

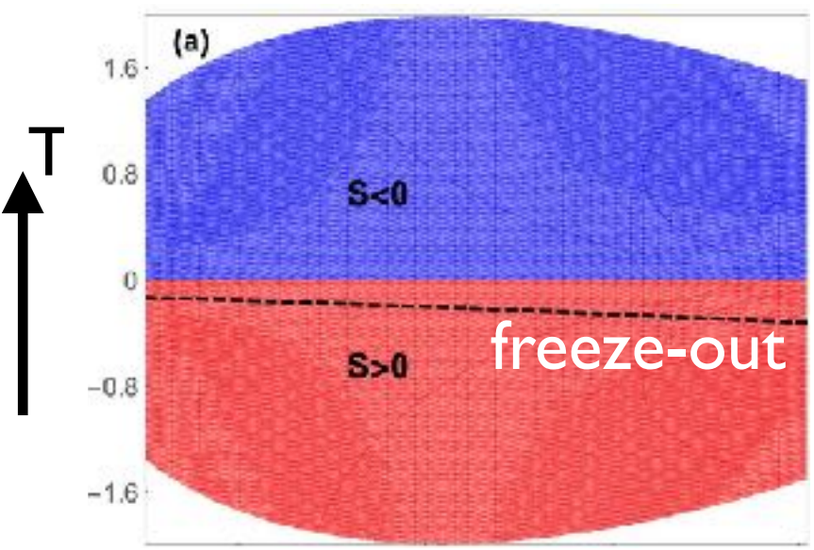


- Off-equil. Gaussian cumulants : **Berdinkov-Rajagopal, 99'**

- Off-equil. Gaussian and non-Gaussian cumulants.

(S. Mukherjee, R. Venugopalan and YY, 15')

Equilibrium



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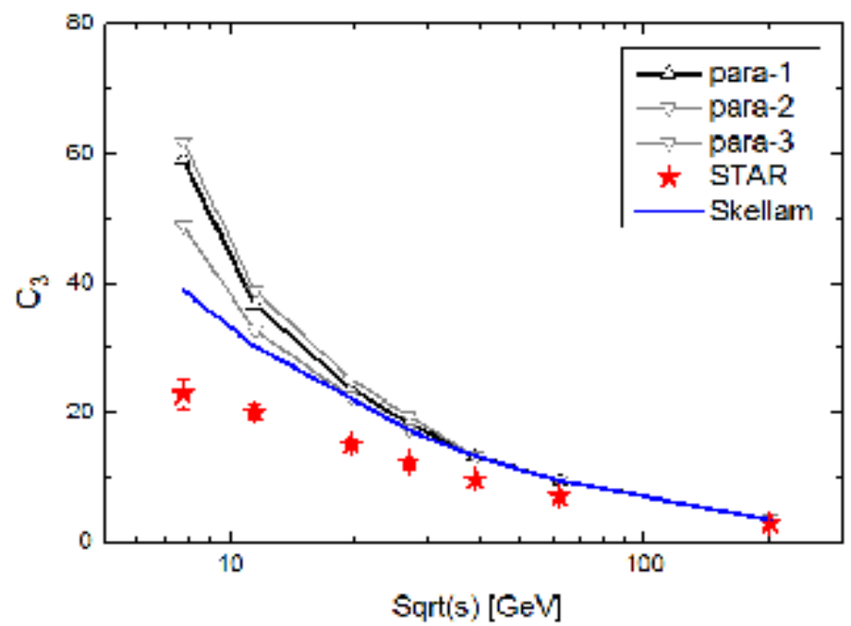
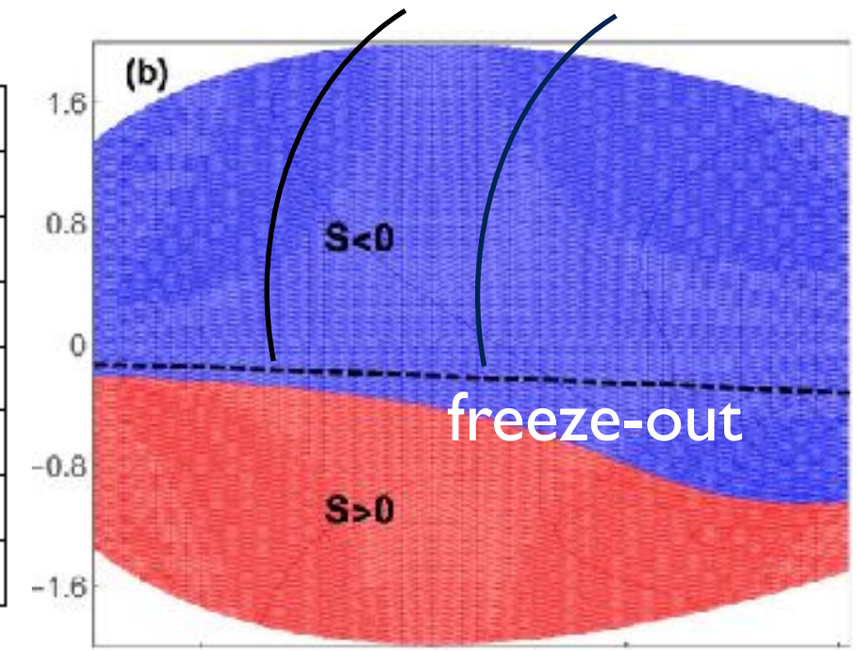


(Lijia Jiang, Pengfei Li, and Huichao Song, 16')

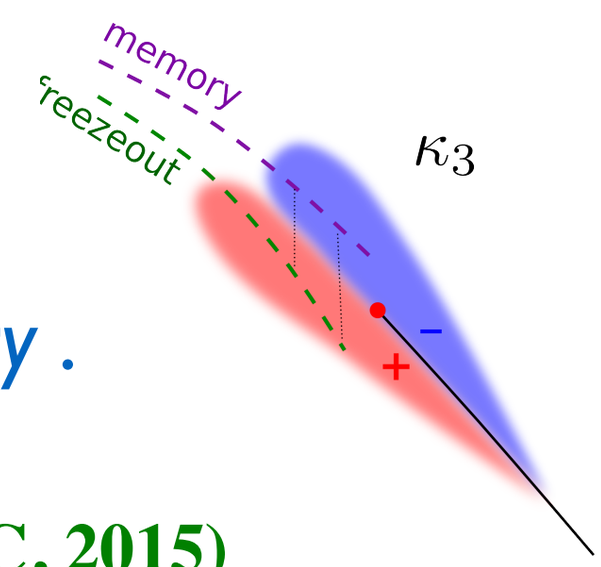
non-equilibrium



Decreasing $\sqrt{s}$

- "Sign puzzle" of skewness:

Equilibrium=Forgetting; Off-equil.=Memory .



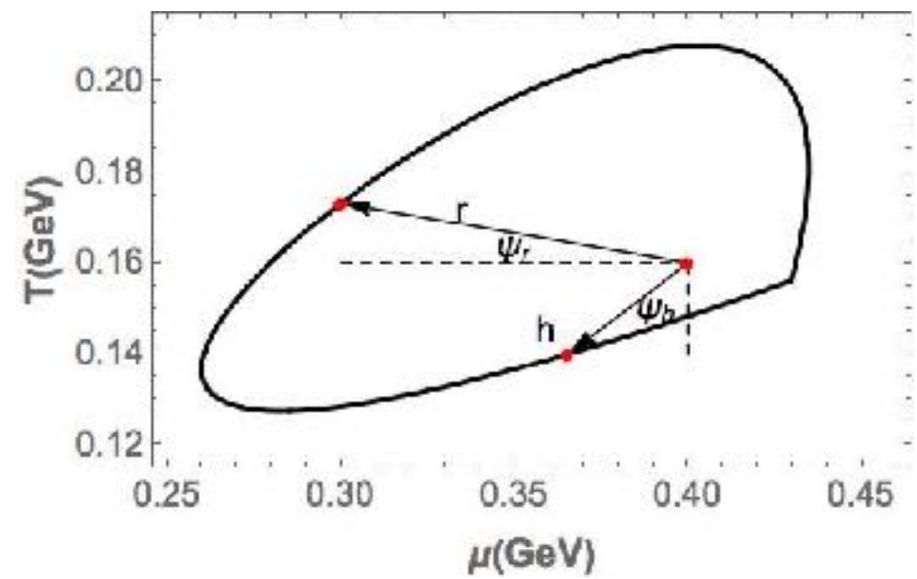
(S. Mukherjee, R. Venugopalan and YY, PRC, 2015)

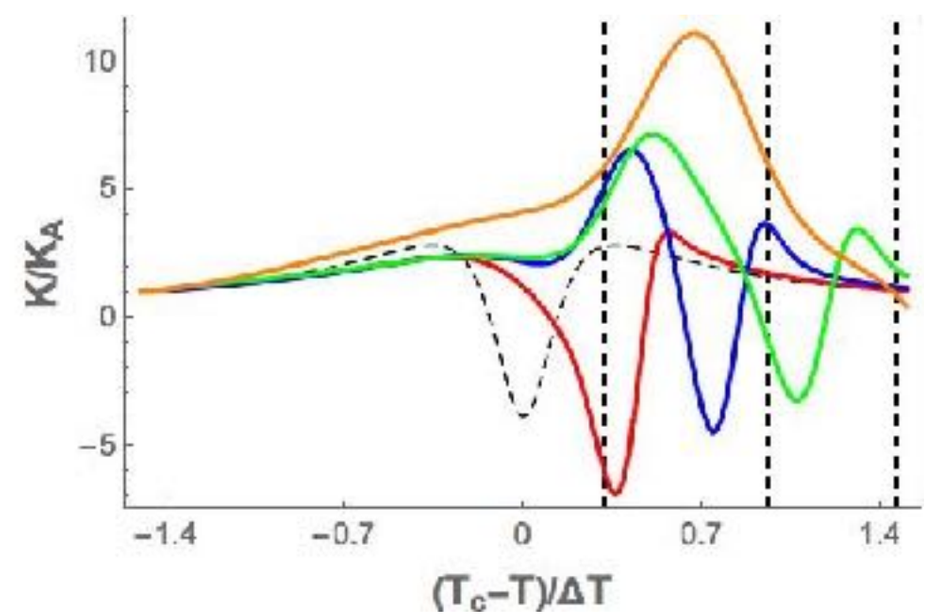
“此情可待成追忆,
只是当时已惘然.”

Memory implies *complexity*!

- Evolution depends on many non-universal inputs:
  - mapping, location of critical point, width of critical regime.
  - Trajectories in phase diagram
  - relaxation rate and expansion rate.
- The non-equilibrium cumulants look complicated.

?





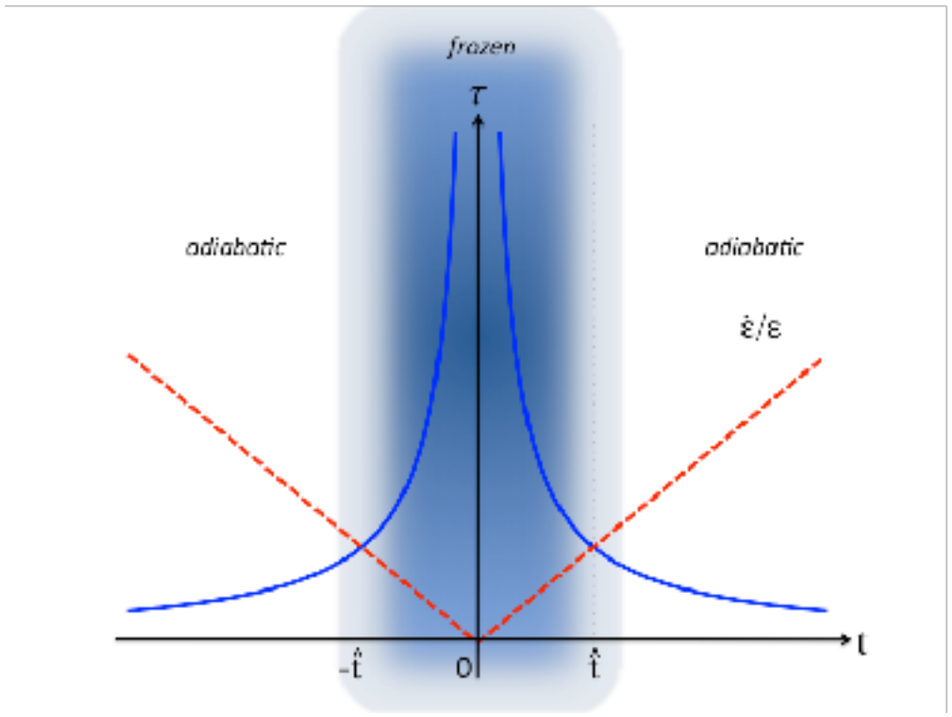
- Questions after QM 15: is universality lost in complexity ?
- Is there a simple way to understand what has been "memorized"?
- Answers to those questions are connected by:

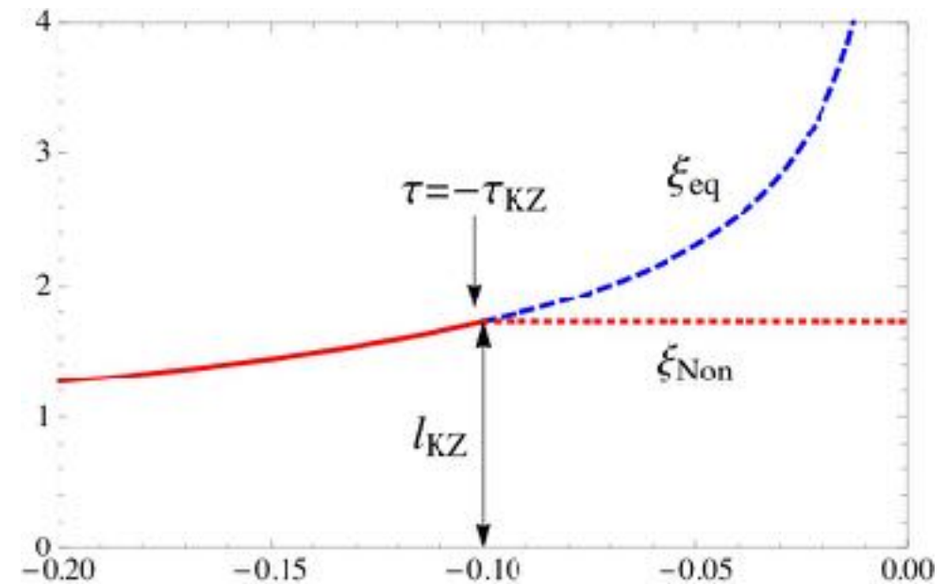
Kibble-Zurek dynamics

Topological defects in cosmological phase transitions. (*T.W. Kibble, Physics Reports 67, 183 (1980))*

Generalized to vortex generation in superfluids. (*W. H. Zurek, "Cosmological experiments in superfluid helium?", Nature 317, 505 (1985))*

Kibble-Zurek dynamics in a "little bang"?





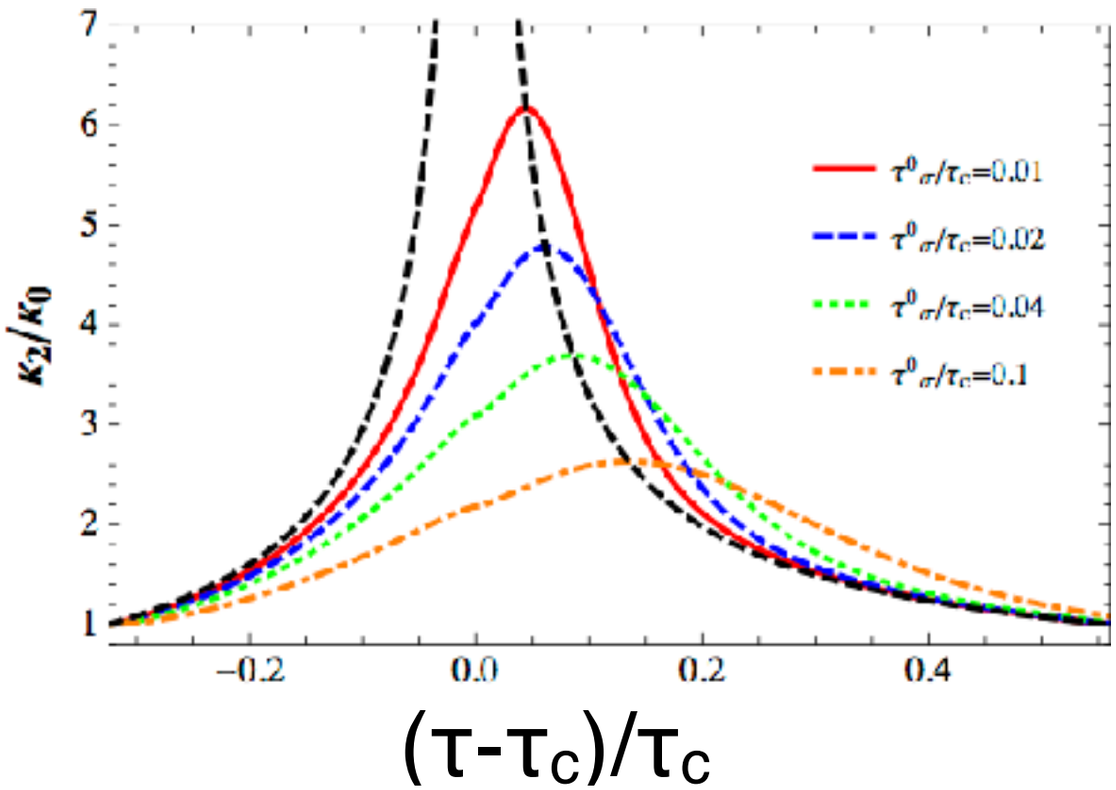
arXiv:1310.1600

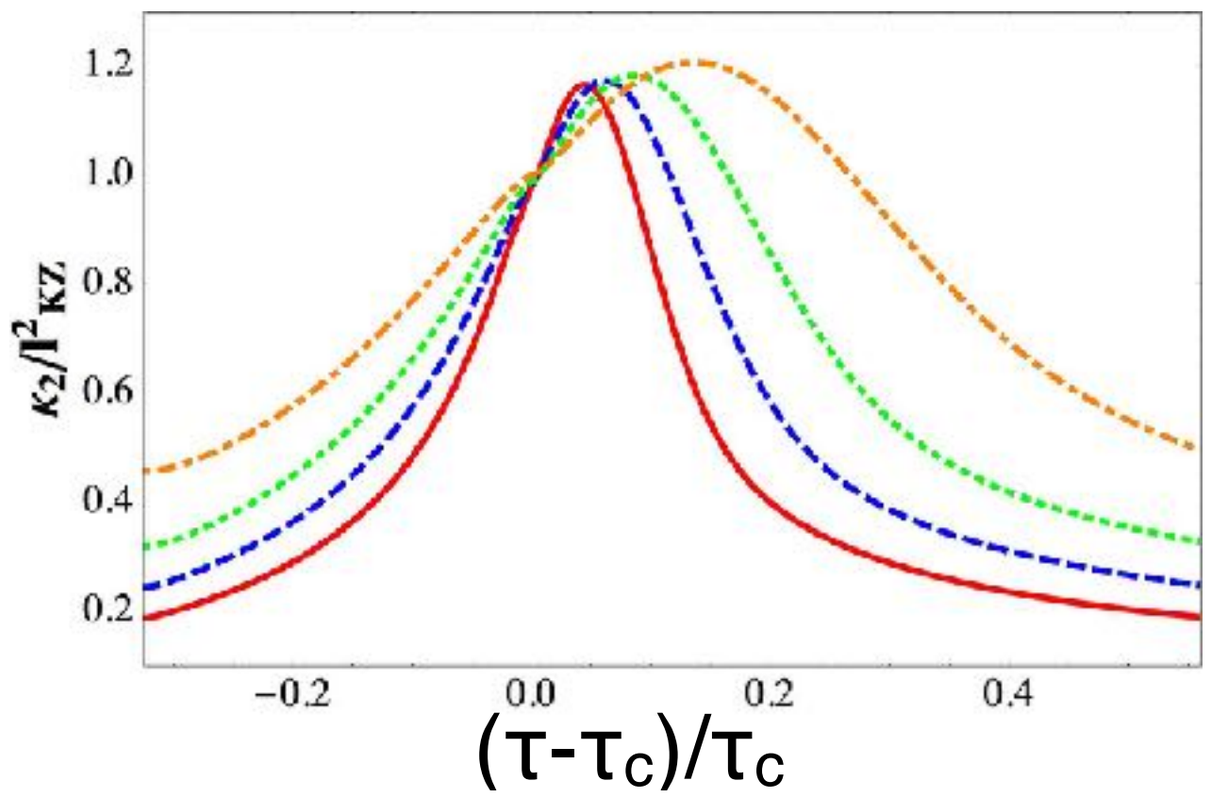
- Critical slowing down: relaxation time grows and becomes shorter than the expansion ("quench") time.
- The correlation length is frozen at the value when:

  Quench (expansion) time = Relaxation time
- Kibble-Zurek dynamics:

$$l_{\mathrm{KZ}} = \xi_{\mathrm{eq}}(\tau_{\mathrm{KZ}})$$ *Magnitude (1980s).*

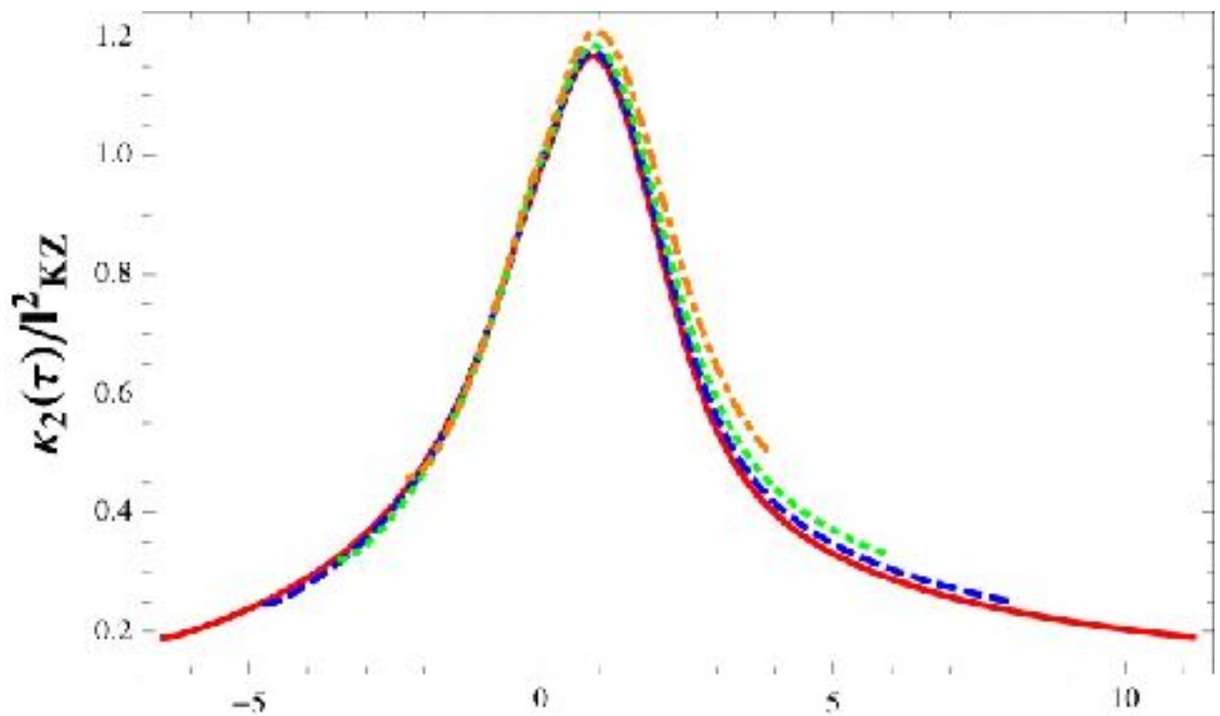
τ_{KZ} *Time evolution (2010s).*





An illustrative example: non-equilibrium evolution of correlation length (Berdnikov-Rajagopal model revisited).

- Rescale Gaussian cumulants by $(l_{kz})^2$.
- Scaling with length is not enough!
- A step forward: rescale time by τ_{kz} !



$(\tau-\tau_c)/\tau_{KZ}$

"you can hide but you can not run."

- Off-equilibrium scaling function !

$$\kappa_2(\tau;\Gamma) \sim l^2_{\mathrm{KZ}}(\Gamma)\ \underbrace{f_2\left(\tau/\tau_{\mathrm{KZ}}(\Gamma)\right)}_{\text{Universal}}$$

(Γ : non-universal inputs)

New physics in an old paper!

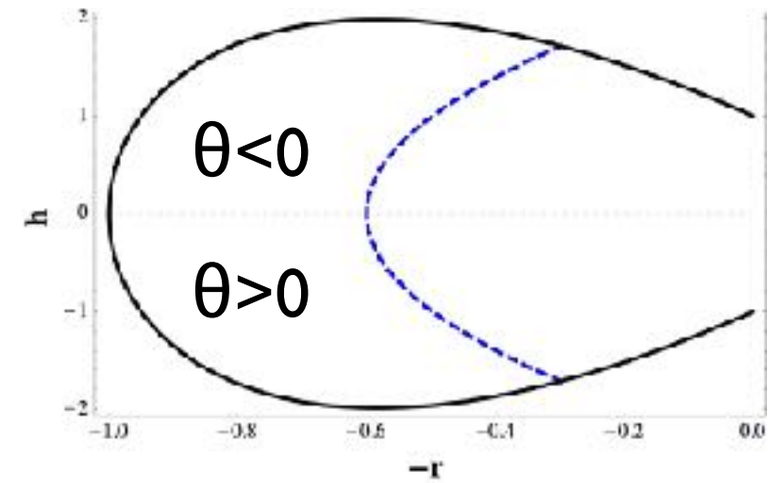
Difficulties when applied to heavy-ion collisions:

- Extending scaling hypothesis for non-Gaussian cumulants. ✓
- Applying non-equilibrium scaling for trajectories away from the critical point. ✓

S. Mukherjee, R. Venugopalan and YY, PRL, Editors' suggestion, 16'

- Non-Gaussian cumulants depends on ξ and θ.

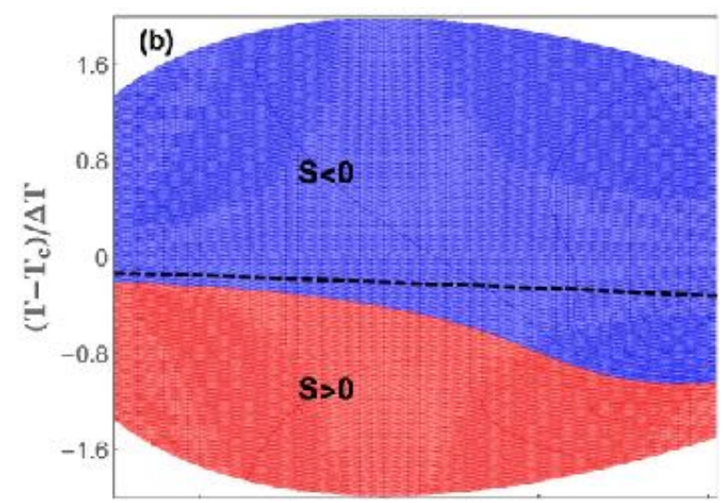
$$\kappa_n^{\mathrm{eq}} \sim \xi_{\mathrm{eq}}^{\#} \times f_n^{\mathrm{eq}}(\theta)$$



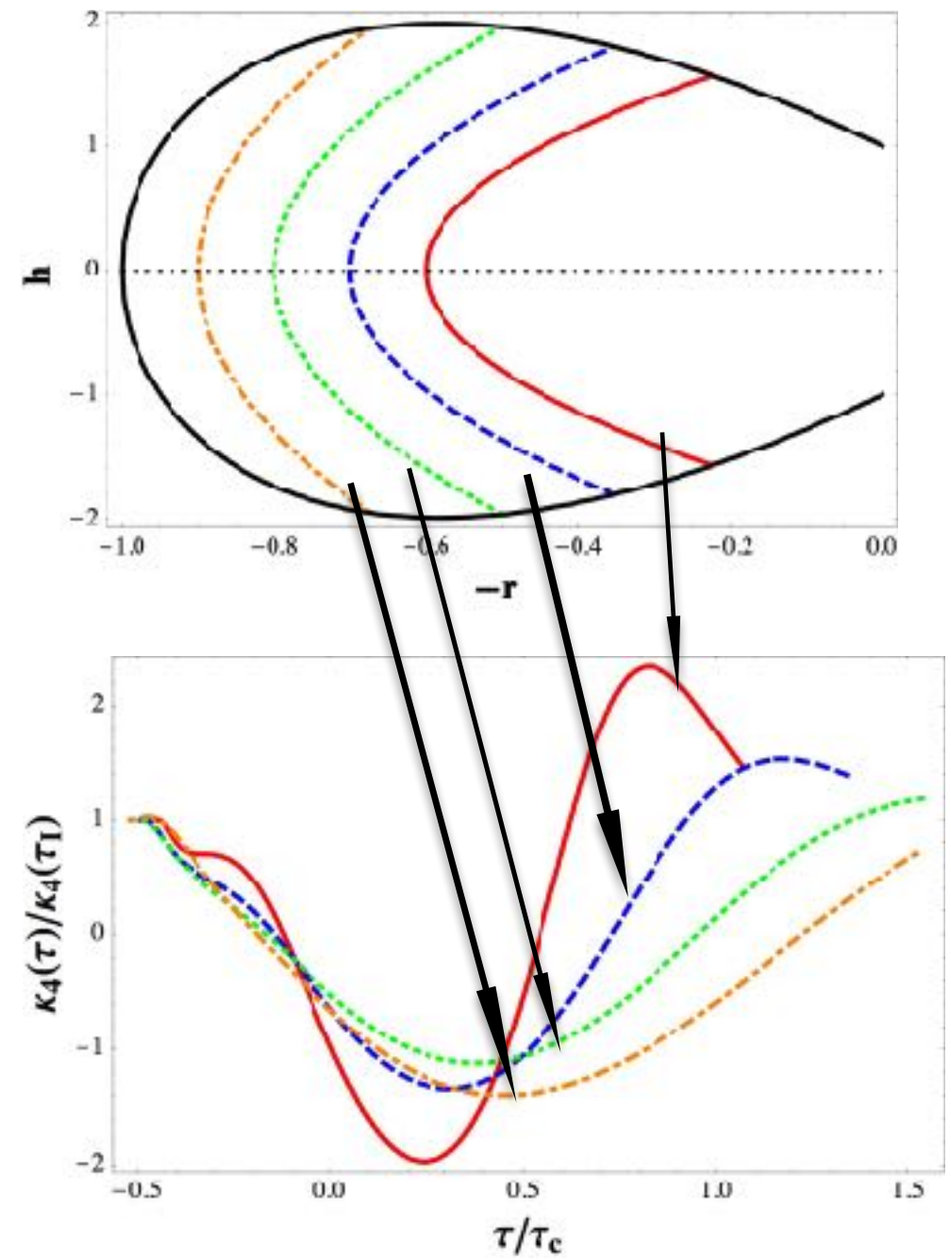
- Additional non-equilibrium scaling variable: ("memory of sign"): $\theta_{KZ} = \theta(\tau_{KZ})$

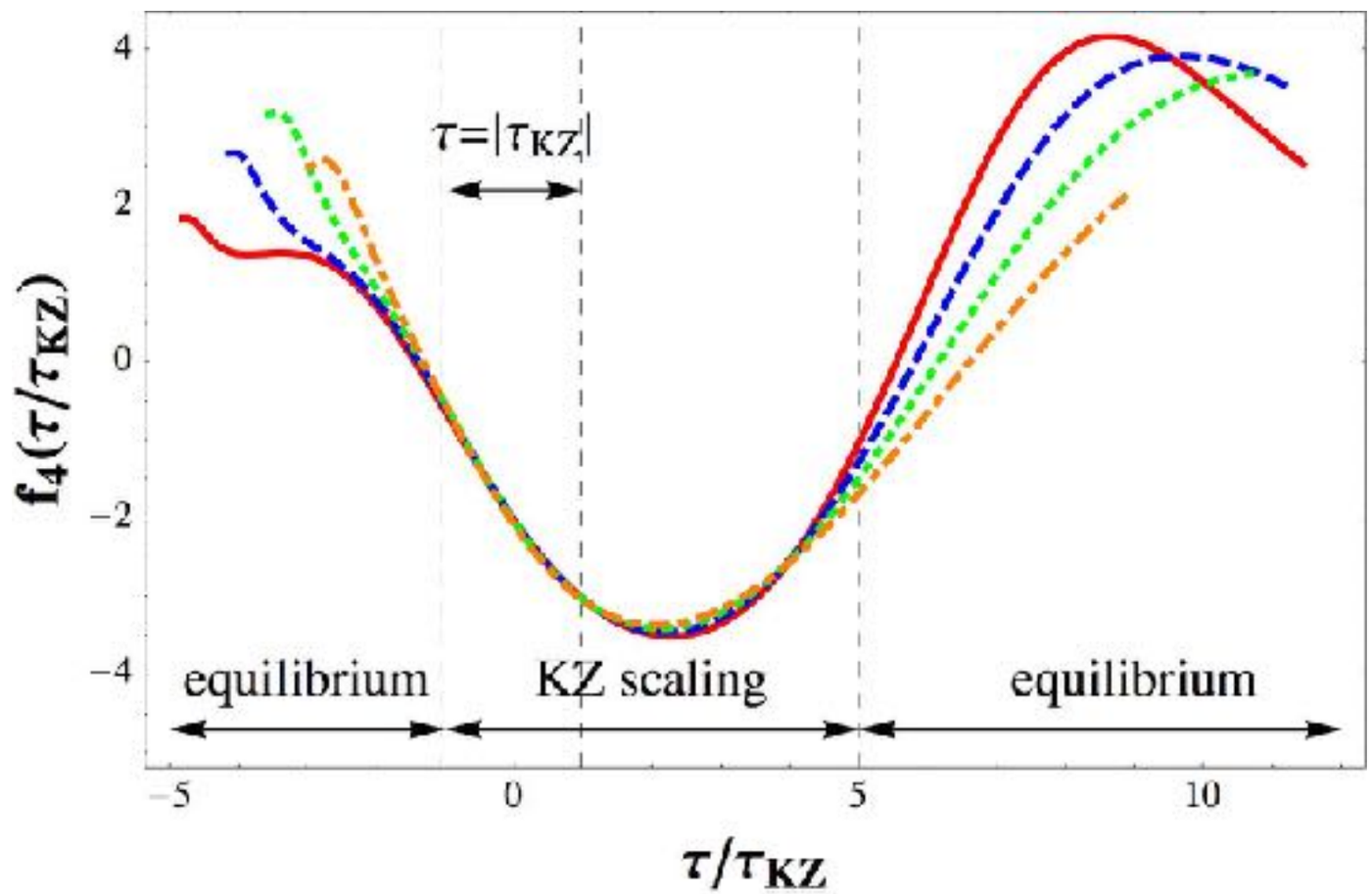
- Generalized scaling hypothesis:

$$\kappa_n(\tau;\Gamma) \sim (l_{KZ}(\Gamma))^{\#} f_n(\tau/\tau_{KZ}\,;\,\theta(\tau_{KZ}))$$



- Ask the key question "what do we expect to see" in a simplified set-up.
- Selecting evolutions with same θ_{KZ} .
- Expectation: rescaled evolutions are independent of trajectories.



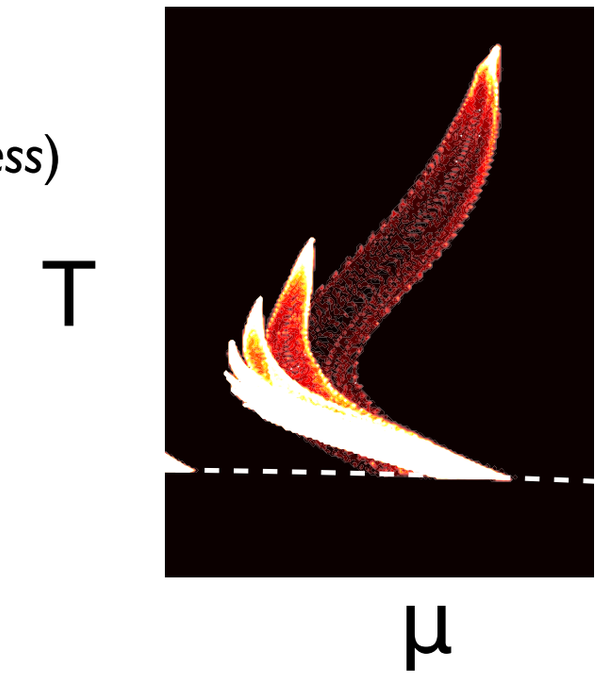


- Universality regained !

S. Mukherjee, R. Venugopalan and YY, PRL, Editors' suggestion, 16'

Search for KZ scaling in experiment data:

(Chun Shen, *S. Mukherjee, B. Schenke, R. Venugopalan and YY, in progress*)



$(\sqrt{s}, b, \eta, \ldots)$

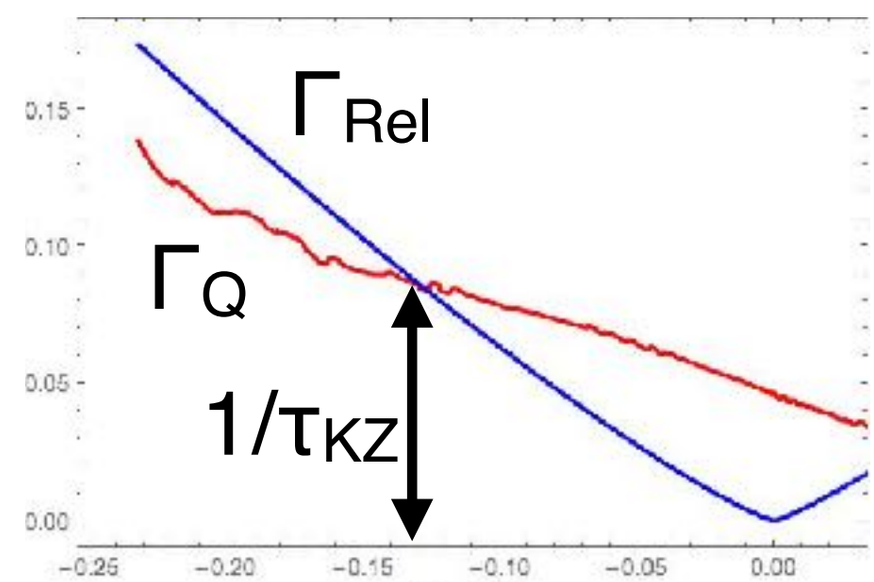
hydro-simulation

$(\mu(\tau), T(\tau))$

expansion rate = Relaxation rate

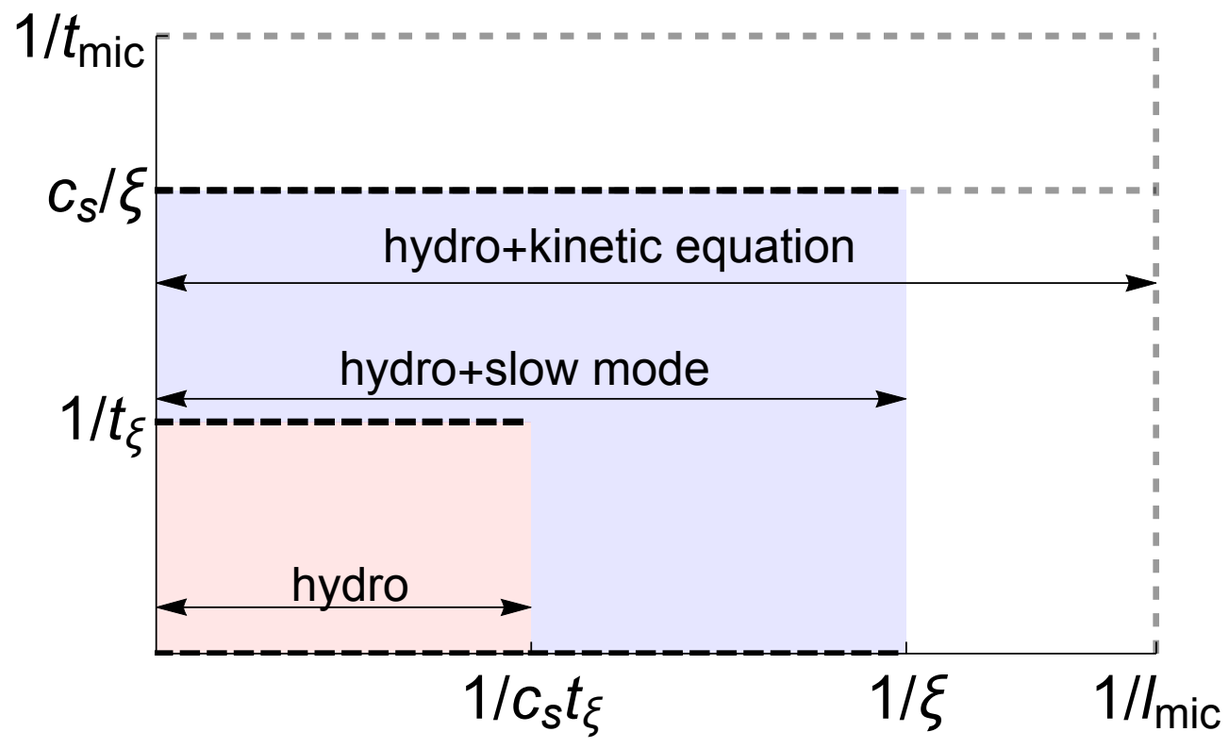
$(l_{KZ}, \tau_{KZ},)$

scale data/$(l_{KZ})^{\#}$



$(T_c - T)/T_c$

Similar τ_f/τ_{KZ}, θ_{KZ} , similar signal : **criticality and universality** !

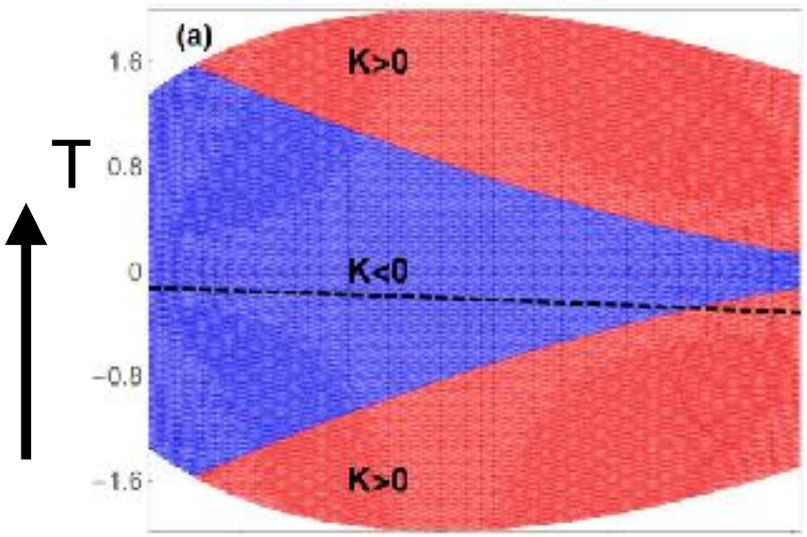


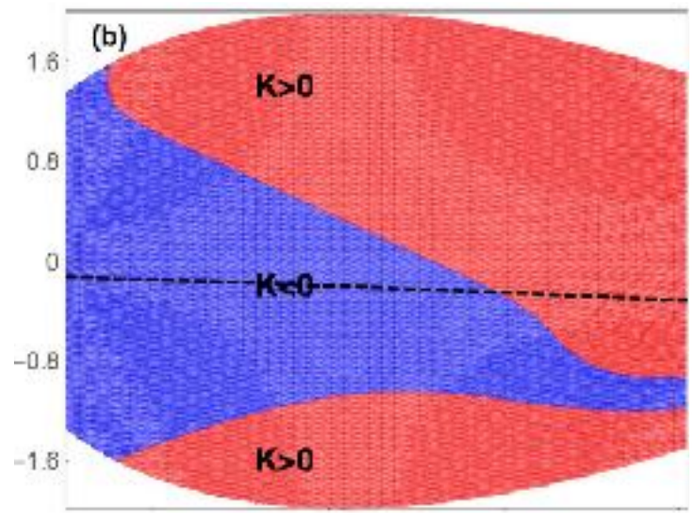
Before: no interplay between critical mode and hydro.

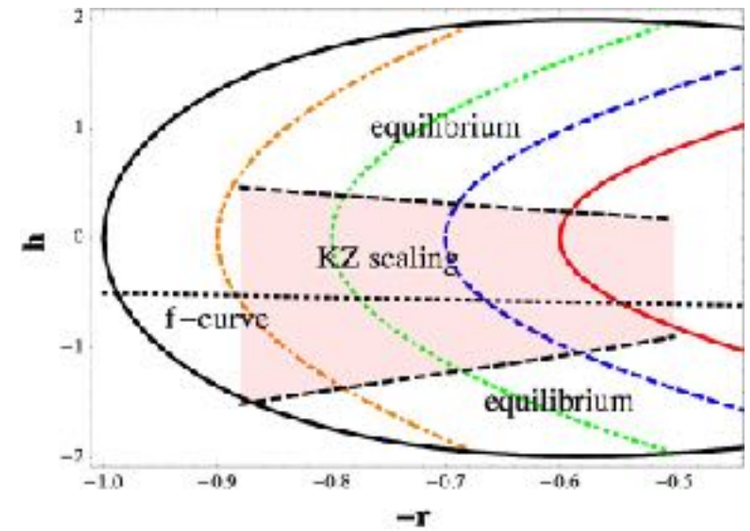
Now and then: an effective theory for critical dynamics (that will be **simulated** and will be **compared** with the data)

(M. Stephanov, YY, to be posted; D. Teaney, F. Yan, Y. A. Akamatsu, YY, in progress; ...).

A shift in paradigm on what do we expect to see







Equilibrium, 11'

non-equilibrium, 15'

Scaling, 16'

(Stephanov, PRL, 11')

(Mukherjee, Venugopalan and YY, PRC, 15')

(Mukherjee, Venugopalan and YY, PRL, Editors' suggestion, 16')

From carton to a quantitive description is under way.

BEST COLLABORATION

Back-up slides

Evolution of critical mode

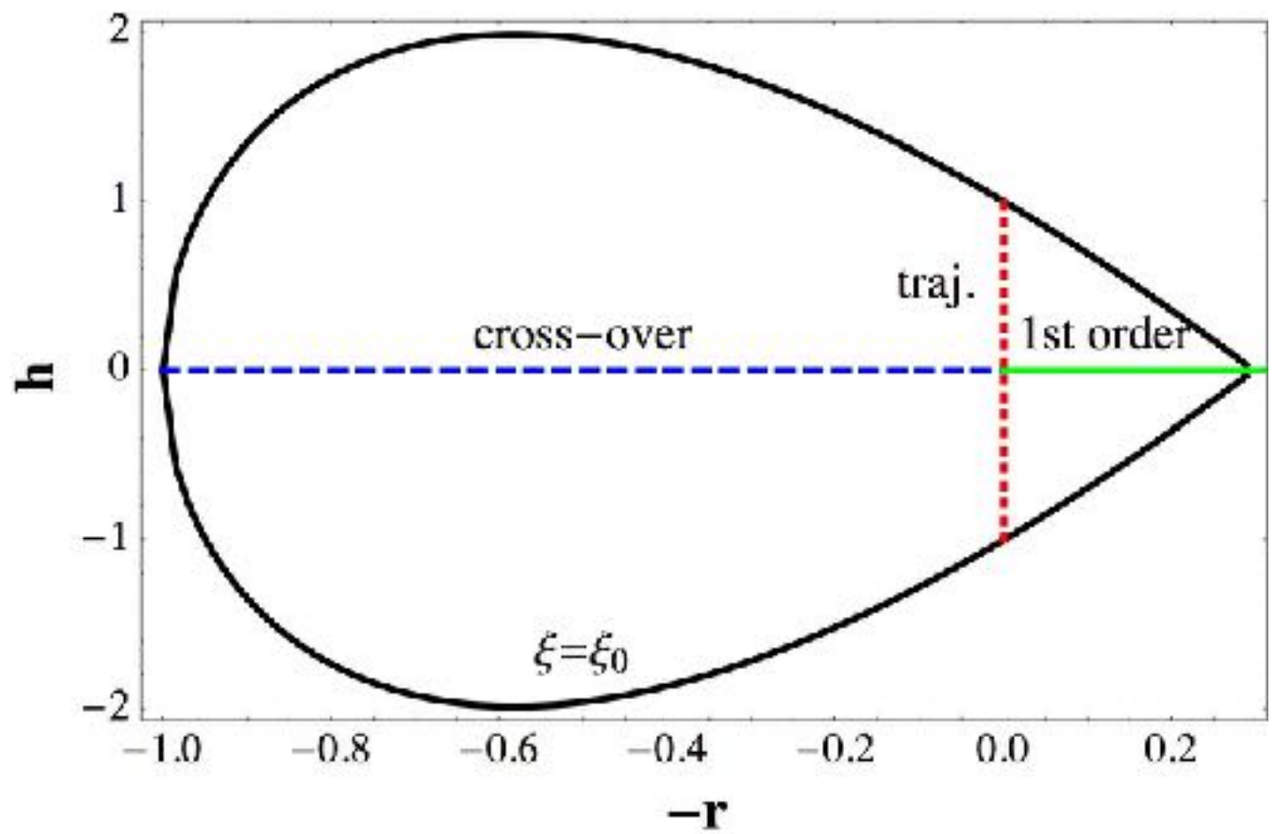
- Critical mode receives "random kicks" from thermalized non-critical modes => Generalized Langevin equation (Halperin-Hohenberg, 77').

- Averaging out the noise => A set of novel evolution equations for cumulants (S. Mukherjee, R. Venugopalan and YY, 1506.00645, PRC).

$$\frac{d\kappa_n(\tau)}{d\tau} = \frac{1}{\tau_\sigma(\tau)} L_n \left[\kappa_1, \kappa_2, \ldots, \kappa_n; \mu_B(\tau), T(\tau), \text{mapping}\right]$$

(S. Mukherjee, R. Venugopalan and YY, PRC, 2015)

Consider a trajectory passing the critical point.



(μ_B, T) ⟷ 3d Ising model (r,h)

The Kibble-Zurek Problem: Universality and the Scaling Limit

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(Dated: September 20, 2012)

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Nonequilibrium Dynamic Critical Scaling of the Quantum Ising Chain

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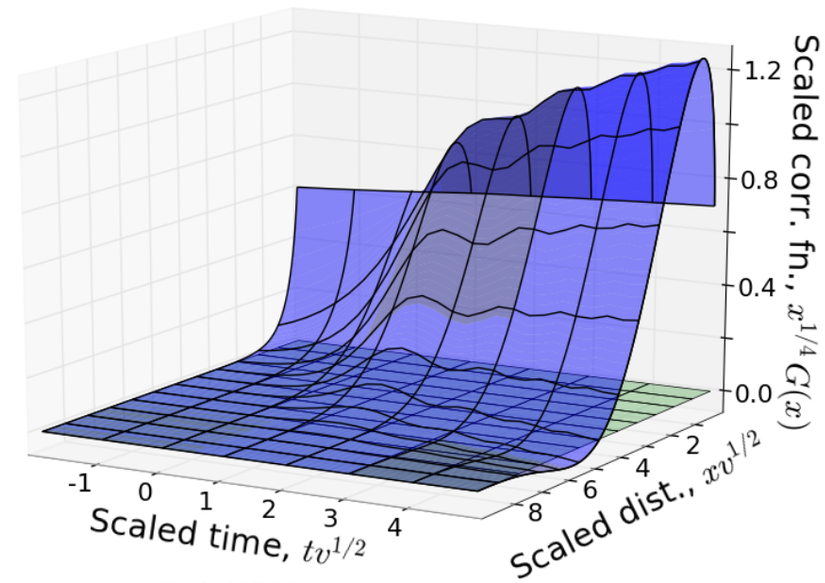
(Received 2 February 2012; published 2 July 2012)

We solve for the time-dependent finite-size scaling functions of the one-dimensional transverse-field Ising chain during a linear-in-time ramp of the field through the quantum critical point. We then simulate Mott-insulating bosons in a tilted potential, an experimentally studied system in the same equilibrium universality class, and demonstrate that universality holds for the dynamics as well. We find qualitatively



PRL 116, 080601 (2016) PHYSICAL REVIEW LETTERS week ending 26 FEBRUARY 2016

Universality in the Dynamics of Second-Order Phase Transitions

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(Received 18 November 2013; revised manuscript received 10 February 2015; published 26 February 2016)

The study of off-equilibrium scaling function is a new frontier in condensed matter community — new physics in an old paper!