Revealing the collision energy dependence of $\eta/s$ in RHIC-BES Au+Au collisions using Bayesian statistics

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Investigate $\sqrt{s_{NN}}$-dependence of $\eta/s$ using statistical analysis

- Possible indication of $\mu_B$-dependence in the transport coefficient
- Statistical analysis $\Rightarrow$ Get both best-fit values and their uncertainties

- Phase A: Find best-fitting model parameters for multiple collision energies independently
- If energy dependence observed in the best-fit parameters

$\Downarrow$

Phase B: parametrize the dependence and find the best fit over all collision energies simultaneously

Transport + hydrodynamics hybrid model


- Initial state described by UrQMD\(^1\) hadron transport
- Start the hydrodynamical evolution at time \(\tau_0\) when the two nuclei have passed through each other
- Convert energy, momentum and baryon number of each particle into 3D Gaussian distributions with width parameters \(W_{\text{trans}}, W_{\text{long}}\)
- (3+1)D viscous hydrodynamics with constant ratio of shear viscosity over entropy density \(\frac{\eta}{s}\) (bulk viscosity ignored)
- Transition from hydro back to UrQMD (“particlization”) when energy density \(\epsilon\) is smaller than the switching condition \(\epsilon_{SW}\)

Bayesian analysis

Model parameters (input): $\vec{x} = (x_1, \ldots, x_n)$

$\begin{align*}
(\tau_0, W_{\text{trans}}, W_{\text{long}}, \eta/s, \epsilon_{SW})
\end{align*}$

$\Downarrow$

Model output $\vec{y} = (y_1, \ldots, y_m) \iff$ Experimental values $\vec{y}^{\text{exp}}$

$\begin{align*}
(N_{\text{ch}}, \langle p_T \rangle, v_2, \ldots)
\end{align*}$

Bayes’ theorem:

\[
\text{Posterior probability } \propto \text{Likelihood} \cdot \text{Prior knowledge}
\]

- Prior knowledge: Range of input parameter values to investigate
- Likelihood: $L \propto \exp \left( -\frac{1}{2} (\vec{y} - \vec{y}^{\text{exp}})^T \Sigma^{-1} (\vec{y} - \vec{y}^{\text{exp}}) \right)$, where $\Sigma$ is the covariance matrix (contains the uncertainties)

Use Markov chain Monte Carlo to sample the posterior probability

- Random walk in input parameter space, constrained by prior
- Use Gaussian process (GP) emulator to estimate model output (based on training data) for likelihood computations in MCMC
$\sqrt{s_{NN}} = 62.4$ GeV

Red: No additional weights on data

Blue: Weight of $v_2$ and $N(\Omega)$ increased by a factor of 5
\[ \sqrt{s_{NN}} = 62.4 \text{ GeV} \]

Increase importance of \( v_2 \) and \( \Omega \) yield by increasing their weight by a factor of 5.
\[ \sqrt{s_{NN}} = 62.4 \text{ GeV} \]

Reweighting has little effect on charged particle yield estimate.

No weighting  
With weighting
\( \sqrt{s_{NN}} = 62.4 \text{ GeV} \)

Pseudorapidity distribution at midrapidity is described better in the reweighted analysis.
\[ \sqrt{s_{NN}} = 62.4 \text{ GeV} \]

Little difference in pseudorapidity distribution at higher centrality
Systematic underestimation of charged particle yield at large rapidities

![Charged particles (20-25)% centrality](image)

No weighting With weighting
\[ \sqrt{s_{NN}} = 62.4 \text{ GeV} \]

Reweighting has no notable effect on pion or kaon $p_T$ spectra; proton yield slightly overestimated.

![Graph showing $dN/dp_T$ for different particle species and centrality](image)

**No weighting**

**With weighting**
\[ \sqrt{s_{NN}} = 62.4 \text{ GeV} \]

Good description of HBT radii (averaged over 4 \( m_T \) bins) regardless of weighting
\( \sqrt{s_{NN}} = 39 \text{ GeV} \)

Red: No additional weights on data

Blue: Weight of \( v_2 \) increased by a factor of 5, \( N(\Omega) \) by a factor of 4; \( N(p) \) and \( N(\bar{p}) \) decreased by 0.5

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$\sqrt{s_{NN}} = 39$ GeV

Notably too few antiprotons (need to estimate the feed-down correction)
Weights of both $p$ and $\bar{p}$ reduced by a factor of 2 in reweighted analysis
$\sqrt{s_{NN}} = 39$ GeV

$\Omega$ $p_T$ distribution better described in reweighted analysis

No weighting  With weighting
\[ \sqrt{s_{NN}} = 39 \text{ GeV} \]

Peak values give very good description of \( v_2 \) in both cases.
\sqrt{s_{NN}} = 39 \text{ GeV}

Meson multiplicities better described in reweighted analysis
\[ \sqrt{s_{NN}} = 39 \text{ GeV} \]

Slightly worse estimate for \( R_{\text{long}} \) from weighted analysis

![Graphs showing \( \langle R \rangle \) for different conditions]
\[ \sqrt{s_{NN}} = 19.6 \, \text{GeV} \]

Red: No additional weights on data

Blue: Weight of \( v_2 \) increased by a factor of 5, \( N(\Omega) \) by a factor of 4; \( N(p) \) and \( N(\bar{p}) \) decreased by 0.5
\[ \sqrt{s_{NN}} = 19.6 \text{ GeV} \]

Weighted analysis gives better fit on \( N(\Omega) \) at low \( p_T \) while peak values from unweighted posterior fit better at high \( p_T \)
\[ \sqrt{s_{NN}} = 19.6 \text{ GeV} \]

Results in the weighted scenario

Charged particles (0-6)% centrality

Charged particles (15-25)% centrality

Model

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\[ \sqrt{s_{NN}} = 19.6 \text{ GeV} \]

Results in the weighted scenario

- \( \pi^+ \) vs. Centrality
- \( K^+ \) vs. Centrality
- \( \rho \) vs. Centrality
- \( \bar{p} \) vs. Centrality

Graphs showing data from STAR, Simulation, GP prediction for simulation, and GP (100 posterior samples) with training data.
$$\sqrt{s_{NN}} = 19.6 \text{ GeV}$$

Results in the weighted scenario
Parameter dependence on collision energy

$\eta/s$ and $\tau_0$ show clear increasing trend towards lower energies (however, minimum of $\tau_0$ increases by construction)

Boxes: 50% confidence range
Whiskers: 95% confidence range
Summary

- Bayesian analysis provides a rigorous method for simultaneous estimation of both the best-fit values and the associated uncertainties.

- Reweighting found necessary for emphasizing relevant physics and compensating for fewer data points:
  - Increased weight on $v_2$ gives better constraints on $\eta/s$.
  - Increased weight on $\Omega$ gives better constraints on switching energy density $\epsilon_{SW}$.
  - Proton and antiproton yields given smaller weights due to the lack of feed-down correction.

- $\eta/s$ shows a clear preference of larger values at lower $\sqrt{s_{NN}}$, strongly suggesting $\mu_B$ dependence.

- 95% confidence ranges still large; in particular, initial state smearing parameters $W_{\text{trans}}$, $W_{\text{long}}$ poorly constrained.
Extra slides
Likelihood function

The likelihood function used in MCMC:

\[
\exp \left( -\frac{1}{2} \sum_{i=1}^{q} \lambda_i \left( \frac{(z_i^* - z_i^{\text{exp}})^2}{(\sigma_{\text{exp}})^2 + \Sigma_i^*} \right) \right)
\]

- \(\lambda_i\) is the variance explained by the \(i\)th principal component
- \(z_i^*\) is the emulator prediction for the \(i\)th principal component at the input parameter point \(\vec{x}^*\)
- \(\vec{z}^{\text{exp}}\) is the experimental data transformed to principal component space
- \(\Sigma_i^*\) is the predictive variance (emulator uncertainty)
- \(\sigma_{\text{exp}}\) is the experimental statistical error (systematic errors ignored)
Investigated parameter ranges

Sample emulator training points evenly over whole parameter space using Latin hypercube method

- Shear viscosity over entropy density $\eta/s$: 0.001 - 0.4
- Transport-to-hydro transition time $\tau_0$: 0.4 - 3.1 fm
- Transverse Gaussian smearing of particles $W_{\text{trans}}$: 0.2 - 2.2 fm
- Longitudinal Gaussian smearing of particles $W_{\text{long}}$: 0.2 - 2.2 fm
- Hydro-to-transport transition energy density $\epsilon_{SW}$: 0.15 - 0.75 GeV/fm$^3$
Principal component analysis

$m$ observables $\Rightarrow m$ Gaussian processes needed for model emulation

However, $m$ can be up to $O(100)$ at top RHIC energies and at the LHC! Number of emulators can be reduced with principal component analysis

First principal component represents the direction of largest variance in output space, second PC the direction of second largest variance, etc.

- Fraction of variance explained by principal component $p_q$: $\text{Var}(p_q) = \frac{\lambda_q}{\sum_{i=1}^{m} \lambda_i}$

- Select the number of principal components which together explain desired fraction of total variance; often only a few PCs are needed to explain 99% of the variance
Analysis procedure

Produce training data

- run simulations with $\mathcal{O}(100)$ different parameter combinations

Preparation for principal component analysis:

1. Scale with experimental values \(\Rightarrow\) Unitless quantities
2. Check that the values are roughly normally distributed; apply a transformation if necessary
3. Center the data; apply possible data weighting

Determine required number of Gaussian processes with PCA

\[\Downarrow\]
Condition the emulators on training data

\[\Downarrow\]
Calibrate on experimental data by running MCMC