Revealing the collision energy dependence of η/s in RHIC-BES Au+Au collisions using Bayesian statistics

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RHIC beam energy scan

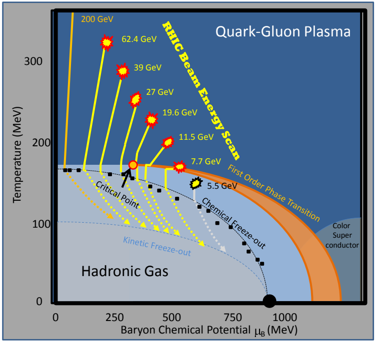
Investigate $\sqrt{s_{NN}}$-dependence of η/s using statistical analysis

- Possible indication of μ_B-dependence in the transport coefficient
- Statistical analysis ⇒ Get both best-fit values and their uncertainties

- Phase A: Find best-fitting model parameters for multiple collision energies independently
- If energy dependence observed in the best-fit parameters

⇓

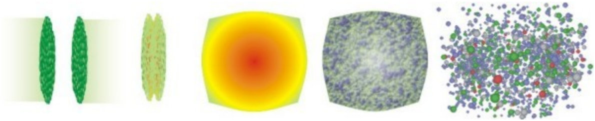
Phase B: parametrize the dependence and find the best fit over all collision energies simultaneously



Picture: G. Odyniec, Acta Phys. Polon. B 43, 627 (2012).

Transport + hydrodynamics hybrid model

Karpenko, Huovinen, Petersen, Bleicher, Phys.Rev.C, 91, 064901 (2015)



- Initial state described by UrQMD[1] hadron transport
- Start the hydrodynamical evolution at time τ_0 when the two nuclei have passed through each other
- Convert energy, momentum and baryon number of each particle into 3D Gaussian distributions with width parameters W_{trans}, W_{long}
- (3+1)D viscous hydrodynamics with constant ratio of shear viscosity over entropy density η/s (bulk viscosity ignored)
- Transition from hydro back to UrQMD ("particlization") when energy density ϵ is smaller than the switching condition ϵ_{SW}

"Cornelius" hypersurface finder, P. Huovinen and H. Petersen, EPJ A48 171 (2012)

[1] S. A. Bass *et al.*, Prog. Part. Nucl. Phys. **41**, 255 (1998), M. Bleicher *et al.*, J. Phys. G **25**, 1859 (1999).

Bayesian analysis

Model parameters (input): $\vec{x} = (x_1, ..., x_n)$

$(\tau_0,\ W_{\text{trans}},\ W_{\text{long}},\ \eta/s,\ \epsilon_{SW})$

$\Downarrow$

Model output $\vec{y} = (y_1, ..., y_m) \Leftrightarrow$ Experimental values $\vec{y}^{\,\text{exp}}$

$(N_{\text{ch}},\ \langle p_T \rangle,\ v_2, ...)$

Bayes' theorem:

Posterior probability $\propto$ Likelihood $\cdot$ Prior knowledge

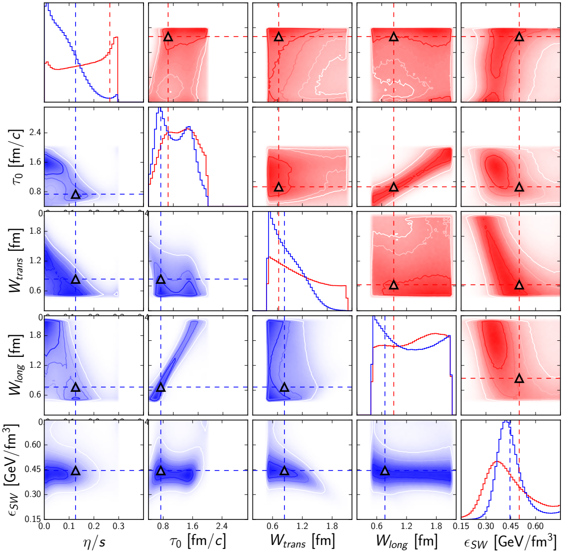
- Prior knowledge: Range of input parameter values to investigate
- Likelihood: $\mathcal{L} \propto \exp\left(-\frac{1}{2}(\vec{y} - \vec{y}^{\,\text{exp}})^T \Sigma^{-1} (\vec{y} - \vec{y}^{\,\text{exp}})\right)$, where Σ is the covariance matrix (contains the uncertainties)

Use Markov chain Monte Carlo to sample the posterior probability

- Random walk in input parameter space, constrained by prior
- Use Gaussian process (GP) emulator to estimate model output (based on training data) for likelihood computations in MCMC

$\sqrt{s_{NN}} = 62.4$ GeV

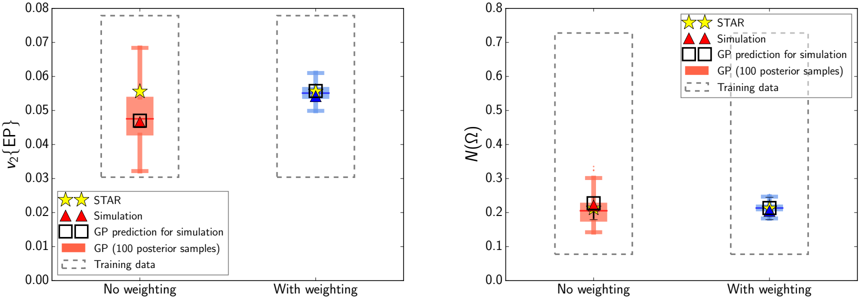
Red: No additional weights on data



Blue: Weight of v_2 and $N(\Omega)$ increased by a factor of 5

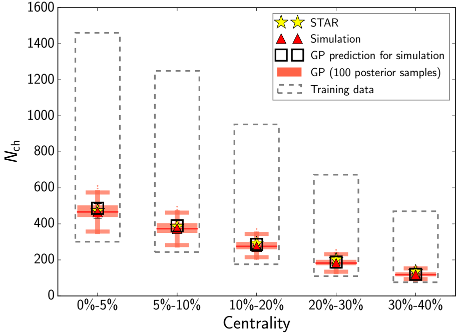
$\sqrt{s_{NN}} = 62.4$ GeV

Increase importance of v_2 and Ω yield by increasing their weight by a factor of 5

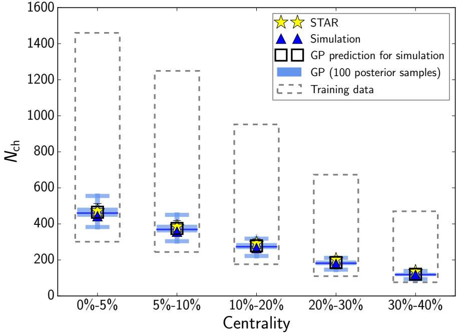


$\sqrt{s_{NN}} = 62.4$ GeV

Reweighting has little effect on charged particle yield estimate



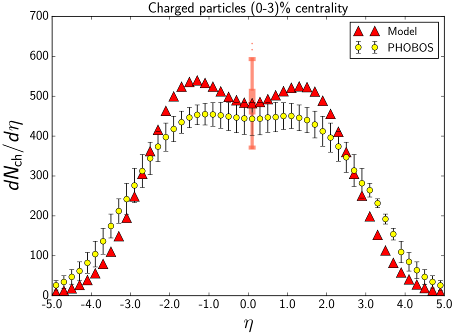
No weighting



With weighting

$\sqrt{s_{NN}} = 62.4$ GeV

Pseudorapidity distribution at midrapidity is described better in the reweighted analysis



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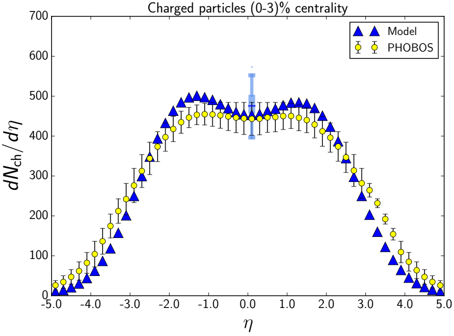
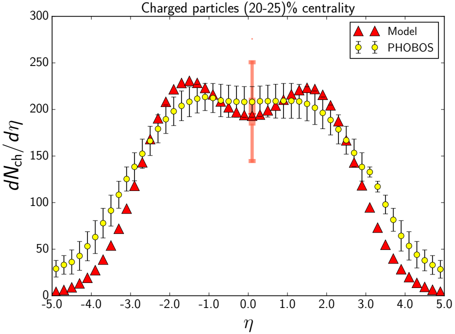


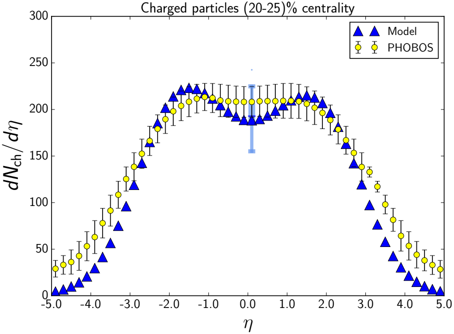
With weighting

$\sqrt{s_{NN}} = 62.4$ GeV

Little difference in pseudorapidity distribution at higher centrality
Systematic underestimation of charged particle yield at large rapidities



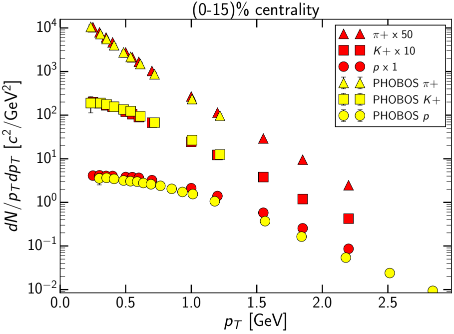
No weighting



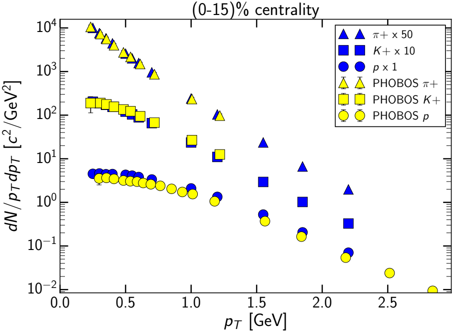
With weighting

$\sqrt{s_{NN}} = 62.4$ GeV

Reweighting has no notable effect on pion or kaon p_T spectra; proton yield slightly overestimated



No weighting

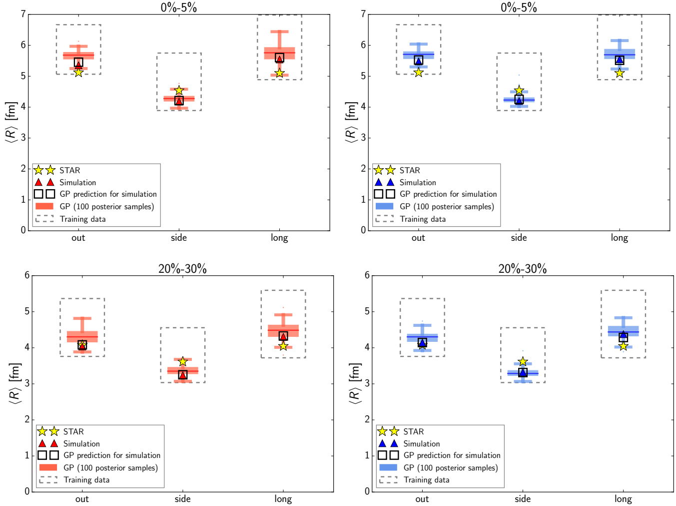


With weighting

$\sqrt{s_{NN}} = 62.4$ GeV

Good description of HBT radii (averaged over 4 m_T bins) regardless of weighting

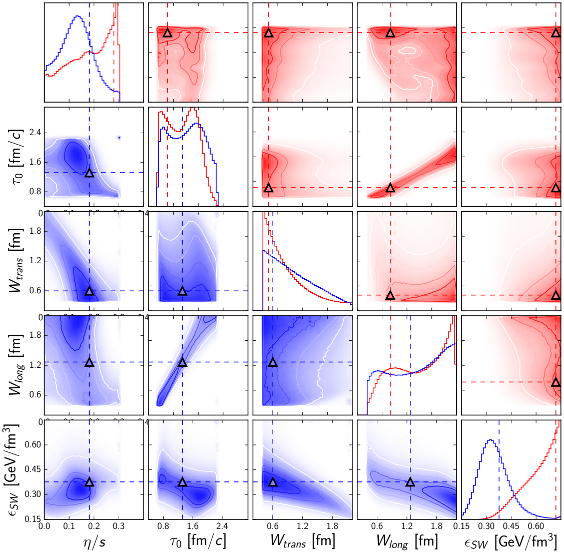
No weighting



With weighting

$\sqrt{s_{NN}} = 39$ GeV

Red: No additional weights on data



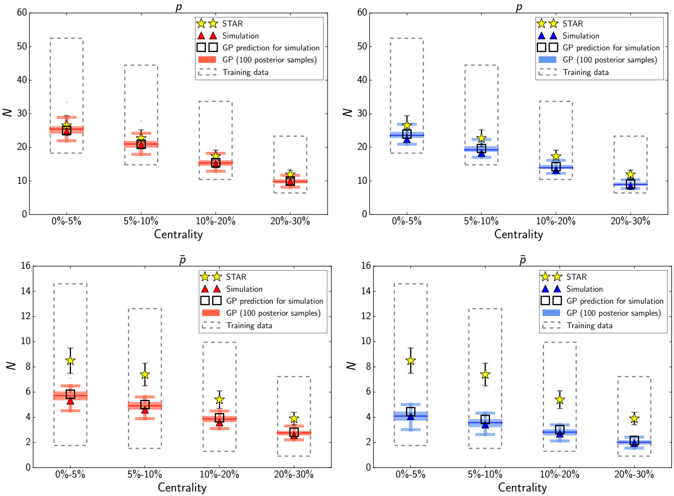
Blue: Weight of v_2 increased by a factor of 5, $N(\Omega)$ by a factor of 4; $N(p)$ and $N(\bar{p})$ decreased by 0.5

$\sqrt{s_{NN}} = 39$ GeV

Notably too few antiprotons (need to estimate the feed-down correction)

Weights of both p and $\bar{p}$ reduced by a factor of 2 in reweighted analysis

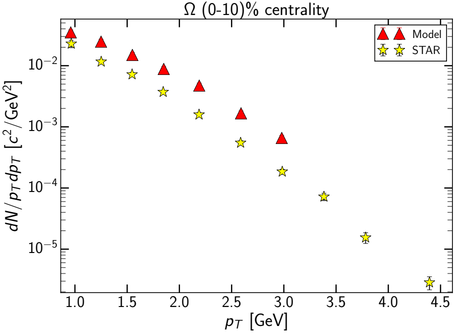
No weighting



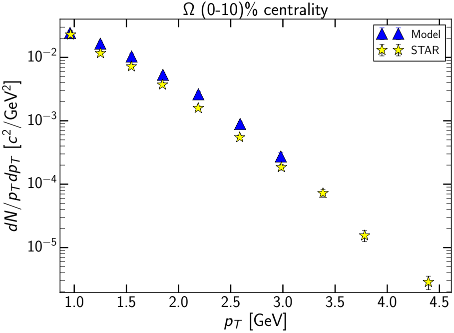
With weighting

$\sqrt{s_{NN}} = 39$ GeV

Ω p_T distribution better described in reweighted analysis



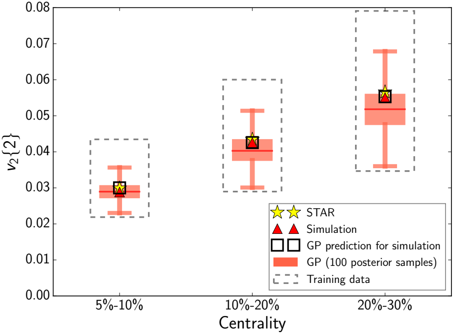
No weighting



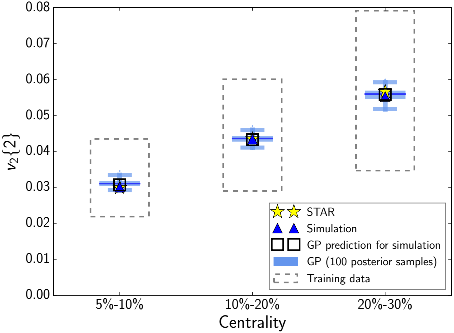
With weighting

$\sqrt{s_{NN}} = 39$ GeV

Peak values give very good description of v_2 in both cases



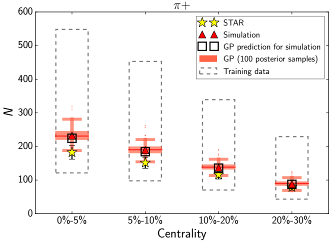
No weighting

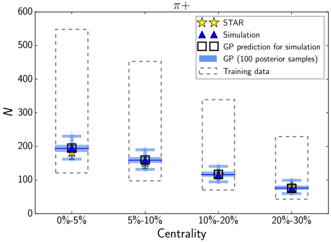


With weighting

$\sqrt{s_{NN}} = 39$ GeV

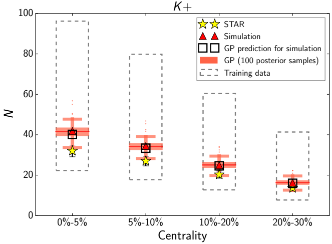
Meson multiplicities better described in reweighted analysis

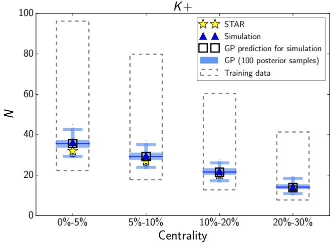




No weighting

With weighting

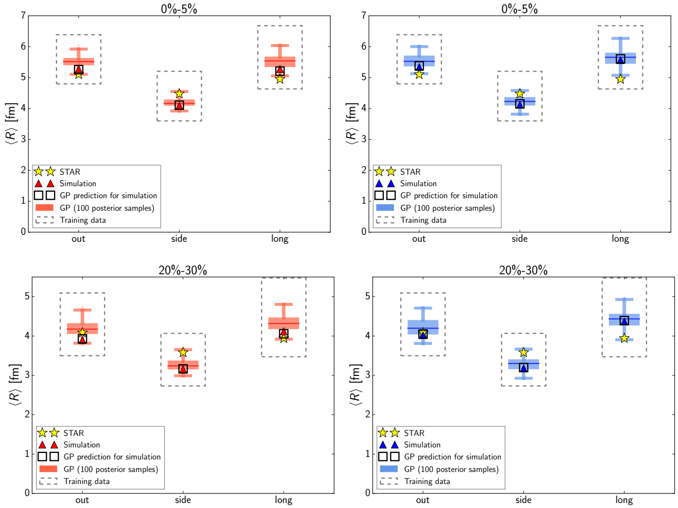




$\sqrt{s_{NN}} = 39$ GeV

Slightly worse estimate for R_{long} from weighted analysis

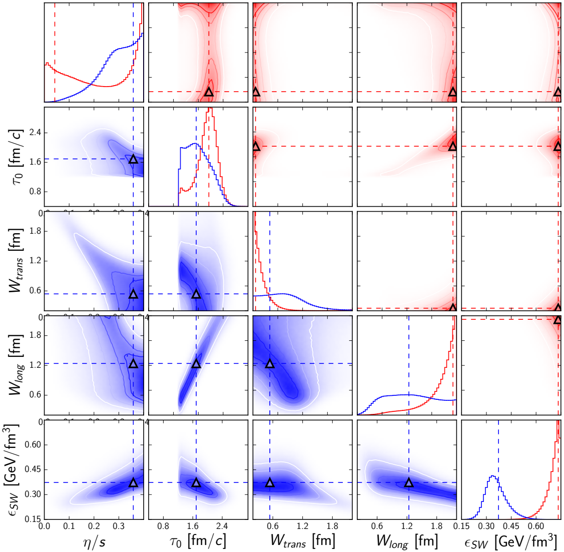
No weighting



With weighting

$\sqrt{s_{NN}} = 19.6$ GeV

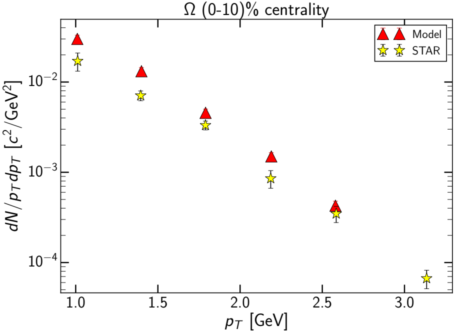
Red: No additional weights on data



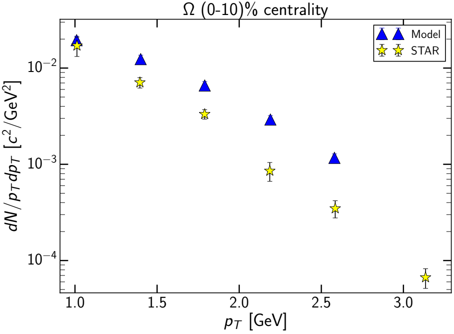
Blue: Weight of v_2 increased by a factor of 5, $N(\Omega)$ by a factor of 4; $N(p)$ and $N(\bar{p})$ decreased by 0.5

$\sqrt{s_{NN}} = 19.6$ GeV

Weighted analysis gives better fit on $N(\Omega)$ at low p_T
while peak values from unweighted posterior fit better at high p_T



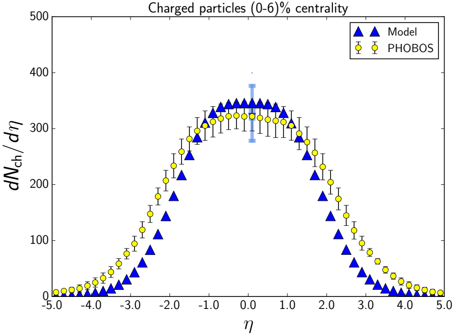
No weighting

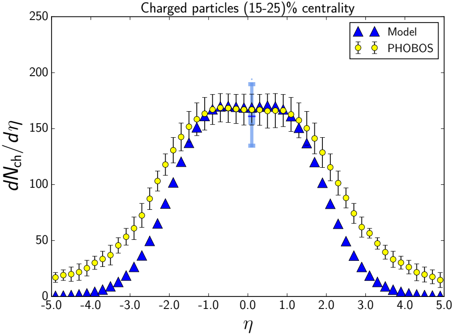


With weighting

$\sqrt{s_{NN}} = 19.6$ GeV

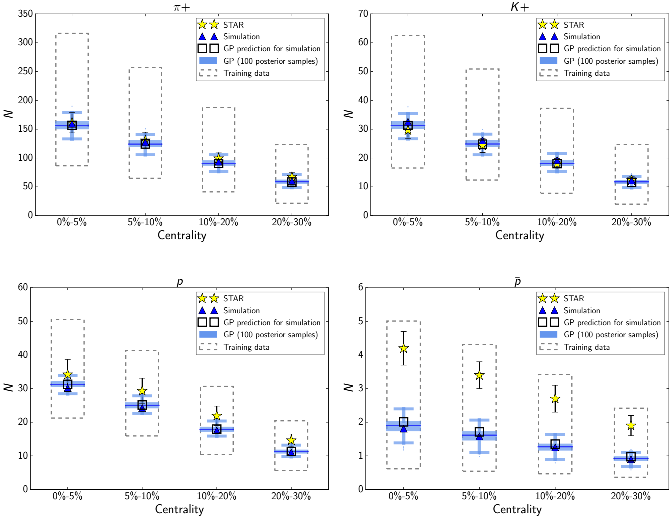
Results in the weighted scenario





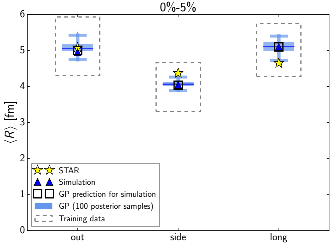
$\sqrt{s_{NN}} = 19.6$ GeV

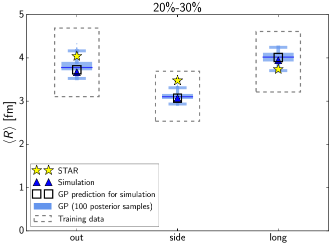
Results in the weighted scenario

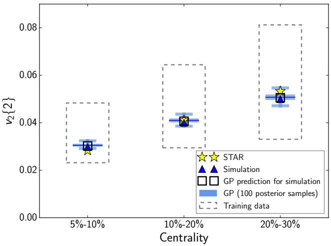


$\sqrt{s_{NN}} = 19.6$ GeV

Results in the weighted scenario

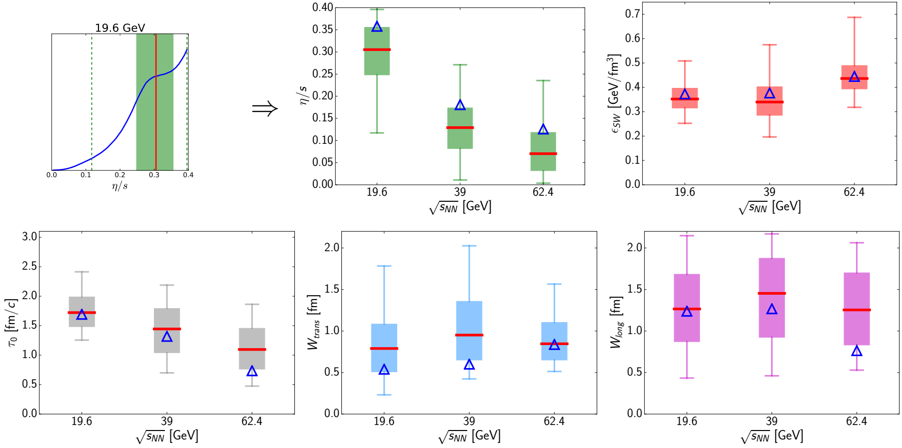






Parameter dependence on collision energy

η/s and τ_0 show clear increasing trend towards lower energies
(however, minimum of τ_0 increases by construction)



Boxes: 50% confidence range

Whiskers: 95% confidence range

Summary

- Bayesian analysis provides a rigorous method for simultaneous estimation of both the best-fit values and the associated uncertainties
- Reweighting found necessary for emphasizing relevant physics and compensating for fewer data points
  - increased weight on v_2 gives better constraints on η/s
  - increased weight on Ω gives better constraints on switching energy density ϵ_{SW}
  - proton and antiproton yields given smaller weights due to the lack of feed-down correction
- η/s shows a clear preference of larger values at lower $\sqrt{s_{NN}}$, strongly suggesting μ_B dependence
- 95% confidence ranges still large; in particular, initial state smearing parameters W_{trans}, W_{long} poorly constrained

Extra slides

Likelihood function

The likelihood function used in MCMC:

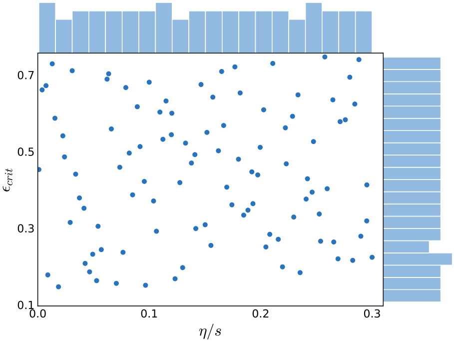
$$\exp\left(-\frac{1}{2}\sum_{i=1}^{q}\lambda_i\frac{(z_i^* - z_i^{\exp})^2}{(\sigma_{\rm stat}^{\exp})^2 + \Sigma_i^*}\right)$$

- λ_i is the variance explained by ith principal component
- z_i^* is the emulator prediction for ith principal component at the input parameter point $\vec{x}^*$
- $\vec{z}^{\exp}$ is the experimental data transformed to principal component space
- Σ_i^* is the predictive variance (emulator uncertainty)
- $\sigma_{\rm stat}^{\exp}$ is the experimental statistical error (systematic errors ignored)

Investigated parameter ranges

Sample emulator training points evenly over whole parameter space using Latin hypercube method

- Shear viscosity over entropy density η/s: 0.001 - 0.4
- Transport-to-hydro transition time τ_0: 0.4 - 3.1 fm
- Transverse Gaussian smearing of particles W_{trans}: 0.2 - 2.2 fm
- Longitudinal Gaussian smearing of particles W_{long}: 0.2 - 2.2 fm
- Hydro-to-transport transition energy density ϵ_{SW}: 0.15 - 0.75 GeV/fm^3



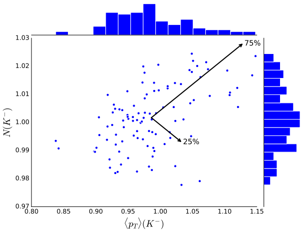
Principal component analysis

m observables $\Rightarrow$ m Gaussian processes needed for model emulation

However, m can be up to $\mathcal{O}(100)$ at top RHIC energies and at the LHC! Number of emulators can be reduced with principal component analysis

First principal component represents the direction of largest variance in output space, second PC the direction of second largest variance, etc.

- Fraction of variance explained by principal component p_q: $\mathrm{Var}(p_q) = \frac{\lambda_q}{\sum_{i=1}^{m} \lambda_i}$
- Select the number of principal components which together explain desired fraction of total variance; often only a few PCs are needed to explain 99% of the variance



Analysis procedure

Produce training data

- run simulations with $\mathcal{O}(100)$ different parameter combinations

Preparation for principal component analysis:

1. Scale with experimental values $\Rightarrow$ Unitless quantities
2. Check that the values are roughly normally distributed; apply a transformation if necessary
3. Center the data; apply possible data weighting

Determine required number of Gaussian processes with PCA

$\Downarrow$

Condition the emulators on training data

$\Downarrow$

Calibrate on experimental data by running MCMC