Measurement of mixed higher order flow harmonics in PbPb collisions

Shengquan Tuo
(Vanderbilt University)
for the CMS Collaboration

Quark Matter 2017

Feb 8, 2017

Motivation

- Azimuthal distribution and flow harmonics

$$P(\phi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} V_n e^{-in\phi}$$

$$V_n \equiv v_n e^{in\Psi_n}$$

Motivation

- Azimuthal distribution and flow harmonics

$$P(\phi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} V_n e^{-in\phi} \qquad V_n \equiv v_n e^{in\Psi_n}$$

- Final state momentum space anisotropy (v_n) = Initial state anisotropy (ε_n) * hydrodynamic response

Motivation

➢ Azimuthal distribution and flow harmonics

$$P(\phi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} V_n e^{-in\phi} \qquad V_n \equiv v_n e^{in\Psi_n}$$

➢ Final state momentum space anisotropy (v_n) = Initial state anisotropy (ε_n) * hydrodynamic response

➢ The response is linear ($v_n \sim k_n * \varepsilon_n$) for n = 2 and 3

Motivation

- Azimuthal distribution and flow harmonics

$$P(\phi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} V_n e^{-in\phi} \qquad V_n \equiv v_n e^{in\Psi_n}$$

- Final state momentum space anisotropy (v_n) = Initial state anisotropy (ε_n) * hydrodynamic response
- The response is linear ($v_n \sim k_n * \varepsilon_n$) for n = 2 and 3
- But for n>3,

$$\begin{aligned} V_4 &= V_{4L} + \chi_{422}(V_2)^2 \\ V_5 &= V_{5L} + \chi_{523}V_2V_3 \\ V_6 &= V_{6L} + \chi_{6222}(V_2)^3 + \chi_{633}(V_3)^2 \\ V_7 &= V_{7L} + \chi_{7223}(V_2)^2V_3 \end{aligned}$$

Phys.Lett. B 744 (2015) 82

"Linear" response (from "ε_n")

Nonlinear response (from ε_2 and ε_3)

Motivation

$$V_4 = V_{4L} + \chi_{422}(V_2)^2$$
$$V_5 = V_{5L} + \chi_{523}V_2V_3$$
$$V_6 = V_{6L} + \chi_{6222}(V_2)^3 + \chi_{633}(V_3)^2$$
$$V_7 = V_{7L} + \chi_{7223}(V_2)^2V_3$$

"Linear" response (from "ε_n")

Nonlinear response (from ε_2 and ε_3)

- Most flow measurements **can not separate** the linear and nonlinear part: $v_n\{\psi_n\}$, $v_n\{2\}$, v_n flucutations, etc.

Motivation

$$
\begin{aligned}
V_4 &= V_{4L} + \chi_{422}(V_2)^2 \\
V_5 &= V_{5L} + \chi_{523}V_2V_3 \\
V_6 &= V_{6L} + \chi_{6222}(V_2)^3 + \chi_{633}(V_3)^2 \\
V_7 &= V_{7L} + \chi_{7223}(V_2)^2V_3
\end{aligned}
$$

"Linear" response (from "ε_n")

Nonlinear response (from ε_2 and ε_3)

- Most flow measurements **can not separate** the "linear" and nonlinear part: $v_n\{\psi_n\}$, $v_n\{2\}$, v_n flucutations, etc.
- Nonlinear part can be studied by: mixed harmonics, nonlinear response coefficients, event plane correlations, etc.

Mixed harmonics:

$$v_5\{\Psi_{23}\} \equiv \frac{Re\left\langle V_5 V_2^* V_3^* \right\rangle}{\sqrt{\left\langle |V_2|^2 |V_3|^2 \right\rangle}}$$

Nonlinear response coefficients:

$$\chi_{523} = \frac{v_5\{\Psi_{23}\}}{\sqrt{\left\langle v_2^2 v_3^2 \right\rangle}}$$

Notations

With $V_n \equiv v_n e^{in\Psi_n}$

Traditional:
(Scalar Product)

$$v_n\{\Psi_n\} \equiv \frac{Re\left\langle V_n V_n^*\right\rangle}{\sqrt{\left\langle \left|V_n\right|^2\right\rangle}} = \frac{\left\langle v_n v_n \overbrace{\cos\left(n\Psi_n - n\Psi_n\right)}^{\boxed{\cos\left(n\phi - n\Psi_n\right)}}\right\rangle}{\sqrt{\left\langle v_n^2\right\rangle}} = \sqrt{\left\langle v_n^2\right\rangle}$$

Notations

With $V_n \equiv v_n e^{in\Psi_n}$

Traditional:
(Scalar Product)

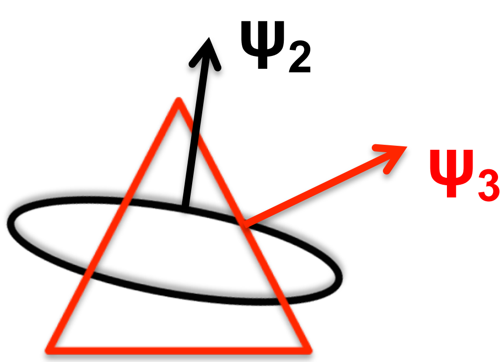
$$v_n\{\Psi_n\} \equiv \frac{Re\langle V_n V_n^*\rangle}{\sqrt{\langle |V_n|^2\rangle}} = \frac{\langle v_n v_n \overbrace{\cos(n\Psi_n - n\Psi_n)}^{\cos(n\phi - n\Psi_n)}\rangle}{\sqrt{\langle v_n^2\rangle}} = \sqrt{\langle v_n^2\rangle}$$

Mixed harmonics:

$$v_5\{\Psi_{23}\} \equiv \frac{Re\langle V_5 V_2^* V_3^*\rangle}{\sqrt{\langle |V_2|^2 |V_3|^2\rangle}} = \frac{\langle v_5 v_2 v_3 \overbrace{\cos(5\Psi_5 - 2\Psi_2 - 3\Psi_3)}^{\cos(5\phi - 2\Psi_2 - 3\Psi_3)}\rangle}{\sqrt{\langle v_2^2 v_3^2\rangle}}$$

V_5 with respect to the direction of V_2 and V_3

Event plane correlations + flow correlations



Notations

With $V_n \equiv v_n e^{in\Psi_n}$

Traditional:
(Scalar Product)

$$v_n\{\Psi_n\} \equiv \frac{Re\langle V_n V_n^*\rangle}{\sqrt{\langle |V_n|^2\rangle}} = \frac{\langle v_n v_n \overbrace{\cos(n\Psi_n - n\Psi_n)}^{\cos(n\phi - n\Psi_n)}\rangle}{\sqrt{\langle v_n^2\rangle}} = \sqrt{\langle v_n^2\rangle}$$

Mixed harmonics:

$$v_5\{\Psi_{23}\} \equiv \frac{Re\langle V_5 V_2^* V_3^*\rangle}{\sqrt{\langle |V_2|^2 |V_3|^2\rangle}} = \frac{\langle v_5 v_2 v_3 \overbrace{\cos(5\Psi_5 - 2\Psi_2 - 3\Psi_3)}^{\cos(5\phi - 2\Psi_2 - 3\Psi_3)}\rangle}{\sqrt{\langle v_2^2 v_3^2\rangle}}$$

V_5 with respect to the direction of V_2 and V_3

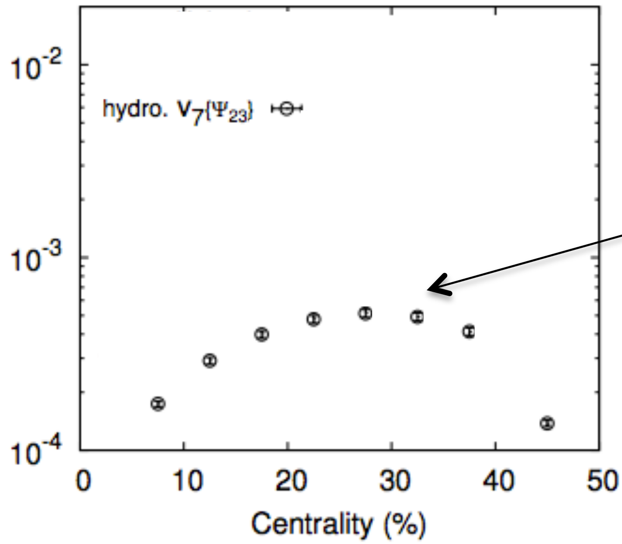
Event plane correlations + flow correlations

From $V_5 = V_{5L} + \chi_{523} V_2 V_3$

$$\chi_{523} = \frac{Re\langle V_5 V_2^* V_3^*\rangle}{\langle |V_2|^2 |V_3|^2\rangle} = \frac{\langle v_5 v_2 v_3 \cos(5\Psi_5 - 2\Psi_2 - 3\Psi_3)\rangle}{\langle v_2^2 v_3^2\rangle} = \frac{v_5\{\Psi_{23}\}}{\sqrt{\langle v_2^2 v_3^2\rangle}}$$

Mixed harmonics divided by lower harmonics including flow correlations

Hydrodynamic predictions

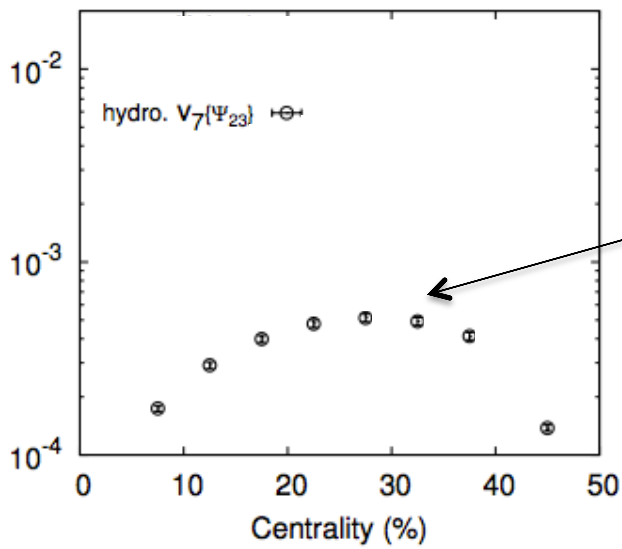


$$v_7\{\Psi_{223}\}$$

η/s=0.08

L. Yan, J. Ollitrault,
Phys.Lett. B744 (2015) 82

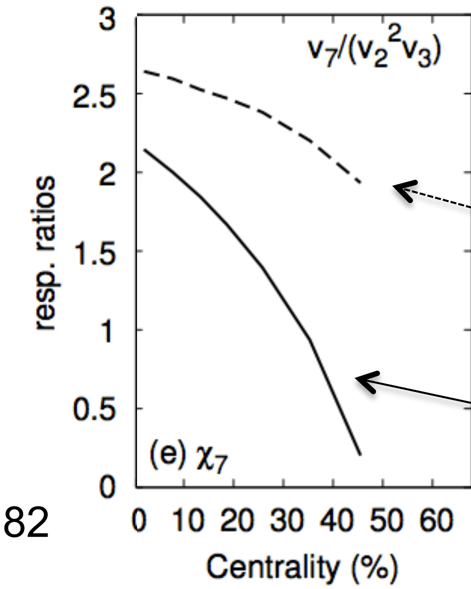
Hydrodynamic predictions



$v_7\{\Psi_{223}\}$

η/s=0.08

L. Yan, J. Ollitrault,
Phys.Lett. B744 (2015) 82

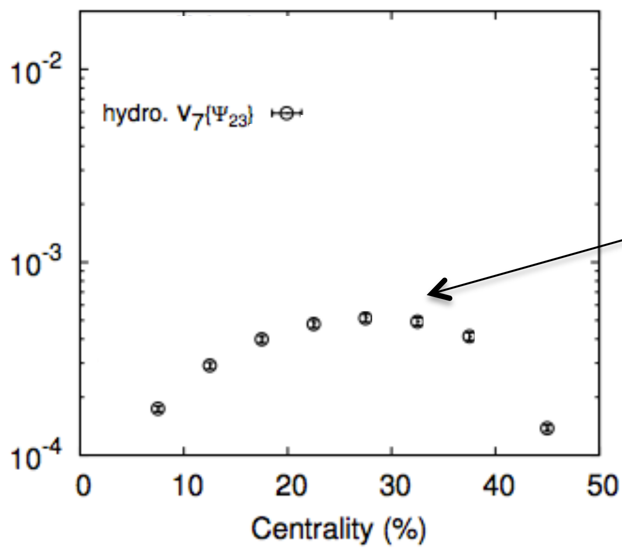


χ_{7223}

η/s=0

η/s=0.08

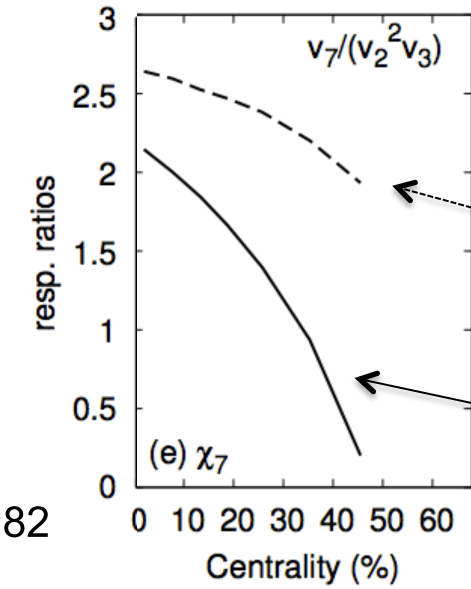
Hydrodynamic predictions



$v_7\{\Psi_{223}\}$

η/s=0.08

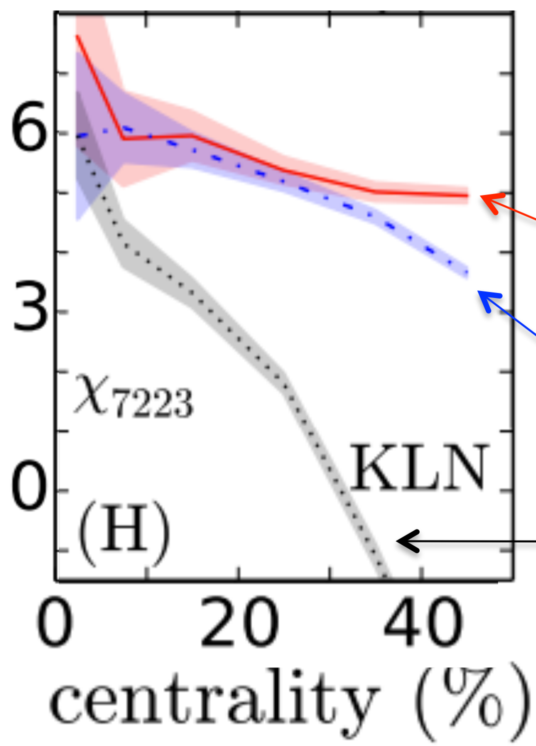
L. Yan, J. Ollitrault,
Phys.Lett. B744 (2015) 82



χ_{7223}

η/s=0

η/s=0.08



χ_{7223}

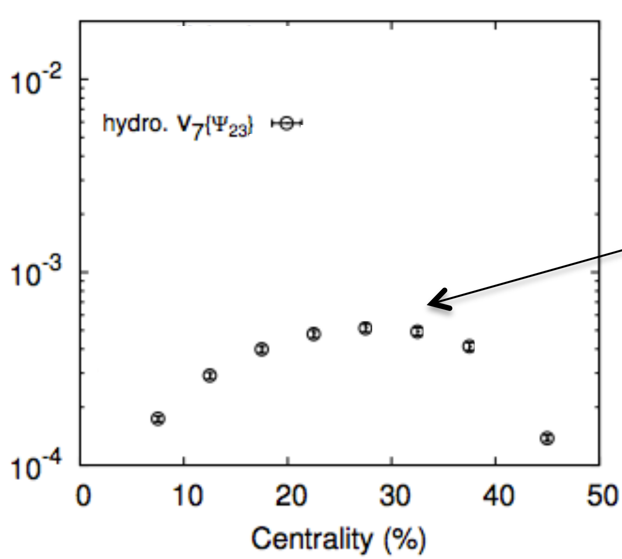
η/s=0

η/s=0.08

η/s=0.2

Q. Jing, U. Heinz, J. Liu, Phys. Rev. C 93, 064901 (2016)

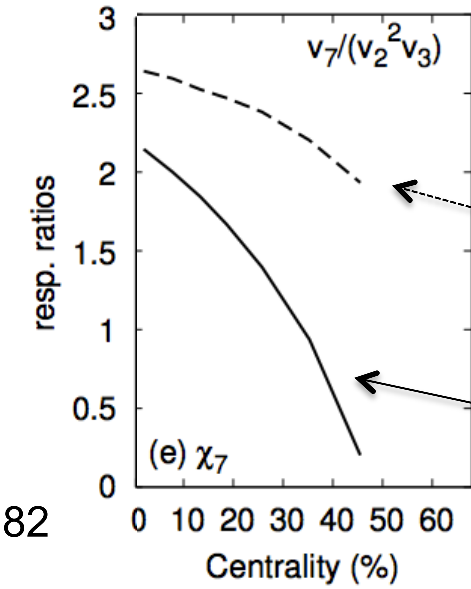
Hydrodynamic predictions



$v_7\{\Psi_{223}\}$

η/s=0.08

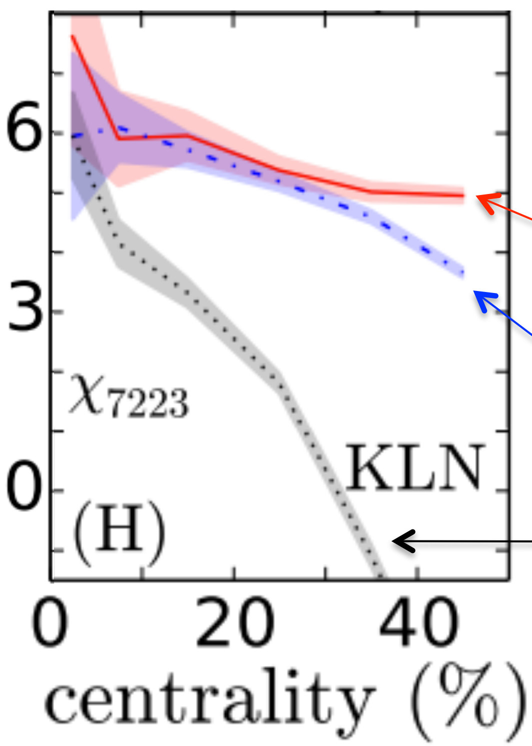
L. Yan, J. Ollitrault, Phys.Lett. B744 (2015) 82



χ_{7223}

η/s=0

η/s=0.08



χ_{7223}

η/s=0

η/s=0.08

η/s=0.2

Nonlinear part has

- significant sensitivity to initial conditions
- weak dependence on η/s during hydro.
- strong sensitivity to η/s at freeze-out

Q. Jing, U. Heinz, J. Liu, Phys. Rev. C 93, 064901 (2016)

List of measurements

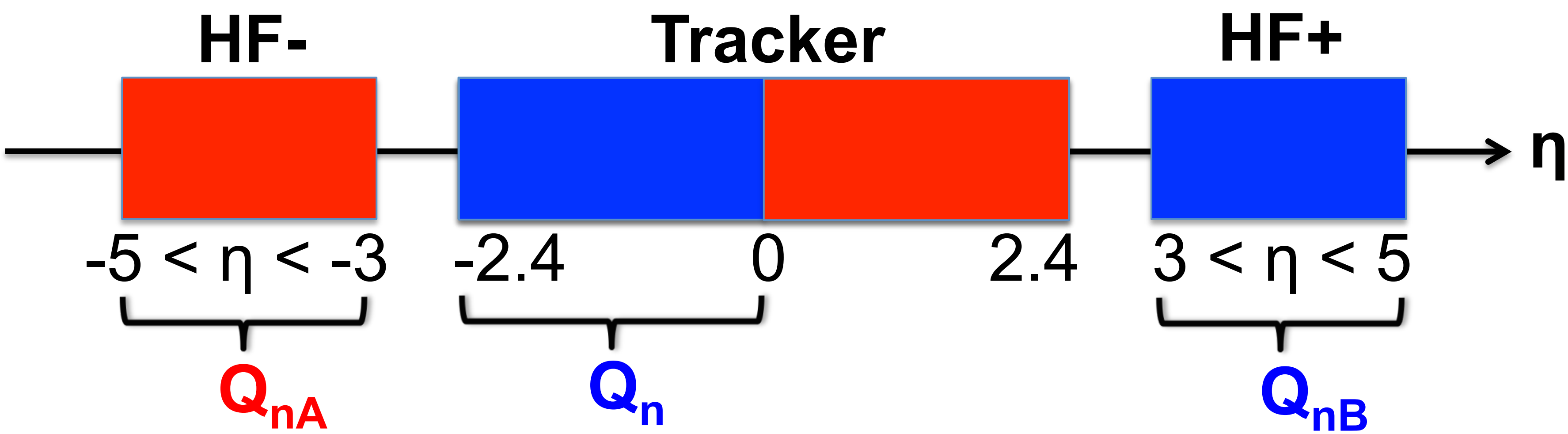
- $v_4\{\Psi_{22}\}$ and $v_6\{\Psi_{222}\}$
- First direct measurement of mixed higher order flow harmonics: $v_5\{\Psi_{23}\}$, $v_6\{\Psi_{33}\}$, and $v_7\{\Psi_{223}\}$
- First direct measurement of nonlinear response coefficients: χ_{422}, χ_{523}, χ_{6222}, χ_{633}, and χ_{7223}
- Comparison of v_n from two-particle correlations (linear + nonlinear) with v_n from mixed harmonics (nonlinear)

Measured in 0-60%, $0.3 < p_T < 8.0$ GeV/c and $|\eta| < 2.4$

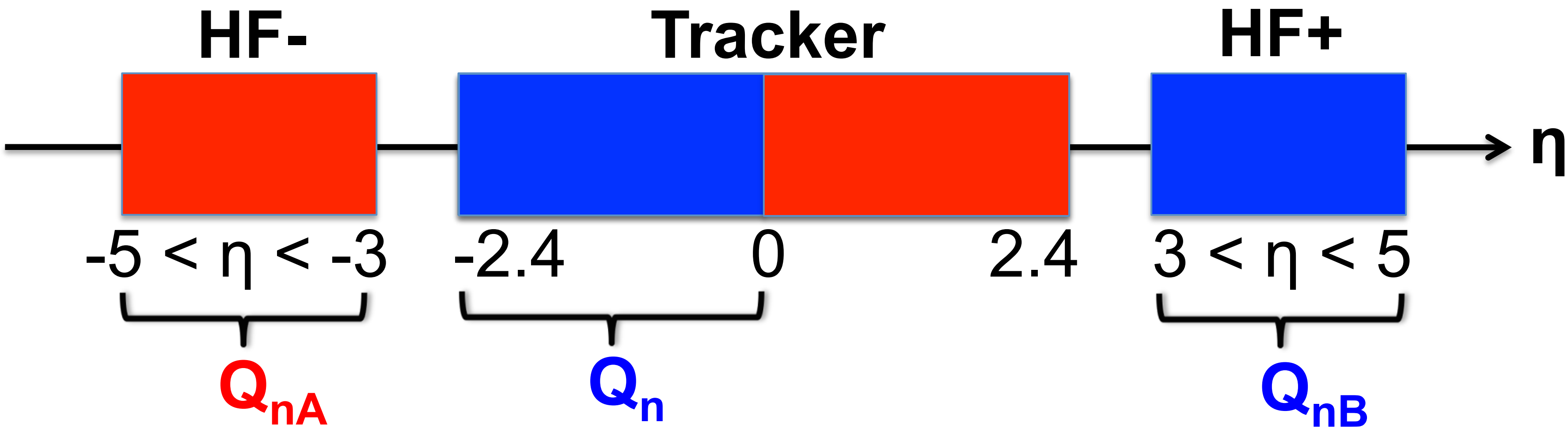
Details: http://cds.cern.ch/record/2244660

Scalar Product Method

Mixed harmonics



Mixed harmonics

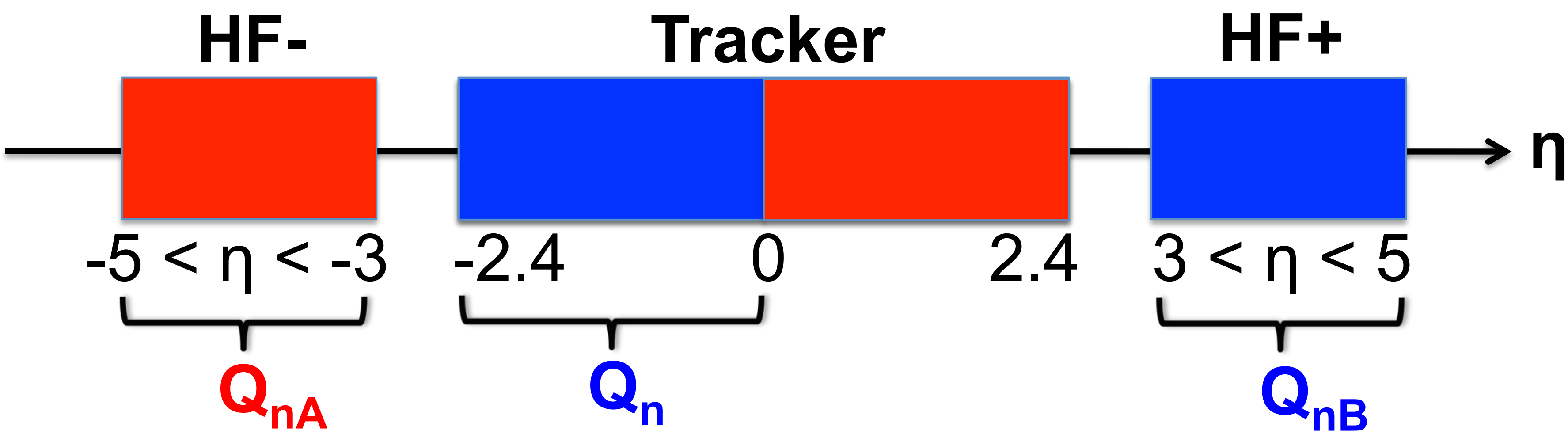


$$V_n \equiv v_n e^{in\Psi_n}$$

$$v_5\{\Psi_{23}\} \equiv \frac{Re\left\langle V_5 V_2^* V_3^* \right\rangle}{\sqrt{\left\langle |V_2|^2 \, |V_3|^2 \right\rangle}}$$

L. Yan, J. Ollitrault,
Phys.Lett. B744 (2015) 82

Mixed harmonics



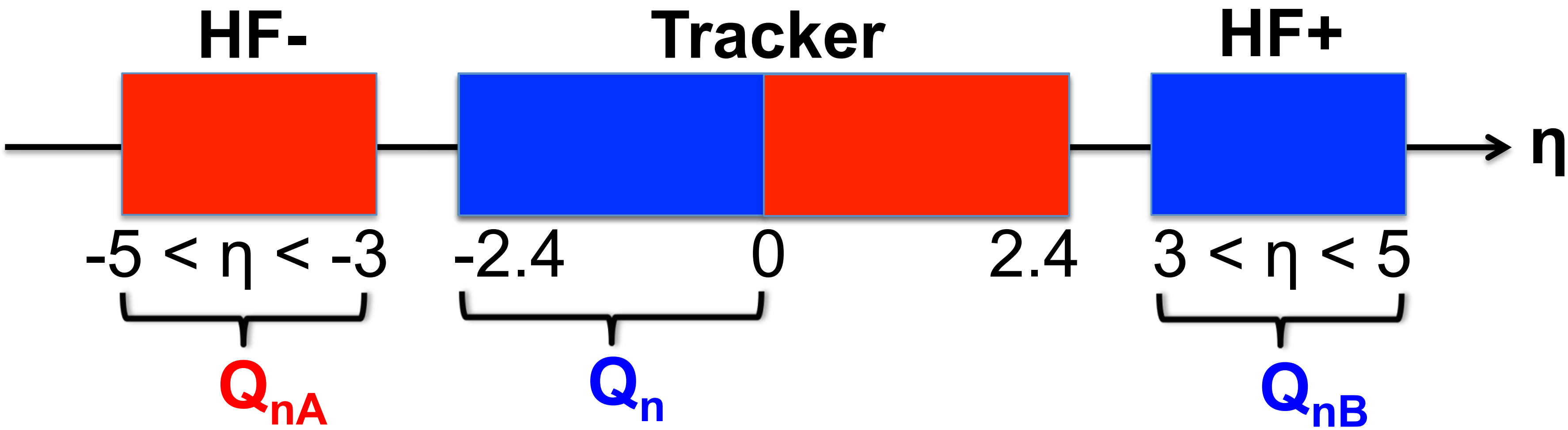
$$V_n \equiv v_n e^{in\Psi_n}$$

$$v_5\{\Psi_{23}\} \equiv \frac{Re\langle V_5 V_2^* V_3^*\rangle}{\sqrt{\langle |V_2|^2 |V_3|^2\rangle}}$$

$$Q_n = \frac{1}{\sum w_j} \sum_j w_j e^{in\phi_j} = |Q_n|\, e^{in\Psi_n}$$

L. Yan, J. Ollitrault, Phys.Lett. B744 (2015) 82

Mixed harmonics



$$V_n \equiv v_n e^{in\Psi_n}$$

$$Q_n = \frac{1}{\sum w_j} \sum_j w_j e^{in\phi_j} = |Q_n| \, e^{in\Psi_n}$$

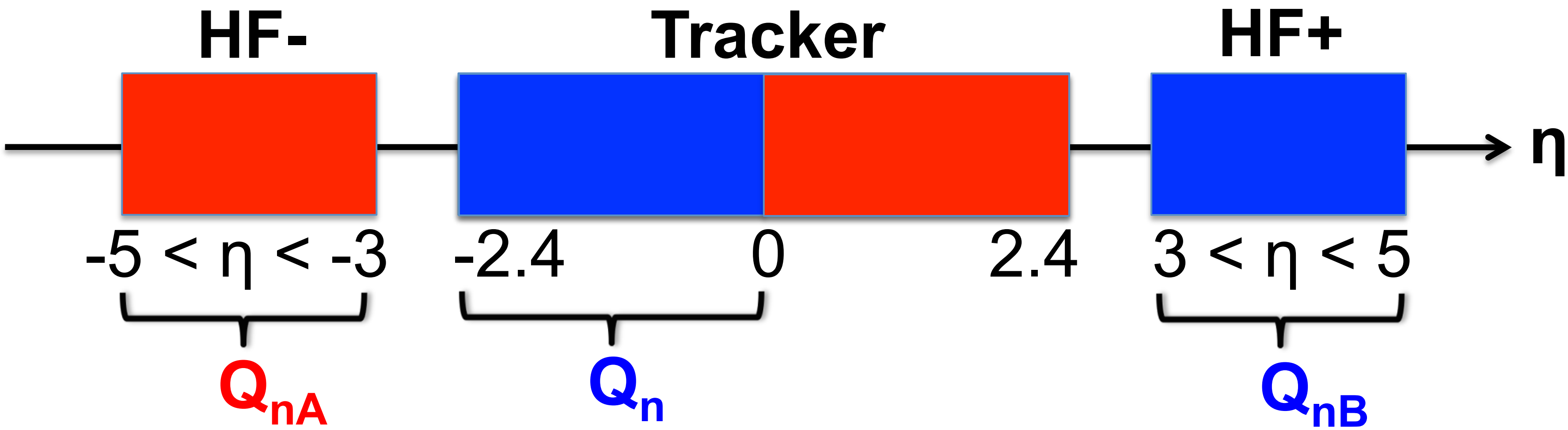
$$v_5\{\Psi_{23}\} \equiv \frac{Re \langle V_5 V_2^* V_3^* \rangle}{\sqrt{\langle |V_2|^2 \, |V_3|^2 \rangle}}$$

$$v_5\{\Psi_{23}\} = \frac{Re\langle Q_5 Q_{2B}^* Q_{3B}^* \rangle}{\sqrt{Re\langle Q_{2A} Q_{3A} Q_{2B}^* Q_{3B}^* \rangle}}$$

L. Yan, J. Ollitrault, Phys.Lett. B744 (2015) 82

- Large η gap (|Δη|>3.0)

Nonlinear response coefficients



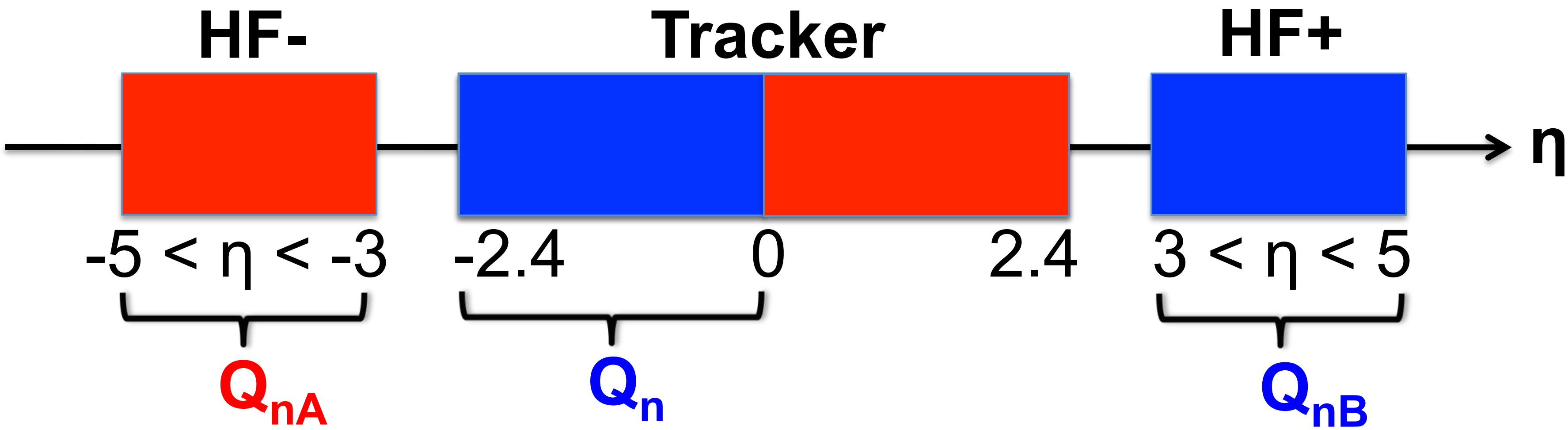
$$V_n \equiv v_n e^{in\Psi_n}$$

$$\chi_{523} = \frac{Re\left\langle V_5 V_2^* V_3^* \right\rangle}{\left\langle \left|V_2\right|^2 \left|V_3\right|^2 \right\rangle}$$

$$Q_n = \frac{1}{\sum w_j} \sum_j w_j e^{in\phi_j} = \left|Q_n\right| e^{in\Psi_n}$$

L. Yan, J. Ollitrault,
Phys.Lett. B744 (2015) 82

Nonlinear response coefficients



$$V_n \equiv v_n e^{in\Psi_n}$$

$$\chi_{523} = \frac{Re\left\langle V_5 V_2^* V_3^* \right\rangle}{\left\langle \left|V_2\right|^2 \left|V_3\right|^2 \right\rangle}$$

$$Q_n = \frac{1}{\sum w_j} \sum_j w_j e^{in\phi_j} = \left|Q_n\right| e^{in\Psi_n}$$

$$\chi_{523} = \frac{Re\langle Q_5 Q_{2B}^* Q_{3B}^* \rangle}{Re\langle Q_2 Q_3 Q_{2B}^* Q_{3B}^* \rangle}$$

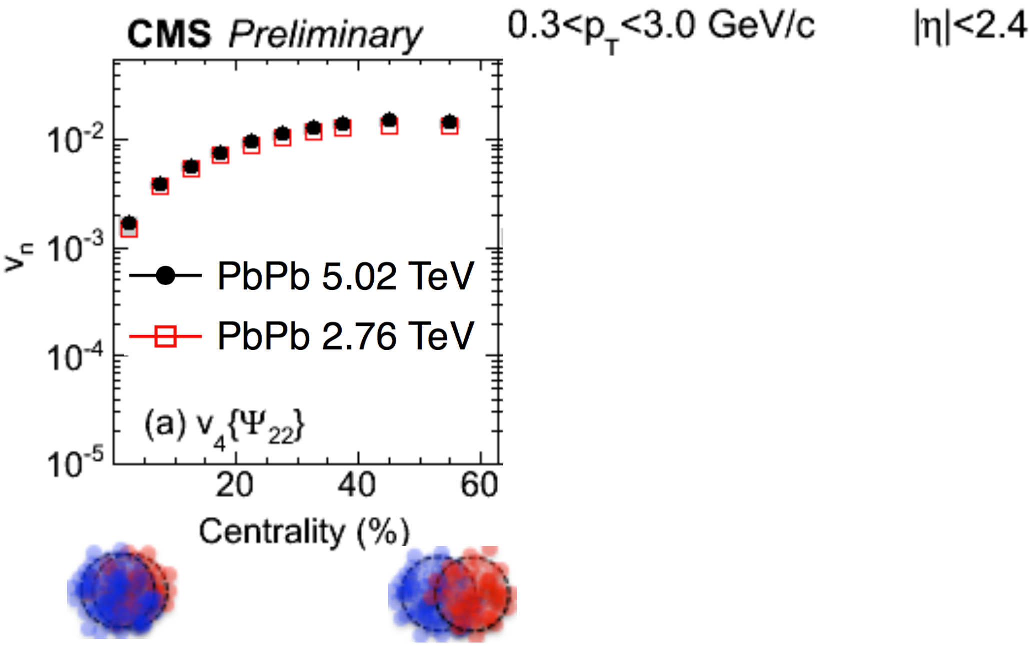
L. Yan, J. Ollitrault,
Phys.Lett. B744 (2015) 82

- Large η gap (|Δη|>3.0)

Results

Mixed harmonics vs. centrality

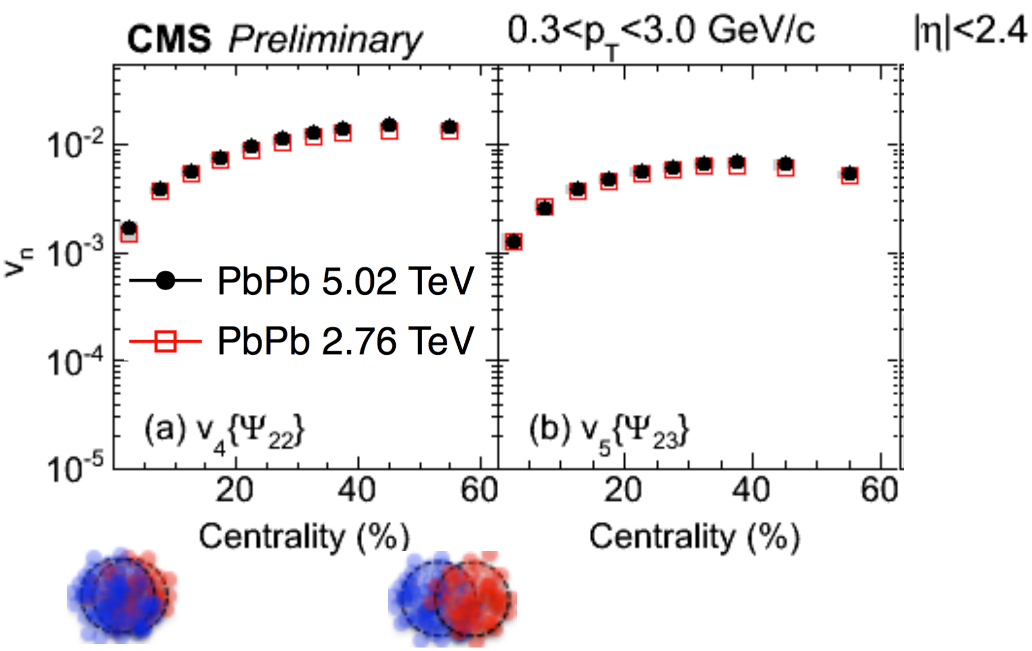
$v_4\{\psi_{22}\}$



- Strong centrality dependence for $v_4\{\psi_{22}\}$

Mixed harmonics vs. centrality

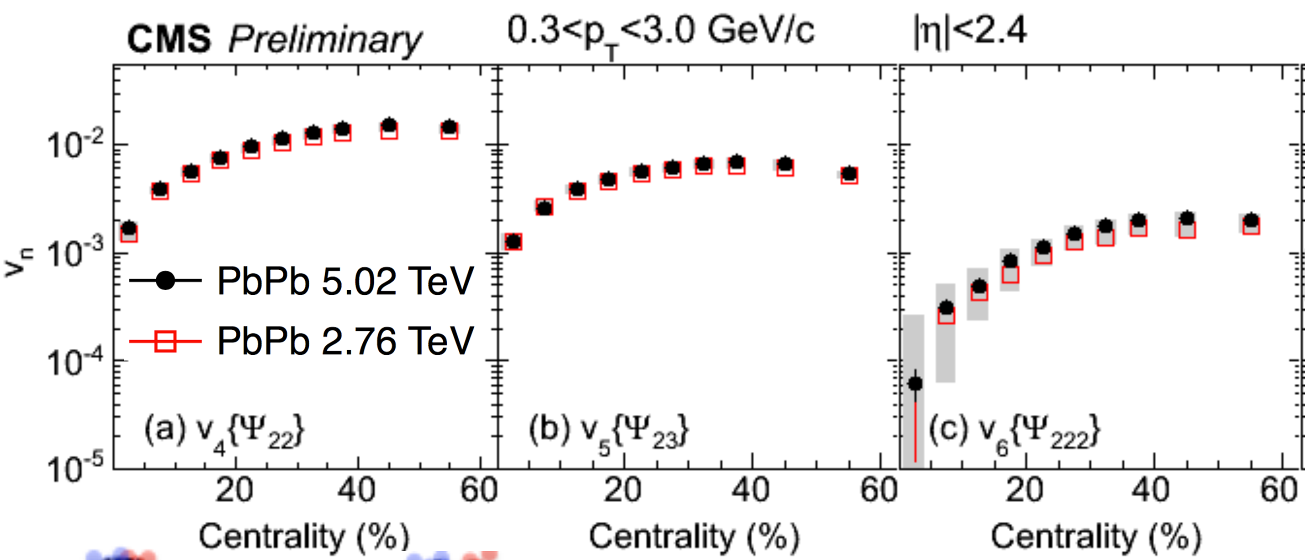
$v_4\{\Psi_{22}\}$ $v_5\{\Psi_{23}\}$

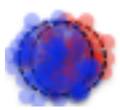


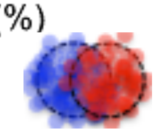
➢ Strong centrality dependence for $v_4\{\Psi_{22}\}$, $v_5\{\Psi_{23}\}$

Mixed harmonics vs. centrality

$v_4\{\Psi_{22}\}$ **$v_5\{\Psi_{23}\}$** **$v_6\{\Psi_{222}\}$**



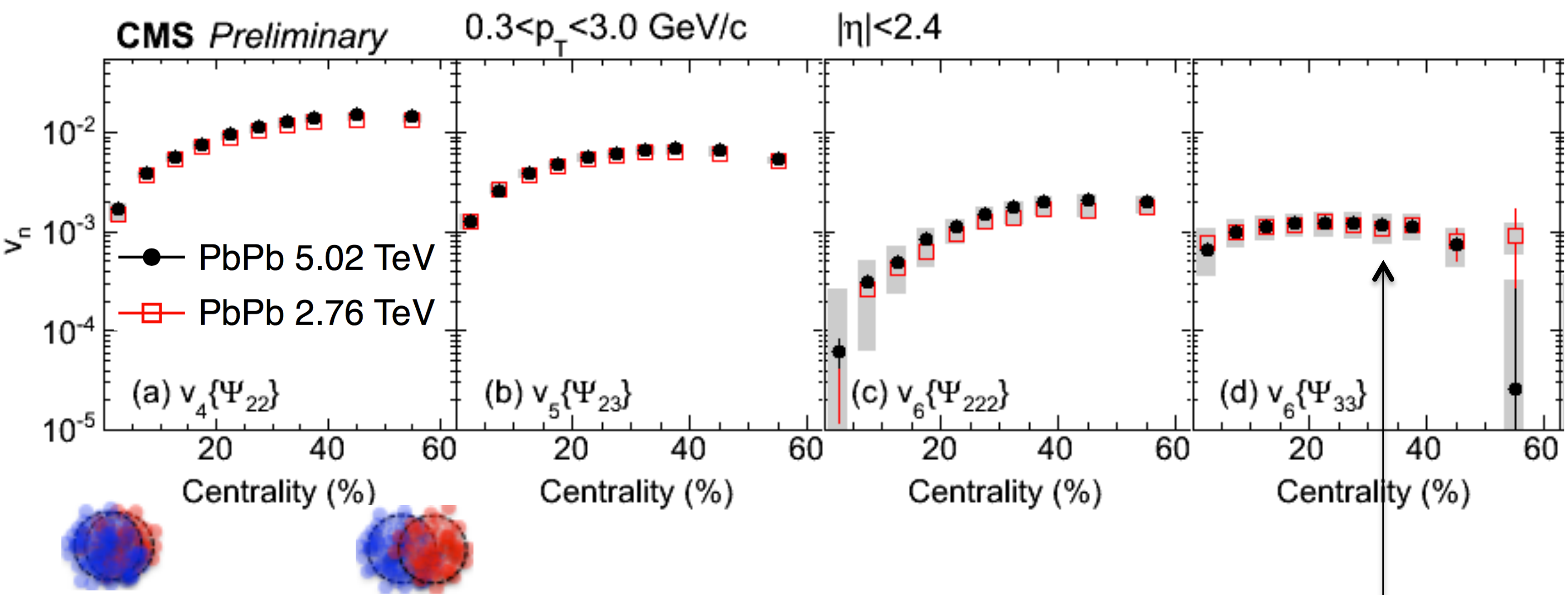




- Strong centrality dependence for $v_4\{\psi_{22}\}$, $v_5\{\psi_{23}\}$, $v_6\{\psi_{222}\}$

Mixed harmonics vs. centrality

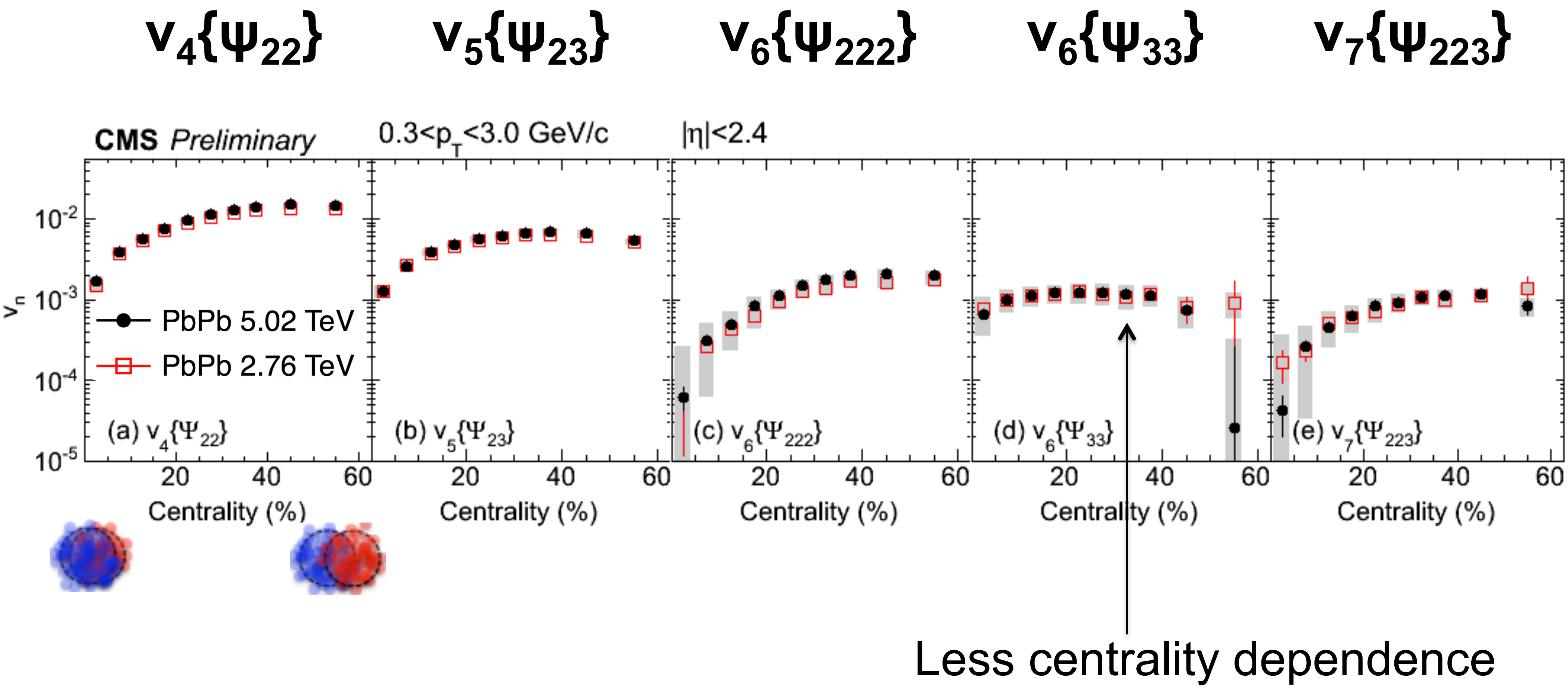
$v_4\{\Psi_{22}\}$ $v_5\{\Psi_{23}\}$ $v_6\{\Psi_{222}\}$ $v_6\{\Psi_{33}\}$



Less centrality dependence

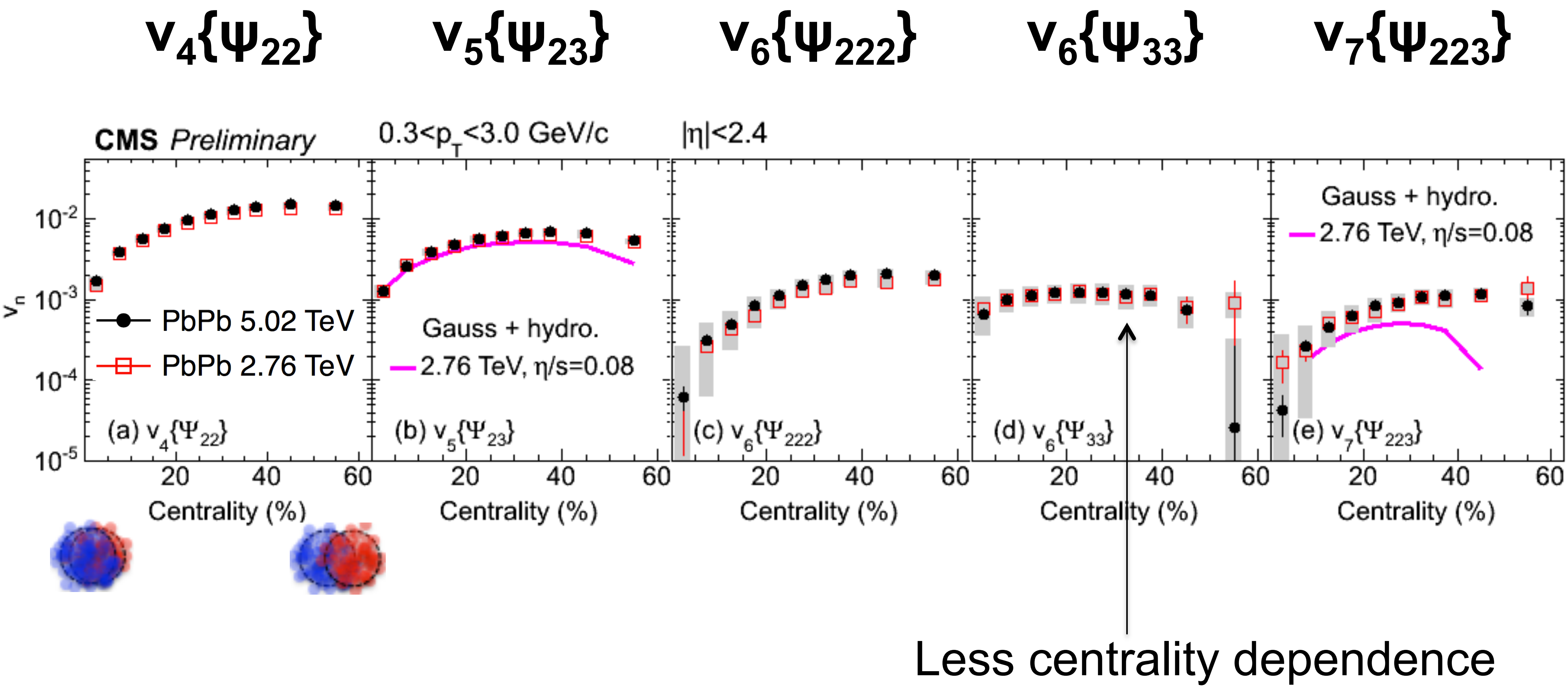
- Strong centrality dependence for $v_4\{\Psi_{22}\}$, $v_5\{\Psi_{23}\}$, $v_6\{\Psi_{222}\}$

Mixed harmonics vs. centrality



➢ Strong centrality dependence for $v_4\{\psi_{22}\}$, $v_5\{\psi_{23}\}$, $v_6\{\psi_{222}\}$ and $v_7\{\psi_{223}\}$

Mixed harmonics vs. centrality

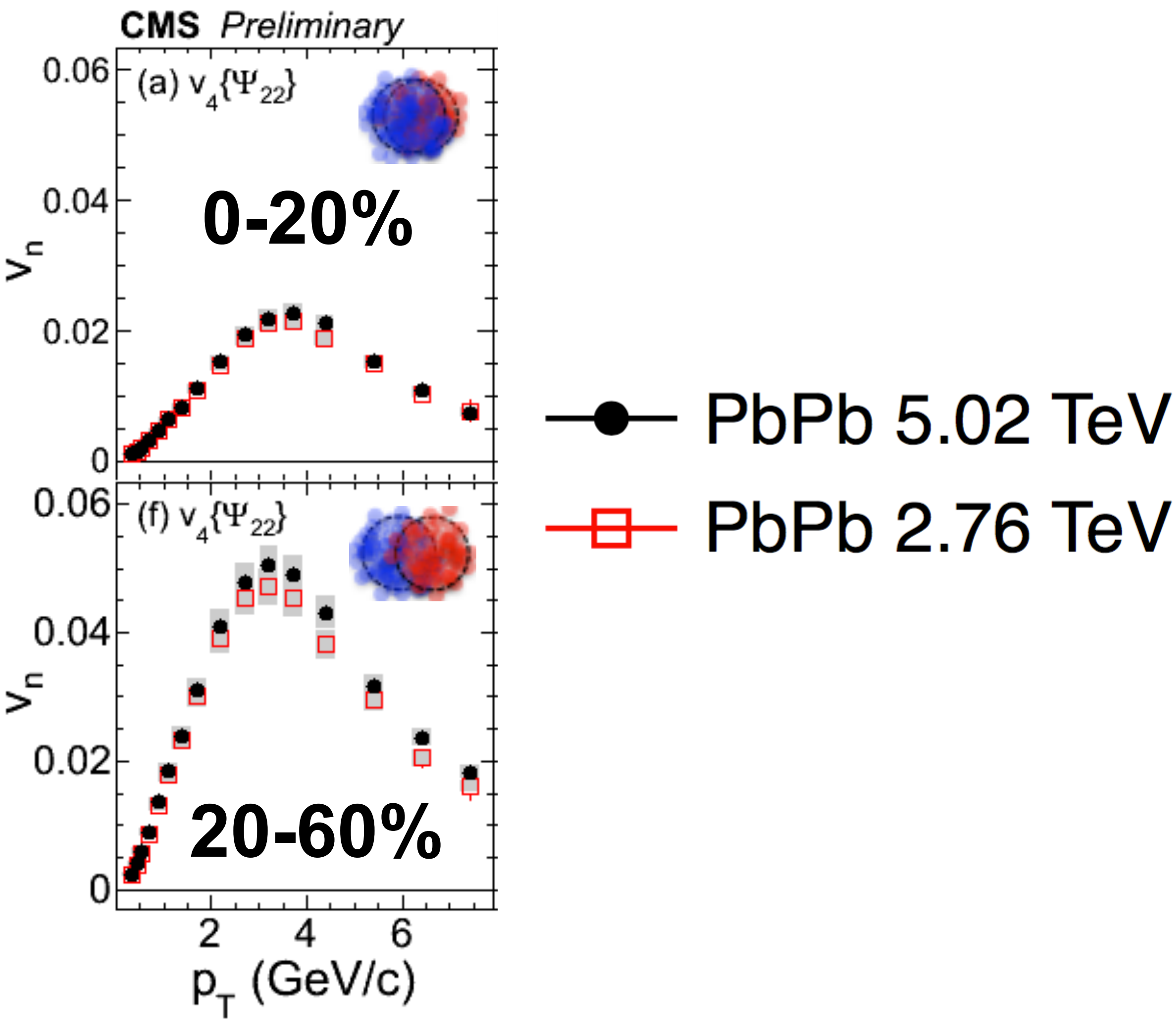


- Strong centrality dependence for $v_4\{\psi_{22}\}$, $v_5\{\psi_{23}\}$, $v_6\{\psi_{222}\}$ and $v_7\{\psi_{223}\}$
- Hydro. with $\eta/s=0.08$ describes the shape of $v_5\{\psi_{23}\}$, but not $v_7\{\psi_{223}\}$

L. Yan, J. Ollitrault, Phys.Lett. B744 (2015) 82

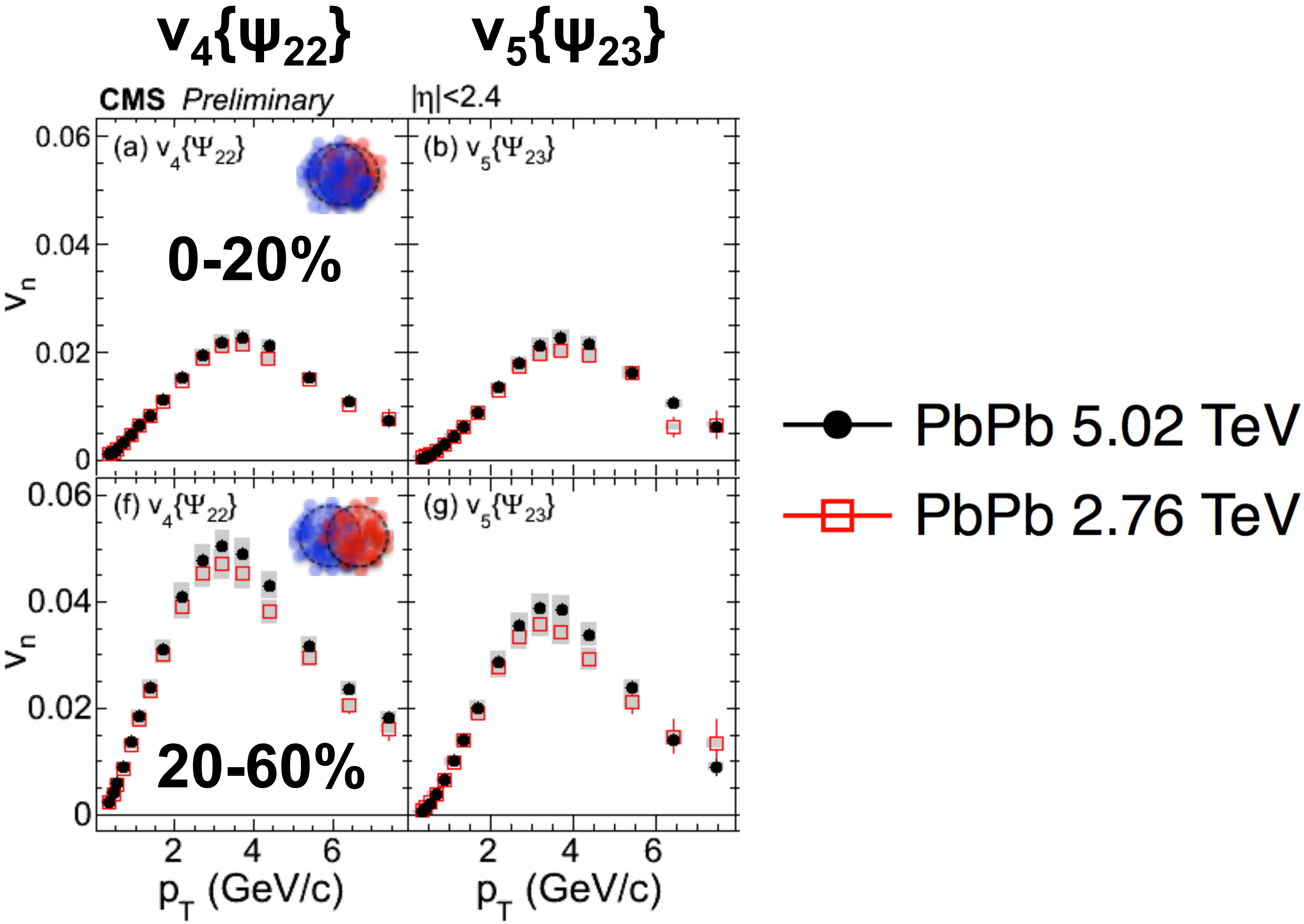
Mixed harmonics vs. p_T

$v_4\{\Psi_{22}\}$



- ➢ No strong energy dependence

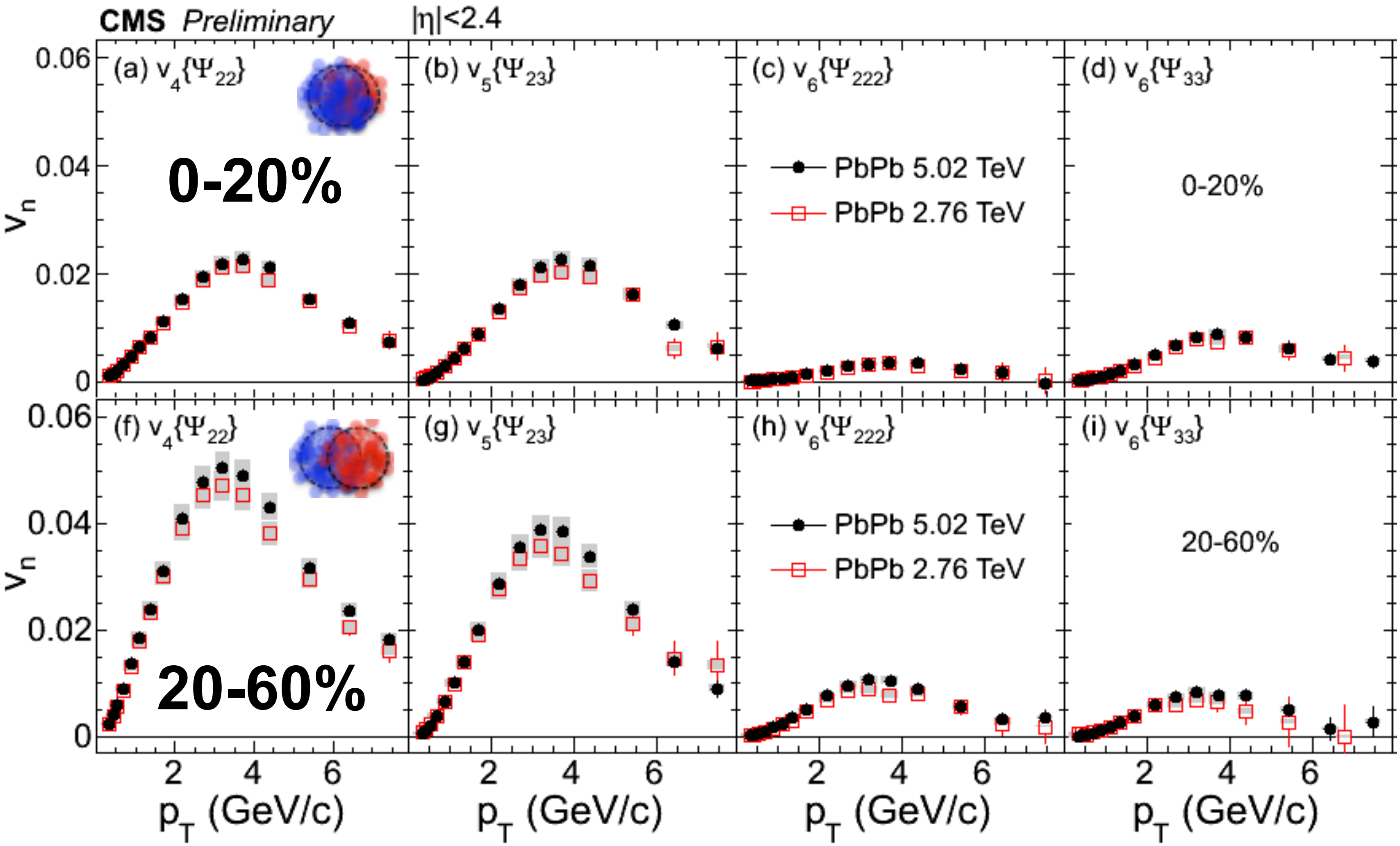
Mixed harmonics vs. p_T



- No strong energy dependence

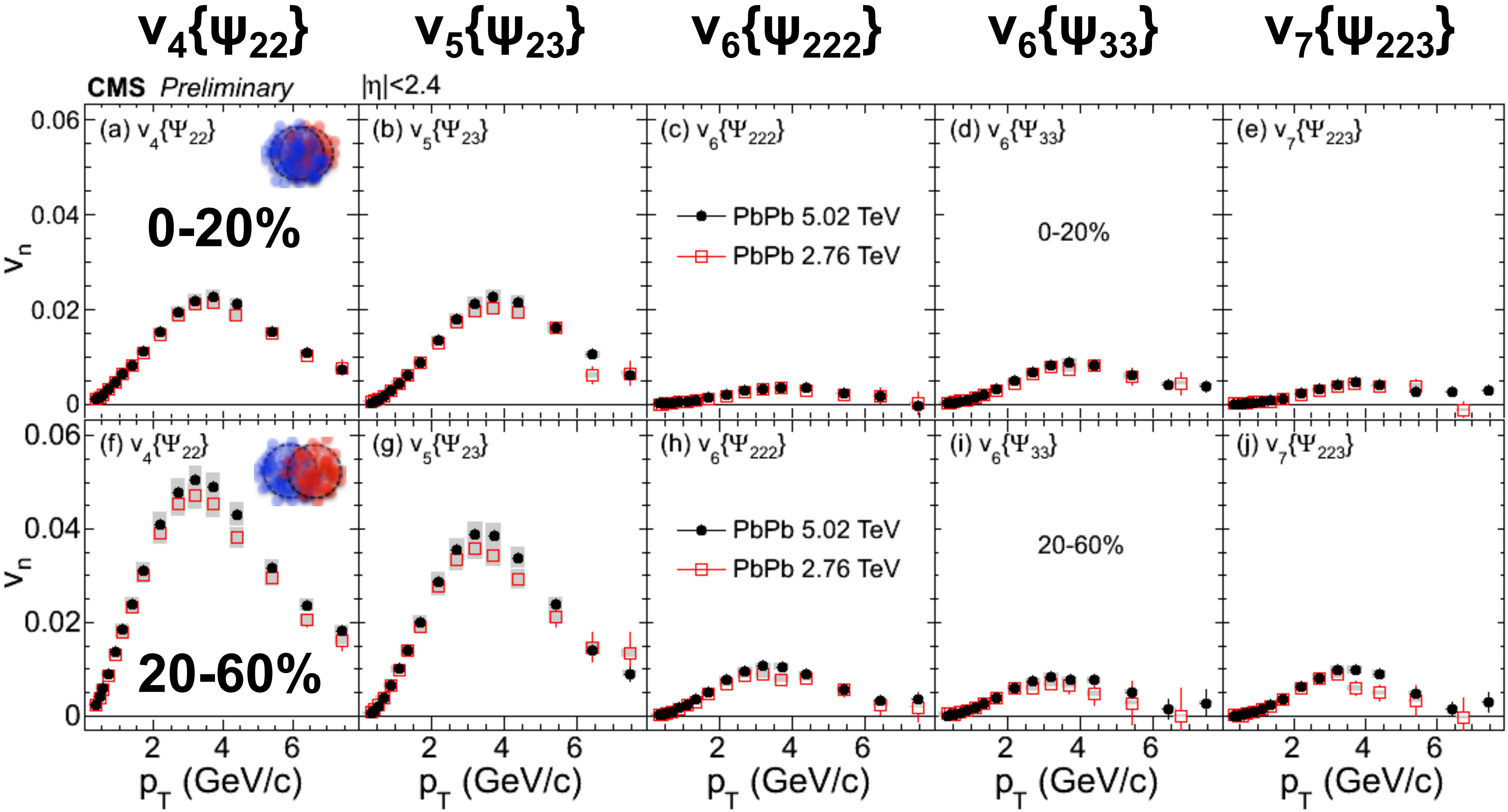
Mixed harmonics vs. p_T

$v_4\{\Psi_{22}\}$ $v_5\{\Psi_{23}\}$ $v_6\{\Psi_{222}\}$ $v_6\{\Psi_{33}\}$



➢ No strong energy dependence

Mixed harmonics vs. p_T

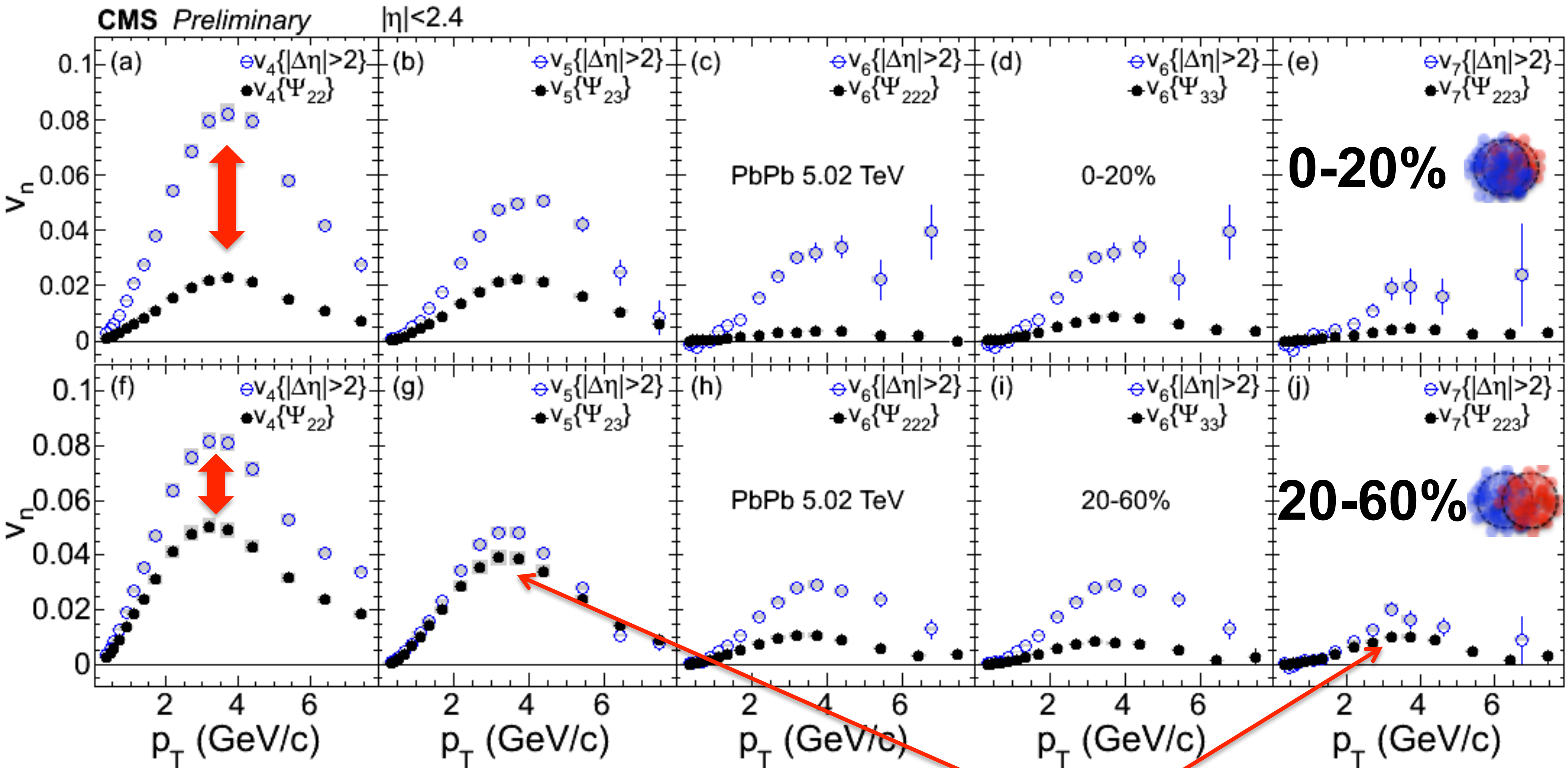


- ➢ No strong energy dependence

(Linear + Nonlinear) vs. Nonlinear

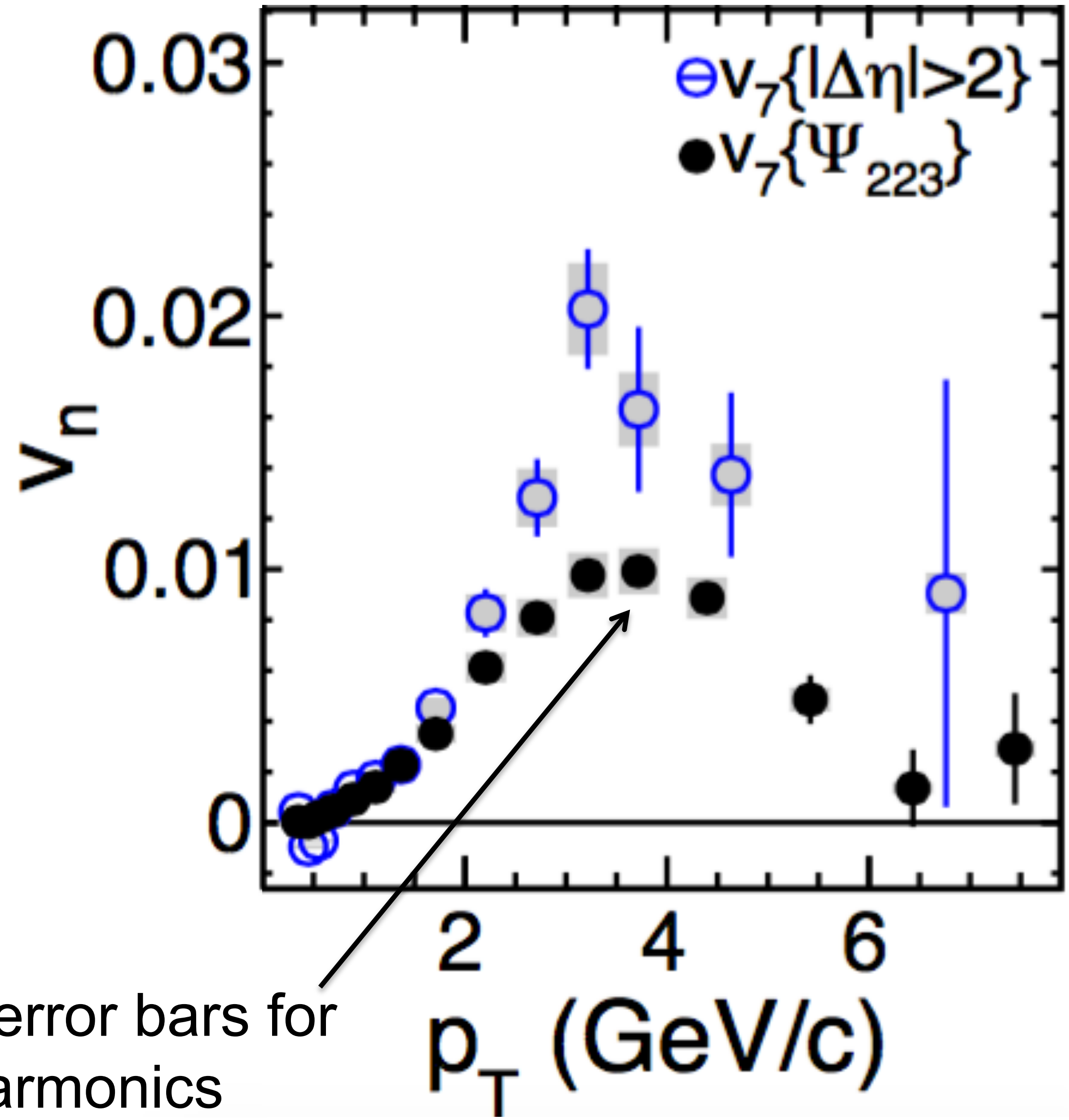
$v_4\{|\Delta\eta|>2\}$ $v_5\{|\Delta\eta|>2\}$ $v_6\{|\Delta\eta|>2\}$ $v_6\{|\Delta\eta|>2\}$ $v_7\{|\Delta\eta|>2\}$

$v_4\{\Psi_{22}\}$ $v_5\{\Psi_{23}\}$ $v_6\{\Psi_{222}\}$ $v_6\{\Psi_{33}\}$ $v_7\{\Psi_{223}\}$



- Larger contribution of nonlinear part for v_5 and v_7

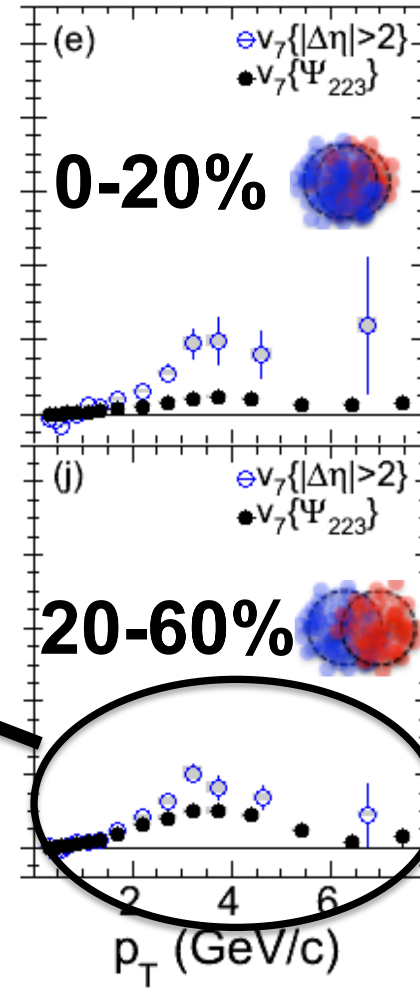
(Linear + Nonlinear) vs. Nonlinear



Smaller error bars for mixed harmonics

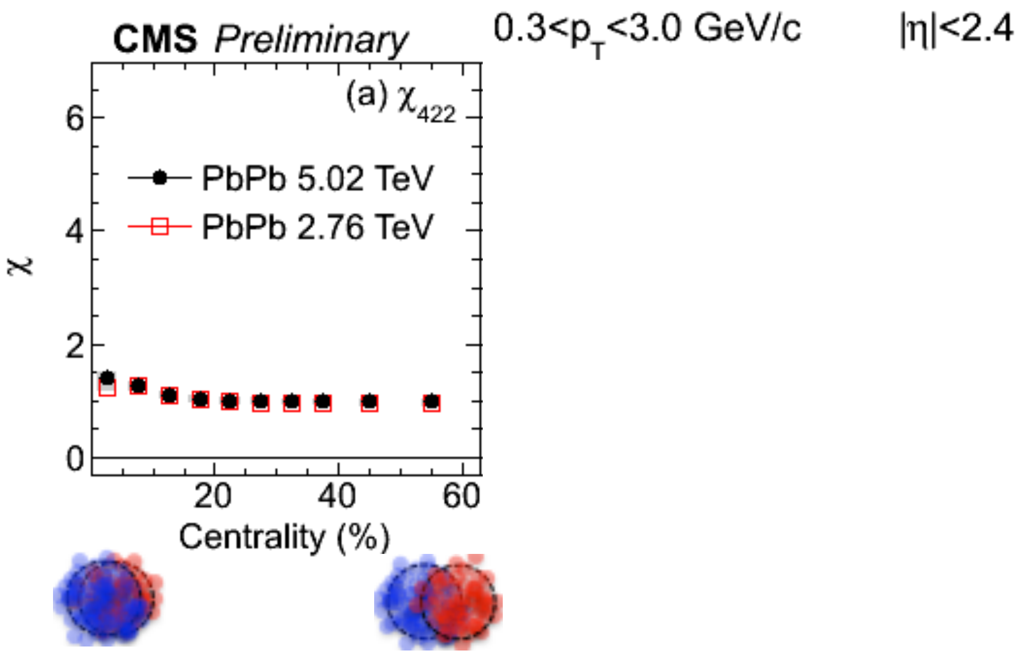
$v_7\{|\Delta\eta|>2\}$

$v_7\{\Psi_{223}\}$



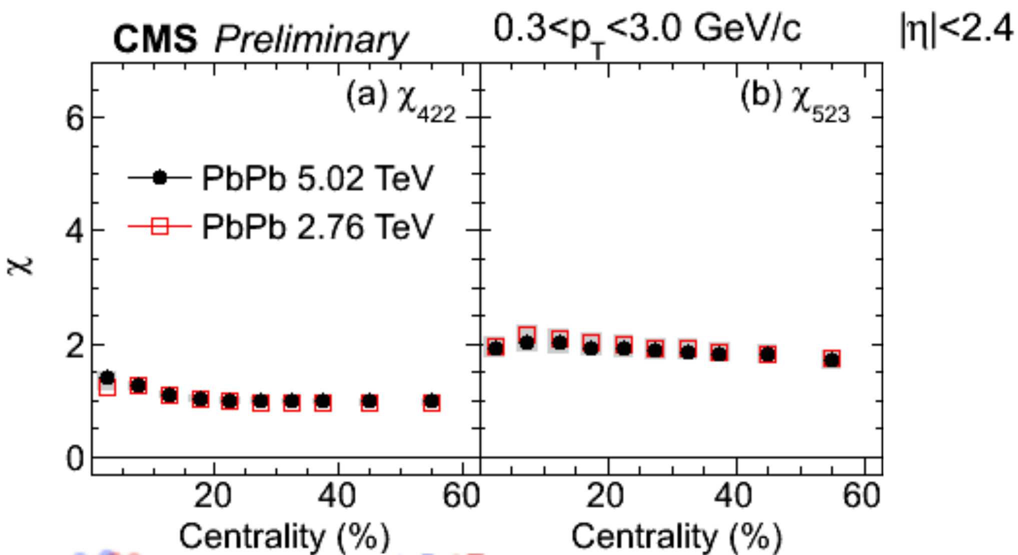
Nonlinear response coefficients vs. centrality

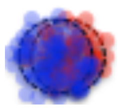
χ_{422}

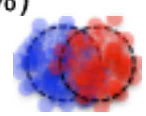


Nonlinear response coefficients vs. centrality

χ_{422} χ_{523}

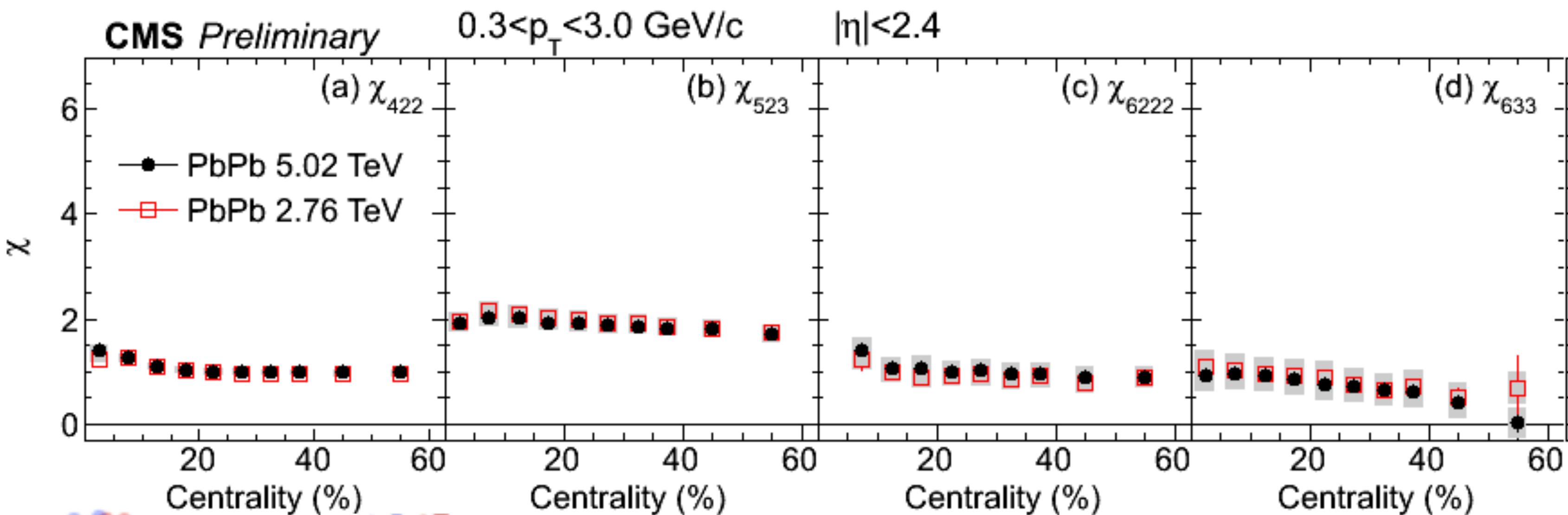


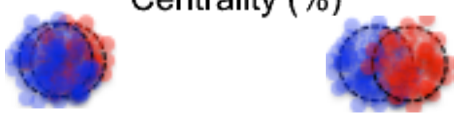




Nonlinear response coefficients vs. centrality

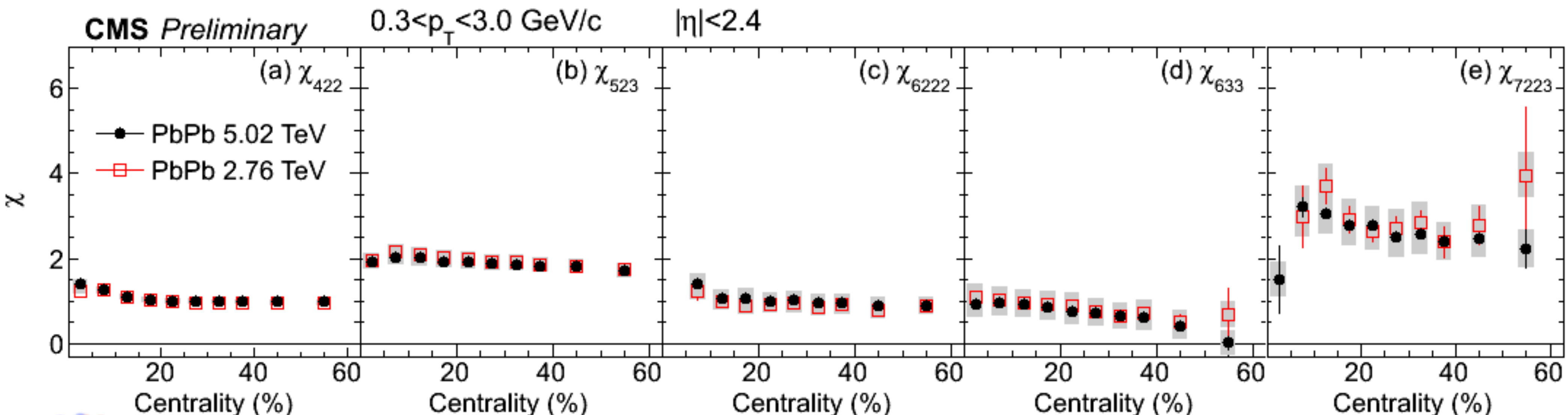
χ_{422} χ_{523} χ_{6222} χ_{633}

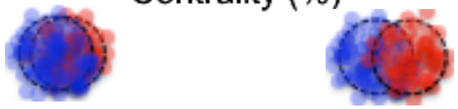




Nonlinear response coefficients vs. centrality

χ_{422} χ_{523} χ_{6222} χ_{633} χ_{7223}

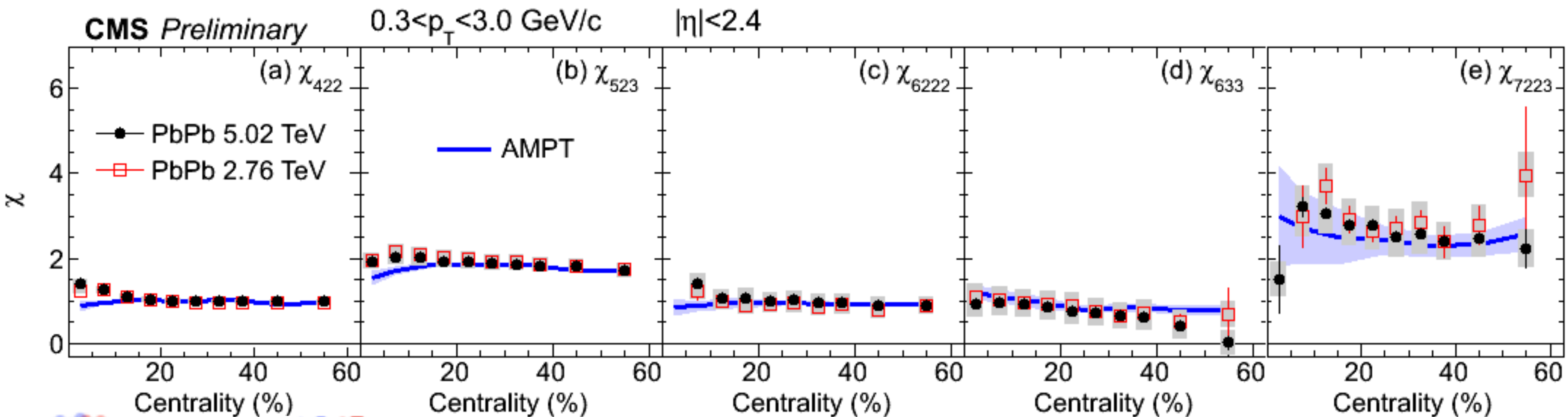


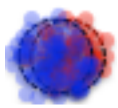


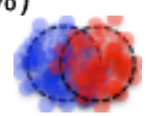
- No strong centrality and energy dependence

Nonlinear response coefficients vs. centrality

χ_{422} χ_{523} χ_{6222} χ_{633} χ_{7223}





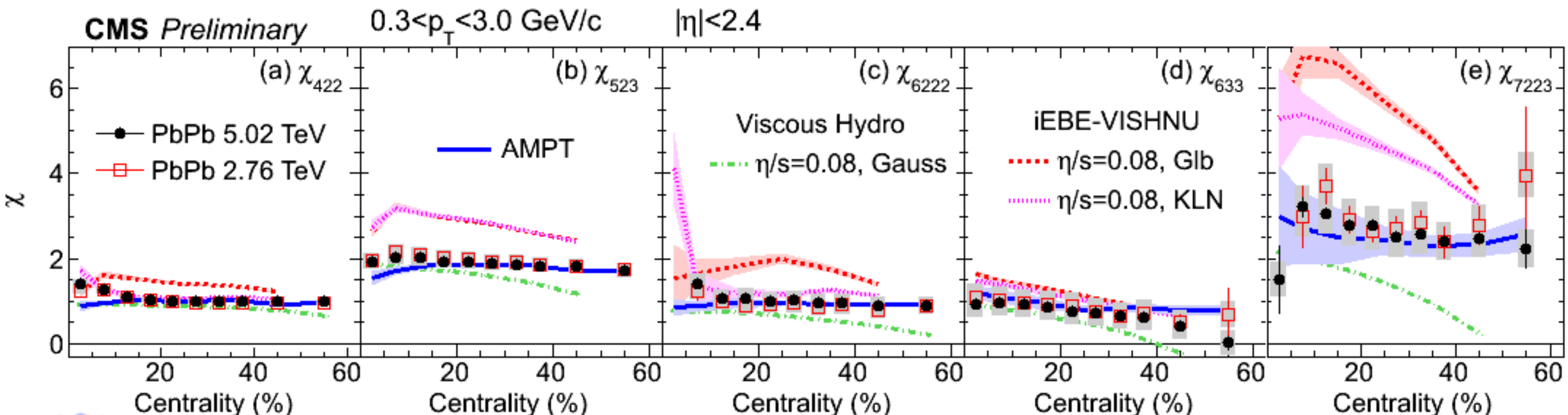


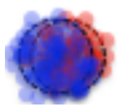
L. Yan, S. Pal, J. Ollitrault, Nucl.Phys. A956 (2016) 340

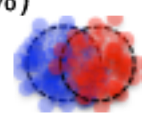
- No strong centrality and energy dependence
- Data agree well with predictions from AMPT

Nonlinear response coefficients vs. centrality

χ_{422} χ_{523} χ_{6222} χ_{633} χ_{7223}







L. Yan, S. Pal, J. Ollitrault, Nucl.Phys. A956 (2016) 340

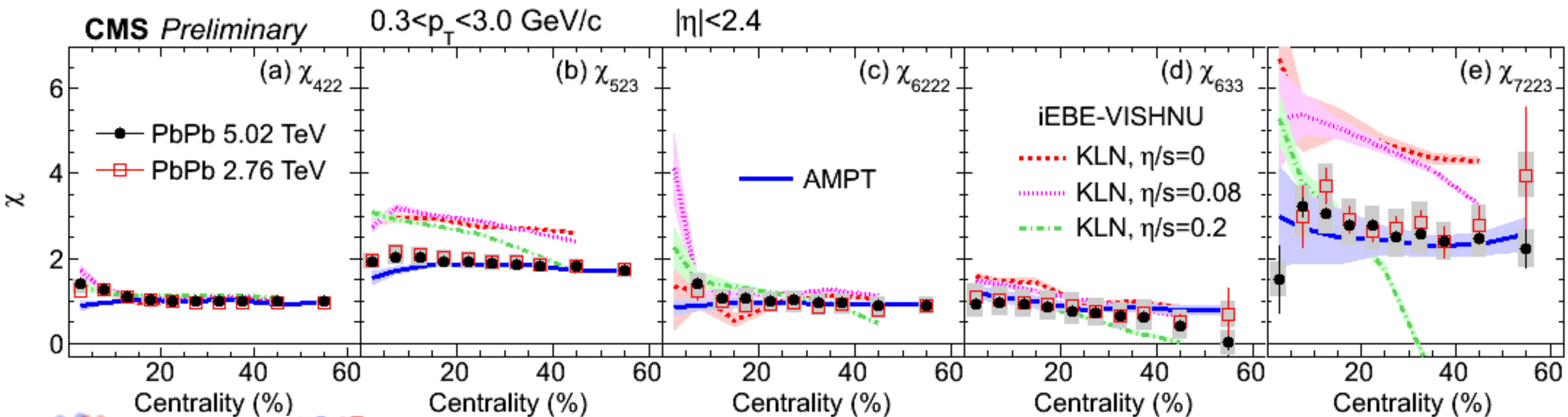
L. Yan, J. Ollitrault, Phys.Lett. B744 (2015) 82

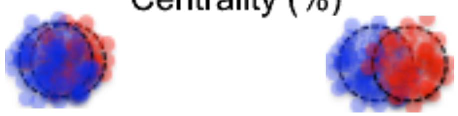
Q. Jing, U. Heinz, J. Liu, Phys. Rev. C 93, 064901 (2016)

- No strong centrality and energy dependence
- Data agree well with predictions from AMPT
- Sensitivity to initial conditions

Nonlinear response coefficients vs. centrality

χ_{422} χ_{523} χ_{6222} χ_{633} χ_{7223}



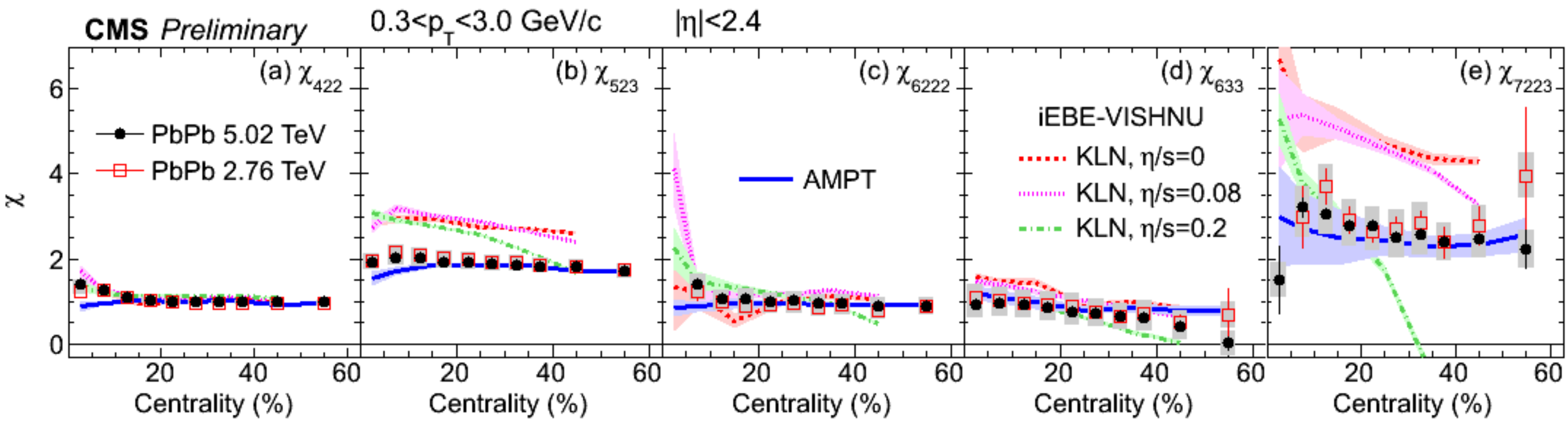


L. Yan, S. Pal, J. Ollitrault, Nucl.Phys. A956 (2016) 340

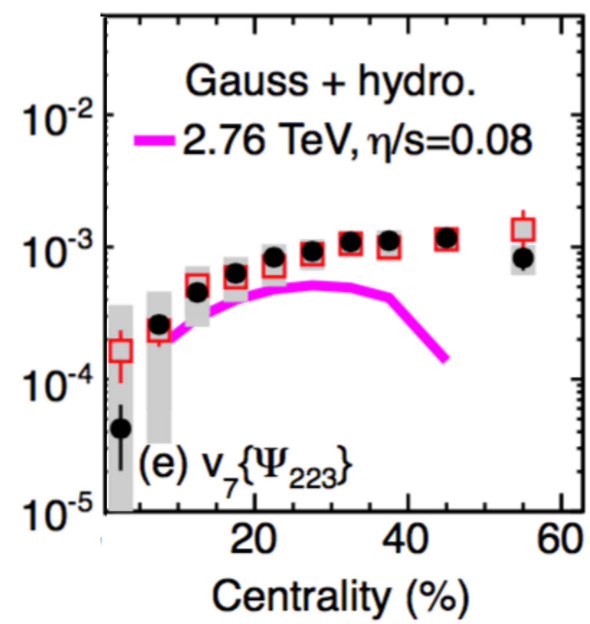
Q. Jing, U. Heinz, J. Liu, Phys. Rev. C 93, 064901 (2016)

- No strong centrality and energy dependence
- Data agree well with predictions from AMPT
- Sensitivity to initial conditions
- Sensitivity to η/s

Summary



- The mixed higher order flow harmonics ($v_5\{\Psi_{23}\}$, $v_6\{\Psi_{33}\}$ and $v_7\{\Psi_{223}\}$), and the nonlinear response coefficients (χ_{422}, χ_{523}, χ_{6222}, χ_{633} and χ_{7223}) are measured as a function of p_T and centrality



- These results are sensitive to initial conditions and η/s at freeze-out, providing constraints on the theoretical description of heavy ion collisions

Backup

iEBE: Nonlinear response coefficients

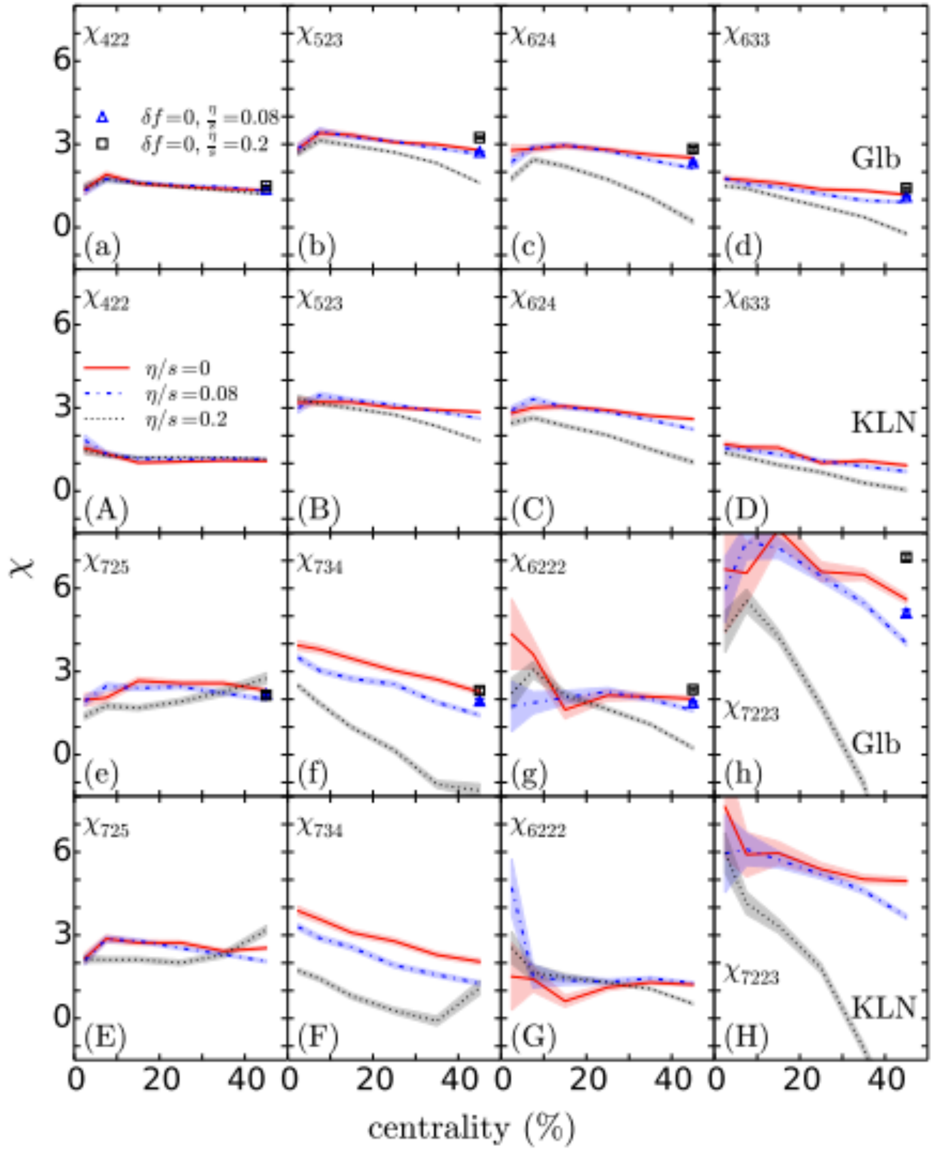


FIG. 6. (Color online) Nonlinear response coefficients defined by Eqs. (8), (13), (16) and (17). Solid red lines correspond to ideal hydrodynamics while dash-dotted blue and dotted black lines correspond to viscous hydrodynamics with $\eta/s = 0.08$ and 0.2, respectively. Panels labeled by lower case (upper case) letters show results from MC-Glb (MC-KLN) initial conditions, respectively. In the MC-Glb panels, the open symbols indicate the values of the mode coupling coefficients in the 40-50% centrality bin if the viscous correction δf at freeze-out is ignored (see text); blue triangles and black squares correspond to $\eta/s = 0.08$ and 0.2, respectively.

Q. Jing, U. Heinz, J. Liu,
Phys. Rev. C 93, 064901 (2016)

ATLAS results from fit and EP correlations

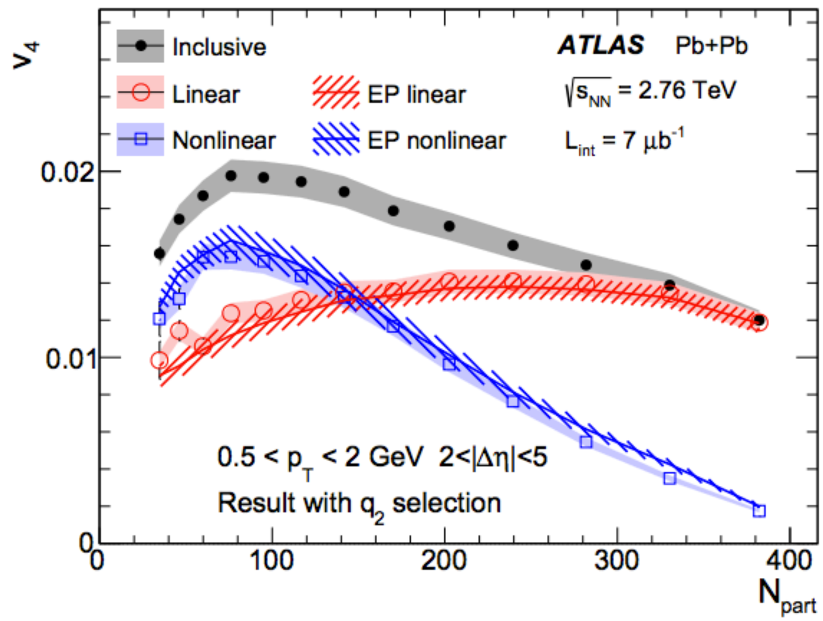


FIG. 11: (Color online) The centrality ($N_{\rm part}$) dependence of the v_4 in $0.5 < p_{\rm T} < 2$ GeV and the associated linear and nonlinear components extracted from the fits in Fig. 10 and Eq. (17). They are compared with the linear and nonlinear component estimated from the previously published event-plane correlations [14] via Eq. (18). The error bars represent the statistical uncertainties, while the shaded bands or hashed bands represent the systematic uncertainties.

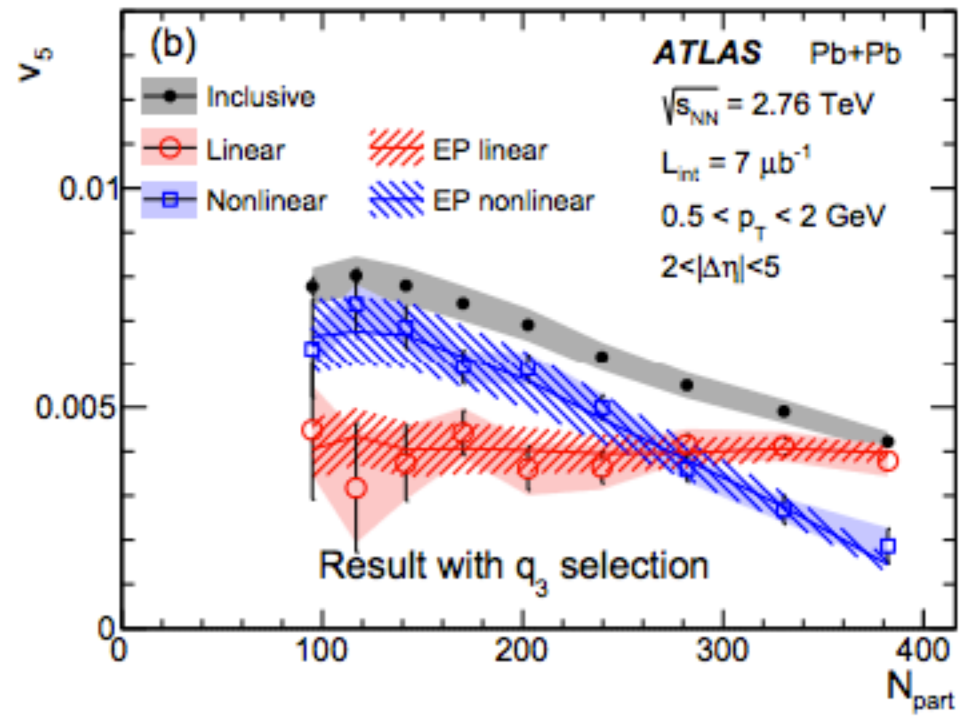


FIG. 15: (Color online) The centrality ($N_{\rm part}$) dependence of the v_5 in $0.5 < p_{\rm T} < 2$ GeV and the associated linear and nonlinear components extracted from the fits in Fig. 13, Fig. 14 and Eq. (20). They are compared with the linear and nonlinear component estimated from the previous published event-plane correlation [14] via Eq. (21). The error bars represent the statistical uncertainties, while the shaded bands or hashed bands represent the systematic uncertainties.

Phys. Rev. C 92, 034903 (2015)