

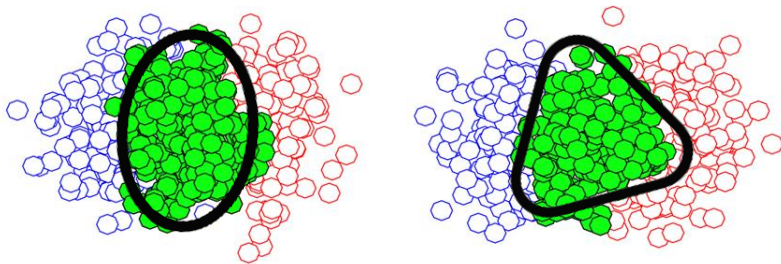
Elliptic Flow Fluctuations in PbPb Collisions at $\sqrt{s_{NN}} = 5.02 \text{ TeV}$

James Castle
(University of Kansas)
On behalf of the CMS collaboration

Quark Matter 2017, Chicago
8 February 2017

A Bit of History

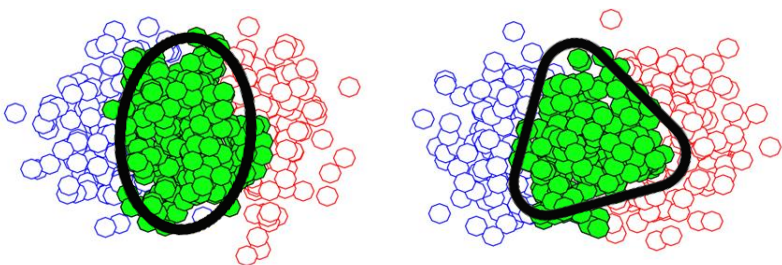
- Observation of non-zero v_3 at RHIC and LHC¹
 - Participant eccentricity fluctuations: $v_n \rightarrow p(v_n)$



¹[Phys.Rev. C81 \(2010\) 054905](#)

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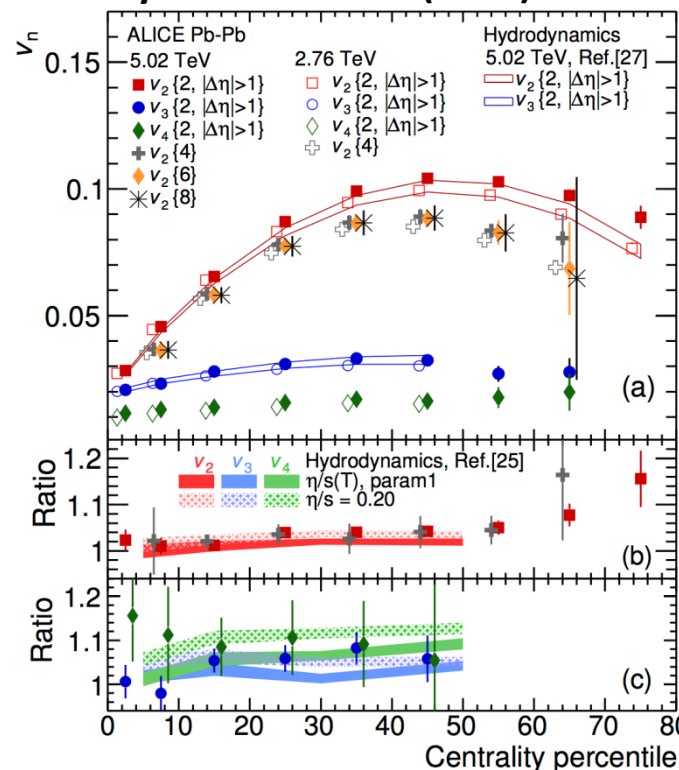
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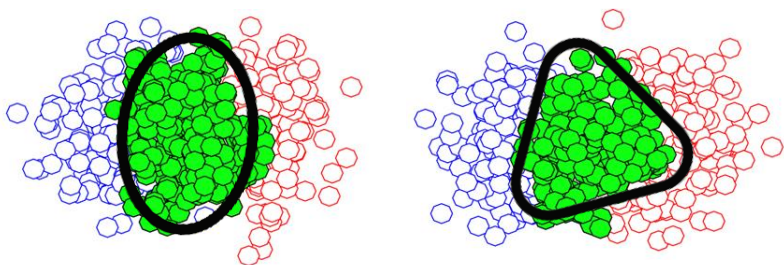
²[Phys.Lett. B659 \(2008\) 537-541](#)

Phys.Rev.Lett. 116 (2016) 132302

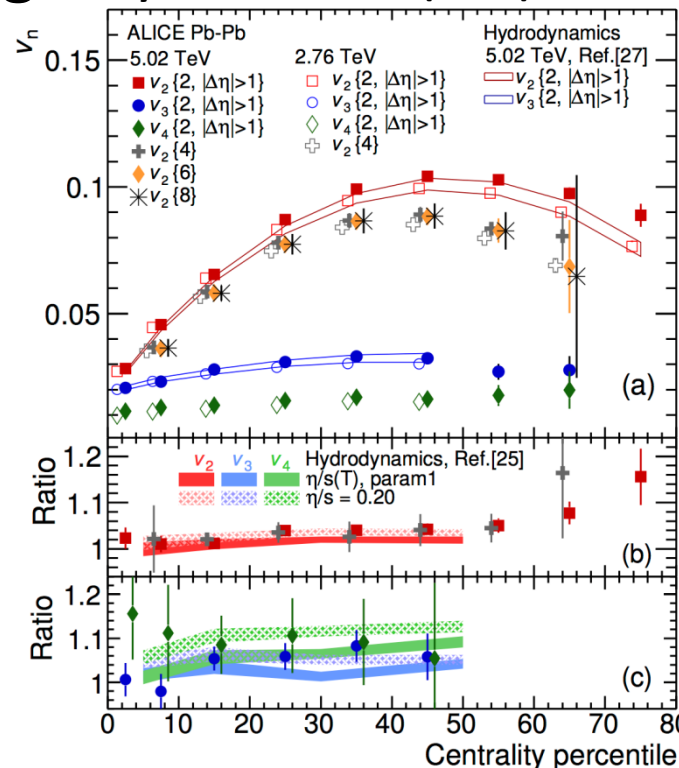


A Bit of History

- Observation of non-zero v_3 at RHIC and LHC¹
 - Participant eccentricity fluctuations: $v_n \rightarrow p(v_n)$
- Multi-particle cumulants² probe the moments of $p(v_n)$
- Measurement of $p(v_n)$ using unfolding³
 - Allows for precision fluctuation studies
 - Obtain all moments of $p(v_n)$
 - Precise cumulants
 - $p(\varepsilon_n)$ inference



Phys.Rev.Lett. 116 (2016) 132302



¹Phys.Rev. C81 (2010) 054905

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³JHEP 1311 (2013) 183

Fluctuation Parametrization

➤ Prevailing eccentricity fluctuation models

- Bessel-Gaussian¹

$$p(\varepsilon_n | \varepsilon_0, \delta) = \frac{\varepsilon_n}{\delta^2} \text{Exp} \left[-\frac{\varepsilon_n^2 + \varepsilon_0^2}{2\delta^2} \right] I_0 \left(\frac{\varepsilon_n \varepsilon_0}{\delta^2} \right), \quad \varepsilon_n \in \mathbb{R}$$

$$\blacksquare \varepsilon_n\{2\} > \varepsilon_n\{4\} = \varepsilon_n\{6\} = \varepsilon_n\{8\}$$

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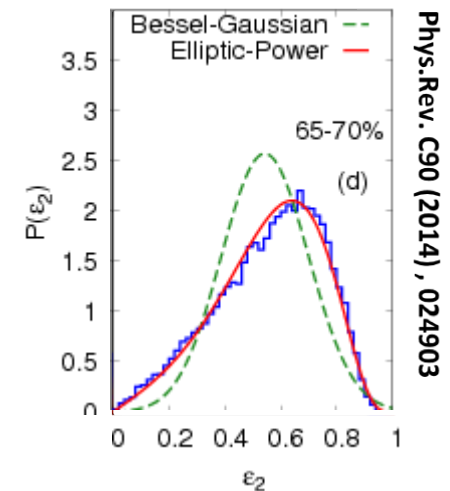
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- $\varepsilon_n\{2\} > \varepsilon_n\{4\} = \varepsilon_n\{6\} = \varepsilon_n\{8\}$

- Elliptic power law²

$$p(\varepsilon_n | \varepsilon_0, \alpha) = \frac{2\alpha\varepsilon_n}{\pi} (1 - \varepsilon_0^2)^{\alpha+1/2} \int_0^\pi \frac{(1 - \varepsilon_n^2)^{\alpha-1} d\phi}{(1 - \varepsilon_0 \varepsilon_n \cos \phi)^{2\alpha+1}}, \quad |\varepsilon_n| \leq 1$$

- $\varepsilon_n\{2\} > \varepsilon_n\{4\} > \varepsilon_n\{6\} > \varepsilon_n\{8\}$

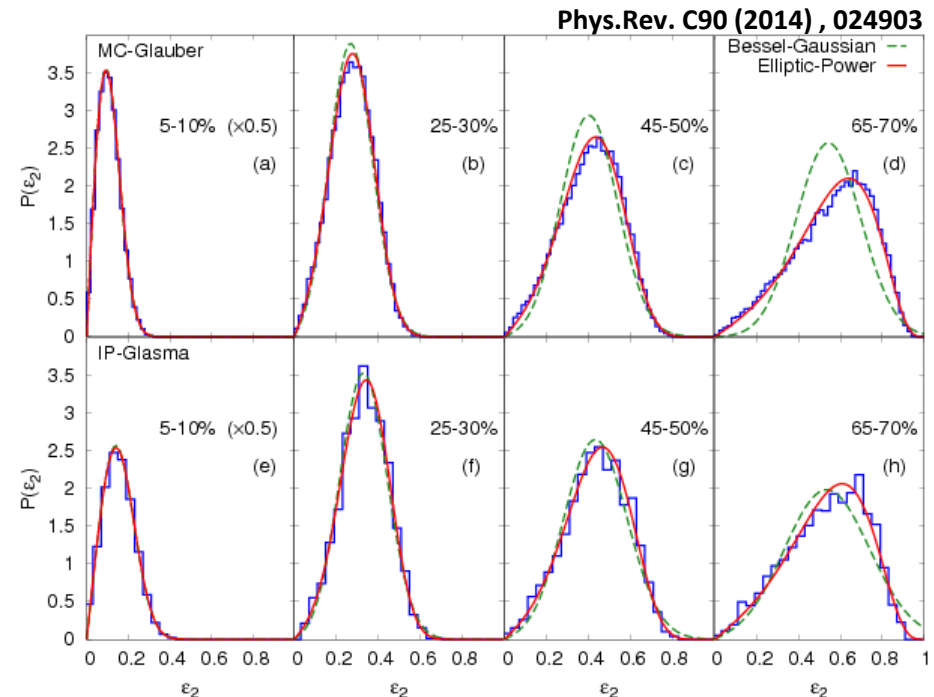


¹[Phys.Lett. B659 \(2008\) 537-541](#)

²[Phys.Rev. C90 \(2014\), 024903](#)

Non-Gaussian Fluctuations

- Fine splitting observed¹ between $v_2\{4\}$ and $v_2\{6\}$
 - Consequence of $p(\varepsilon_2)$ being skewed with respect to the reaction plane²

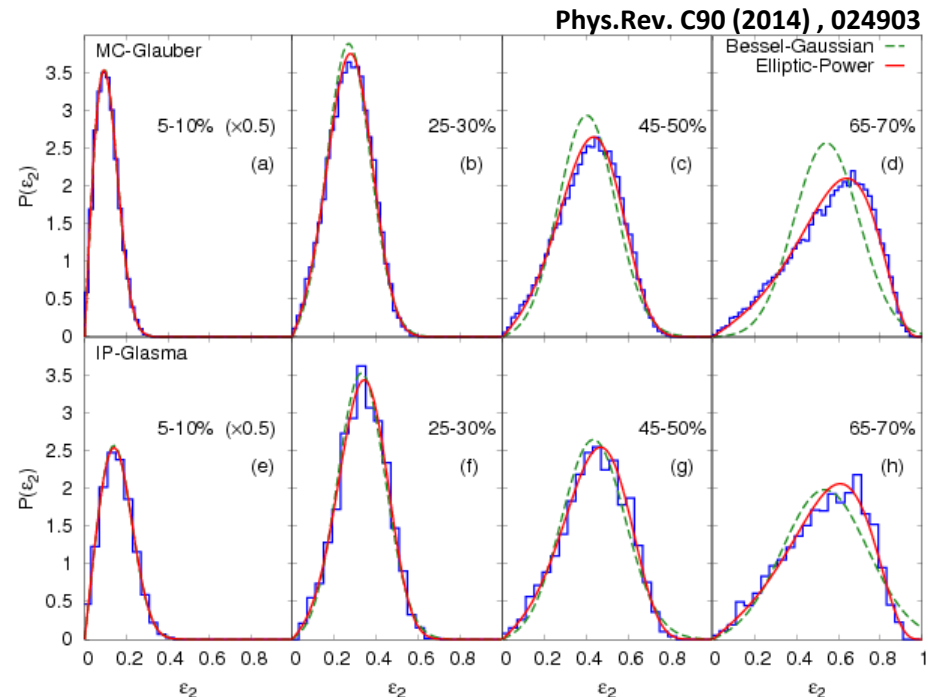
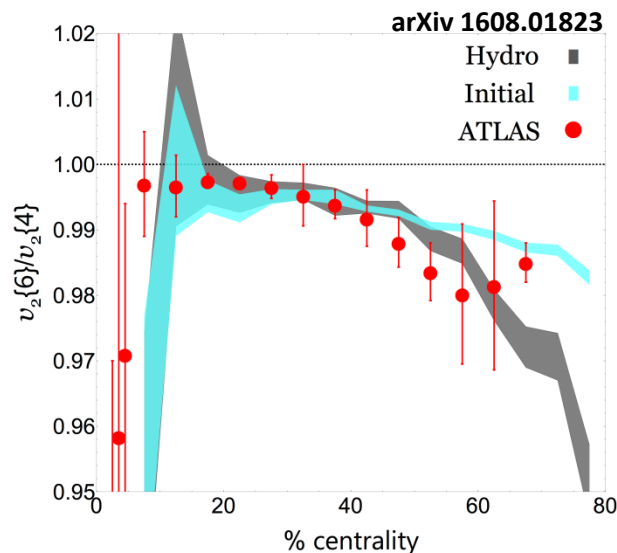


¹[Eur.Phys.J. C74 \(2014\), 3157](#)

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Non-Gaussian Fluctuations

- Fine splitting observed¹ between $v_2\{4\}$ and $v_2\{6\}$
 - Consequence of $p(\varepsilon_2)$ being skewed with respect to the reaction plane²
- Hydrodynamic calculations² show that $v_2\{6\}/v_2\{4\} \approx \varepsilon_2\{6\}/\varepsilon_2\{4\}$
 - Observable helps to constrain initial stages of the QGP



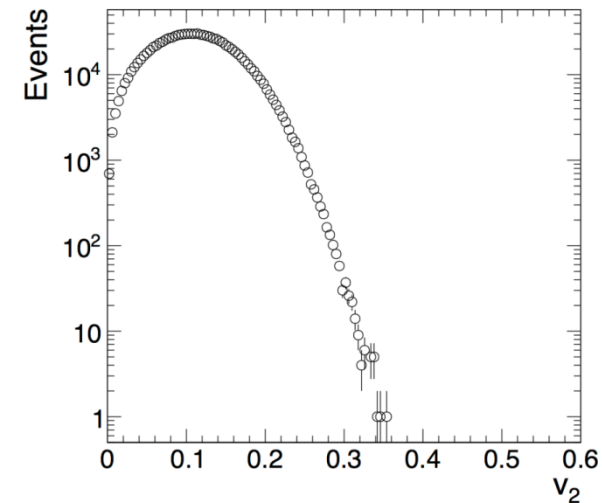
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Observed Flow Harmonic Distributions

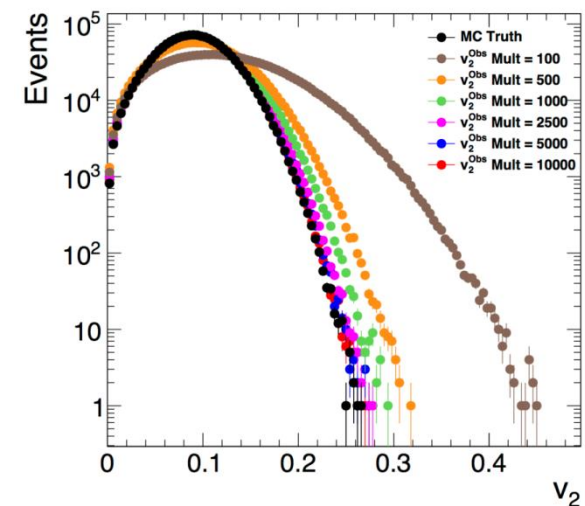
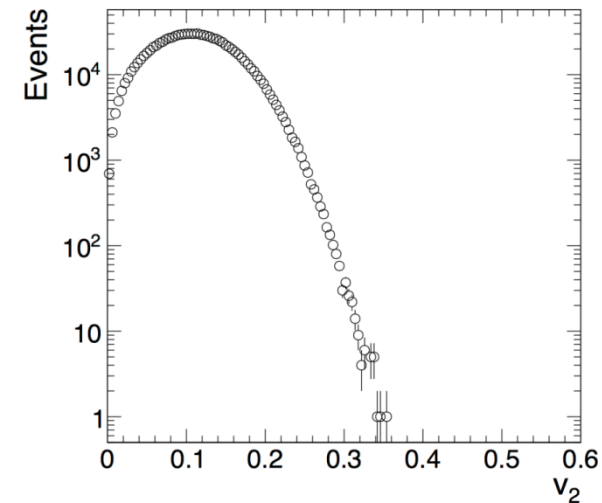
➤ Construct an event-by-event $p(v_n^{obs})$:

- $\vec{v}_n^{obs} = \left(\frac{\sum \cos n\phi_i/\varepsilon_i}{\sum 1/\varepsilon_i}, \frac{\sum \sin n\phi_i/\varepsilon_i}{\sum 1/\varepsilon_i} \right) - \langle \vec{v}_n^{obs} \rangle$



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- Must account for statistical resolution



Observed Flow Harmonic Distributions

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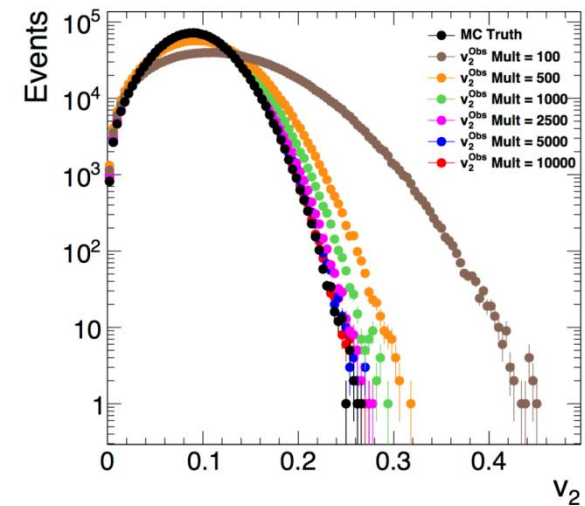
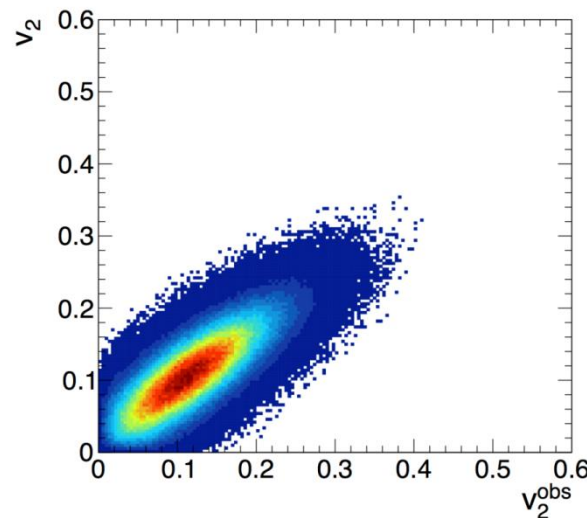
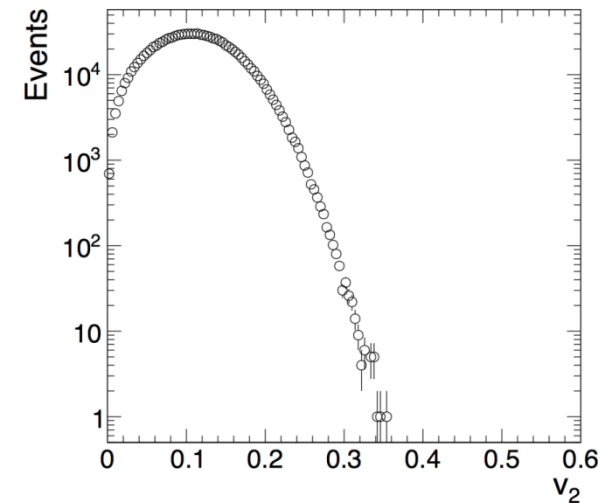
- $$\vec{v}_n^{obs} = \left(\frac{\sum \cos n\phi_i / \varepsilon_i}{\sum 1/\varepsilon_i}, \frac{\sum \sin n\phi_i / \varepsilon_i}{\sum 1/\varepsilon_i} \right) - \langle \vec{v}_n^{obs} \rangle$$

➤ Must account for statistical resolution

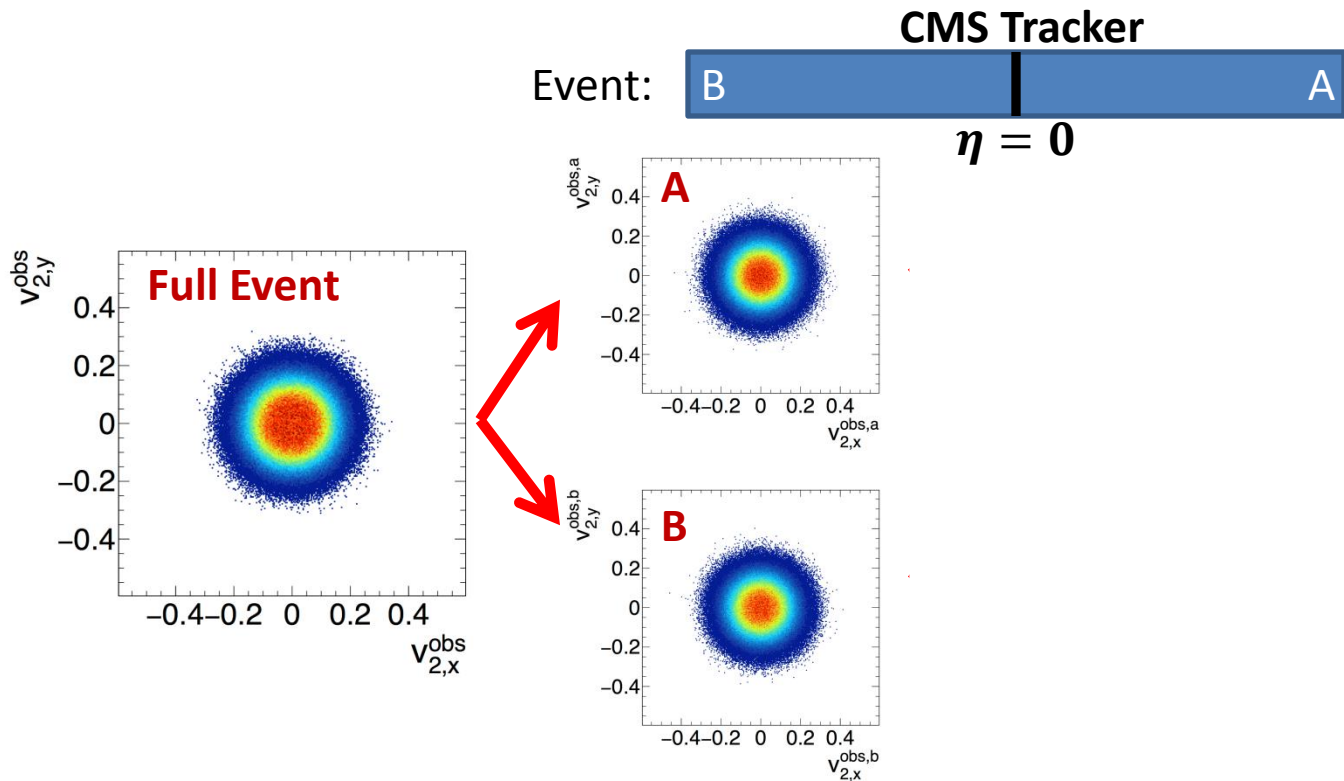
- Quantified by a response function

- $$p(v_n^{obs} | v_n) \times p(v_n) = p(v_n^{obs})$$

- Use unfolding

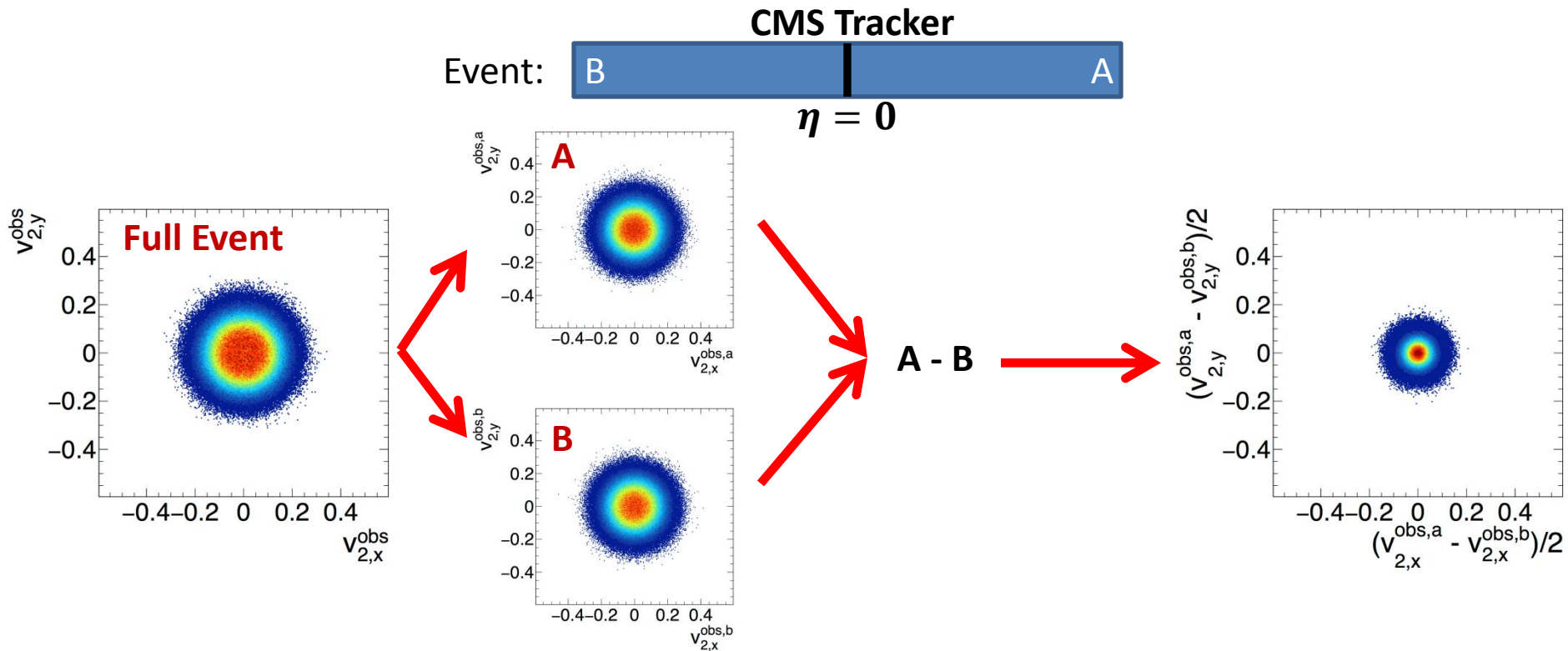


Building the Response Function



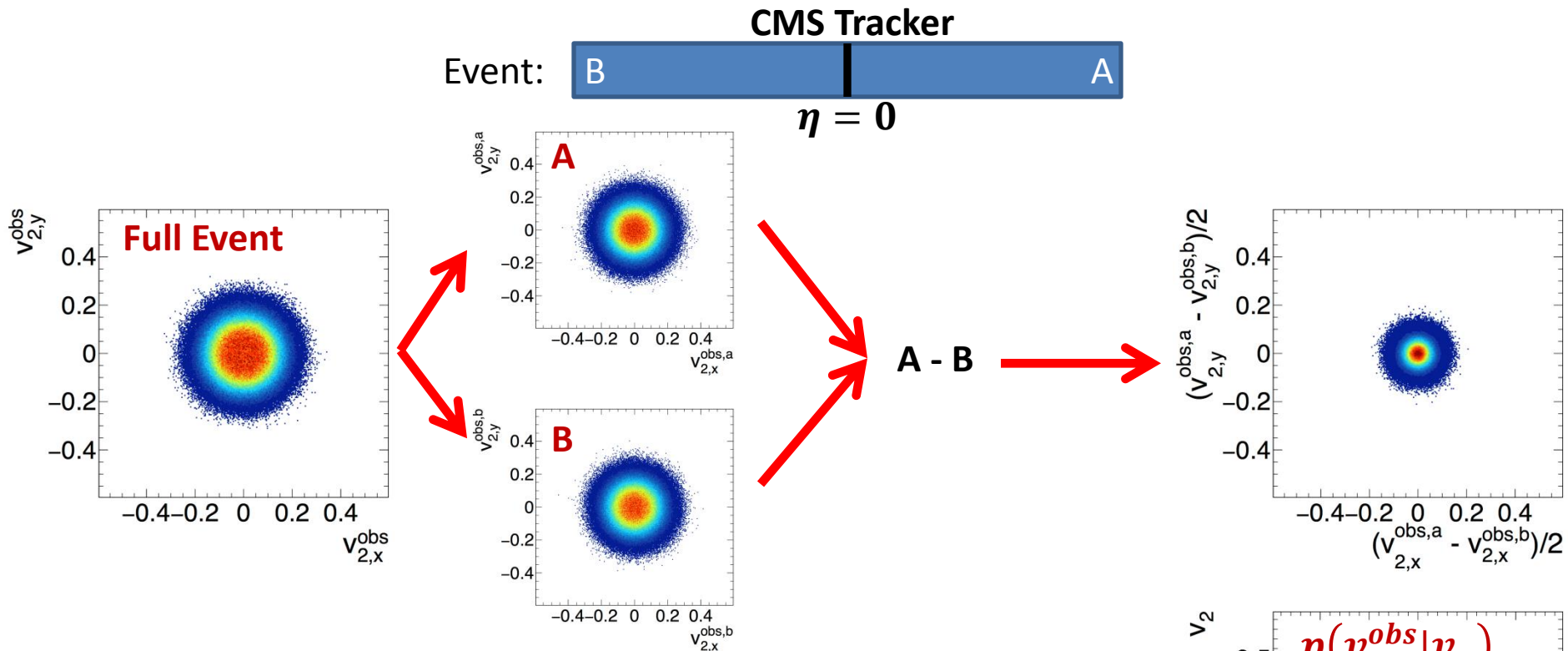
➤ Divide event into two, symmetric subevents

Building the Response Function



- Divide event into two, symmetric subevents
- Physical flow signal cancels in distribution of $(\vec{v}_n^{obs,a} - \vec{v}_n^{obs,b})/2$
 - Remaining effects are those from statistical smearing and non-flow

Building the Response Function

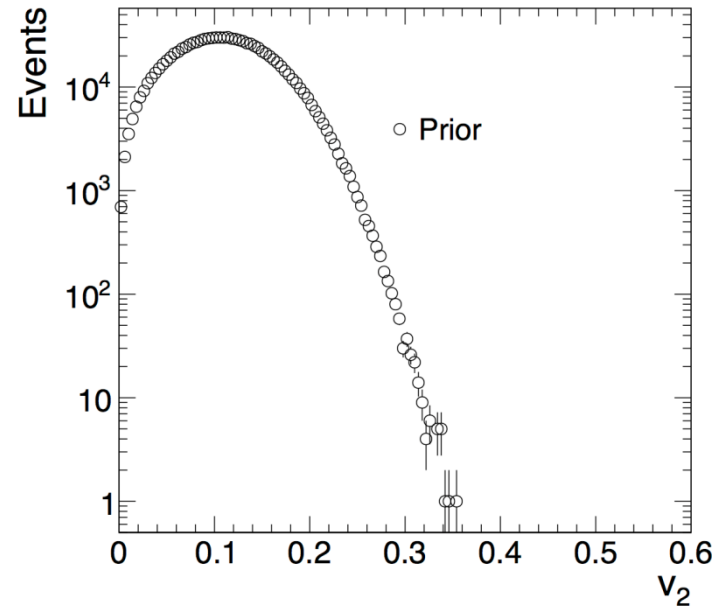


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- Physical flow signal cancels in distribution of $(\vec{v}_n^{obs,a} - \vec{v}_n^{obs,b})/2$
 - Remaining effects are those from statistical smearing and non-flow
- Subevent difference distribution is used to build the response function

Unfolding

➤ Iteratively remove smearing using D'Agostini iteration unfolding¹

- $\hat{c}_i^{iter+1} = \hat{M}_{ij}^{iter} e_j$
 - $\hat{M}_{ij}^{iter} = \frac{A_{ji}\hat{c}_i^{iter}}{\sum_{m,k} A_{mi}A_{jk}\hat{c}_k^{iter}}$
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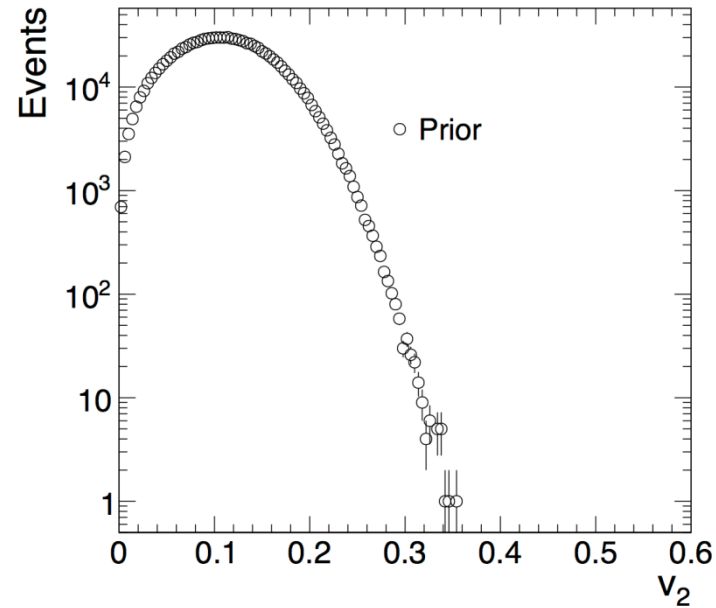


¹G. D'Agostini, Nucl. Instrum. Meth. A362, 487 (1995)

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- Regularization necessary
 - Smeared space χ^2/NDF
 - $p(v_n^{obs} | v_n) \times p(v_n)^{iter} = p(v_n^{obs})$

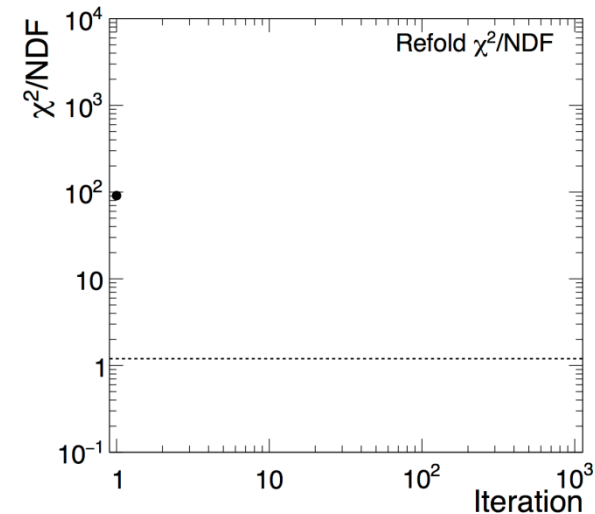
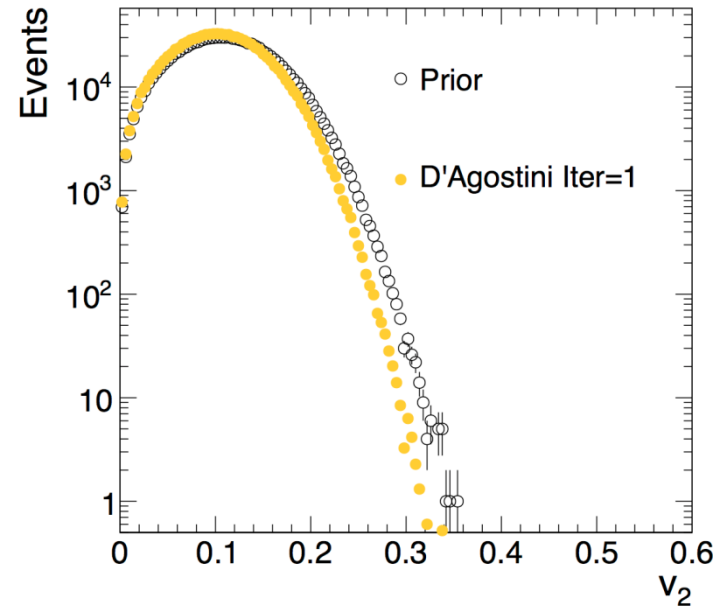


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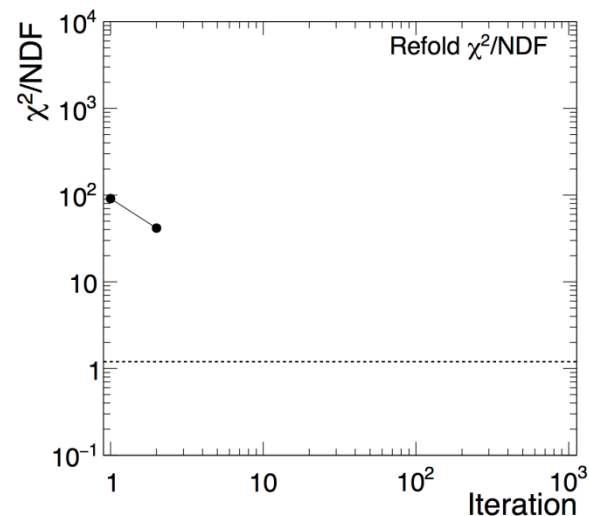
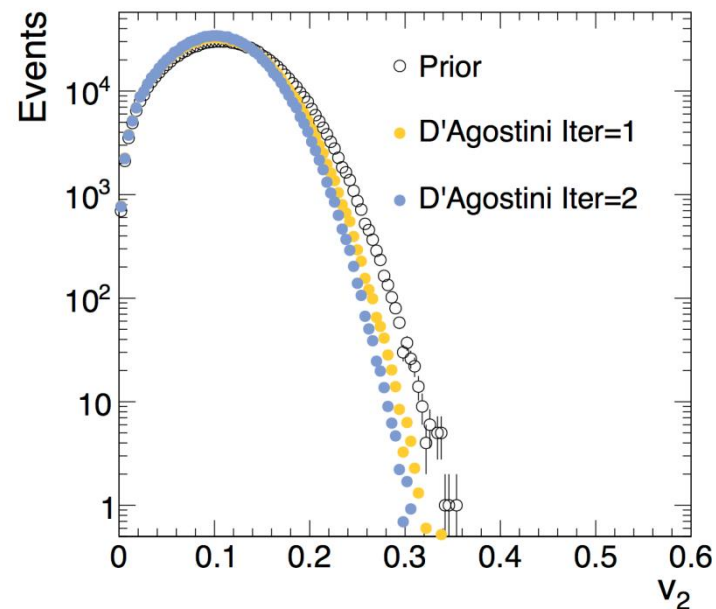


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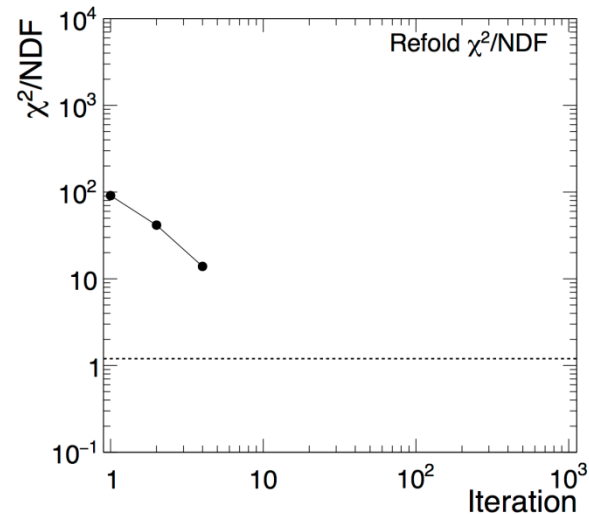
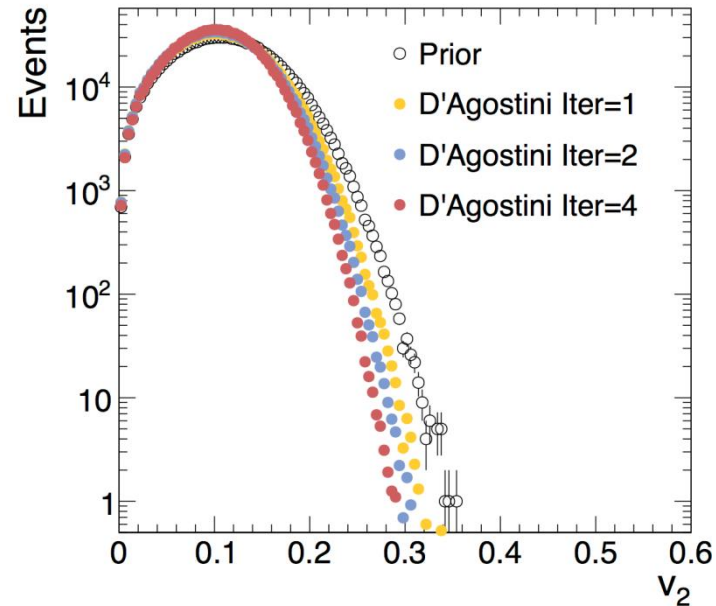


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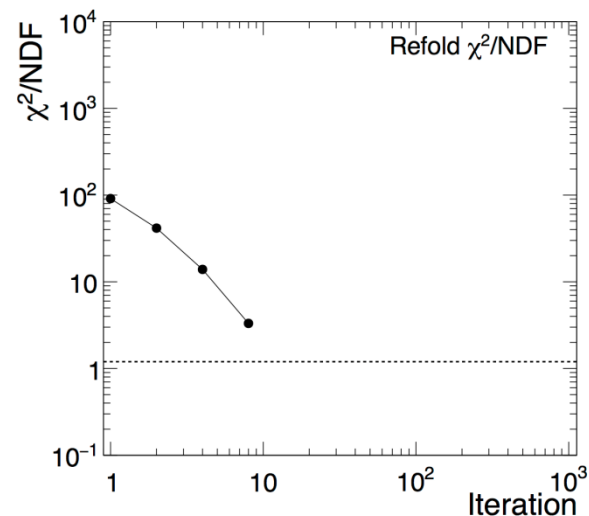
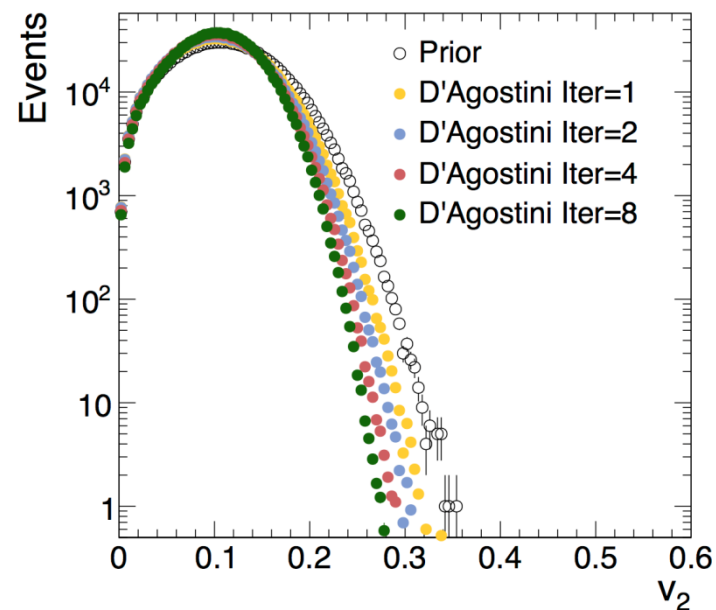


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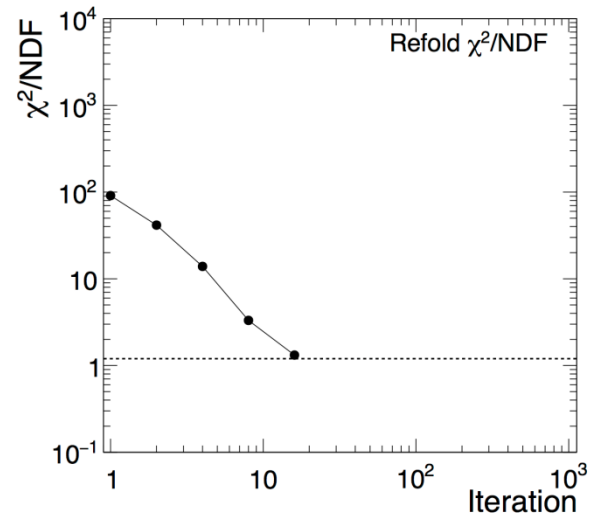
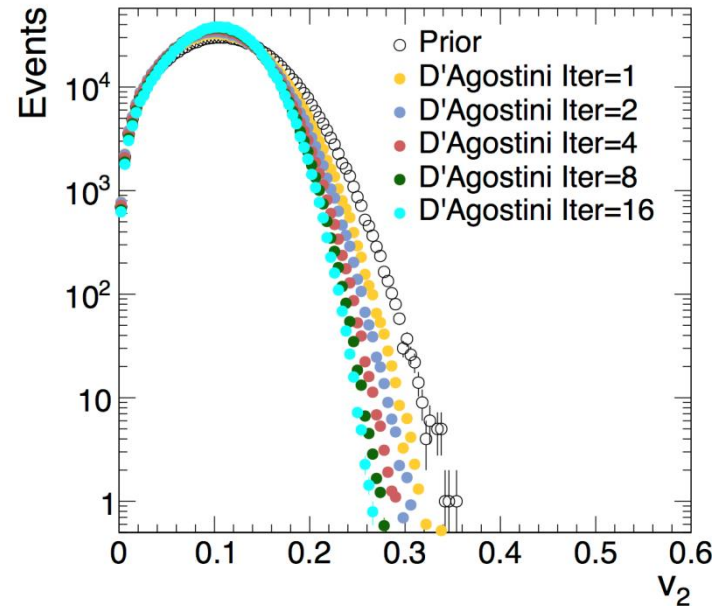


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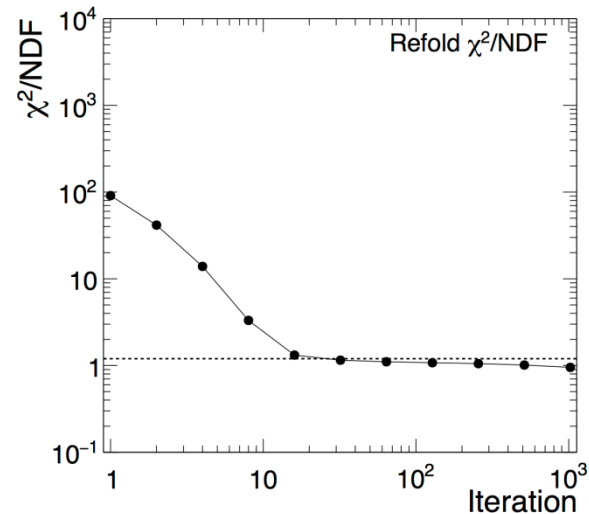
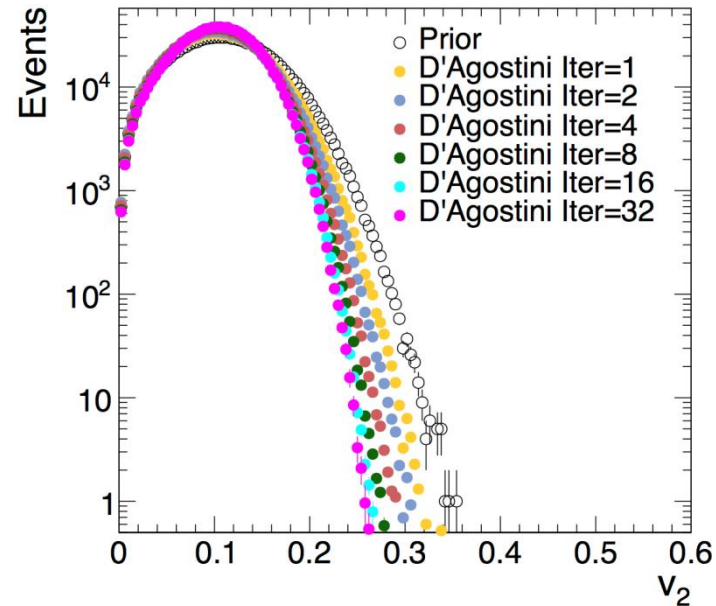


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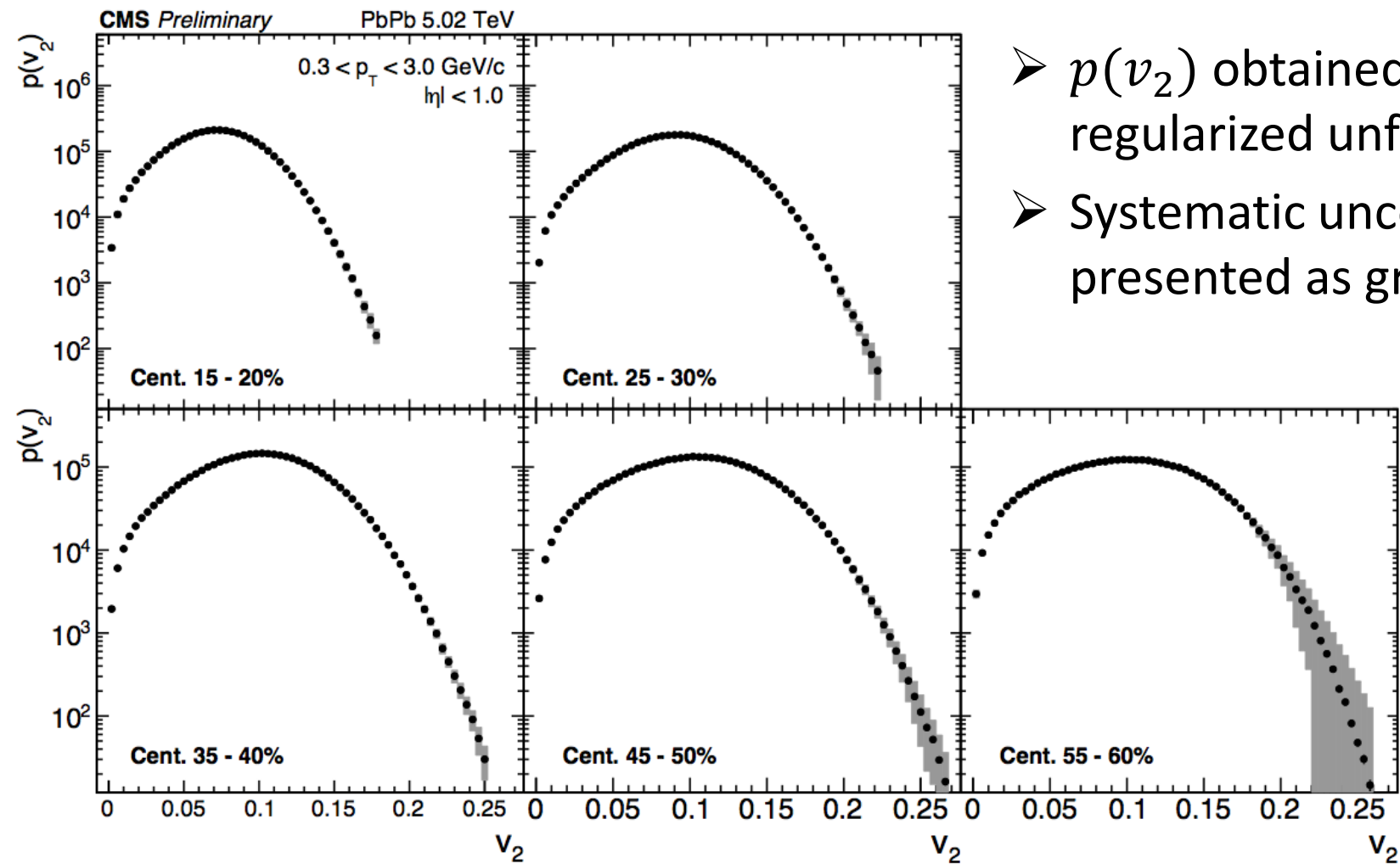
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Results: $p(v_2)$ Distributions



- $p(v_2)$ obtained from regularized unfolding
- Systematic uncertainties presented as gray bands

Fluctuation Parametrization

➤ Elliptic power law parametrization:

$$p(\varepsilon_n | \varepsilon_0, \alpha) = \frac{2\alpha\varepsilon_n}{\pi} (1 - \varepsilon_0^2)^{\alpha+1/2} \int_0^\pi \frac{(1 - \varepsilon_n^2)^{\alpha-1} d\phi}{(1 - \varepsilon_0\varepsilon_n \cos \phi)^{2\alpha+1}}$$
$$p(v_n) = \frac{1}{k_n} p\left(\frac{v_n}{k_n}\right)$$

Fluctuation Parametrization

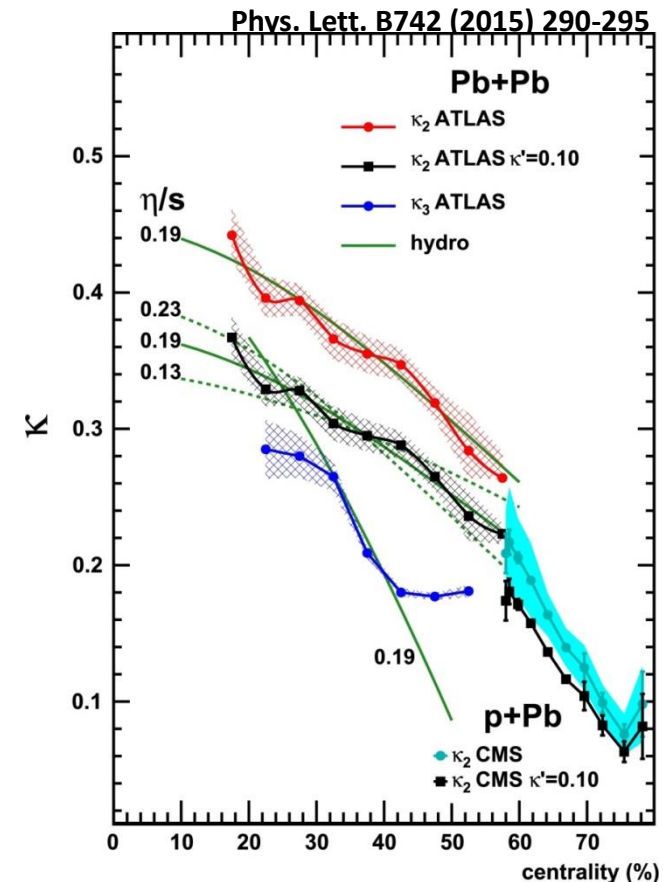
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➤ k_n

- Response coefficient
- Factorizes when fluctuations are non-Gaussian



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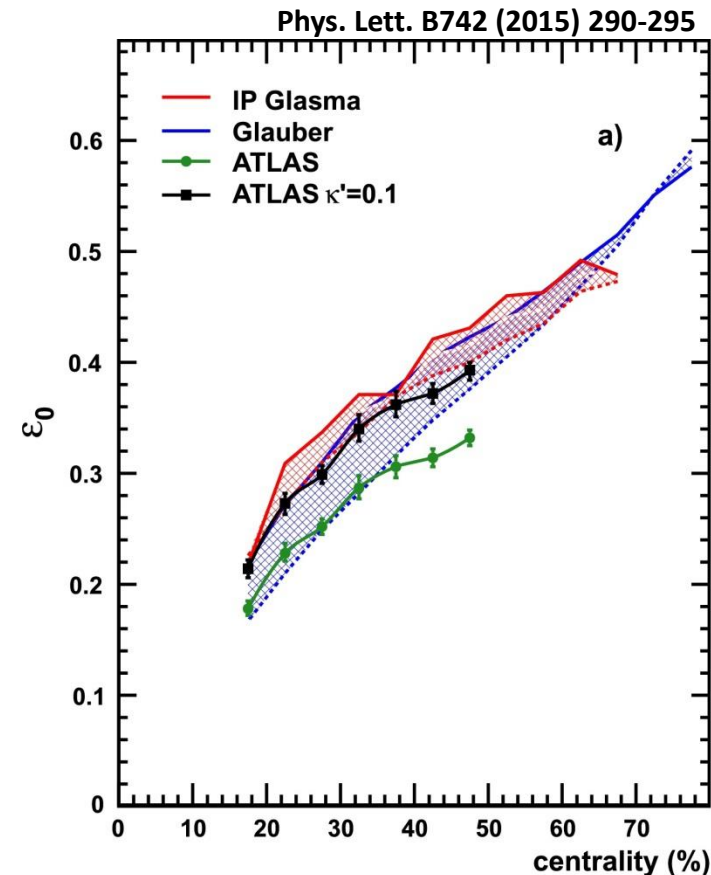
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➤ ε_0

- Approximately equal to the mean reaction plane eccentricity



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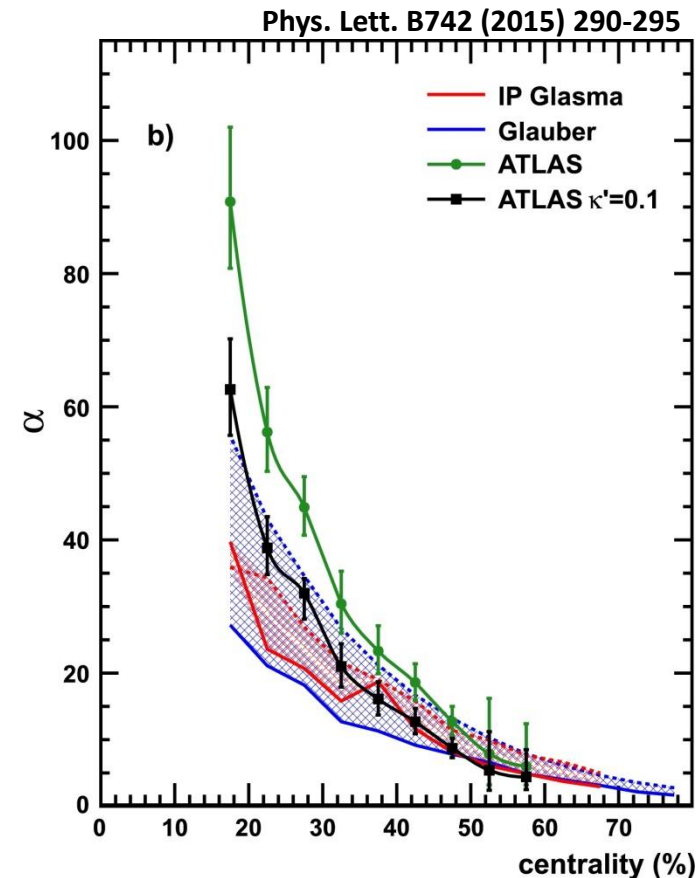
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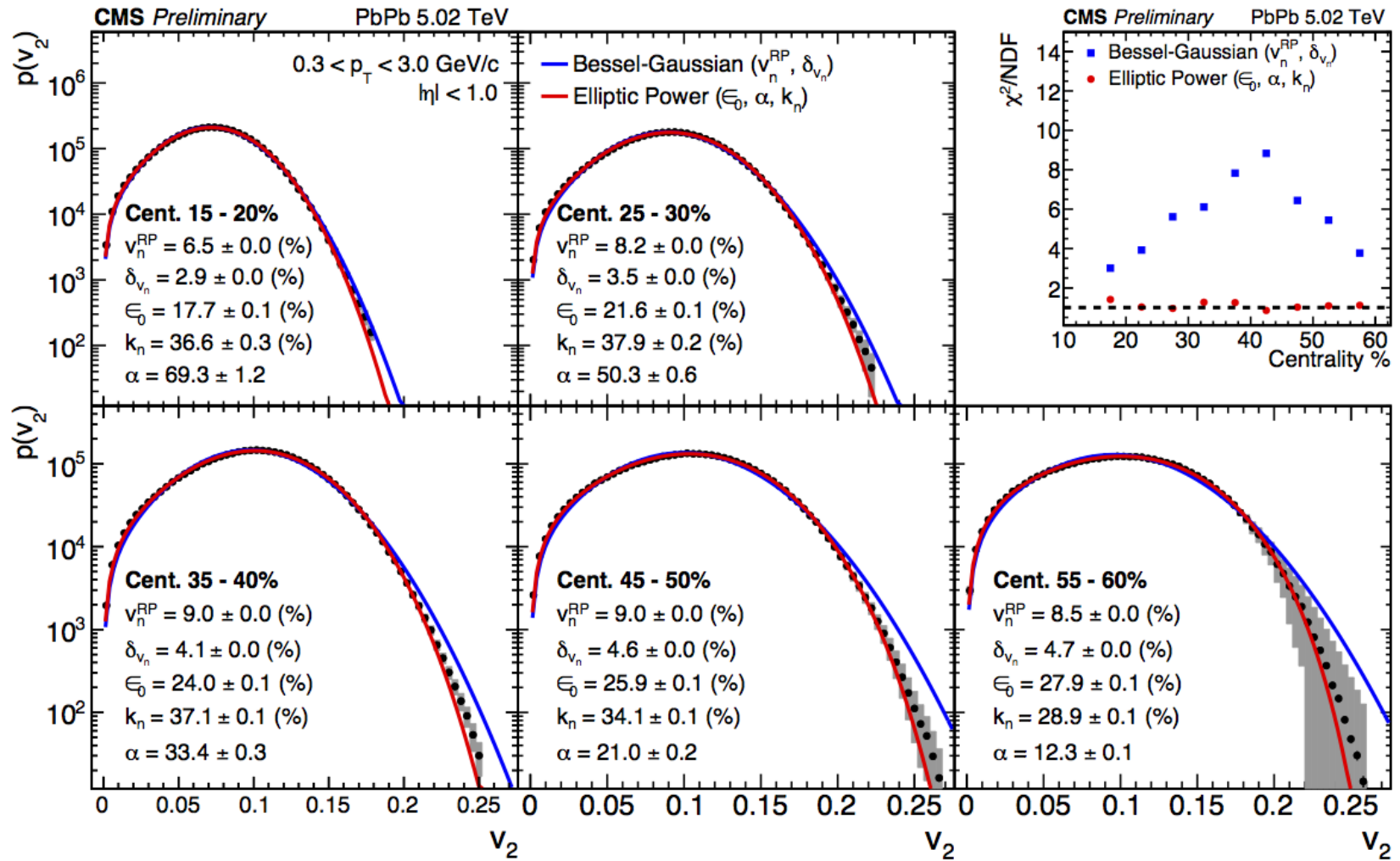
- Approximately equal to the mean reaction plane eccentricity

- α

- Describes fluctuations magnitude

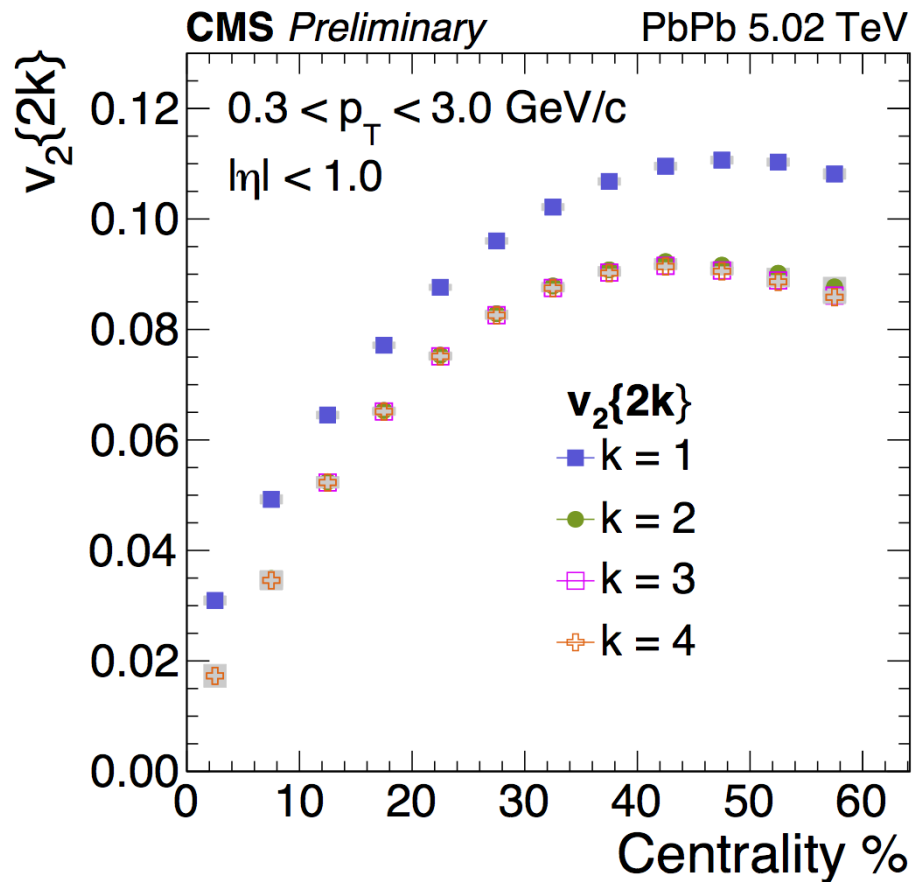


Results: Fitting $p(v_2)$ with Fluctuation Models



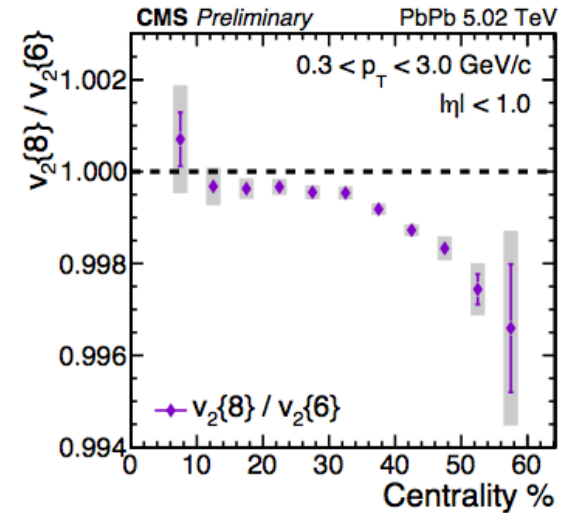
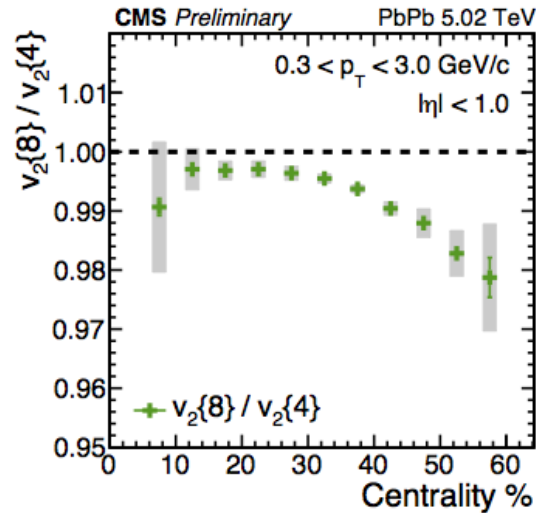
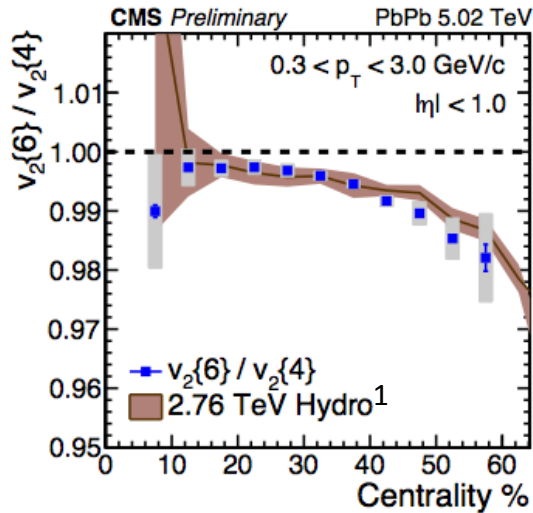
- Elliptic power parametrization consistently describes data better than Bessel-Gaussian
- Elliptic power fit allows for extraction k_n without an assumed initial-state model

Results: Cumulants



- Exhibits the expected $v_2\{2\} > v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$ behavior
- Statistical uncertainties obtained using Jackknife resampling
- Splitting of the higher-order cumulants more pronounced in peripheral events

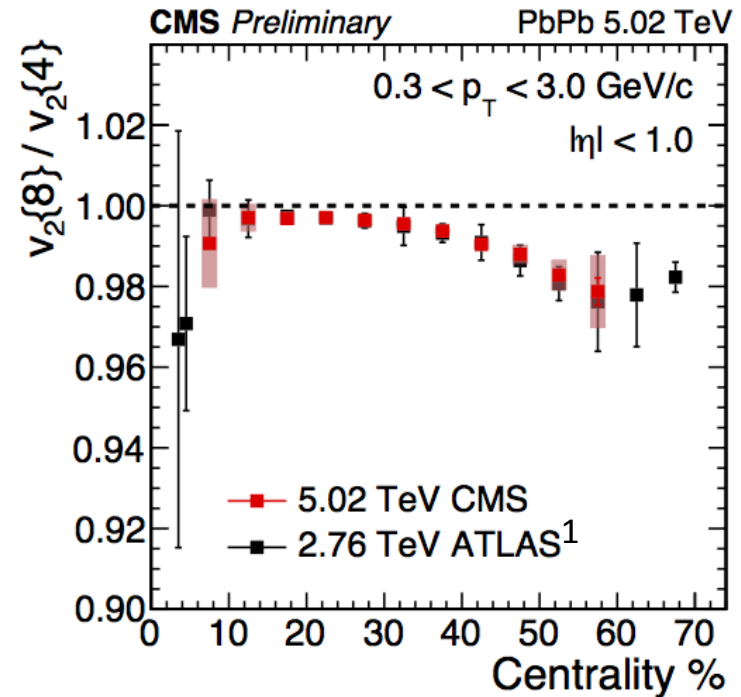
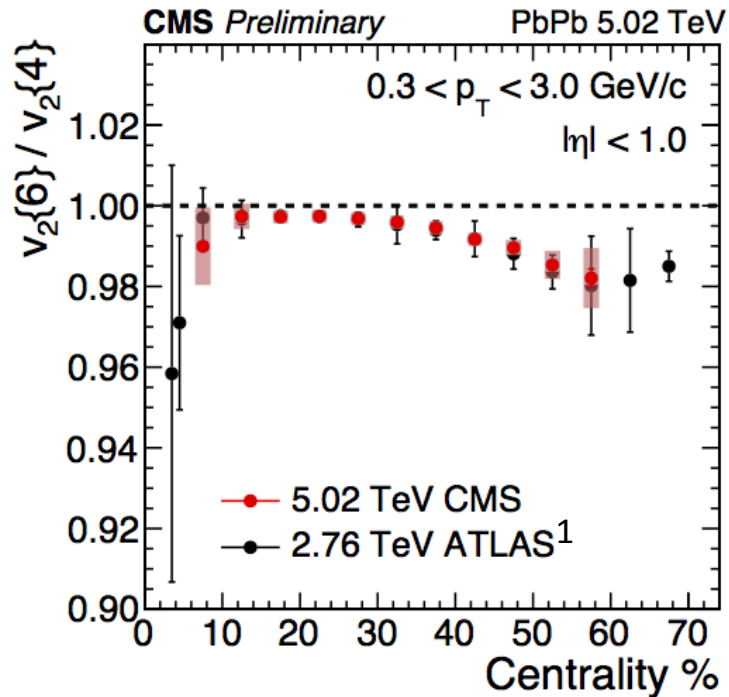
Results: Higher-Order Cumulant Ratios



- Fine-level splitting observed between higher-order cumulants
 - $v_2\{4\} > v_2\{6\} > v_2\{8\}$
- Hydrodynamic predictions¹ for 2.76 TeV consistent with 5.02 TeV measurement

¹[arXiv 1608.01823](https://arxiv.org/abs/1608.01823)

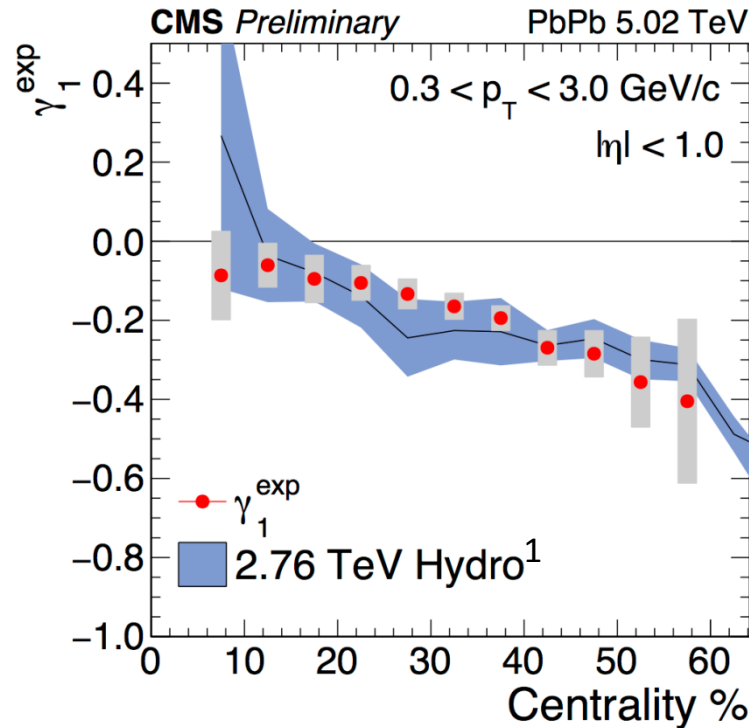
Results: Compare to Other Measurements



- Higher-order cumulant ratios consistent between 2.76 and 5.02 TeV measurements
- Suggests no strong energy dependence between 2.76 and 5.02 TeV

¹[Eur. Phys. J. C74 \(2014\), 3157](#)

Results: Skewness



$$\gamma_1^{exp} = -6\sqrt{2}v_2\{4\}^2 \frac{v_2\{4\} - v_2\{6\}}{(v_2\{2\}^2 - v_2\{4\}^2)^{3/2}} \approx \frac{\langle (v_2^{RP} - \langle v_2^{RP} \rangle)^3 \rangle}{\left(\sqrt{\langle (v_2^{RP})^2 \rangle - \langle v_2^{RP} \rangle^2} \right)^3}$$

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Summary

➤ By measuring $p(v_2)$ we have observed

- $v_2\{4\} > v_2\{6\} > v_2\{8\}$
- Skewness is negative
- $p(v_2)$ described well by the elliptic power law parametrization
 - Allows for extraction of initial anisotropy and response coefficient without an assumed model of initial state
 - Bessel-Gaussian fails because it does not incorporate $|\varepsilon_n| \leq 1$ constraint