

hydrodynamic predictions for mixed harmonic correlations in 200 GeV Au+Au collisions

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with

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based on [arXiv 1608.02982](https://arxiv.org/abs/1608.02982) (accepted in PRC)

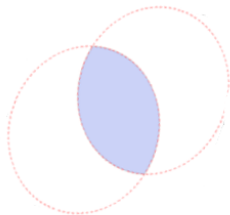


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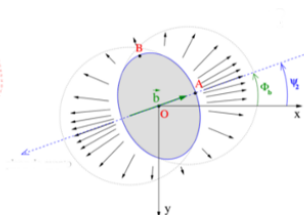
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introduction

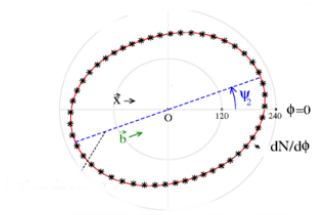
introduction



Relativistic Heavy Ion
Collisions

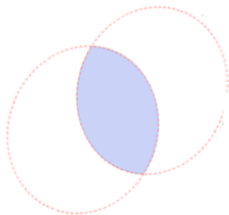


Thermalized I.C.

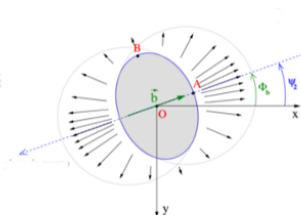


Cools Down
(Particle Emission)

introduction



Relativistic Heavy Ion Collisions

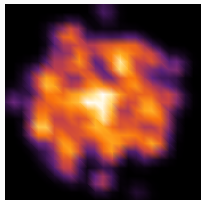


Thermalized I.C.



Cools Down
(Particle Emission)

Initial Conditions are not smooth



Initial Anisotropy \Rightarrow Collective Anisotropic Flow
(coordinate space) (momentum space)

Studying flow indirect probe of initial anisotropy

introduction

Study anisotropic azimuthal distribution by a Fourier series:

$$P(\phi) = \frac{1}{2\pi} \sum_n v_n e^{in\Psi_n} e^{-in\phi}, \quad \text{where } V_n = v_n e^{in\Psi_n} (\text{flow vector}).$$

Where

$v_n \rightarrow$ flow harmonic } fluctuate in event-by-event, even within a

$\Psi_n \rightarrow$ symmetry plane } centrality class

$V_n = \langle e^{in\phi} \rangle \rightarrow \{v_n, \Psi_n\}$ (large set of statistical properties)

$\phi \equiv$ azimuthal angle of emitted particle;

$\langle \dots \rangle \equiv$ mean in one event.

- Experimental data: v_n carries information of the whole event (flow and non-flow).
- Theoretical models: v_n comparison gives few information about the event.

method - mixed harmonic correlation

- (a) V_n for different harmonics aligned?
- (b) Correlation of 2 particles? ("ridge")
- (c) 3-particle correlation? (probe non-linear hydrodynamics response to the medium*, η/s .)

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Method - Mixed Harmonic Correlation

When only collective **flow** is presented, particles are independently emitted:

$$P(\phi_1, \dots, \phi_n) \stackrel{(\text{flow})}{=} P(\phi_1) \dots P(\phi_n)$$

Distribution factorize for **flow**!

Basic Building Block of correlation measurement: *m-particle correlator*

$$\langle m \rangle_{n_1, n_2, \dots, n_m} \equiv \left\langle \left\langle \cos(n_1 \phi^{a_1} + n_2 \phi^{a_2} \dots + n_m \phi^{a_m}) \right\rangle_{m \text{ particles}} \right\rangle$$

$\langle \dots \rangle_{m \text{ particles}}$: average for each event, all possible groups of m particles.

mixed harmonic correlation

$$\langle m \rangle_{n_1, n_2, \dots, n_m} \stackrel{\text{(flow)}}{=} \langle V_{n_1}^{a_1} V_{n_2}^{a_2} \dots V_{n_m}^{a_m} \rangle$$
$$\stackrel{\text{(flow)}}{=} \left\langle v_{|n_1|}^{a_1} v_{|n_2|}^{a_2} \dots v_{|n_m|}^{a_m} \cos(n_1 \Psi_{n_1}^{a_1} + n_2 \Psi_{n_2}^{a_2} \dots n_m \Psi_{n_m}^{a_m}) \right\rangle .$$

$$\langle \dots \rangle \equiv \frac{\sum_{\text{events}} W \dots}{\sum_{\text{events}} W} := \text{average over all events}$$

- $W = 1$ simple average, or $W \equiv W(p_T)$ or $W \equiv W(\eta)$.

In this work W is related with the number of charged hadrons (event).

So there are W combinations, $W = \frac{M!}{(M-m)!}$.

- $W_{\langle 2 \rangle} = M(M-1)$
- $W_{\langle 4 \rangle} = M(M-1)(M-2)(M-3)$.
- Each collision has a uncontrolled (random) azimuthal orientation.
- Only rotation invariant quantities can be measured.

$$n_1 + n_2 + \dots + n_k = 0$$

applications

- LHC has results for m-particle correlations.
- Predictions for RHIC top energy*

(a) Simplest Measurement: 2-particle cumulants

$$v_n\{2\} = \sqrt{\langle 2 \rangle_{n,-n}} \stackrel{\text{(flow)}}{=} \sqrt{\langle v_n^2 \rangle}, \text{ (RMS of } v_n \text{ - absence of non-flow)}$$



non-flow correlations can be important!

imposing rapidity gap, non-flow can be suppressed.

(b) How to perform non-flow measurements?

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(b) How to perform non-flow measurements?

suppressing small number of particles → correlations between more than 2 particles

m-particle cumulants $v_n\{m\}$

* 01/23/17 two papers STAR
arXiv:1701.06496 and 1701.06497

m-particle cumulants - and the symmetric cumulants

$$-v_n\{4\}^4 \equiv \langle 4 \rangle_{n,n,-n,-n} - 2\langle 2 \rangle_{n,-n}^2 \stackrel{\text{(flow)}}{=} \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2.$$

$$4v_n\{6\}^6 \equiv \langle 6 \rangle_{n,n,n,-n,-n,-n} - 9\langle 4 \rangle_{n,n,-n,-n} \langle 2 \rangle_{n,-n} + 12\langle 2 \rangle_{n,-n}^3 \\ \stackrel{\text{(flow)}}{=} \langle v_n^6 \rangle - 9\langle v_n^4 \rangle \langle v_n^2 \rangle + 12\langle v_n^2 \rangle^3$$

Suppressing non-flow correlations we can obtain information about hydro IC.

Symmetric Cumulants

Besides, give information about how flow harmonics fluctuates in E-b-E.

$$\text{SC}(n, m) \equiv \langle 4 \rangle_{n,m,-n,-m} - \langle 2 \rangle_{n,-n} \langle 2 \rangle_{m,-m} \\ \stackrel{\text{(flow)}}{=} \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle.$$

”based on 4-particle correlations”

symmetric cumulants

Anyway, the information can be best viewed in a normalized version.

$$\text{NSC}(n, m) \equiv \frac{\text{SC}(n, m)}{\langle 2 \rangle_{n,-n} \langle 2 \rangle_{m,-m}} \stackrel{\text{(flow)}}{=} \frac{\langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle}{\langle v_n^2 \rangle \langle v_m^2 \rangle}.$$

- Sensitive to magnitude of the flow vector $|V_n|^2 = v_n^2$.
- How to obtain information about the direction of V_n ?

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Inspired by correlations measured by ATLAS, we can obtain 2 and 3 plane correlators, for instance:

$$\begin{aligned} & \langle \cos 4(\Phi_2 - \Phi_4) \rangle \\ & \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle \end{aligned}$$

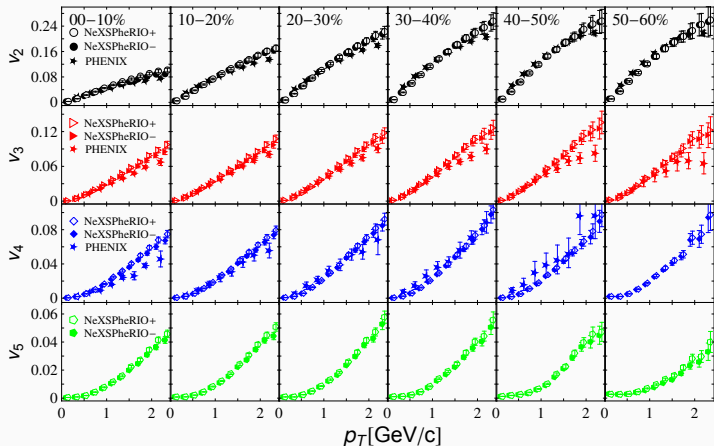
NeXSPheRIO provides reasonable description of top RHIC energy:

1. rapidity and
transverse
momentum spectra
2. directed-elliptic flow
3. anisotropic flow
4. 2-particle
correlation
5. trigger angle
dependence
6. rapidity flow
fluctuation

M. Luzum-Wed

11:20

NeXSPheRIO fits RHIC data extraordinary well, without the necessity of viscosity!



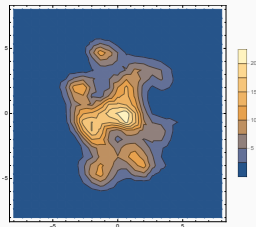
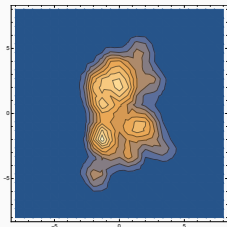
FGG, F.Grassi, M. Luzum and JY Ollitrault, PRL 109 202302

NeXus+SPheRIO \rightarrow NeXSPheRIO

- **NeXus**: initial condition generator 3D;
- **SPheRIO**: 3+1D relativistic ideal hydrodynamics code.

Strategy:

- Au+Au at $\sqrt{s} = 200$ AGeV
- Initial Conditions (NeXuS) - 6 Centralities (Total 00 – 60%);
- Ideal Hydrodynamic Expansion (SPheRIO);
- "Detected" only charged particles with $|\eta| < 1$;



initial conditions and flow harmonics

It is known that some properties of the IC are related with v_n (FGG et. al. arXiv:1111.6538)

- v_2 understood as a response to the almond-shaped overlap area ε_2 :
 $v_2 \propto \varepsilon_2$;
- The triangularity ε_3 is a very good predictor to v_3 : $v_3 \propto \varepsilon_3$;
- Non-linear terms are necessary to predict v_4 (and v_5) from initial energy density: $v_4 \propto k\varepsilon_4 + k'\varepsilon_2^2$ and $v_5 \propto k\varepsilon_5 + k'\varepsilon_2\varepsilon_3$,

These properties still hold even with bulk and shear viscosity (FGG et. al. arXiv:1411.2574)

where eccentricities ε_n are computed as

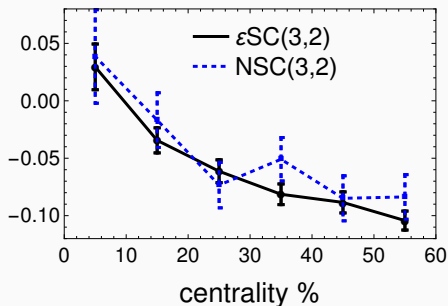
$$\varepsilon_n \equiv \left| \frac{\int d^2r r^n e^{in\phi} \rho(r, \phi)}{\int d^2r r^n \rho(r, \phi)} \right|, \quad (1)$$

$\rho(r, \phi)$ is the energy density and (r, ϕ) the, spatial coordinates of the IC.

initial conditions and flow harmonics

As $v_n \propto \varepsilon_n$ for $n \leq 3$, symmetric cumulants for ε_n should be \approx NSC(3,2).

$$\varepsilon SC(3,2) \equiv \frac{\langle \varepsilon_3^2 \varepsilon_2^2 \rangle - \langle \varepsilon_3^2 \rangle \langle \varepsilon_2^2 \rangle}{\langle \varepsilon_3^2 \rangle \langle \varepsilon_2^2 \rangle}.$$



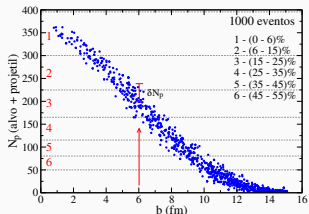
small difference between NSC(3,2) and $\varepsilon SC(3,2)$!

One can study NSC(3,2) without need hydro events - more statistics and vary models and parameters.

results - mixed harmonic correlations

How is measurement performed? We need to mimic experiment.

1. Centrality binning



- v_n for peripheral collisions are larger than central collisions.
- Large bins has large fluctuations on impact parameters, i.e. large v_n fluctuations.

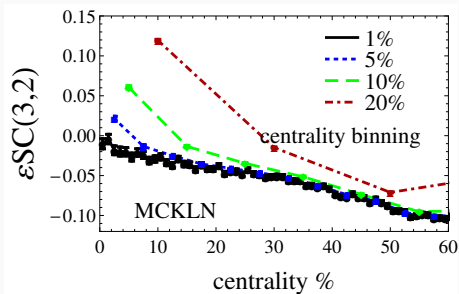
2. Event-weight: LHC (ALICE) use non-unity event-weight (W).

Recombination for SC in a cen centrality bin.

$$NSC_{10\%}^{cen} = \frac{\sum_{c=1}^{10} NSC_{1\%}^c \sum_{\text{events}} W_{\langle m \rangle}^c}{\sum_{c=1}^{10} \sum_{\text{events}} W_{\langle m \rangle}^c}.$$

$NSC_{1\%}^c$ is computed in a c sub-bin for a centrality bin cen .

centrality binning effect - sc



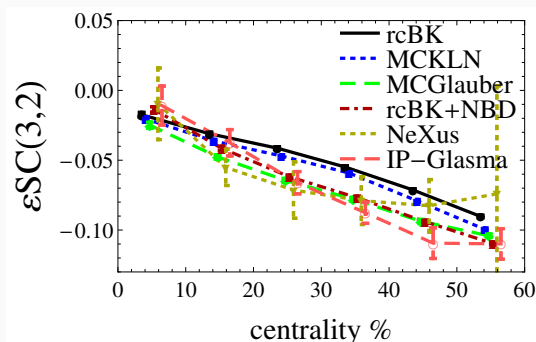
- This effect does not represent interesting unknown physics

In this work 1% recombine 10%: Predictions to 200 A GeV Au-Au, will be performed using NeXSPheRIO with centrality class recombination with event weighing

This procedure is what is done experimentally, if we want compare models with data we have to do this.

symmetric cumulants results

For other models of IC, we can compare $\varepsilon NS(3,2)$

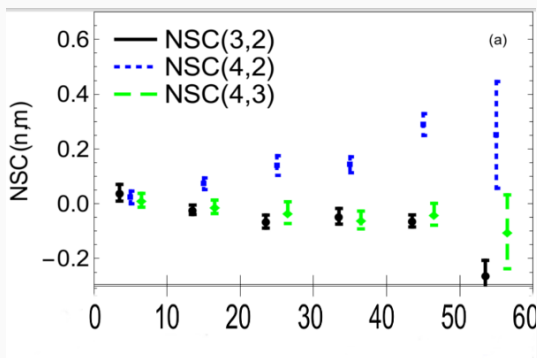


Models quite different. $\varepsilon NS(3,2)$ does not vary by a large of amount

symmetric cumulants results

NeXSPheRIO result for $NSC(n, m)$

Au-Au 200 AGeV, $p_T > 0.2 \text{ GeV}$ $|\eta| < 1$.



- Non-linear effect is important
- $NSC(4, 2)$: $v_4 \propto k \epsilon_4 + k' \epsilon_2^2$

results - event plane correlations

Some event plane correlations that can be measured in RHIC

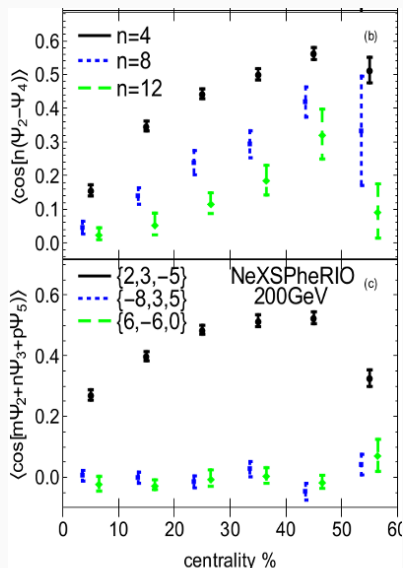
$$\langle \cos 4(\Phi_2 - \Phi_4) \rangle \equiv \frac{\langle 3 \rangle_{2,2,-4}}{\sqrt{\langle 4 \rangle_{2,2,-2,-2} \langle 2 \rangle_{4,-4}}} \stackrel{\text{(flow)}}{=} \frac{\langle v_2^2 v_4 \cos 4(\Psi_2 - \Psi_4) \rangle}{\sqrt{\langle v_2^4 \rangle \langle v_4^2 \rangle}}$$

$$\langle \cos 6(\Phi_2 - \Phi_3) \rangle \equiv \frac{\langle 5 \rangle_{2,2,2,-3,-3}}{\sqrt{\langle 6 \rangle_{2,2,2,-2,-2,-2} \langle 4 \rangle_{3,3,-3,-3}}} \stackrel{\text{(flow)}}{=} \frac{\langle v_2^3 v_3^2 \cos 6(\Psi_2 - \Psi_3) \rangle}{\sqrt{\langle v_2^6 \rangle \langle v_4^4 \rangle}}$$

$$\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle \equiv \frac{\langle 3 \rangle_{2,3,-5}}{\sqrt{\langle 2 \rangle_{2,-2} \langle 2 \rangle_{3,-3} \langle 2 \rangle_{5,-5}}} \stackrel{\text{(flow)}}{=} \frac{\langle v_2 v_3 v_5 \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle}{\sqrt{\langle v_2^2 \rangle \langle v_3^2 \rangle \langle v_5^2 \rangle}}.$$

results - event plane correlations

Au-Au 200 AGeV - NeXSPheRIO, $p_T > 0.2 \text{ GeV}$ $|\eta| < 1$.

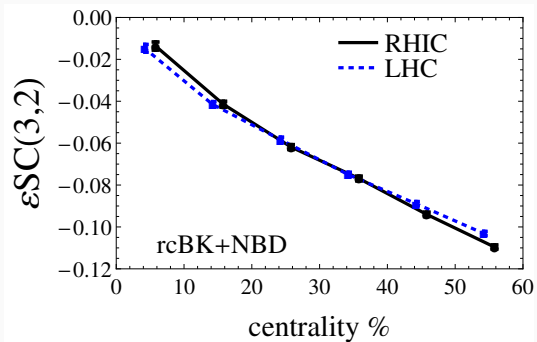


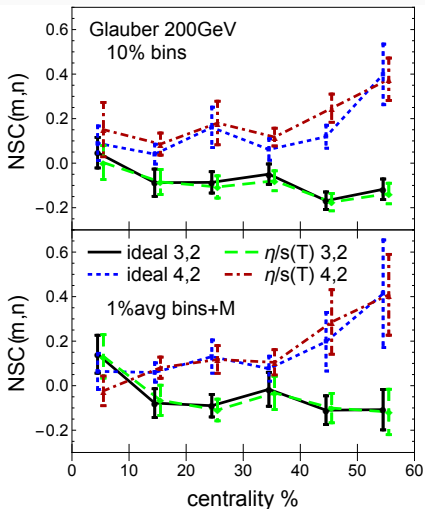
- $\langle \cos n(\Phi_2 - \Phi_4) \rangle$:
 $v_4 \propto k\varepsilon_4 + k'\varepsilon_2^2$
- $\langle \cos(5\Psi_5 - 3\Psi_3 - 2\Psi_2) \rangle$: $v_5 \propto$
 $k\varepsilon_5 + k'\varepsilon_2\varepsilon_3$,
- Ψ_2 and Ψ_3 are not correlated.

summary

- New observables add constraints to theoretical models and probe aspects of the system that are independent of the traditional single-harmonic measurements.
- Many of these new observables have not yet been measured at RHIC as a test of models across energies.
- NeXSPheRIO for Symmetric Cumulants and Event Plane Correlations made for RHIC.
- Provide an important baseline for comparison to correlations of flow harmonics, which contain nontrivial information about the initial state as well as QGP transport properties.
- We also point out significant biases that can appear when using wide centrality bins and non-trivial event weighting, necessitating care in performing experimental analyses and in comparing theoretical calculations to these measurements.

backup slides





sc(3,2) recombined

