hydrodynamic predictions for mixed harmonic correlations in 200 gev au+au collisions

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with
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based on arXiv 1608.02982 (accepted in PRC)
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introduction
introduction

Initial Conditions are not smooth

Initial Anisotropy

⇒

Collective Anisotropic Flow

(study flow indirect probe of initial anisotropy)

Relativistic Heavy Ion Collisions

Thermallyzed I.C.

Cools Down

(Particle Emission)
Initial Conditions are not smooth

Initial Anisotropy $\Rightarrow$ Collective Anisotropic Flow
(coordinate space) (momentum space)

Studying flow indirect probe of initial anisotropy
Study anisotropic azimuthal distribution by a Fourier series:

\[ P(\phi) = \frac{1}{2\pi} \sum_n v_n e^{in\Psi_n} e^{-in\phi}, \text{ where } V_n = v_n e^{in\Psi_n} \text{ (flow vector)}. \]

Where
- \( v_n \rightarrow \text{flow harmonic} \)
- \( \Psi_n \rightarrow \text{symmetry plane} \)
- \( V_n = \langle e^{in\phi} \rangle \rightarrow \{v_n, \Psi_n\} \text{(large set of statistical properties)} \)

\( \phi \equiv \text{azimuthal angle of emitted particle}; \)
\( \langle \cdots \rangle \equiv \text{mean in one event}. \)

- Experimental data: \( v_n \) carries information of the whole event (flow and non-flow).
- Theoretical models: \( v_n \) comparison gives few information about the event.
(a) $V_n$ for different harmonics aligned?
(b) Correlation of 2 particles? ("ridge")
(c) 3-particle correlation? (probe non-linear hydrodynamics response to the medium*, $\eta/s$.)
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Method - **Mixed Harmonic Correlation**

When only collective **flow** is presented, particles are independently emitted:

$$P(\phi_1, \ldots, \phi_n)^{\text{(flow)}} = P(\phi_1) \ldots P(\phi_n)$$

Distribution factorize for **flow**!

Basic Building Block of correlation measurement: **$m$-particle correlator**

$$\langle m \rangle_{n_1, n_2, \ldots, n_m} \equiv \left\langle \cos(n_1 \phi_a^1 + n_2 \phi_a^2 \ldots + n_m \phi_a^m) \right\rangle_m \text{ particles}$$

$$\langle \cdots \rangle_m \text{ particles} : \text{ average for each event, all possible groups of } m \text{ particles.}$$

* arXiv:1104.4740
mixed harmonic correlation

\[ \langle m \rangle_{n_1, n_2, \ldots, n_m} \stackrel{(\text{flow})}{=} \langle V^{a_1}_{n_1} V^{a_2}_{n_2} \ldots V^{a_m}_{n_m} \rangle \]

\( (\text{flow}) \)

\[ \langle \prod \rangle \equiv \sum_{\text{events}} \frac{W}{\sum_{\text{events}}} := \text{average over all events} \]

- \( W = 1 \) simple average, or \( W \equiv W(\rho_T) \) or \( W \equiv W(\eta) \).

In this work \( W \) is related with the number of charged hadrons (event). So there are \( W \) combinations, \( W = \frac{M!}{(M-m)!} \).

- \( W_{\langle 2 \rangle} = M(M-1) \)
- \( W_{\langle 4 \rangle} = M(M-1)(M-2)(M-3) \).

- Each collision has a uncontrolled (random) azimuthal orientation.
- Only rotation invariant quantities can be measured.

\[ n_1 + n_2 + \cdots + n_k = 0 \]
applications
• LHC has results for m-particle correlations.
• Predictions for RHIC top energy*

(a) Simplest Measurement: 2-particle cumulants
\[ v_n\{2\} = \sqrt{\langle 2 \rangle_{n, -n}^{(\text{flow})}} = \sqrt{\langle v_n^2 \rangle}, \text{(RMS of } v_n \text{ - absence of non-flow)} \]
\[ \Downarrow \]
non-flow correlations can be important!
imposing rapidity gap, non-flow can be suppressed.

(b) How to perform non-flow measurements?
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(a) Simplest Measurement: 2-particle cumulants

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↓

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(b) How to perform non-flow measurements?

suppressing small number of particles \(\rightarrow\) correlations between more than 2 particles

m-particle cumulants \(v_n\{m\}\)

* 01/23/17 two papers STAR

arXiv:1701.06496 and 1701.06497
m-particle cumulants - and the symmetric cumulants

\[-v_n\{4\}^4 \equiv \langle 4 \rangle_{n,n,-n,-n} - 2\langle 2 \rangle_{n,-n} \overset{\text{(flow)}}{=} \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2.\]

\[4v_n\{6\}^6 \equiv \langle 6 \rangle_{n,n,n,-n,-n,-n} - 9\langle 4 \rangle_{n,n,-n,-n}\langle 2 \rangle_{n,-n} + 12\langle 2 \rangle_{n,-n}^3 \overset{\text{(flow)}}{=} \langle v_n^6 \rangle - 9\langle v_n^4 \rangle\langle v_n^2 \rangle + 12\langle v_n^2 \rangle^3.\]

Suppressing non-flow correlations we can obtain information about hydro IC.

**Symmetric Cumulants**

Besides, give information about how flow harmonics fluctuates in E-b-E.

\[SC(n, m) \equiv \langle 4 \rangle_{n,m,-n,-m} - \langle 2 \rangle_{n,-n}\langle 2 \rangle_{m,-m} \overset{\text{(flow)}}{=} \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle\langle v_m^2 \rangle.\]

"based on 4-particle correlations"
symmetric cumulants
Anyway, the information can be best viewed in a normalized version.

\[ \text{NSC}(n, m) \equiv \frac{\text{SC}(n, m)}{\langle 2 \rangle_{n,-n} \langle 2 \rangle_{m,-m}} \stackrel{\text{(flow)}}{=} \frac{\langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle}{\langle v_n^2 \rangle \langle v_m^2 \rangle}. \]

- Sensitive to magnitude of the flow vector \(|V_n|^2 = v_n^2\).
- How to obtain information about the direction of \(V_n\)?
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\]

- Sensitive to magnitude of the flow vector \(|V_n|^2 = v_n^2\).
- How to obtain information about the direction of \(V_n\)?

Inspired by correlations measured by ATLAS, we can obtain 2 and 3 plane correlators, for instance:

\[
\langle \cos 4 (\Phi_2 - \Phi_4) \rangle \\
\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle
\]
NeXSPheRIO provides reasonable description of top RHIC energy:

1. rapidity and transverse momentum spectra
2. directed-elliptic flow
3. anisotropic flow
4. 2-particle correlation
5. trigger angle dependence
6. rapidity flow fluctuation

M. Luzum-Wed 11:20
NeXSPheRIO fits RHIC data extraordinary well, without the necessity of viscosity!

FGG, F.Grassi, M. Luzum and JY Ollitrault, PRL 109 202302
NeXus+SPheRIO → NeXSPheRIO

- **NeXus**: initial condition generator 3D;
- **SPheRIO**: 3+1D relativistic ideal hydrodynamics code.

**Strategy:**

- **Au+Au at** $\sqrt{s} = 200$ AGeV
- **Initial Conditions (NeXuS)** - 6 Centralities (Total 00 – 60%);
- **Ideal Hydrodynamic Expansion (SPheRIO)**;
- **”Detected”** only charged particles with $|\eta| < 1$;
initial conditions and flow harmonics

It is known that some properties of the IC are related with $v_n$ (FGG et. al. arXiv:1111.6538)

- $v_2$ understood as a response to the almond-shaped overlap area $\varepsilon_2$: $v_2 \propto \varepsilon_2$;

- The triangularity $\varepsilon_3$ is a very good predictor to $v_3$: $v_3 \propto \varepsilon_3$;

- Non-linear terms are necessary to predict $v_4$ (and $v_5$) from initial energy density: $v_4 \propto k\varepsilon_4 + k'\varepsilon_2^2$ and $v_5 \propto k\varepsilon_5 + k'\varepsilon_2\varepsilon_3$.

These properties still hold even with bulk and shear viscosity (FGG et. al. arXiv:1411.2574)

where eccentricities $\varepsilon_n$ are computed as

$$
\varepsilon_n \equiv \left| \frac{\int d^2r \ r^n e^{in\phi} \rho(r, \phi)}{\int d^2r \ r^n \rho(r, \phi)} \right|,
$$

(1)

$\rho(r, \phi)$ is the energy density and $(r, \phi)$ the, spatial coordinates of the IC.
As \( \nu_n \propto \varepsilon_n \) for \( n \leq 3 \), symmetric cumulants for \( \varepsilon_n \) should be \( \approx \text{NSC}(3,2) \).

\[
\varepsilon_{SC}(3, 2) \equiv \frac{\langle \varepsilon_3 \varepsilon_2 \rangle - \langle \varepsilon_3 \rangle \langle \varepsilon_2 \rangle}{\langle \varepsilon_3 \rangle \langle \varepsilon_2 \rangle}.
\]

small difference between NSC(3,2) and \( \varepsilon_{SC}(3, 2) \)!

One can study NSC(3,2) without need hydro events - more statistics and vary models and parameters.
results - mixed harmonic correlations
How is measurement performed? We need to mimic experiment.

1. Centrality binning

- $v_n$ for peripheral collisions are larger than central collisions.
- Large bins have large fluctuations on impact parameters, i.e. large $v_n$ fluctuations.

2. Event-weight: LHC (ALICE) use non-unity event-weight ($W$).

Recombination for SC in a $cen$ centrality bin.

$$NSC_{10\%}^{cen} = \frac{\sum_{c=1}^{10} NSC_{1\%}^c \sum_{\text{events}} W_c^c}{\sum_{c=1}^{10} \sum_{\text{events}} W_c^c}. $$

$NSC_{1\%}^c$ is computed in a $c$ sub-bin for a centrality bin $cen$. 
• This effect does not represent interesting unknown physics

In this work 1% recombine 10%: Predictions to 200 A GeV Au-Au, will be performed using NeXSPheRIO with centrality class recombination with event weighing

This procedure is what is done experimentally, if we want compare models with data we have to do this.
For other models of IC, we can compare $\varepsilon_{NS}(3, 2)$

Models quite different. $\varepsilon_{NS}(3, 2)$ does not vary by a large of amount.
symmetric cumulants results

NeXSPheRIO result for $NSC(n, m)$
Au-Au 200 AGeV, $p_T > 0.2\text{GeV}$ $|\eta| < 1$.

- Non-linear effect is important
- $NSC(4,2)$: $v_4 \propto k \varepsilon_4 + k' \varepsilon_2^2$
Some event plane correlations that can be measured in RHIC

\[
\langle \cos 4 (\Phi_2 - \Phi_4) \rangle \equiv \frac{\langle 3 \rangle_{2,2,-4}}{\sqrt{\langle 4 \rangle_{2,2,-2,-2} \langle 2 \rangle_{4,-4}}} \quad \text{(flow)} \quad \frac{\langle v_2^2 v_4 \cos 4(\Psi_2 - \Psi_4) \rangle}{\sqrt{\langle v_2^4 \rangle \langle v_4^2 \rangle}}
\]

\[
\langle \cos 6 (\Phi_2 - \Phi_3) \rangle \equiv \frac{\langle 5 \rangle_{2,2,2,-3,-3}}{\sqrt{\langle 6 \rangle_{2,2,2,-2,-2,-2} \langle 4 \rangle_{3,3,-3,-3}}} \quad \text{(flow)} \quad \frac{\langle v_2^3 v_3^2 \cos 6(\Psi_2 - \Psi_3) \rangle}{\sqrt{\langle v_2^6 \rangle \langle v_4^4 \rangle}}
\]

\[
\langle \cos (2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle \equiv \frac{\langle 3 \rangle_{2,3,-5}}{\sqrt{\langle 2 \rangle_{2,-2} \langle 2 \rangle_{3,-3} \langle 2 \rangle_{5,-5}}} \quad \text{(flow)} \quad \frac{\langle v_2 v_3 v_5 \cos (2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle}{\sqrt{\langle v_2^2 \rangle \langle v_3^2 \rangle \langle v_5^2 \rangle}}.
\]
results - event plane correlations

Au-Au 200 AGeV - NeXSPheRIO, $p_T > 0.2\,GeV \mid \eta \mid < 1$.

- $\langle \cos n(\Phi_2 - \Phi_4) \rangle$: $v_4 \propto k\varepsilon_4 + k'\varepsilon_2^2$
- $\langle \cos(5\Psi_5 - 3\Psi_3 - 2\Psi_2) \rangle$: $v_5 \propto k\varepsilon_5 + k'\varepsilon_2\varepsilon_3$,
- $\Psi_2$ and $\Psi_3$ are not correlated.
summary
New observables add constraints to theoretical models and probe aspects of the system that are independent of the traditional single-harmonic measurements.

Many of these new observables have not yet been measured at RHIC as a test of models across energies.

NeXSPheRIO for Symmetric Cumulants and Event Plane Correlations made for RHIC.

Provide an important baseline for comparison to correlations of flow harmonics, which contain nontrivial information about the initial state as well as QGP transport properties.

We also point out significant biases that can appear when using wide centrality bins and non-trivial event weighting, necessitating care in performing experimental analyses and in comparing theoretical calculations to these measurements.
backup slides
$\varepsilon_{SC(3,2)}$ vs centrality %

- $\varepsilon_{SC(3,2)}$ is plotted against the centrality percentage.
- Two lines are shown:
  - Solid line for RHIC
  - Dotted line for LHC

The plot shows the dependence of $\varepsilon_{SC(3,2)}$ on the centrality percentage for both RHIC and LHC experiments, with LHC data denoted as 'rcBK+NBD'.
sc(3,2) recombined

![Graph showing the comparison between εSC(3,2) and NSC(3,2) across various centrality percentages. The graph plots the values on the y-axis from -0.30 to 0.05, with the x-axis representing centrality percentages from 0 to 60. The εSC(3,2) line is depicted in black, while the NSC(3,2) line is shown in blue with error bars indicating variability.]
pb-pb 2.76 gev

ALICE 2.76 TeV

MCKLN

EXP. bins

EXP. bins+M

1% avg bins+M

NSC(4,2)

NSC(3,2)

NSC(4,3)

centrality %