RAPIDITY DEPENDENT FLOW FLUCTUATIONS
AT RHIC

Matthew Luzum
(with F. Gardim, F. Grassi, J. Noronha-Hostler, J-Y Oliotraut)

Universidade de São Paulo

Quark Matter 2017
February 8, 2017
Rapidity dependent fluctuations

- Fluctuations are important
  - Essential for understanding measurements
  - Suggests new measurements
- Fluctuations depend on rapidity in addition to transverse coordinates
- \( \Rightarrow \) must take into account when analyzing measurements
- \( \Rightarrow \) can themselves be studied
Fluctuations are important

- Essential for understanding measurements
- Suggests new measurements

Fluctuations depend on rapidity in addition to transverse coordinates

⇒ must take into account when analyzing measurements

⇒ can themselves be studied
Fluctuations are important
- Essential for understanding measurements
- Suggests new measurements

Fluctuations depend on rapidity in addition to transverse coordinates

⇒ must take into account when analyzing measurements

⇒ can themselves be studied
Fluctuations are important
- Essential for understanding measurements
- Suggests new measurements

Fluctuations depend on rapidity in addition to transverse coordinates

⇒ must take into account when analyzing measurements

⇒ can themselves be studied
Rapidity fluctuations

- Fluctuations are important
  - Essential for understanding measurements
  - Suggests new measurements
- Fluctuations depend on rapidity in addition to transverse coordinates
  - \( \implies \) must take into account when analyzing measurements
  - \( \implies \) can themselves be studied
Fluctuations are important
- Essential for understanding measurements
- Suggests new measurements

Fluctuations depend on rapidity in addition to transverse coordinates

⇒ must take into account when analyzing measurements

⇒ can themselves be studied
NeXSPheRIO

- NeXus is a model for hydrodynamic initial conditions that includes dynamic, rapidity-dependent fluctuations
- Fits RHIC data well with ideal hydro “NeXSPheRIO”
- Useful baseline for comparison to new data
- Extensive results for $v_3$ from STAR (Phys. Rev. C 88, 014904 (2013))
NeXus is a model for hydrodynamic initial conditions that includes dynamic, rapidity-dependent fluctuations.

- Fits RHIC data well with ideal hydro “NeXSPheRIO”
- Use useful baseline for comparison to new data
- Extensive results for $v_3$ from STAR (Phys. Rev. C 88, 014904 (2013))

\begin{center}
\includegraphics[width=\textwidth]{nexspherio.png}
\end{center}

NeXSPheRIO

- NeXus is a model for hydrodynamic initial conditions that includes dynamic, rapidity-dependent fluctuations
- Fits RHIC data well with ideal hydro “NeXSPheRIO”
- Useful baseline for comparison to new data
- Extensive results for $v_3$ from STAR (Phys. Rev. C 88, 014904 (2013))

NeXSPheRIO

- NeXus is a model for hydrodynamic initial conditions that includes dynamic, rapidity-dependent fluctuations
- Fits RHIC data well with ideal hydro “NeXSPheRIO”
- → useful baseline for comparison to new data
- Extensive results for $v_3$ from STAR (Phys. Rev. C 88, 014904 (2013))

CORRELATIONS

- Prerequisite notation:

\[ P(\phi) = \sum_{n=-\infty}^{\infty} V_n(\eta) e^{-in\phi} \]

- In hydro picture, particles are independent
- Azimuthal measurements are multiparticle correlations.

\[ \langle m \rangle_{n_1, n_2, ..., n_m} \equiv \left\langle \langle \cos(n_1 \phi + n_2 \phi + ... + n_m \phi) \rangle_m \text{ particles} \right\rangle \]

\[ \left( \text{flow} \right) = \left\langle V_{n_1} V_{n_2} \cdots V_{n_m} \right\rangle \]

\[ \sum n_i = 0 \]

- E.g., pairs: \( \langle V^n_A V^{B*}_n \rangle = f(\eta^A, \eta^B) \)
CORRELATIONS

- Prerequisite notation:

\[ P(\phi) = \sum_{n=-\infty}^{\infty} V_n(\eta) e^{-in\phi} \]

- In hydro picture, particles are independent
  Azimuthal measurements are multiparticle correlations.

\[ \langle m \rangle_{n_1,n_2,\ldots,n_m} \equiv \left\langle \langle \cos(n_1\phi + n_2\phi \ldots + n_m\phi) \rangle_m \text{ particles} \right\rangle \]

\[ \text{flow} \equiv \langle V_{n_1} V_{n_2} \ldots V_{n_m} \rangle \]

\[ \sum n_l = 0 \]

- E.g., pairs: \( \langle V^A_n V^{B*}_n \rangle = f(\eta^A, \eta^B) \)
CORRELATIONS

- Prerequisite notation:

\[ P(\phi) = \sum_{n=-\infty}^{\infty} V_n(\eta) e^{-in\phi} \]

- In hydro picture, particles are independent
- Azimuthal measurements are multiparticle correlations.

\[ \langle m \rangle_{n_1,n_2,...,n_m} \equiv \langle \cos(n_1\phi + n_2\phi + ... + n_m\phi) \rangle_{m \text{ particles}} \]

\[ \langle \text{flow} \rangle \equiv \langle V_{n_1} V_{n_2} \cdots V_{n_m} \rangle \]

\[ \sum n_i = 0 \]

- E.g., pairs: \[ \langle V_n^A V_n^B \rangle = f(\eta^A, \eta^B) \]
CORRELATIONS

- Prerequisite notation:

\[ P(\phi) = \sum_{n=-\infty}^{\infty} V_n(\eta) e^{-in\phi} \]

- In hydro picture, particles are independent

- Azimuthal measurements are multiparticle correlations.

\[ \langle m \rangle_{n_1, n_2, \ldots, n_m} \equiv \left\langle \langle \cos(n_1\phi + n_2\phi + \ldots + n_m\phi) \rangle \right\rangle_m \text{ particles} \]

\[ \equiv \left\langle V_{n_1} V_{n_2} \ldots V_{n_m} \right\rangle \]

\[ \sum n_i = 0 \]

- E.g., pairs: \[ \langle V_n^A V_{n}^{B} \rangle = f(\eta^A, \eta^B) \]
CORRELATIONS

- Prerequisite notation:

\[ P(\phi) = \sum_{n=-\infty}^{\infty} V_n(\eta) e^{-in\phi} \]

- In hydro picture, particles are independent
- Azimuthal measurements are multiparticle correlations.

\[ \langle m \rangle_{n_1,n_2,\ldots,n_m} \equiv \langle \cos(n_1\phi + n_2\phi + \ldots + n_m\phi) \rangle_m \text{ particles} \]

\[ \equiv \langle V_{n_1} V_{n_2} \ldots V_{n_m} \rangle \]

\[ \sum n_i = 0 \]

- E.g., pairs: \[ \langle V_n^A V_{n^*}^B \rangle = f(\eta^A, \eta^B) \]
Correlations

- Prerequisite notation:

\[ P(\phi) = \sum_{n=-\infty}^{\infty} V_n(\eta) e^{-in\phi} \]

- In hydro picture, particles are independent
- Azimuthal measurements are multiparticle correlations.

\[ \langle m \rangle_{n_1,n_2,\ldots,n_m} \equiv \left\langle \left\langle \cos(n_1\phi + n_2\phi + \ldots + n_m\phi) \right\rangle_{m \text{ particles}} \right\rangle 
= \langle V_{n_1} V_{n_2} \ldots V_{n_m} \rangle 
\sum n_i = 0 \]

E.g., pairs: \[ \langle V_n^A V_n^{B*} \rangle = f(\eta^A, \eta^B) = f(\frac{\eta^A + \eta^B}{2}, \eta^A - \eta^B) \]
Differential $\nu_2\{2\}(\eta)$ involves correlation between particle of interest (POI) and a particle in a reference detector (Ref)

\[
\nu_3\{2\}(\eta) = \nu_3\{EP\}(\eta) \equiv \frac{\langle V_3^{POI} V_3^{*\text{Ref}} \rangle}{\sqrt{\langle V_3^{\text{Ref}A} V_3^{*\text{Ref}B} \rangle}}
\]
Differential $v_2\{2\}(\eta)$ involves correlation between particle of interest (POI) and a particle in a reference detector (Ref)

$$v_3\{2\}(\eta) = v_3\{EP\}(\eta) \equiv \frac{\langle V_{3}^{\text{POI}} V_{3}^{\text{Ref}} \rangle}{\sqrt{\langle V_{3}^{\text{Ref}A} V_{3}^{\text{Ref}B} \rangle}}$$
Differential $v_2\{2\}(\eta)$ involves correlation between particle of interest (POI) and a particle in a reference detector (Ref)

$$v_3\{2\}(\eta) = v_3\{EP\}(\eta) \equiv \frac{\langle V_{3}^{\text{POI}} V_{3}^{*\text{Ref}} \rangle}{\sqrt{\langle V_{3}^{\text{Ref}A} V_{3}^{*\text{Ref}B} \rangle}}$$
Differential $v_2\{2\}(\eta)$ involves correlation between particle of interest (POI) and a particle in a reference detector (Ref)

\[
v_3\{2\}(\eta) = v_3\{EP\}(\eta) \equiv \frac{\langle V_3^{\text{POI}} V_3^{\ast \text{Ref}} \rangle}{\sqrt{\langle V_3^{\text{Ref}A} V_3^{\ast \text{Ref}B} \rangle}}
\]
Differential $v_2\{2\}(\eta)$ involves correlation between particle of interest (POI) and a particle in a reference detector (Ref)

$$v_3\{2\}(\eta) = v_3\{EP\}(\eta) \equiv \frac{\langle V_{3}^{POI} V_{3}^{*\text{Ref}} \rangle}{\sqrt{\langle V_{3}^{\text{Ref}^A} V_{3}^{*\text{Ref}^B} \rangle}}$$
Differential $v_2\{2\}(\eta)$ involves correlation between particle of interest (POI) and a particle in a reference detector (Ref).

$$v_3\{2\}(\eta) = v_3\{EP\}(\eta) \equiv \frac{\langle V_{3}^{\text{POI}} V_{3}^{*\text{Ref}} \rangle}{\sqrt{\langle V_{3}^{\text{Ref}^A} V_{3}^{*\text{Ref}^B} \rangle}}$$
Differential $v_2\{2\}(\eta)$ involves correlation between particle of interest (POI) and a particle in a reference detector (Ref)

\[
v_3\{2\}(\eta) = v_3\{EP\}(\eta) \equiv \frac{\langle V_3^{POI} V_3^{*Ref} \rangle}{\sqrt{\langle V_3^{RefA} V_3^{*RefB} \rangle}}
\]
**Differential $v_3 \{ 2 \}(\eta)$**


- $v_3(\eta)$ has small dependence on $\eta$, similar to data
Differential $v_3\{2\}(\eta)$


- $v_3(\eta)$ has small dependence on $\eta$, similar to data
Differential $v_3^{\{2\}}(\eta)$

$\bullet$ $v_3(\eta)$ has small dependence on $\eta$, similar to data

Differential $v_3(\eta)$

$\eta$

- $v_3(\eta)$ has small dependence on $\eta$, similar to data

Differential $v_3(\eta)$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$

$\eta$
**Differential $v_3\{2\}(\eta)$**


- $v_3(\eta)$ has small dependence on $\eta$, similar to data
\( \eta \)

**Ref.**

**POI**

\[
v_3\{EP\}_{\text{MAX}} \equiv \frac{\langle V_{3}^{\text{POI}} V_{3}^{\text{Ref}} \rangle}{\sqrt{\langle | V_{3}^{\text{Ref}} |^2 \rangle}} = \sqrt{\langle | V_{3}^{\text{POI}} |^2 \rangle}
\]
\[ v_3 \{EP\}_{\text{MAX}} = \frac{\langle V_3^{\text{POI}} V_3^{\text{Ref}}^* \rangle}{\sqrt{\langle |V_3^{\text{Ref}}|^2 \rangle}} = \sqrt{\langle |V_3^{\text{POI}}|^2 \rangle} \]
\[ v_3^{\{EP\}}_{\text{MAX}} = \frac{\langle V^\text{POI}_3 V^\text{Ref}_3^\ast \rangle}{\sqrt{\langle |V^\text{Ref}_3|^2 \rangle}} = \sqrt{\langle |V^\text{POI}_3|^2 \rangle} \]
\[ v_{3\{EP\}}^{MAX} \equiv \frac{\langle V_{3}^{POI} V_{3}^{*Ref} \rangle}{\sqrt{\langle |V_{3}^{Ref}|^2 \rangle}} = \sqrt{\langle |V_{3}^{POI}|^2 \rangle} \]
\[ v_3 \{EP\}_{MAX} \equiv \frac{\langle V_3^{POI} V_3^{*Ref} \rangle}{\sqrt{\langle |V_3^{Ref}|^2 \rangle}} = \sqrt{\langle |V_3^{POI}|^2 \rangle} \]
Differential $v_3\{2\}(\eta)$

- Average magnitude of $v_3$ does not depend on $\eta$ within TPC
- $\Rightarrow v_3\{EP\}(\eta)$ doesn’t show dependence on absolute rapidity, but relative rapidity
- Magnitude depends on placement of reference detector
Average magnitude of $v_3$ does \textit{not} depend on $\eta$ within TPC

$\implies v_3\{EP\}(\eta)$ doesn’t show dependence on absolute rapidity, but \textit{relative} rapidity

Magnitude depends on placement of reference detector
Average magnitude of $v_3$ does not depend on $\eta$ within TPC

$\implies v_3\{EP\}(\eta)$ doesn’t show dependence on absolute rapidity, but relative rapidity

Magnitude depends on placement of reference detector
**Average magnitude of** $v_3$ **does not depend on** $\eta$ **within TPC**

$\implies v_3\{EP\}(\eta)$ **doesn’t show dependence on absolute rapidity, but relative rapidity**

**Magnitude depends on placement of reference detector**
**Differential $v_3\{2\}(\eta)$**

- Average magnitude of $v_3$ does *not* depend on $\eta$ within TPC
- $\implies v_3\{EP\}(\eta)$ doesn’t show dependence on absolute rapidity, but *relative* rapidity
- Magnitude depends on placement of reference detector
Average magnitude of $v_3$ does not depend on $\eta$ within TPC

$\Rightarrow$ $v_3\{EP\}(\eta)$ doesn’t show dependence on absolute rapidity, but relative rapidity

Magnitude depends on placement of reference detector
Differential $v_3\{2\}(\eta)$

- Average magnitude of $v_3$ does not depend on $\eta$ within TPC
- $\implies v_3\{EP\}(\eta)$ doesn’t show dependence on absolute rapidity, but relative rapidity
- Magnitude depends on placement of reference detector
$
u_3\{2\}^2$ vs Relative Rapidity


- Can investigate $\Delta \eta$ dependence explicitly
  - Data fit by narrow Gaussian plus wide Gaussian
  - Calculation not Gaussian; small error bars $\Rightarrow$ too wide, despite reasonable fit to data
  - $\Rightarrow$ use experimental error bars with Gaussian fit to calculation
$v_3(2)^2$ vs relative rapidity

- Can investigate $\Delta \eta$ dependence explicitly
  - Data fit by narrow Gaussian plus wide Gaussian
  - Calculation not Gaussian; small error bars $\Rightarrow$ too wide, despite reasonable fit to data
  - $\Rightarrow$ use experimental error bars with Gaussian fit to calculation

Can investigate $\Delta \eta$ dependence explicitly

Data fit by narrow Gaussian plus wide Gaussian

Calculation not Gaussian; small error bars $\Rightarrow$ too wide, despite reasonable fit to data

$\Rightarrow$ use experimental error bars with Gaussian fit to calculation
$v_3^2 \{ 2 \}^2$ vs Relative Rapidity

- Can investigate $\Delta \eta$ dependence explicitly
- Data fit by narrow Gaussian plus wide Gaussian
- Calculation not Gaussian; small error bars $\Rightarrow$ too wide, despite reasonable fit to data
- $\Rightarrow$ use experimental error bars with Gaussian fit to calculation
$v_3^2 \{2\}^2$ vs relative rapidity

- Can investigate $\Delta \eta$ dependence explicitly
- Data fit by narrow Gaussian plus wide Gaussian
- Calculation not Gaussian; small error bars $\Rightarrow$ too wide, despite reasonable fit to data
- $\Rightarrow$ use experimental error bars with Gaussian fit to calculation

Gaussian fit parameters vs. centrality

- Experimental weighting improves agreement
- NeXSPheRIO still too wide in peripheral collisions
Higher Cumulants

- Can also measure cumulants of >2 particles

\[ v_n\{2\} = \left\langle |V_n|^2 \right\rangle^{1/2} \]

\[ v_n\{4\} = \left( 2\left\langle |V_n|^2 \right\rangle - \left\langle |V_n|^4 \right\rangle \right)^{1/4} \]

- \( v_n\{2\} \) vs. \( v_n\{4\} \) is measure of EBE \( v_n \) fluctuations
- Higher cumulants (including SC) measured with no rapidity gap
- Not comparing apples to apples
Higher Cumulants

- Can also measure cumulants of >2 particles

\[ v_n\{2\} = \left\langle \left| V_n \right|^2 \right\rangle^{1/2} \]

\[ v_n\{4\} = \left( 2\left\langle \left| V_n \right|^2 \right\rangle^2 - \left\langle \left| V_n \right|^4 \right\rangle \right)^{1/4} \]

\[ \implies v_n\{2\} \text{ vs. } v_n\{4\} \text{ is measure of EBE } v_n \text{ fluctuations} \]

- Higher cumulants (including SC) measured with no rapidity gap
- Not comparing apples to apples
Higher Cumulants

- Can also measure cumulants of >2 particles
  \[ v_n\{2\} = \left\langle |V_n|^2 \right\rangle^{1/2} \]
  \[ v_n\{4\} = \left(2\left\langle |V_n|^2 \right\rangle^2 - \left\langle |V_n|^4 \right\rangle\right)^{1/4} \]

- \( v_n\{2\} \) vs. \( v_n\{4\} \) is measure of EBE \( v_n \) fluctuations
- Higher cumulants (including SC) measured with no rapidity gap
- Not comparing apples to apples
Can also measure cumulants of >2 particles

\[ v_n\{2\} = \left\langle V_n^A V_n^{B*} \right\rangle^{1/2} \]

\[ v_n\{4\} = \left( 2\left\langle |V_n|^2\right\rangle^2 - \left\langle |V_n|^4\right\rangle \right)^{1/4} \]

\[ \implies v_n\{2\} \text{ vs. } v_n\{4\} \text{ is measure of EBE } v_n \text{ fluctuations} \]

Higher cumulants (including SC) measured with no rapidity gap

Not comparing apples to apples
**Higher Cumulants**

- Can also measure cumulants of >2 particles

\[ v_n\{2\} = \left( V_n^A V_n^{B*} \right)^{1/2} \]

\[ v_n\{4\} = \left( 2\langle |V_n|^2 \rangle^2 - \langle |V_n|^4 \rangle \right)^{1/4} \]

- \( v_n\{2\} \) vs. \( v_n\{4\} \) is measure of EBE \( v_n \) fluctuations
- Higher cumulants (including SC) measured with no rapidity gap
- Not comparing apples to apples

\[ \eta \]

[Diagram of A and B with \( \eta \) notation]
**Gapped Cumulants**

- Can measure “gapped” cumulants: (see, e.g., Jia, Zhou, Trzupek, arXiv:1701.03830)

\[
\nu_n\{2\}^2 = \langle V_n^A V_n^{B*} \rangle \\
- \nu_n\{4\}^{(1)}_{(1)} = \langle V_n^A V_n^A V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle^2 \\
- \nu_n\{4\}^{(2)}_{(2)} = \langle V_n^A V_n^B V_n^{A*} V_n^{B*} \rangle - \langle V_n^A V_n^{B*} \rangle^2 - \langle |V_n^A|^2 \rangle \langle |V_n^B|^2 \rangle \\
- \nu_n\{4\}^{(3)}_{(3)} = \langle V_n^A V_n^B V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle \langle |V_n^B|^2 \rangle \\
\]

![Diagram of A and B regions with rapidity \(\eta\) and \(\Delta \eta\)]
Gapped Cumulants

- Can measure “gapped” cumulants: (see, e.g., Jia, Zhou, Trzupek, arXiv:1701.03830)

\[
\nu_n\{2\}^2 = \langle V_n^A V_n^{B*} \rangle \\
- \nu_n\{4\}_{(1)}^4 = \langle V_n^A V_n^A V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle^2 \\
- \nu_n\{4\}_{(2)}^4 = \langle V_n^A V_n^B V_n^{A*} V_n^{B*} \rangle - \langle V_n^A V_n^{B*} \rangle^2 - \langle |V_n^A|^2 \rangle \langle |V_n^B|^2 \rangle \\
- \nu_n\{4\}_{(3)}^4 = \langle V_n^A V_n^B V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle \langle |V_n^B|^2 \rangle \\
\]
Can measure “gapped” cumulants: (see, e.g., Jia, Zhou, Trzupek, arXiv:1701.03830)

\[
\begin{align*}
\nu_n\{2\}^2 &= \langle V_n^A V_n^{B*} \rangle \\
- \nu_n\{4\}_{(1)}^4 &= \langle V_n^A V_n^A V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle^2 \\
- \nu_n\{4\}_{(2)}^4 &= \langle V_n^A V_n^B V_n^{A*} V_n^{B*} \rangle - \langle V_n^A V_n^{B*} \rangle^2 - \langle |V_n^A|^2 \rangle \langle |V_n^B|^2 \rangle \\
- \nu_n\{4\}_{(3)}^4 &= \langle V_n^A V_n^B V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle \langle |V_n^B|^2 \rangle
\end{align*}
\]
Can measure “gapped” cumulants: (see, e.g., Jia, Zhou, Trzupek, arXiv:1701.03830)

\[ v_n\{2\}^2 = \langle V_n^A V_n^{B*} \rangle \]

\[ - v_n\{4\}^{(1)} = \langle V_n^A V_n^B V_n^{A*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle^2 \]

\[ - v_n\{4\}^{(2)} = \langle V_n^A V_n^B V_n^{A*} V_n^{B*} \rangle - \langle V_n^A V_n^{B*} \rangle^2 - \langle |V_n|^2 \rangle \langle |V_n|^2 \rangle \]

\[ - v_n\{4\}^{(3)} = \langle V_n^A V_n^B V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle \langle |V_n|^2 \rangle \]

GAPPED CUMULANTS

RAPIDITY DEPENDENT FLUCTUATIONS

Matthew Luzum (USP)

02/08/2017 12 / 14
Can measure “gapped” cumulants: (see, e.g., Jia, Zhou, Trzupek, arXiv:1701.03830)

\[
v_n\{2\}^2 = \langle V_n^A V_n^{B*} \rangle
- v_n\{4\}_{(1)}^4 = \langle (V_n^A V_n^{B*})^2 \rangle - 2\langle V_n^A V_n^{B*} \rangle^2
- v_n\{4\}_{(2)}^4 = \langle V_n^A V_n^B V_n^{A*} V_n^{B*} \rangle - \langle V_n^A V_n^{B*} \rangle^2 - \langle |V_n^A|^2 \rangle \langle |V_n^B|^2 \rangle
- v_n\{4\}_{(3)}^4 = \langle V_n^A V_n^B V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle \langle |V_n^B|^2 \rangle
\]
Can measure “gapped” cumulants: (see, e.g., Jia, Zhou, Trzupek, arXiv:1701.03830)

\[ v_n\{2\}^2 = \langle V_n^A V_n^{B*} \rangle \]

\[ - v_n\{4\}_1^4 = \langle (V_n^A V_n^{B*})^2 \rangle - 2\langle V_n^A V_n^{B*} \rangle^2 \]

\[ - v_n\{4\}_2^4 = \langle V_n^A V_n^B V_n^{A*} V_n^{B*} \rangle - \langle V_n^A V_n^{B*} \rangle^2 - \langle \| V_n^A \| \rangle \langle \| V_n^B \| \rangle \]

\[ - v_n\{4\}_3^4 = \langle V_n^A V_n^B V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle \langle \| V_n^B \| \rangle \]
Gapped Cumulants

- Can measure “gapped” cumulants: (see, e.g., Jia, Zhou, Trzupek, arXiv:1701.03830)

\[ v_n\{2\}^2 = \langle V_n^A V_n^{B*} \rangle \]
\[ - v_n\{4\}^{(1)} = \langle (V_n^A V_n^{B*})^2 \rangle - 2\langle V_n^A V_n^{B*}\rangle^2 \]
\[ - v_n\{4\}^{(2)} = \langle V_n^A V_n^B V_n^{A*} V_n^{B*} \rangle - \langle V_n^A V_n^{B*}\rangle^2 - \langle |V_n^A|^2\rangle\langle |V_n^B|^2\rangle \]
\[ - v_n\{4\}^{(3)} = \langle V_n^A V_n^B V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*}\rangle\langle |V_n^B|^2\rangle \]
Gapped Cumulants

- Can measure “gapped” cumulants: (see, e.g., Jia, Zhou, Trzupek, arXiv:1701.03830)

\[
\begin{align*}
  v_n\{2\}^2 &= \langle V_n^A V_n^{B*} \rangle \\
  - v_n\{4\}_{(1)}^4 &= \langle (V_n^A V_n^{B*})^2 \rangle - 2\langle V_n^A V_n^{B*} \rangle^2 \\
  - v_n\{4\}_{(2)}^4 &= \langle V_n^A V_n^B V_n^{A*} V_n^{B*} \rangle - \langle V_n^A V_n^{B*} \rangle^2 - \langle |V_n|^2 \rangle \langle |V_n|^2 \rangle \\
  - v_n\{4\}_{(3)}^4 &= \langle V_n^A V_n^B V_n^{B*} V_n^{B*} \rangle - 2\langle V_n^A V_n^{B*} \rangle \langle |V_n|^2 \rangle
\end{align*}
\]
Gapped Cumulants

- Can measure “gapped” cumulants: (see, e.g., Jia, Zhou, Trzupek, arXiv:1701.03830)

\[ \nu_n\{2\}^2 = \left\langle V_n^A V_n^{B*} \right\rangle \]

\[ - \nu_n\{4\}^{(1)}_4 = \left\langle (V_n^A V_n^{B*})^2 \right\rangle - 2\left\langle V_n^A V_n^{B*} \right\rangle^2 \]

\[ - \nu_n\{4\}^{(2)}_4 = \left\langle V_n^A V_n^B V_n^{A*} V_n^{B*} \right\rangle - \left\langle V_n^A V_n^{B*} \right\rangle^2 - \left\langle |V_n|^2 \right\rangle \left\langle |V_n|^2 \right\rangle \]

\[ - \nu_n\{4\}^{(3)}_4 = \left\langle V_n^A V_n^B V_n^{B*} V_n^{B*} \right\rangle - 2\left\langle V_n^A V_n^{B*} \right\rangle \left\langle |V_n|^2 \right\rangle \]
- \( -\eta_0 - W < \eta_a < -\eta_0 + W \) and \( \eta_0 - W < \eta_b < \eta_0 + W \)

\[
\eta_0 \sim 0.2, 0.4, 0.6, 0.8
\]

\( v_2\{2\}(\eta_a, \eta_b) \)

- \( v_2\{4\} \) has smaller dependence on rapidity gap than \( v_2\{2\} \)
- \( \Rightarrow \) Ratio \( v_2\{4\}/v_2\{2\} \) has dependence on \( \Delta \eta = 2\eta_0 \) only from denominator
$v_2\{4\}$ has smaller dependence on rapidity gap than $v_2\{2\}$

$\Rightarrow$ Ratio $v_2\{4\}/v_2\{2\}$ has dependence on $\Delta \eta = 2\eta_0$ only from denominator
\( v_2\{4\} \) has smaller dependence on rapidity gap than \( v_2\{2\} \)

\[ \frac{v_2\{4\}(\eta_a, \eta_b)}{v_2\{2\}} \] has dependence on \( \Delta \eta = 2\eta_0 \) only from denominator
\( -\eta_0 - W < \eta_a < -\eta_0 + W \) and \( \eta_0 - W < \eta_b < \eta_0 + W \)

- \( v_2\{4\}\) has smaller dependence on rapidity gap than \( v_2\{2\}\)

\[ \Rightarrow \text{Ratio } v_2\{4\} / v_2\{2\} \text{ has dependence on } \Delta \eta = 2\eta_0 \text{ only from denominator} \]
- $-\eta_0 - W < \eta_a < -\eta_0 + W$ and $\eta_0 - W < \eta_b < \eta_0 + W$

$20-30\%$

- $v_2\{4\}$ has smaller dependence on rapidity gap than $v_2\{2\}$

$\Rightarrow$ Ratio $v_2\{4\}/v_2\{2\}$ has dependence on $\Delta \eta = 2\eta_0$ only from denominator
\( v_2\{4\} \) has smaller dependence on rapidity gap than \( v_2\{2\} \)

\( \Rightarrow \) Ratio \( v_2\{4\}/v_2\{2\} \) has dependence on \( \Delta \eta = 2\eta_0 \) only from denominator
- $-\eta_0 - W < \eta_a < -\eta_0 + W$ and $\eta_0 - W < \eta_b < \eta_0 + W$

- $20-30\%$

- $v_2\{4\}$ has smaller dependence on rapidity gap than $v_2\{2\}$

- $\implies$ Ratio $v_2\{4\}/v_2\{2\}$ has dependence on $\Delta \eta = 2\eta_0$ only from denominator
Heavy-ion system has dependence on rapidity

Need to take into account for precise understanding of data (even “fixed rapidity” measurements)

Rapidity-dependent fluctuations can be studied in detail

The rapidity-dependent fluctuations in NeXus give reasonable description of data

Dependence on relative rapidity more important than absolute rapidity

Higher order “gapped cumulants” can be used to study the nature of short-range hydrodynamic fluctuations and/or non-flow correlations
Summary

- Heavy-ion system has dependence on rapidity
- Need to take into account for precise understanding of data (even “fixed rapidity” measurements)
- Rapidity-dependent fluctuations can be studied in detail
- The rapidity-dependent fluctuations in NeXus give reasonable description of data
- Dependence on relative rapidity more important than absolute rapidity
- Higher order “gapped cumulants” can be used to study the nature of short-range hydrodynamic fluctuations and/or non-flow correlations
SUMMARY

- Heavy-ion system has dependence on rapidity
- Need to take into account for precise understanding of data (even “fixed rapidity” measurements)
- Rapidity-dependent fluctuations can be studied in detail
  - The rapidity-dependent fluctuations in NeXus give reasonable description of data
  - Dependence on relative rapidity more important than absolute rapidity
  - Higher order “gapped cumulants” can be used to study the nature of short-range hydrodynamic fluctuations and/or non-flow correlations
**Summary**

- Heavy-ion system has dependence on rapidity
- Need to take into account for precise understanding of data (even “fixed rapidity” measurements)
- Rapidity-dependent fluctuations can be studied in detail
- The rapidity-dependent fluctuations in NeXus give reasonable description of data
- Dependence on relative rapidity more important than absolute rapidity
- Higher order “gapped cumulants” can be used to study the nature of short-range hydrodynamic fluctuations and/or non-flow correlations
Summary

- Heavy-ion system has dependence on rapidity
- Need to take into account for precise understanding of data (even "fixed rapidity" measurements)
- Rapidity-dependent fluctuations can be studied in detail
- The rapidity-dependent fluctuations in NeXus give reasonable description of data
- Dependence on relative rapidity more important than absolute rapidity
- Higher order “gapped cumulants” can be used to study the nature of short-range hydrodynamic fluctuations and/or non-flow correlations
Summary

- Heavy-ion system has dependence on rapidity
- Need to take into account for precise understanding of data (even “fixed rapidity” measurements)
- Rapidity-dependent fluctuations can be studied in detail
- The rapidity-dependent fluctuations in NeXus give reasonable description of data
- Dependence on relative rapidity more important than absolute rapidity
- Higher order “gapped cumulants” can be used to study the nature of short-range hydrodynamic fluctuations and/or non-flow correlations
\( \nu_3\{4\} \) from NeXSPheRIO consistent with STAR

- \( \nu_3\{4\} \) is inconsistent between RHIC and LHC, but ratio \( \nu_3\{4\}/\nu_3\{2\} \) is consistent