Dynamical critical fluctuations near the QCD critical point

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Outline

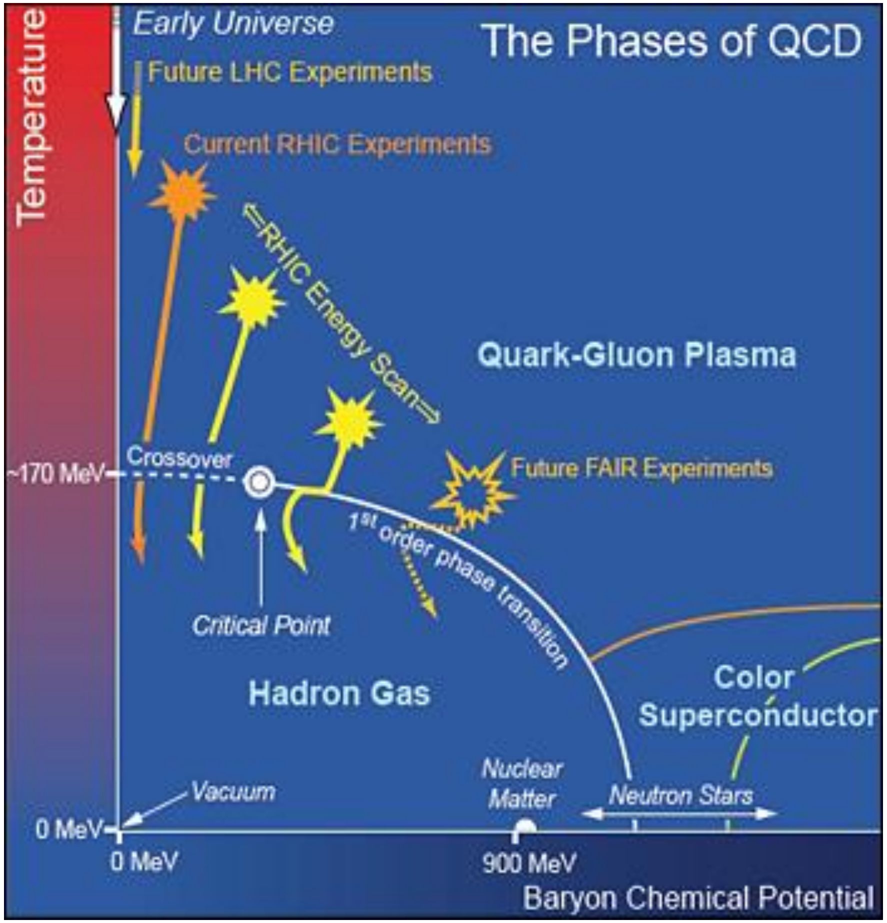
I. Introduction

II. **Equilibrium** critical fluctuations along the freeze-out surface

III. **Dynamical** critical fluctuations from Langevin dynamics

IV. Summary and outlook

Critical Point --- the landmark of the QCD phase diagram.



- **Theoretical analysis, CP is predicted.**
- **Lattice simulation :**
  - small μ, finite T
  - **crossover**
- **Effective theories**
  - (P)NJL, QM, FRG, DSE, etc)
  - finite T and large μ
  - **1st order**

- **Experimental facilities:**
  - RHIC (BES)
  - FAIR, NICA

➢ **The location of CP? The signals?**

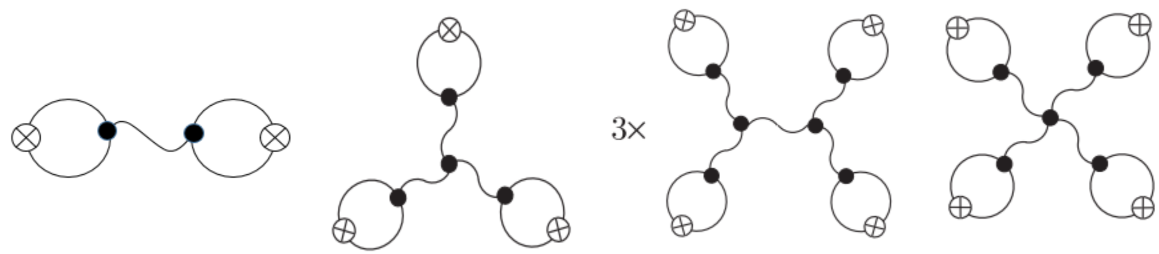
M. Stephanov, PRL 102, 032301(2009)

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots\right].$$

$$\langle\sigma_0^2\rangle = \frac{T}{V}\xi^2 \qquad \langle\sigma_0^3\rangle = \frac{2\lambda_3 T}{V}\xi^6; \qquad \langle\sigma_0^4\rangle_c = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\xi^8$$

- Non-Gaussian fluctuations:

$$\left\langle(\delta N)^2\right\rangle \sim \xi^2$$
$$\left\langle(\delta N)^3\right\rangle \sim \xi^{4.5}$$
$$\left\langle(\delta N)^4\right\rangle \sim \xi^7$$



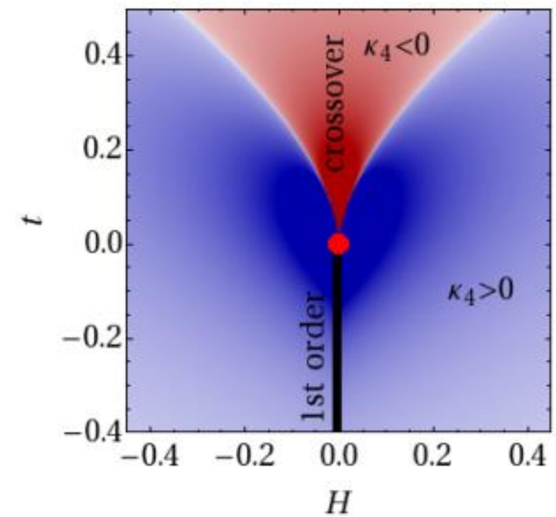
- **kurtosis:**

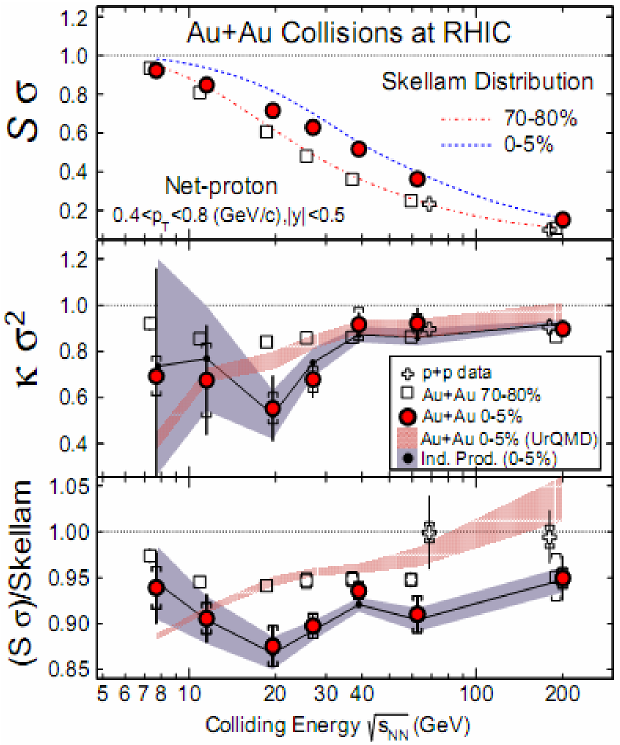
$\kappa_4 < 0$, from the crossover side

$\kappa_4 > 0$, from the 1st order side

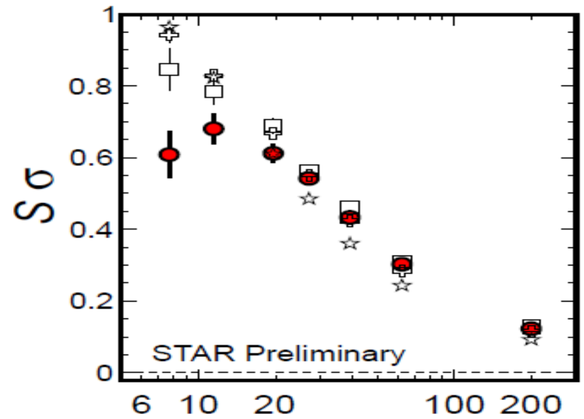
Nonmonotonic in the vicinity of CP

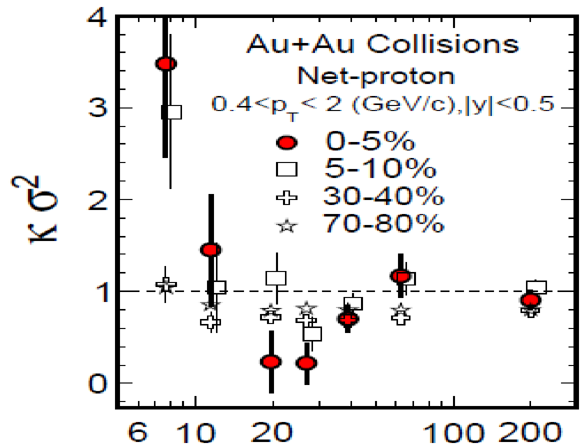
M. Stephanov, PRL 107, 052301 (2011)





STAR Collaboration, PRL, 112, 032302 (2013)





$$S\,\sigma = \frac{C_3}{C_2} \sim \chi_B^{(3)}/\chi_B^{(2)}$$

$$\kappa\sigma^2 = \frac{C_4}{C_2} \sim \chi_B^{(4)}/\chi_B^{(2)}$$

Xiaofeng Luo(for the STAR Collaboration), PoS(CPOD2014)019

Indications from experimental data:

- **Deviations from statistical baselines.**
- **Nonmonotonic at $\sqrt{s_{NN}} \sim 20\ GeV$.**

Equilibrium critical fluctuations?

Dynamical critical fluctuations?

Equilibrium fluctuations

Equilibrium critical fluctuations along the freeze out surface

The formalism

Particle emissions in HIC near Critical Point, Cooper-Frye formula: Jiang, Li & Song , PRC, 94, 024918

$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p_\mu d\sigma^\mu}{2\pi^3} f(x,p) \xrightarrow{M \to g(\bar{\sigma}+\sigma(x))} \begin{aligned} f(x,p) &= f_0(x,p)\left[1 - g\sigma(x)/(\gamma T)\right] \\ &= f_0 + \delta f \end{aligned}$$

Critical fluctuations for particles on the freeze-out surface

$$\langle(\delta N)^n\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^n \left(\prod_{i=1,\ldots,n} \int \frac{1}{E_i} d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu\right) \left(-\frac{g}{T}\right)^n \frac{f_{01}\ldots f_{0n}}{\gamma_1\ldots\gamma_n} \langle\sigma_1\ldots\sigma_n\rangle_c$$

with n=2,3,4, the spatial correlators of sigma are written as

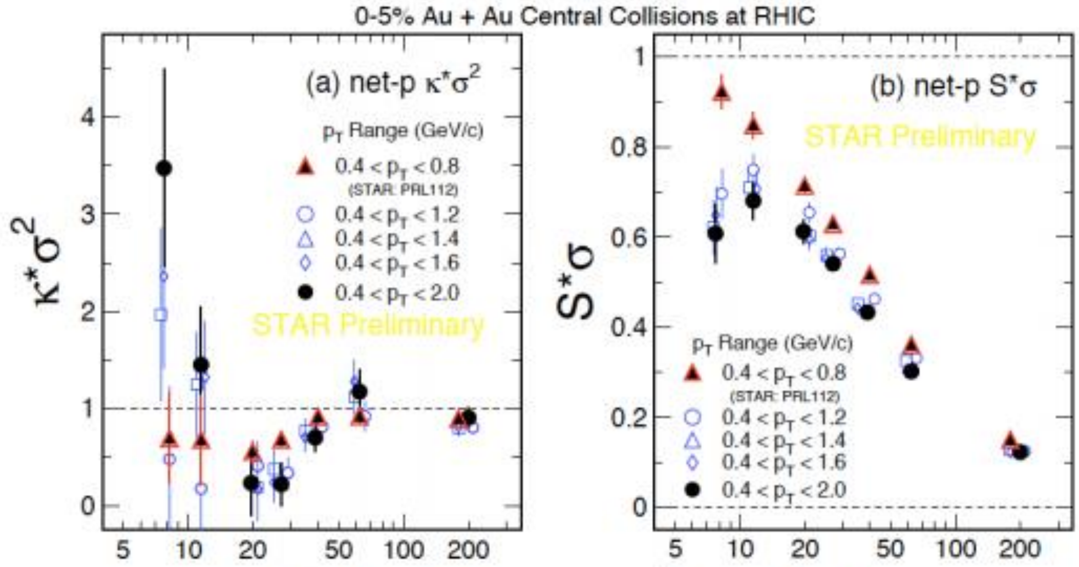
$$\begin{aligned}
\langle\sigma_1\sigma_2\rangle_c &= TD(x_1-x_2), \\
\langle\sigma_1\sigma_2\sigma_3\rangle_c &= -2T^2\lambda_3 \int d^3z D(x_1-z) D(x_2-z) D(x_3-z), \\
\langle\sigma_1\sigma_2\sigma_3\sigma_4\rangle_c &= -6T^3\lambda_4 \int d^3z D(x_1-z) D(x_2-z) D(x_3-z) D(x_4-z) \\
&\quad +12T^3\lambda_3^2 \int d^3u \int d^3v D(x_1-u) D(x_2-u) D(x_3-v) D(x_4-v) D(u-v).
\end{aligned}$$

In an isothermal and equilibrated system, integrate over coordinate space, the results in Stephanov's PRL09 are reproduced.

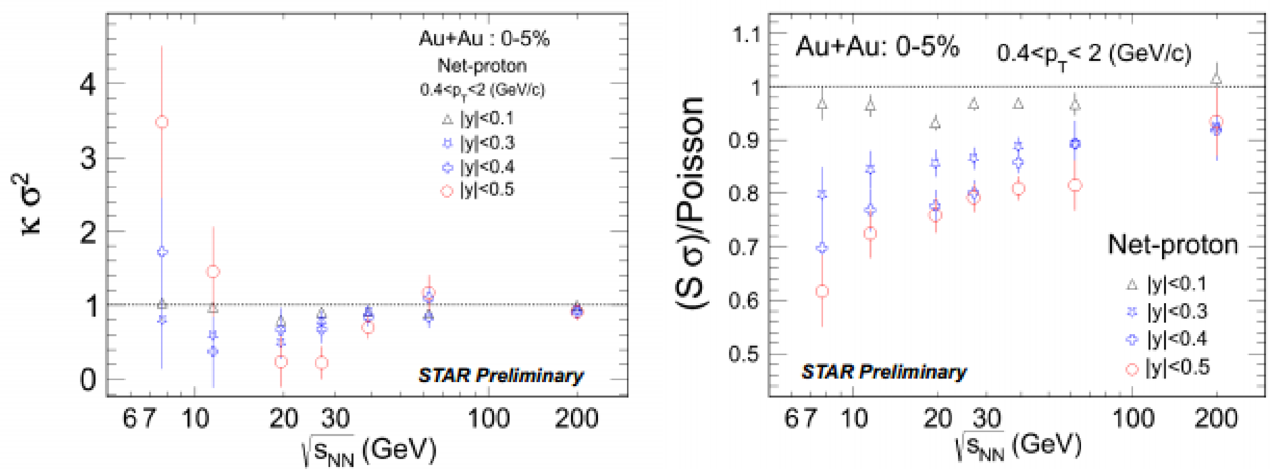
Acceptance dependence of BES data

Xiaofeng Luo(for the STAR Collaboration), PoS(CPOD2014)019

p_T dependence:



y dependence:



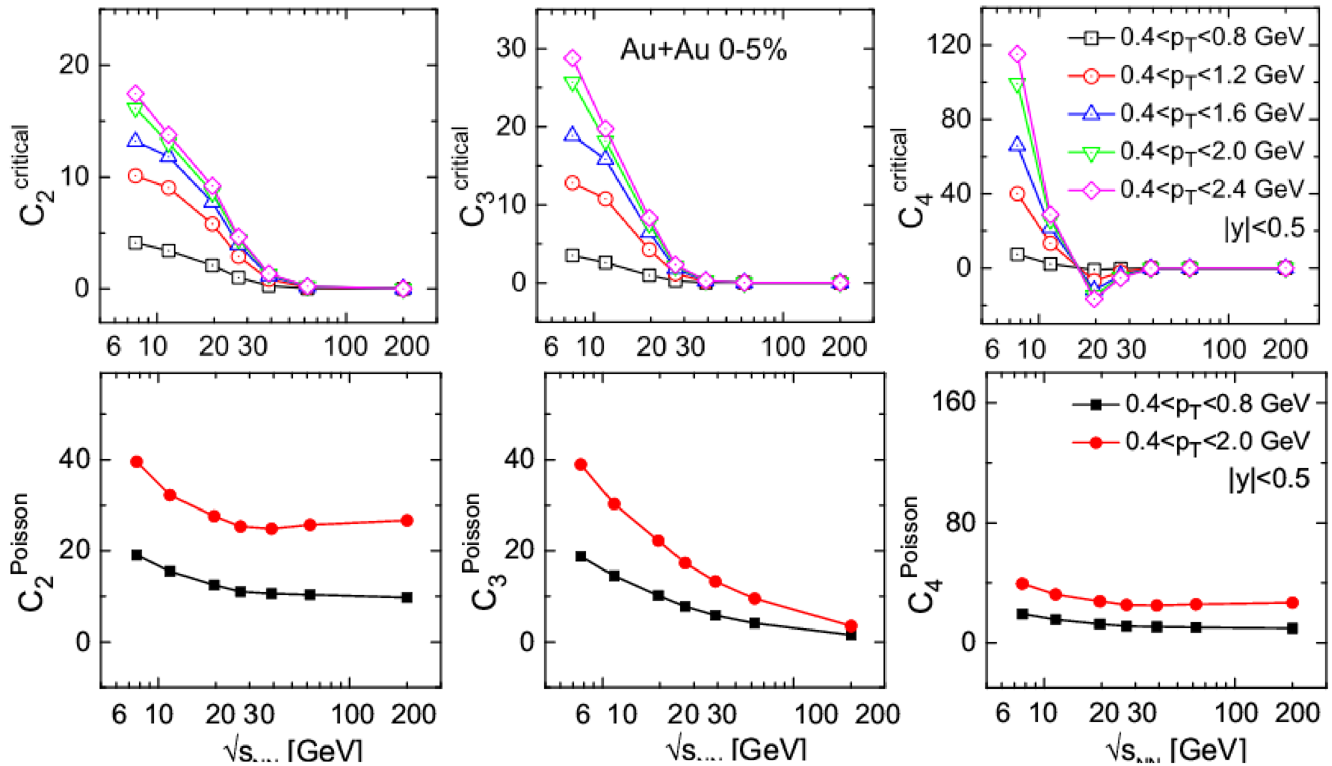
The signals are significantly enhanced when the pt and y acceptance are increased.

Acceptance dependence of critical fluctuations

p_T dependence:

Jiang, Li & Song , PRC, 94, 024918

Notes from Jiang & Song



y dependence

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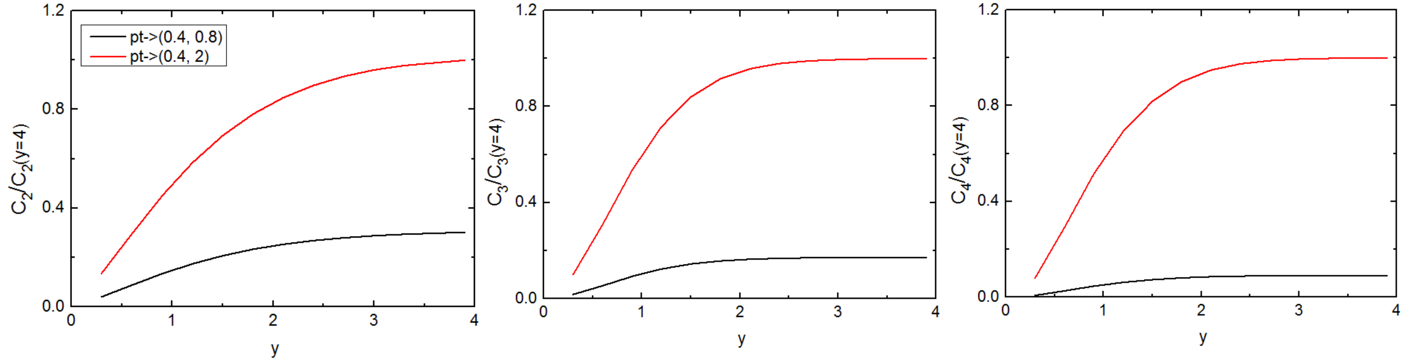
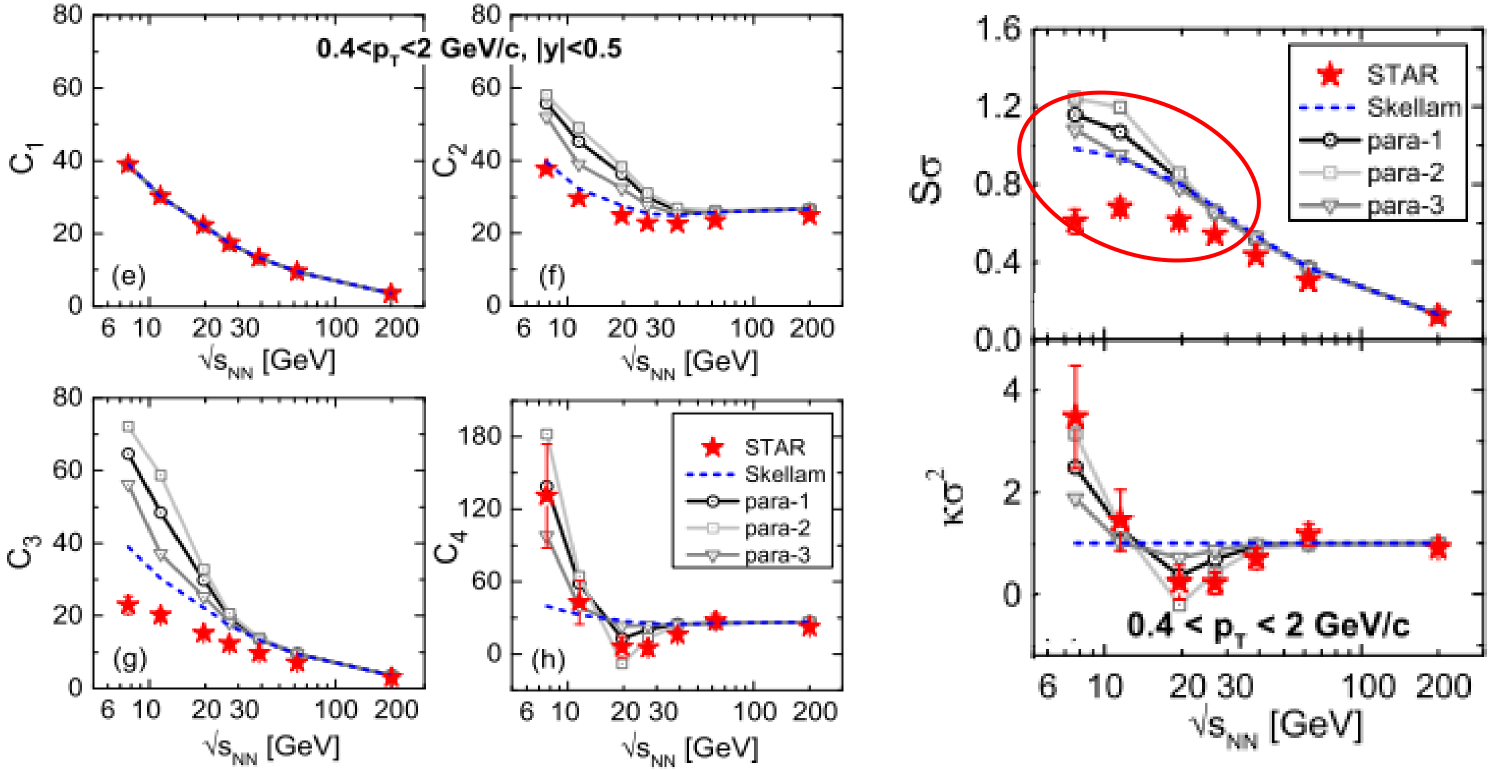


Similar behaviors for critical fluctuations with blast wave model. [Ling & Stephanov PRC 93, 034915]

Net Protons 0-5%

Jiang, Li & Song , PRC, 94, 024918



- Theoretical results = statistical baselines + critical fluctuations.
- The C4 and $\kappa\sigma^2$ are approximately described.
- over predict C2 and C3 due to positive contribution of critical fluctuations.

Short summary on equilibrium fluctuations

- **Equilibrium critical fluctuations on the freeze-out surface**

  - The acceptance dependence of experimental data can be qualitatively explained.

  - C_4 and $\kappa\sigma^2$ can be approximately described, over predict C_2 and C_3.

Short summary on equilibrium fluctuations

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dynamical critical fluctuations?

Real time evolution from Fokker-Plank equation

The Fokker-Plank equation

Mukherjee, Venugopalan & Yin PRC 2015

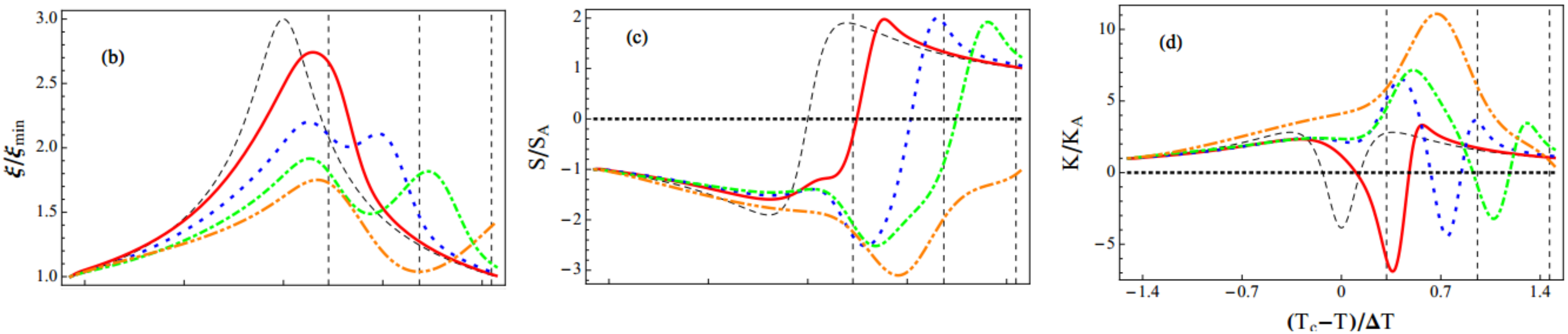
$$\partial_\tau P(\sigma;\tau) = \frac{1}{(m_\sigma^2 \tau_{\rm eff})}\left\{\partial_\sigma\left[\partial_\sigma \Omega_0(\sigma) + V_4^{-1}\partial_\sigma\right] P(\sigma;\tau)\right\},$$

The higher order cumulants

$$\partial_\tau \kappa_2(\tau) = -2\,\tau_{\rm eff}^{-1}\,(b^2)\left[\left(\frac{\kappa_2}{b^2}\right)F_2(M) - 1\right]\left[1+\mathcal{O}(\epsilon^2)\right],$$

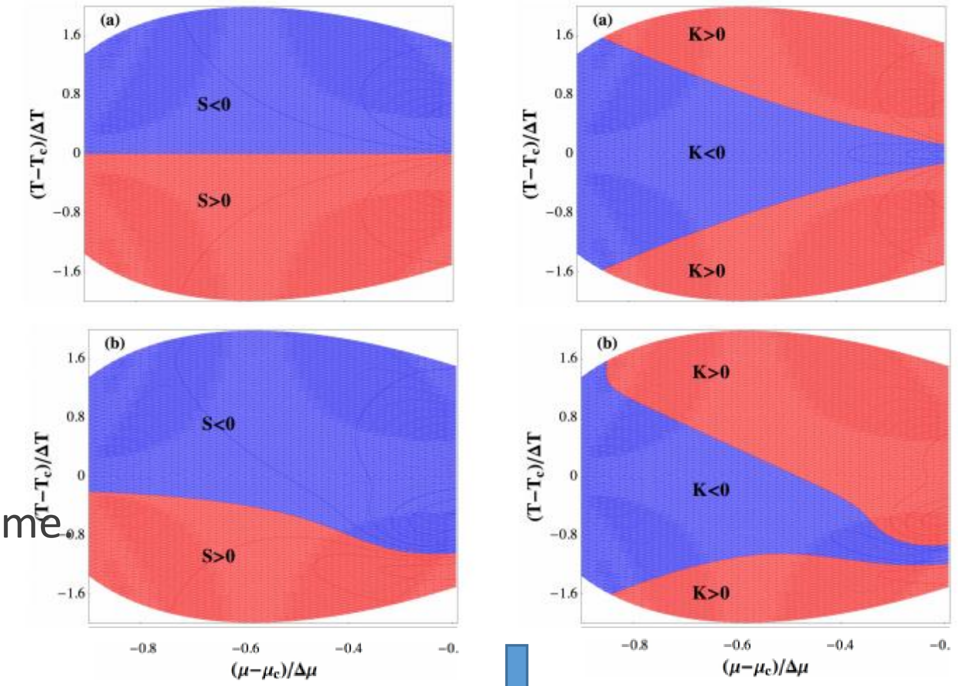
$$\partial_\tau \kappa_3(\tau) = -3\,\tau_{\rm eff}^{-1}\,(\epsilon\, b^3)\left[\left(\frac{\kappa_3}{\epsilon\, b^3}\right)F_2(M) + \left(\frac{\kappa_2}{b^2}\right)^2 F_3(M)\right] \times \left[1+\mathcal{O}(\epsilon^2)\right],$$

$$\partial_\tau \kappa_4(\tau) = -4\,\tau_{\rm eff}^{-1}\,(\epsilon^2\, b^4)\left\{\left(\frac{\kappa_4}{\epsilon^2\, b^4}\right)F_2(M) + 3\left(\frac{\kappa_2}{b^2}\right)\left(\frac{\kappa_3}{\epsilon\, b^3}\right)F_3(M) + \left(\frac{\kappa_2}{b^2}\right)^3 F_4\right\} \times \left[1+\mathcal{O}(\epsilon^2)\right].$$



- **Memory effects from dynamical evolution**
- **The sign and value of Skewness and Kurtosis can be different from equilibrated ones.**

Mukherjee, Venugopalan & Yin PRC 2015



- **Fokker-Plank equation:**

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$

However,

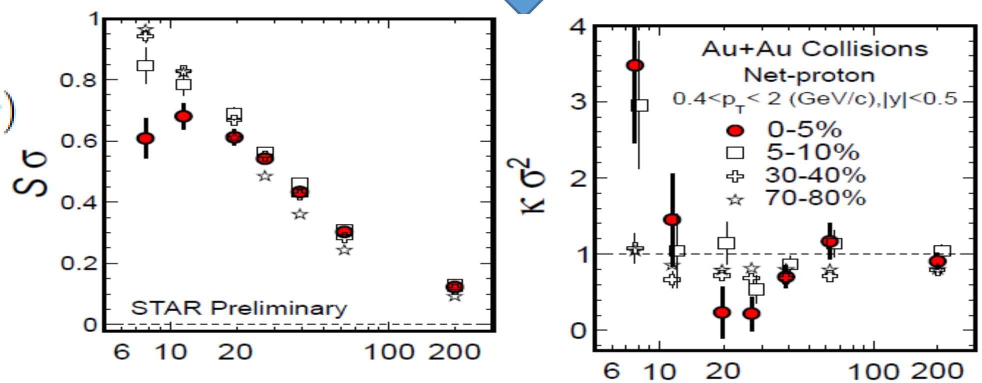
$$\sigma = \frac{1}{V}\int d^3x\sigma(x)$$

information for events integrated out,
Could not combine with a freeze-out scheme.

- **Langevin equation:**

$$\partial^\mu\partial_\mu\sigma\left(t,x\right) + \eta\partial_t\sigma\left(t,x\right) + V'_{eff}\left(\sigma\right) = \xi\left(t,x\right)$$

Spatial information reserved



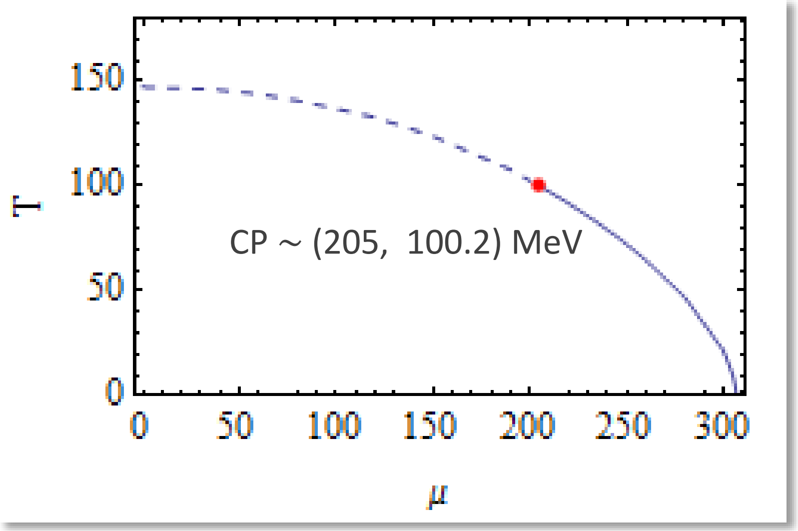
Dynamical critical fluctuations from Langevin dynamics

e-b-e Langevin dynamics

Jiang, Wu, Song, in preparation

- **The Lagrangian of linear sigma model**

$$\mathcal{L} = \bar{q}\left[i\gamma^{\mu}\partial_{\mu} - g\left(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}\right) + \gamma_0\mu\right]q + \frac{1}{2}\left(\partial_{\mu}\sigma\partial^{\mu}\sigma + \partial_{\mu}\vec{\pi}\partial^{\mu}\vec{\pi}\right) - U\left(\sigma,\vec{\pi}\right)$$



- **Effective potential:** ($<\vec{\pi}>=0$)

$$V_{eff}\left(\sigma\right) = U\left(\sigma\right) + \Omega_{q\bar{q}}\left(\sigma;T,\mu\right)$$

$$U\left(\sigma\right) = \frac{1}{4}\lambda^2\left(\sigma^2 - v^2\right)^2 - h\sigma - U_0$$

$$\Omega_{q\bar{q}}\left(\sigma;T,\mu\right) = -d_q\int\frac{d^3p}{\left(2\pi\right)^3}\left\{E + T\ln\left[1 + e^{-(E-\mu)/T}\right] + T\ln\left(1 + e^{-(E+\mu)/T}\right)\right\}$$

- **Langevin equation：** $\partial^{\mu}\partial_{\mu}\sigma\left(t,x\right) + \eta\partial_t\sigma\left(t,x\right) + V'_{eff}\left(\sigma\right) = \xi\left(t,x\right)$

- **Isothermal system, with the decreasing of temperature**

$$\frac{T\left(t\right)}{T_0} = \left(\frac{t}{t_0}\right)^{-0.45} \quad \text{(Hubble like)}$$

- **Chiral hydrodynamics, refer to**

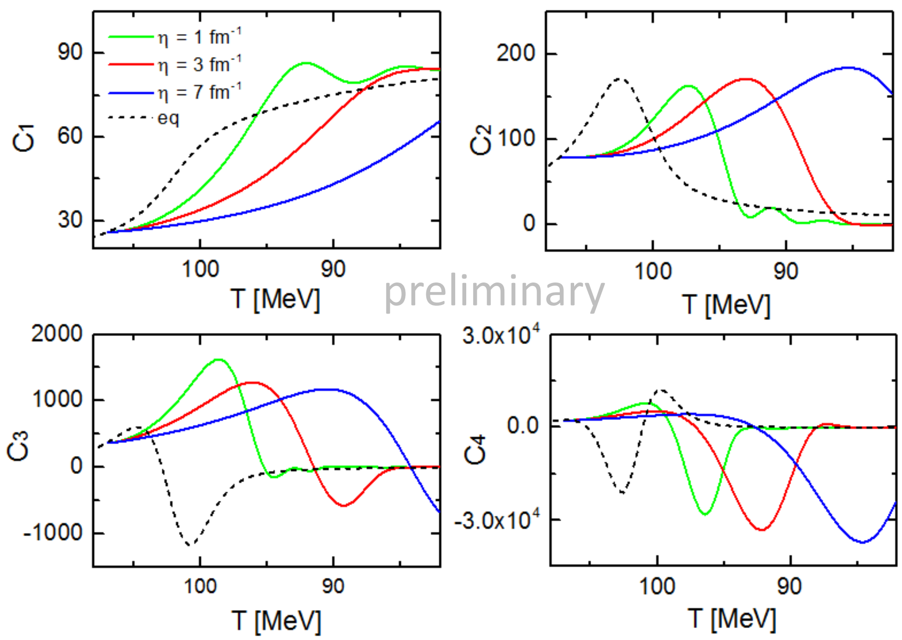
K. Peach et al, PRC68 (2003), M. Nahrgang et al, PRC84 (2011), C. Herold et al, PRC93 (2016),

Dynamical evolution of sigma's cumulants

Jiang, Wu, Song, in preparation

- **On the crossover side ($\mu = 200\ MeV$)**

1. $T_0 = 107\ MeV$, different damping



- The cumulants are from statistics on 100,000 events
- The memory effects ,the sign and value of C3, C4 different from equilibrium ones.
- The magnitude of critical fluctuations strongly depends on the initial conditions.

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Jiang, Wu, Song, in preparation

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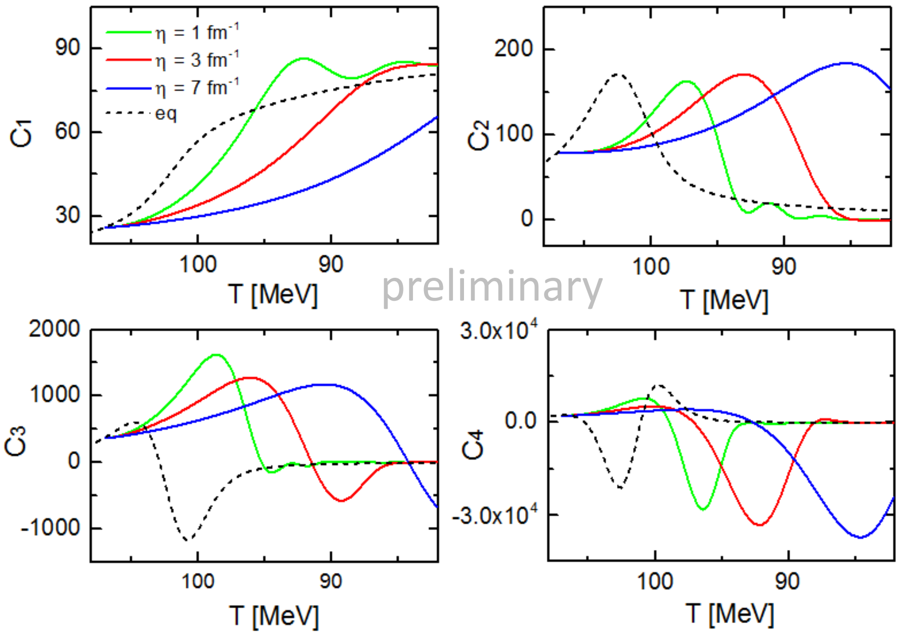
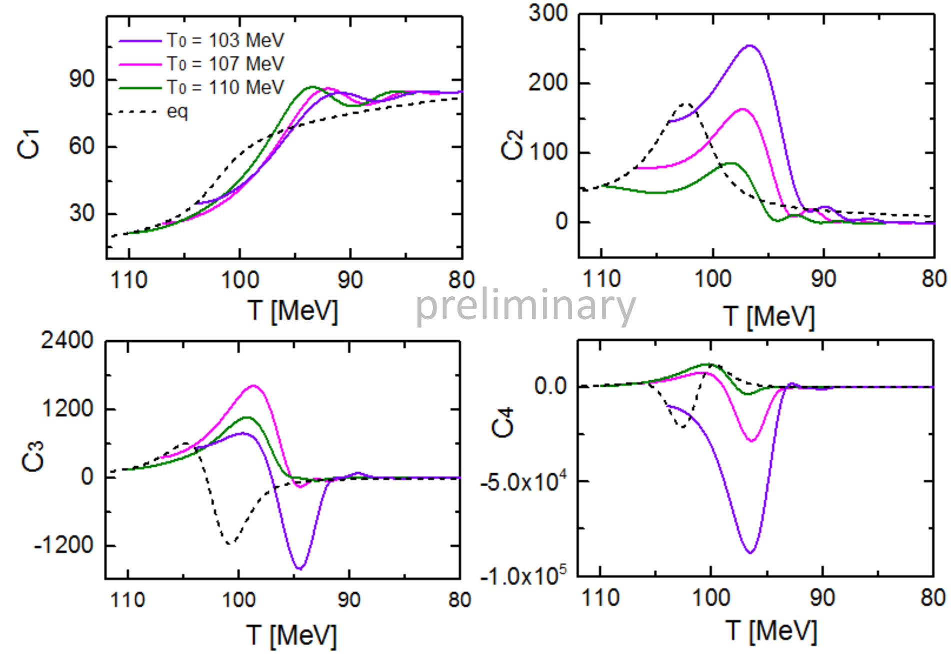


2. $\eta = 1\ fm^{-1}$, different T_0 (ξ_0)



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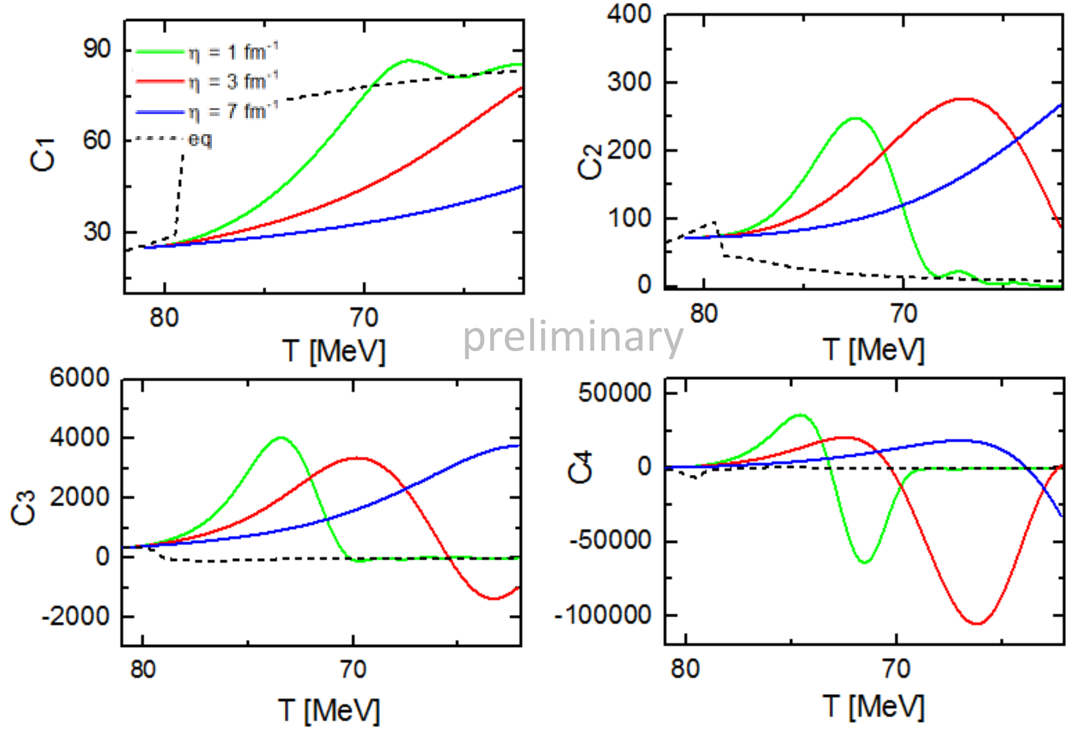
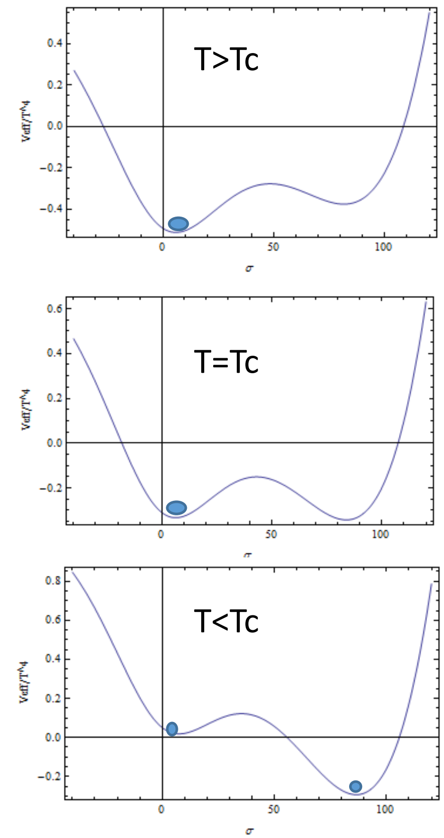
- The memory effects ,the sign and value of C3, C4 different from equilibrium ones.
- The magnitude of critical fluctuations strongly depends on the initial conditions.

Super cooling in 1st order phase transition

Jiang, Wu, Song, in preparation

- **On the 1st order side ($\mu = 240\ MeV$)**





Super cooling — Dynamical critical fluctuations much larger than equilibrium fluctuations

Sigma's cumulants at freeze-out

Jiang, Wu, Song, in preparation

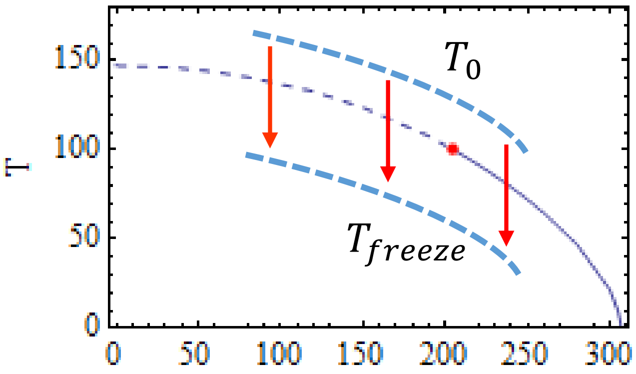
- **Freeze-out**

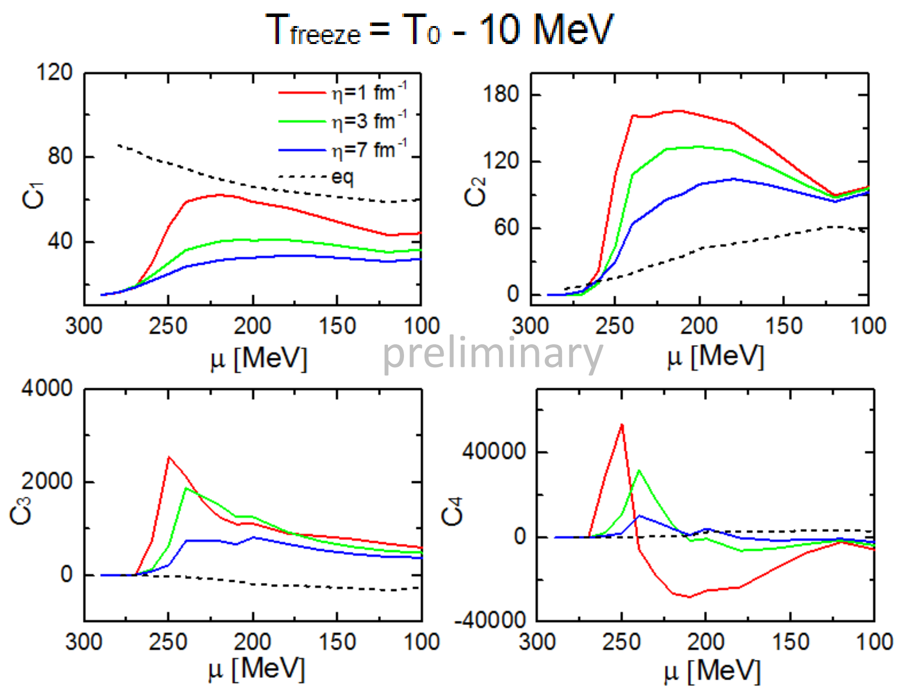
The starting points:

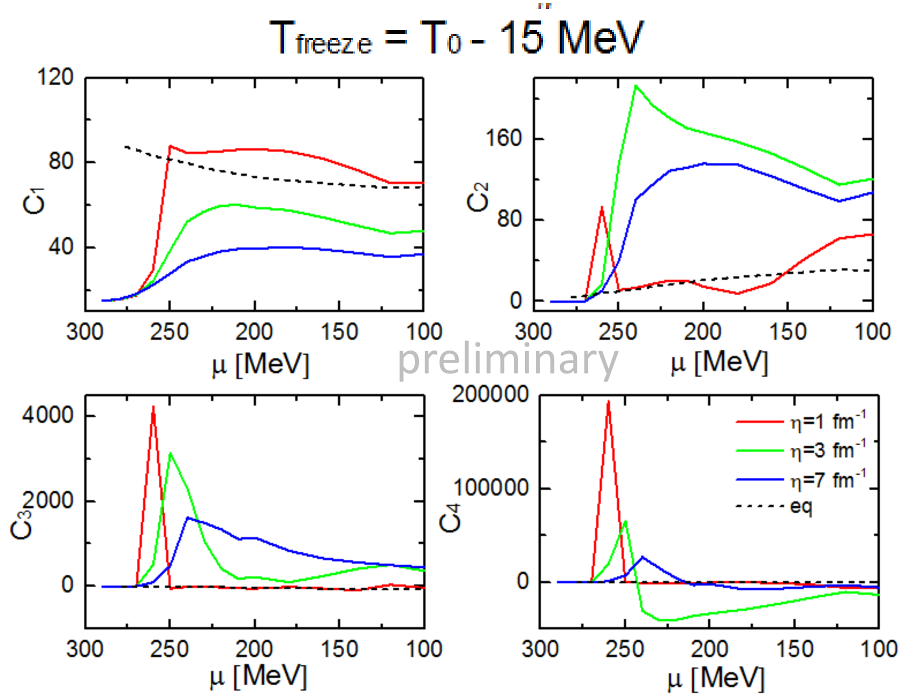
$$T_0 = T_c + 4\ MeV,\ \text{where}\ m_\sigma \sim 1\ fm.$$

The assumed freeze-out line:

$$T_{freeze} < T_c$$







Particle's critical fluctuations at freeze-out

Jiang, Wu, Song, in preparation

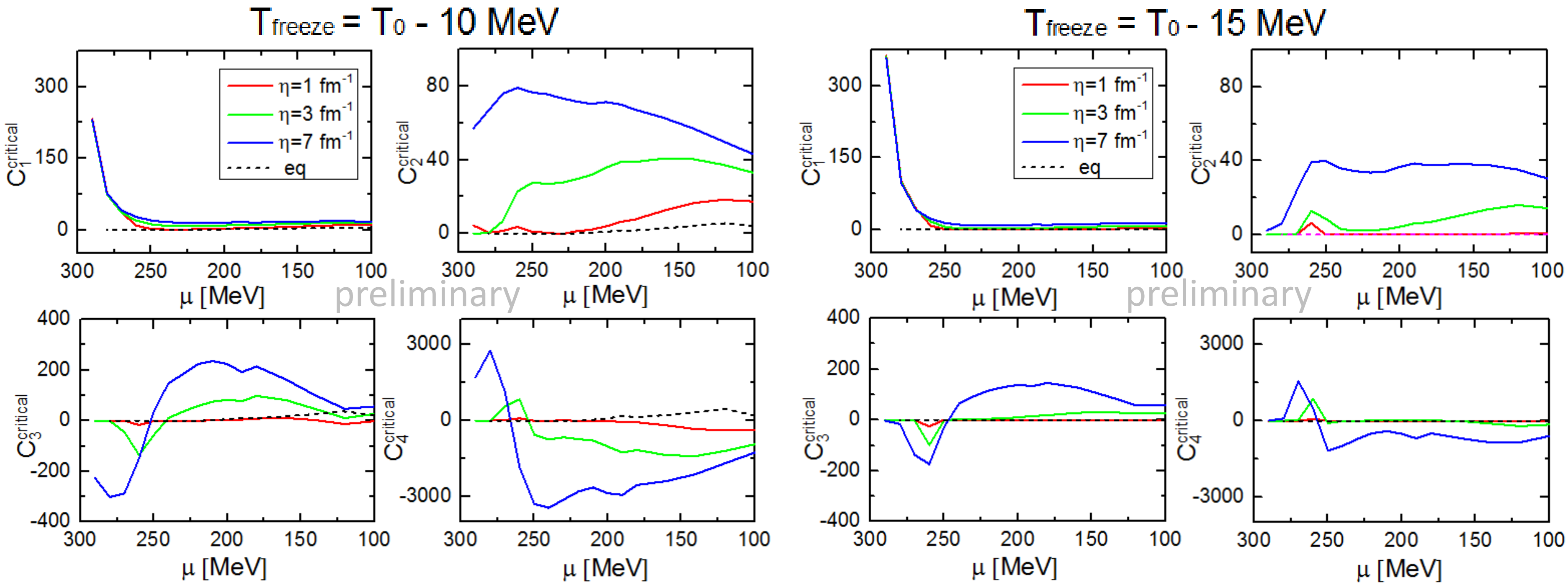
- **Particle number fluctuations:**

Assume the effective mass of particles is $M = m_0 + g\sigma_{i-th}$

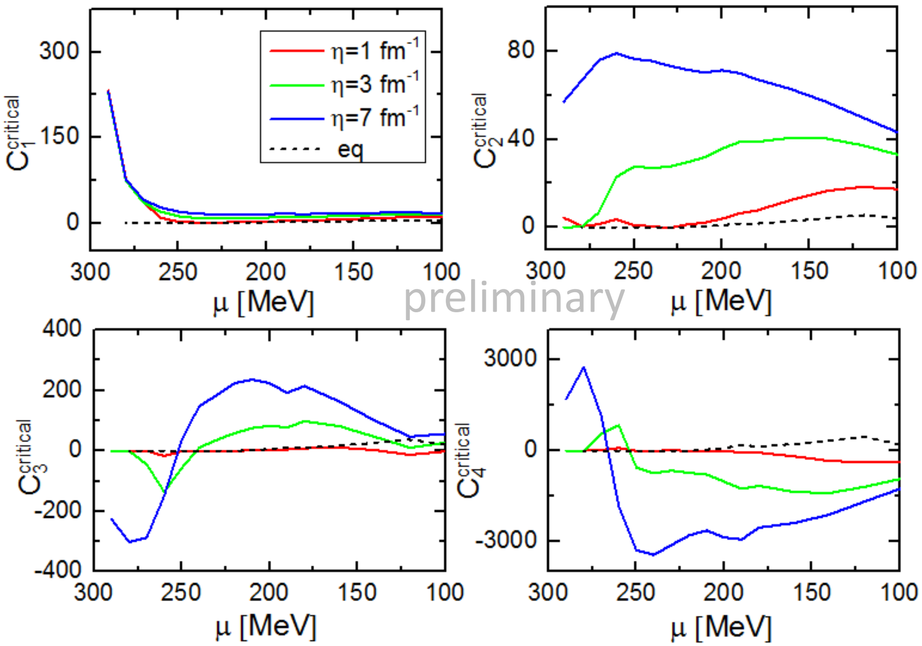
The particle number for i-th event is

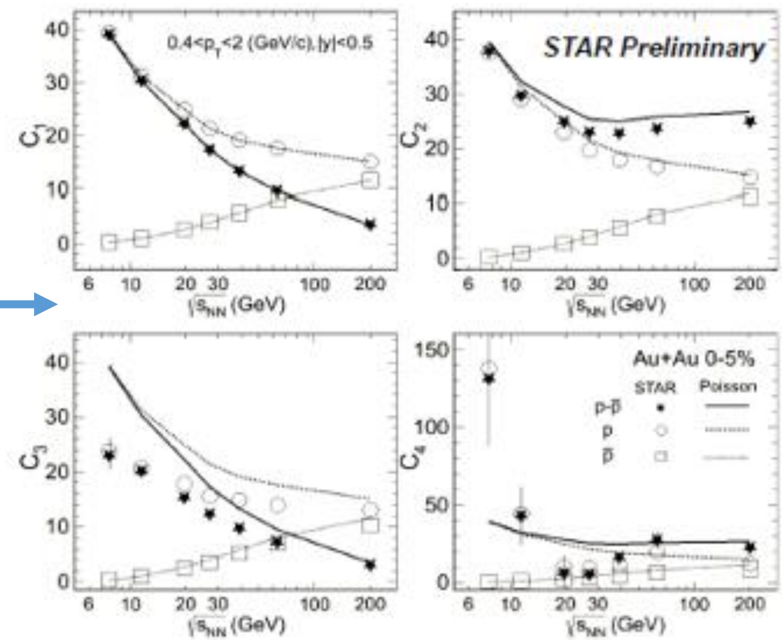
$$N_{i-th} = \frac{g}{(2\pi)^3}\int d^3p \sum dx^3 \exp\left[-\sqrt{p^2 + M\left(\sigma_{i-th}\right)^2}/T\right]$$

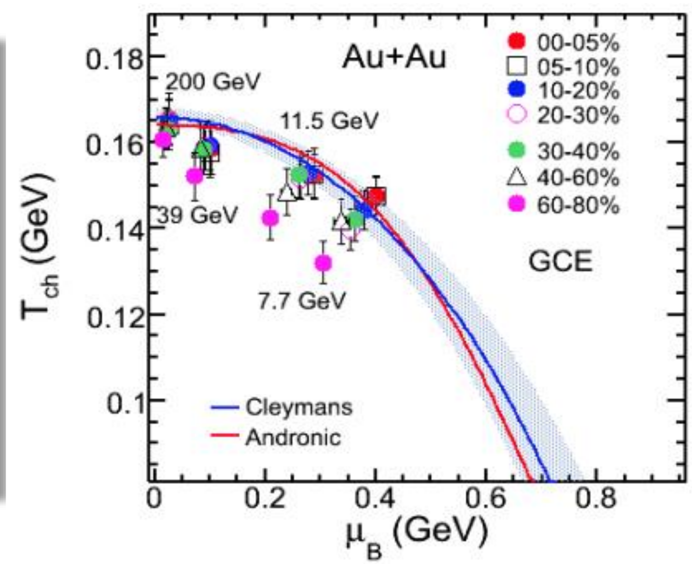
The cumulants of particle number are statistics on $\{N_1, N_2, ..., N_m\}$



For certain damping coefficient, the critical fluctuations of particles are negative for C_3, and positive for C_4 at large chemical potential. Besides, C_4 are nonmonotonic near the critical chemical potentials







Towards the comparison with ex-data:

- **Location of the critical point**
  the current model: $(\mu_c, T_c) \sim$ (205, 100.2) MeV
  3D Ising mapping
- **T_0 and T_{freeze}?**
- **Damping coefficient $\eta(T)$**
- **Realistic evolution of the system**

Summary and outlook

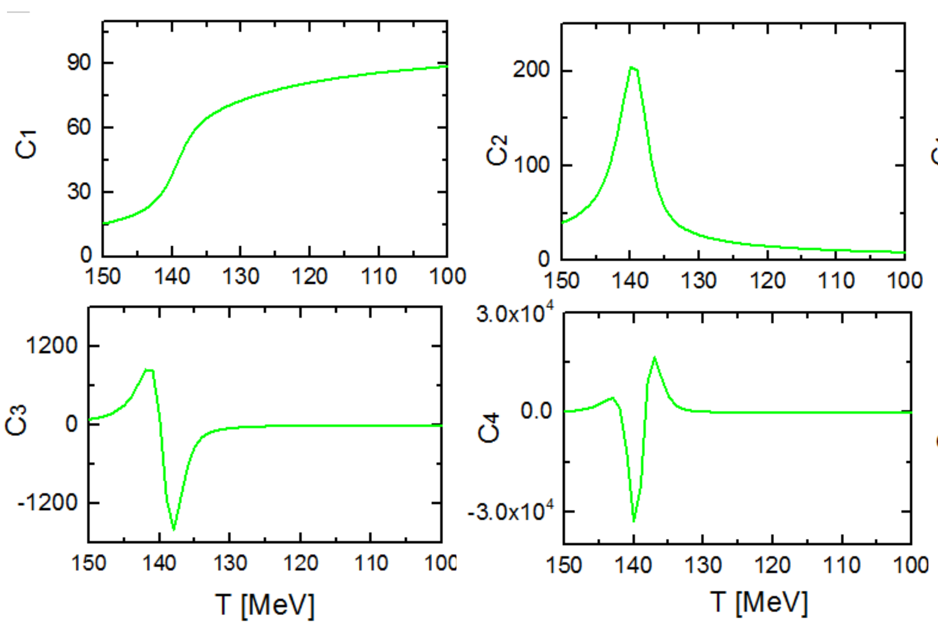
- **STAR BES provided exciting measurements on cumulants for net protons.**
- **Equilibrium critical fluctuations on the freeze-out surface,**
  - qualitatively describe the acceptance dependence.
  - C_4 and $\kappa\sigma^2$ can be approximately described, over predict C_2 and C_3.
- **Dynamical critical fluctuations from Langevin dynamics**
  - Memory effects on crossover side and 1st order phase transition side
  - Dynamical critical fluctuations on possible freeze out line present potentials to explain the experimental data
- **Future work**
  - dynamical Critical fluctuations in non-isothermal system (on going)
  - combine with realistic micro/macroscopic evolution (on going)
  - …

Thank you!

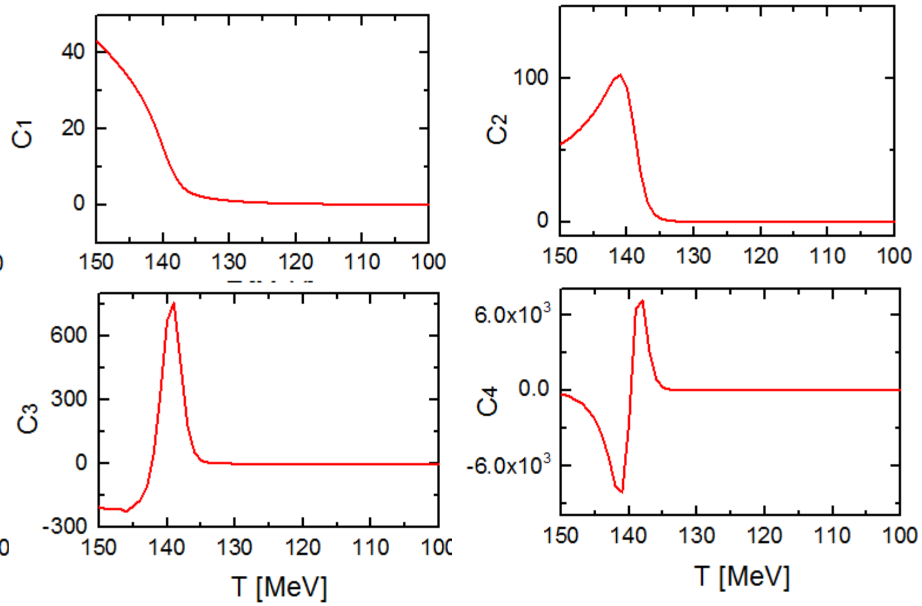
Back up

Equilibrium particle number fluctuations

The equilibrium cumulants from sigma to particles are

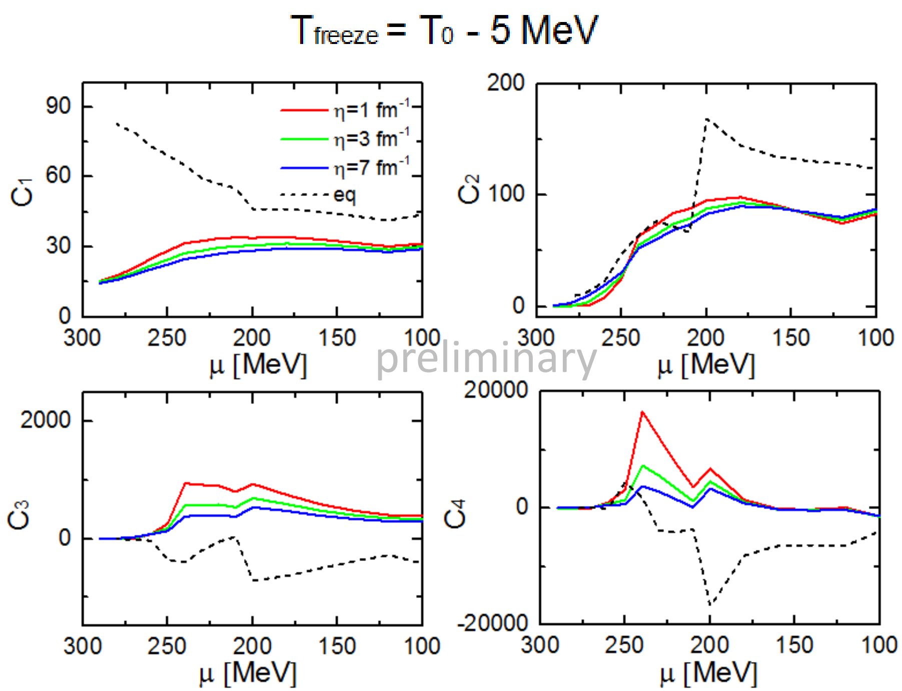


Sigma's fluctuations

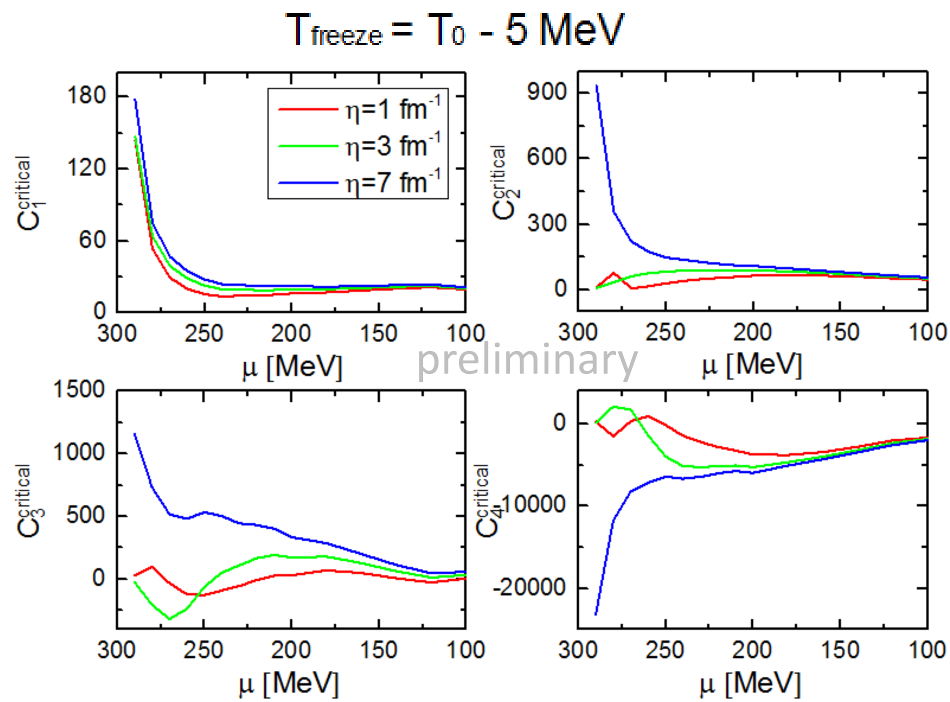


particles' fluctuations

T_{freeze}=T_0-5



Sigma's fluctuations



particles' fluctuations