Dynamical critical fluctuations near the QCD critical point

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I. Introduction

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Theoretical analysis, CP is predicted.

- Lattice simulation:
  - small $\mu$, finite $T$
  - crossover

- Effective theories:
  - (P)NJL, QM, FRG, DSE, etc
  - finite $T$ and large $\mu$
  - 1st order

Experimental facilities:
- RHIC (BES)
- FAIR, NICA

The location of CP? The signals?
Non-Gaussian fluctuations:

\[
\left\langle (\delta N)^2 \right\rangle \sim \xi^2 \\
\left\langle (\delta N)^3 \right\rangle \sim \xi^{4.5} \\
\left\langle (\delta N)^4 \right\rangle \sim \xi^7
\]

kurtosis:

\[\kappa_4 < 0\], from the crossover side

\[\kappa_4 > 0\], from the 1st order side

Nonmonotonic in the vicinity of CP

\[P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \quad \Omega = \int d^3x \left[ \frac{1}{2} (\nabla \sigma)^2 + \frac{m_0^2}{2} \sigma^2 + \frac{\lambda_3}{3} \sigma^3 + \frac{\lambda_4}{4} \sigma^4 + \cdots \right].\]

\[
\left\langle \sigma_0^2 \right\rangle = \frac{T}{V} \xi^2 \\
\left\langle \sigma_0^3 \right\rangle = \frac{2\lambda_3 T}{V} \xi^6; \\
\left\langle \sigma_0^4 \right\rangle_c = \frac{6T}{V} [2(\lambda_3 \xi)^2 - \lambda_4] \xi^8.
\]
STAR BES: cumulants ratios


\[ S \sigma = \frac{c_3}{c_2} \sim \frac{\chi_B^{(3)}}{\chi_B^{(2)}} \]

\[ \kappa \sigma^2 = \frac{c_4}{c_2} \sim \frac{\chi_B^{(4)}}{\chi_B^{(2)}} \]

Xiaofeng Luo (for the STAR Collaboration), PoS(CPOD2014)019

Indications from experimental data:

- Deviations from statistical baselines.
- Nonmonotonic at \( \sqrt{s_{NN}} \sim 20 \text{ GeV} \).

Equilibrium critical fluctuations?

Dynamical critical fluctuations?
Equilibrium critical fluctuations along the freeze out surface
Particle emissions in HIC near Critical Point, Cooper-Frye formula: Jiang, Li & Song, PRC, 94, 024918

\[
E \frac{dN}{d^3p} = \int_{\Sigma} \frac{p_{\mu} d\sigma^\mu}{2\pi^3} f(x, p) \quad M \rightarrow g(\bar{\sigma} + \sigma(x)) \quad \Rightarrow \quad f(x, p) = f_0(x, p) \left[ 1 - g\sigma(x) / (\gamma T) \right] = f_0 + \delta f
\]

Critical fluctuations for particles on the freeze-out surface

\[
\langle (\delta N)^n \rangle_c = \left( \frac{g_i}{(2\pi)^3} \right)^n \left( \prod_{i=1, \ldots, n} \int \frac{1}{E_i} d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu \right) \left( -\frac{g}{T} \right)^n \frac{f_{01} \ldots f_{0n}}{\gamma_1 \ldots \gamma_n} \langle \sigma_1 \ldots \sigma_n \rangle_c
\]

with \( n=2,3,4 \), the spatial correlators of sigma are written as

\[
\langle \sigma_1 \sigma_2 \rangle_c = TD(x_1 - x_2), \\
\langle \sigma_1 \sigma_2 \sigma_3 \rangle_c = -2T^2\lambda_3 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z), \\
\langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c = -6T^3\lambda_4 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z) D(x_4 - z) + 12T^3\lambda_3^2 \int d^3u \int d^3v D(x_1 - u) D(x_2 - u) D(x_3 - v) D(x_4 - v) D(u - v).
\]

In an isothermal and equilibrated system, integrate over coordinate space, the results in Stephanov’s PRL09 are reproduced.
Acceptance dependence of BES data

$p_T$ dependence:

The signals are significantly enhanced when the $p_T$ and $y$ acceptance are increased.
Acceptance dependence of critical fluctuations

$p_T$ dependence:

Jiang, Li & Song, PRC, 94, 024918

Notes from Jiang & Song

$y$ dependence

Notes from Jiang & Song

Similar behaviors for critical fluctuations with blast wave model. [Ling & Stephanov PRC 93, 034915]
Theoretical results = statistical baselines + critical fluctuations.

The $C_4$ and $\kappa \sigma^2$ are approximately described.

over predict $C_2$ and $C_3$ due to positive contribution of critical fluctuations.
Equilibrium critical fluctuations on the freeze-out surface

- The acceptance dependence of experimental data can be qualitatively explained.
- $C_4$ and $\kappa \sigma^2$ can be approximately described, over predict $C_2$ and $C_3$. 
Short summary on equilibrium fluctuations

- Equilibrium critical fluctuations on the freeze-out surface
  - The acceptance dependence of experimental data can be qualitatively explained.
  - $C_4$ and $\kappa \sigma^2$ can be approximately described, over predict $C_2$ and $C_3$.

**dynamical** critical fluctuations?
The Fokker-Plank equation

\[
\partial_\tau P(\sigma;\tau) = \frac{1}{m^2_{\sigma\tau_{\text{eff}}}} \left\{ \partial_\sigma \left[ \partial_\sigma \Omega_0(\sigma) + V_4^{-1} \partial_\sigma \right] P(\sigma;\tau) \right\},
\]

The higher order cumulants

\[
\partial_\tau \kappa_2(\tau) = -2 \tau_{\text{eff}}^{-1} (b^2) \left[ \left( \frac{\kappa_2}{b^2} \right) F_2(M) - 1 \right] \left[ 1 + \mathcal{O}(\epsilon^2) \right],
\]

\[
\partial_\tau \kappa_3(\tau) = -3 \tau_{\text{eff}}^{-1} (\epsilon b^3) \left[ \left( \frac{\kappa_3}{\epsilon b^3} \right) F_2(M) + \left( \frac{\kappa_2}{b^2} \right)^2 F_3(M) \right] \times \left[ 1 + \mathcal{O}(\epsilon^2) \right],
\]

\[
\partial_\tau \kappa_4(\tau) = -4 \tau_{\text{eff}}^{-1} (\epsilon^2 b^4) \left\{ \left( \frac{\kappa_4}{\epsilon^2 b^4} \right) F_2(M) + 3 \left( \frac{\kappa_2}{b^2} \right) \left( \frac{\kappa_3}{\epsilon b^3} \right) F_3(M) + \left( \frac{\kappa_2}{b^2} \right)^3 F_4 \right\} \times \left[ 1 + \mathcal{O}(\epsilon^2) \right].
\]

- Memory effects from dynamical evolution
- The sign and value of Skewness and Kurtosis can be different from equilibrated ones.
Different dynamical equations

- **Fokker-Plank equation:**
  \[
  \frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]
  \]
  However,
  \[
  \sigma = \frac{1}{V} \int d^3x \sigma(x)
  \]
  information for events integrated out, Could not combine with a freeze-out scheme.

- **Langevin equation:**
  \[
  \partial^\mu \partial_\mu \sigma(t, x) + \eta \partial_t \sigma(t, x) + V'_{\text{eff}}(\sigma) = \xi(t, x)
  \]
  Spatial information reserved

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**Dynamical critical fluctuations from Langevin dynamics**
e-b-e Langevin dynamics

Jiang, Wu, Song, in preparation

• The Lagrangian of linear sigma model

\[ \mathcal{L} = \bar{q} \left[ i\gamma^\mu \partial_\mu - g (\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\pi}) + \gamma_0 \mu \right] q + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi} \right) - U (\sigma, \vec{\pi}) \]

• Effective potential: \( \langle \vec{\pi} \rangle = 0 \)

\[ V_{eff} (\sigma) = U (\sigma) + \Omega_{q\bar{q}} (\sigma; T, \mu) \]

\[ U (\sigma) = \frac{1}{4} \lambda^2 (\sigma^2 - \nu^2)^2 - \hbar \sigma - U_0 \]

\[ \Omega_{q\bar{q}} (\sigma; T, \mu) = -d_q \int \frac{d^3p}{(2\pi)^3} \left\{ E + T \ln \left[ 1 + e^{-(E-\mu)/T} \right] + T \ln \left( 1 + e^{-(E+\mu)/T} \right) \right\} \]

• Langevin equation:

\[ \partial^\mu \partial_\mu \sigma (t, x) + \eta \partial_t \sigma (t, x) + V'_{eff} (\sigma) = \xi (t, x) \]

• Isothermal system, with the decreasing of temperature

\[ \frac{T(t)}{T_0} = \left( \frac{t}{t_0} \right)^{-0.45} \] (Hubble like)

• Chiral hydrodynamics, refer to

On the crossover side ($\mu = 200\ MeV$)

1. $T_0 = 107\ MeV$, different damping

- The cumulants are from statistics on 100,000 events
- The memory effects, the sign and value of $C_3, C_4$ different from equilibrium ones.
- The magnitude of critical fluctuations strongly depends on the initial conditions.
• **On the crossover side** \((\mu = 200 \text{ MeV})\)

1. \(T_0 = 107 \text{ MeV}\), different damping

2. \(\eta = 1 \text{ fm}^{-1}\), different \(T_0 (\xi_0)\)

- The cumulants are from statistics on 100,000 events
- The memory effects, the sign and value of \(C_3, C_4\) different from equilibrium ones.
- The magnitude of critical fluctuations strongly depends on the initial conditions.
Super cooling in 1\textsuperscript{st} order phase transition

- On the 1\textsuperscript{st} order side ($\mu = 240$ MeV)

Super cooling — Dynamical critical fluctuations much larger than equilibrium fluctuations
**Freeze-out**

The starting points:

\[ T_0 = T_c + 4 \text{ MeV}, \text{ where } m_\sigma \sim 1 \text{ fm}. \]

The assumed freeze-out line:

\[ T_{\text{freeze}} < T_c \]

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**Sigma’s cumulants at freeze-out**

\[ T_{\text{freeze}} = T_0 - 10 \text{ MeV} \]

\[ T_{\text{freeze}} = T_0 - 15 \text{ MeV} \]
Particle’s critical fluctuations at freeze-out

**Particle number fluctuations:**

Assume the effective mass of particles is $M = m_0 + g\sigma_{i-th}$

The particle number for $i$-th event is

$$N_{i-th} = \frac{g}{(2\pi)^3} \int d^3p \sum dx^3 \exp \left[-\sqrt{p^2 + M (\sigma_{i-th})^2}/T\right]$$

The cumulants of particle number are statistics on $\{N_1, N_2, ..., N_m\}$

For certain damping coefficient, the critical fluctuations of particles are negative for $C_3$, and positive for $C_4$ at large chemical potential. Besides, $C_4$ are nonmonotonic near the critical chemical potentials.
Towards the comparison with ex-data:

- **Location of the critical point**
  - the current model: $(\mu_c, T_c) \sim (205, 100.2)$ MeV
  - 3D Ising mapping

- **$T_0$ and $T_{\text{freeze}}$?**

- **Damping coefficient $\eta(T)$**

- **Realistic evolution of the system**
Summary and outlook

- **STAR BES provided exciting measurements on cumulants for net protons.**

- **Equilibrium critical fluctuations on the freeze-out surface,**
  - qualitatively describe the acceptance dependence.
  - $C_4$ and $\kappa \sigma^2$ can be approximately described, over predict $C_2$ and $C_3$.

- **Dynamical critical fluctuations from Langevin dynamics**
  - Memory effects on crossover side and 1st order phase transition side
  - Dynamical critical fluctuations on possible freeze out line present potentials to explain the experimental data

- **Future work**
  - dynamical Critical fluctuations in non-isothermal system (on going)
  - combine with realistic micro/macroscopic evolution (on going)
  - ...

**Thank you!**
Back up
The equilibrium cumulants from sigma to particles are:

Sigma's fluctuations

Particles' fluctuations
$T_{\text{freeze}} = T_0 - 5$

Sigma’s fluctuations

particles’ fluctuations