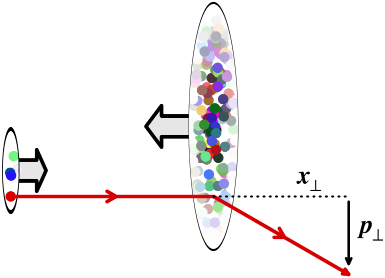
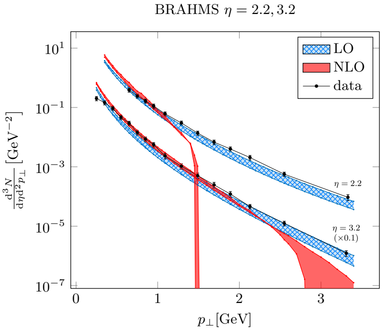
# *Particle production in $pA$ collisions beyond leading order*

**Edmond Iancu**
IPhT Saclay & CNRS

w/ A.H. Mueller and D.N. Triantafyllopoulos, arXiv:1608.05293



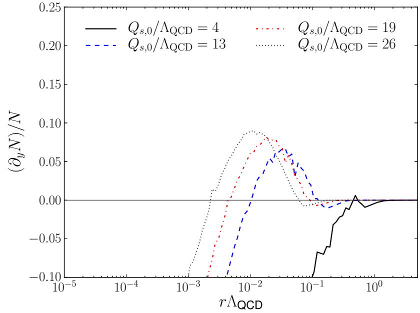



# Introduction

- Particle production in $pp$ and $pA$ collisions at forward rapidities explores the physics of high gluon densities at small-$x$
  - non-linear phenomena: gluon saturation, multiple scattering
  - resummations based on the eikonal approximation (Wilson lines)
  - non-linear evolution equations: BK, B-JIMWLK
- Effective theory derived in pQCD: Color Glass Condensate
- The CGC formalism is now being promoted to NLO
  - NLO versions for the BK and B-JIMWLK equations *(Balitsky and Chirilli, 2008, 2013; Kovner, Lublinsky, and Mulian, 2013)*
  - NLO impact factor for particle production in $pA$ collisions *(Chirilli, Xiao, and Yuan, 2012; Mueller and Munier, 2012)*
- But the strict NLO approximations turned out to be problematic

# NLO BK evolution

- "Negative growth" of the dipole scattering amplitude




Lappi, Mäntysaari, arXiv:1502.02400

- Not really a surprise
  - similar problems for NLO BFKL
  - large transverse logarithms
  - collinear resummations
  - Mellin representation

*(Salam, Ciafaloni, Colferai, Stasto, 98-03; Altarelli, Ball, Forte, 00-03)*

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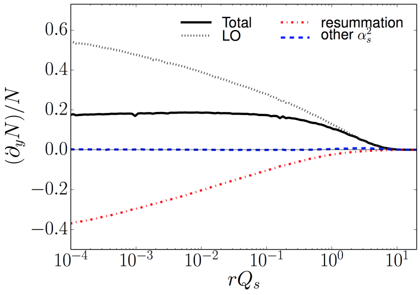


Lappi, Mäntysaari, arXiv:1601.06598

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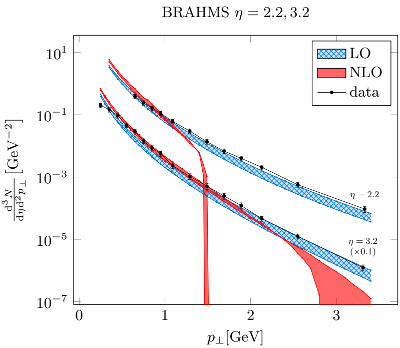
- Collinear improvement for NLO BK (transverse coordinates)

  *(E.I., J. Madrigal, A. Mueller, G. Soyez, and D. Triantafyllopoulos, 2015)*

- Evolution becomes stable with promising phenomenology
  - excellents fits to DIS *(Iancu et al, 2015; Albacete, 2015)*

# Particle production in d+Au collisions (RHIC)

- Very good agreement at low $p_\perp$ ☺ ... but negative at larger $p_\perp$ ☹




Stasto, Xiao, and Zaslavsky, arXiv:1307.4057

- Is this a real problem ?
  - "small-$x$ resummations do not apply at large $p_\perp$"
  - but $p_\perp \sim Q_s$ is not that large !
- Likely related to the rapidity subtraction in NLO impact factor

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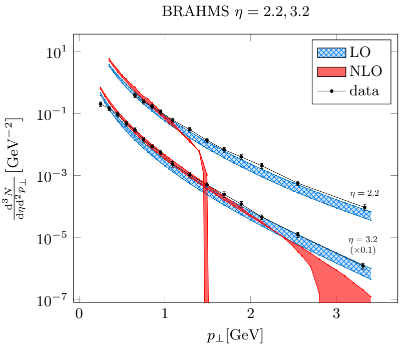


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- Various proposals which alleviate the problem (pushed to higher $p_\perp$)
  - *Kang, Vitev, and Xing, arXiv:1403.5221*
  - *Altinoluk, Armesto, Beuf, Kovner, and Lublinsky, arXiv:1411.2869*
  - *Ducloué, Lappi, and Zhu, arXiv:1604.00225*

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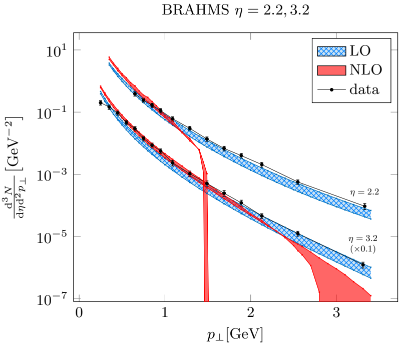


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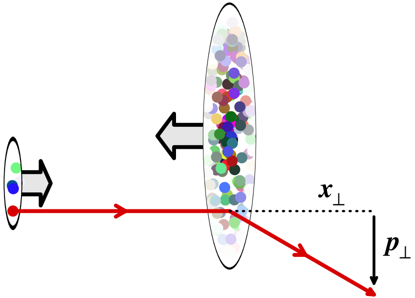
- A reorganization of the perturbative expansion which avoids the rapidity subtraction *(E.I., A. Mueller and D. Triantafyllopoulos, 2016)*
- Ongoing numerical implementation which looks promising *(Ducloué, Lappi, and Zhu, w.i.p.)*

# Forward quark production in $pA$ collisions

- A quark initially collinear with the proton acquires a transverse momentum $p_\perp$ via multiple scattering off the saturated gluons




$$x_p \equiv \frac{p^+}{q^+} = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{\eta}$$

$$X_g \equiv \frac{p^-}{P^-} = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{-\eta}$$

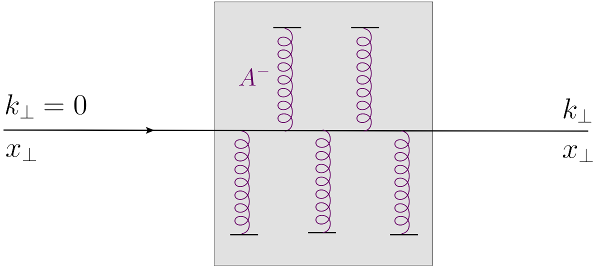
$X_g \ll x_p$ when $\eta > 0$

  - $\eta$ : quark rapidity in the COM frame
  - $x_p$ : longitudinal fraction of the quark in the proton
  - $X_g$ : longitudinal fraction of the gluon in the target

- Gluons in the nucleus have a typical transverse momentum $k_\perp \sim Q_s(X_g)$

# Multiple scattering

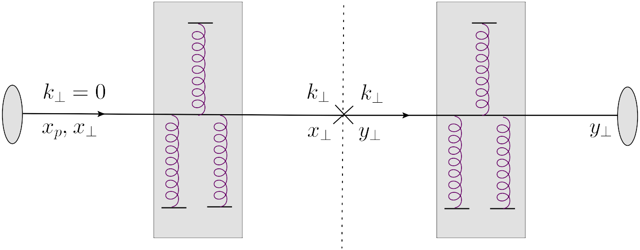
- Eikonal approximation $\Longrightarrow$ the transverse coordinate representation




Amplitude: $$\mathcal{M}_{ij}(\boldsymbol{k}_\perp) \equiv \int \mathrm{d}^2\boldsymbol{x}_\perp \, \mathrm{e}^{-i\boldsymbol{x}_\perp\cdot\boldsymbol{k}_\perp} \, V_{ij}(\boldsymbol{x}_\perp)$$

Wilson line: $$V(\boldsymbol{x}_\perp) = \mathrm{P}\exp\left\{ ig \int \mathrm{d}x^+ A_a^-(x^+, \boldsymbol{x}_\perp) t^a \right\}$$

- $A_a^-$ : color field representing small-$x$ gluons in the nucleus

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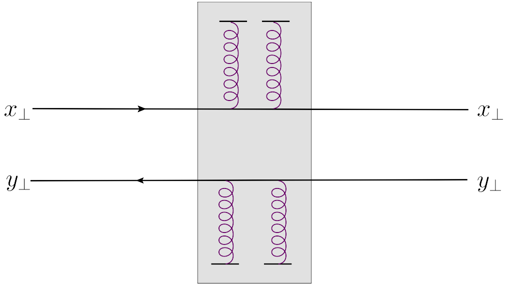
Cross-section: $$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{k}_\perp} \simeq x_p q(x_p, Q^2)\,\frac{1}{N_c}\left\langle \sum_{ij} |\mathcal{M}_{ij}(\boldsymbol{k}_\perp)|^2 \right\rangle_{X_g}$$

- Average over the color fields $A^-$ in the target (CGC)
- Two Wilson lines at different transverse coordinates, traced over color

# Dipole picture

- Equivalently: the elastic $S$-matrix for a $q\bar{q}$ color dipole




$$S(\boldsymbol{x},\boldsymbol{y};X_g)\equiv\frac{1}{N_c}\left\langle \mathrm{tr}\left[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\right]\right\rangle_{X_g}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{k}}\simeq x_p q(x_p)\int_{\boldsymbol{x},\boldsymbol{y}}\mathrm{e}^{-\mathrm{i}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{k}}\,S(\boldsymbol{x},\boldsymbol{y};X_g)$$

- The Fourier transform $\mathcal{S}(\boldsymbol{k},X_g)$ : "unintegrated gluon distribution"

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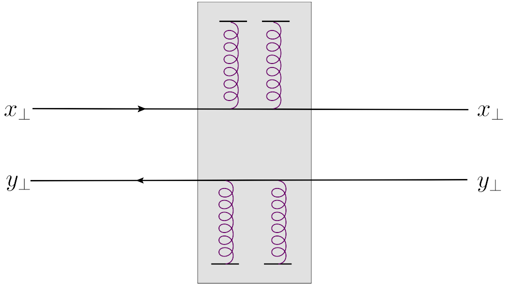


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- 'Hybrid factorization': collinear fact. for $p$ & CGC fact. for $A$ *(Dumitru, Hayashigaki, and Jalilian-Marian, arXiv:hep-ph/0506308)*.

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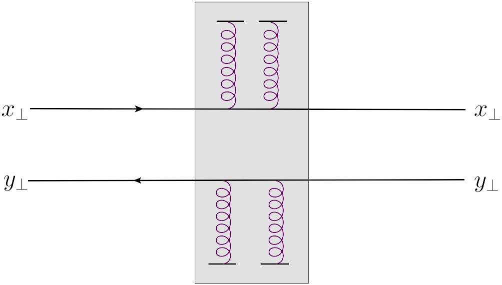


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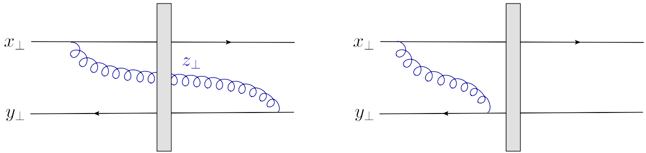
- The dipole picture is preserved by the high-energy evolution up to NLO *(Kovchegov and Tuchin, 2002; Mueller and Munier, 2012)*

# BK equation *(leading order)*

- Probability $\sim \alpha_s \ln \frac{1}{x}$ to radiate a soft gluon with $x \equiv \frac{p^+}{k^+} \ll 1$

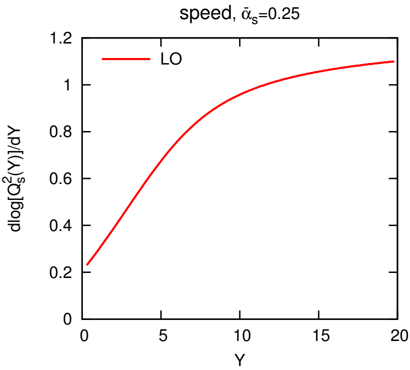


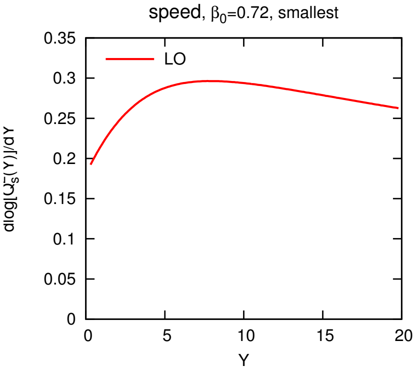

- When $\alpha_s \ln \frac{1}{x} \sim 1$ : resummation to all orders (part of LO)
- Evolution equation for the dipole $S$–matrix $S_{\boldsymbol{xy}}(Y)$ with $Y \equiv \ln(1/x)$

$$\frac{\partial S_{\boldsymbol{xy}}}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2\boldsymbol{z} \, \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \left[ S_{\boldsymbol{xz}} S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}} \right]$$

  - dipole kernel: probability for the dipole to emit a soft gluon at $\boldsymbol{z}$
  - large-$N_c$ approximation to the Balitsky-JIMWLK hierarchy
  - saturation momentum $Q_s(Y)$: $S(r, Y) = 0.5$ when $r = 1/Q_s(Y)$

- The evolution speed: saturation exponent $\lambda_s \equiv \mathrm{d}\ln Q_s^2/\mathrm{d}Y$
- At LO, $\lambda_s \sim 1$ is way too large: $\lambda_{\text{HERA}} = 0.2 \div 0.3$

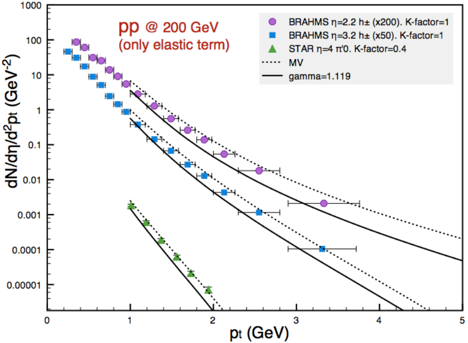


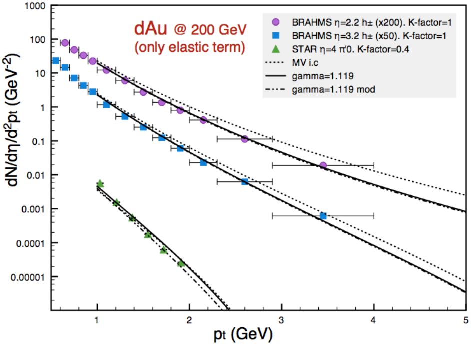




- Including running coupling dramatically slows down the evolution
- ... but there are other, equally important, NLO corrections !

*(Albacete, Dumitru, Fujii, Nara, arXiv:1209:2001)*

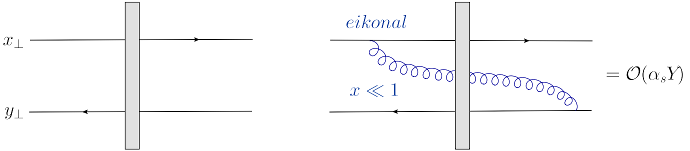
- Fit parameters: initial condition for the rcBK equation + $K$-factors



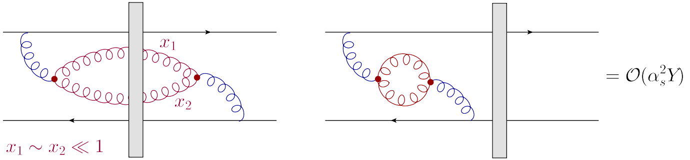




$$\frac{\mathrm{d}N}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{k}}\bigg|_{\mathrm{LO}} = K^h \int_{x_p}^{1} \frac{\mathrm{d}z}{z^2}\,\frac{x_p}{z} q\left(\frac{x_p}{z}\right) \mathcal{S}\left(\frac{\boldsymbol{k}}{z}, X_g\right) D_{h/q}(z)$$

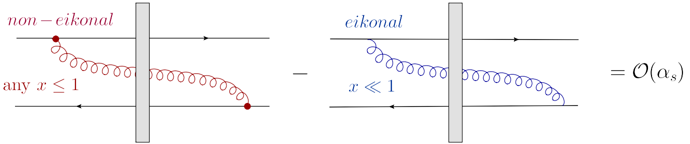
- LO approximation: any number $n \geq 0$ of soft emissions $\Longrightarrow (\alpha_s Y)^n$




- NLO corrections to the evolution: 2 soft gluons, with similar values of $x$

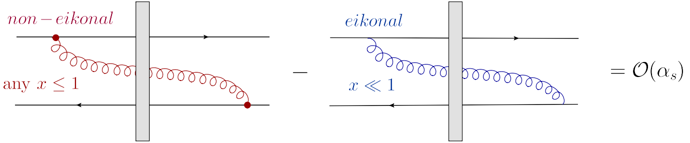



- NLO correction to impact factor: the first gluon can be hard



# Towards NLO factorization in $pA$

- The first gluon contributes both to the evolution (when $x \ll 1$) and to the NLO impact factor (generic $x$) : How to avoid over counting ?
- $k_\perp$-factorization : use a 'rapidity subtraction'

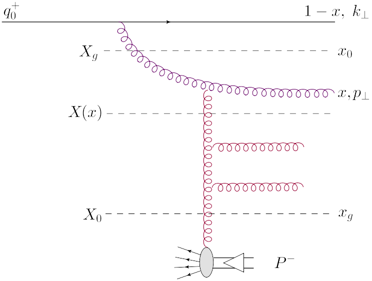



  - the method used by *Chirilli, Xiao, and Yuan (arXiv:1203.6139)*
  - leads to a negative cross-section at semi-hard $k_\perp$
- Our proposal *(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)*
  - separate the first gluon emission from the evolution and compute it with the exact kinematics
- The integral representation of the BK equation is useful in that sense

# BK evolution in integral form

$$S_{\boldsymbol{xy}}(X_g) = S_{\boldsymbol{xy}}(X_0) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \int_{\boldsymbol{z}} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \Big[ S_{\boldsymbol{xz}} S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}} \Big] (X(x))$$

- N.B. Except for the first gluon, the evolution is associated with the nucleus




- $X(x)$ : energy fraction in the target

$$X(x) \simeq \frac{k_\perp^2}{xs} = \frac{X_g}{x}$$

$$X \leq 1 \Longrightarrow x \geq X_g$$

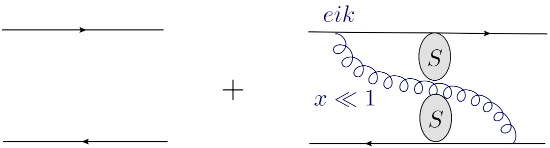
- Even at LO, a running coupling is generally understood
  - $\bar{\alpha}_s \to \bar{\alpha}_s(r_{\min})$, where $r_{\min} \equiv \min\left\{|\boldsymbol{x}-\boldsymbol{y}|, |\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}|\right\}$

# BK evolution in integral form

- In more compact, but formal, notations (coordinate or momentum space)

$$\mathcal{S}(X_g) = \mathcal{S}_0 + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \mathcal{K}(0)\, \mathcal{S}\big(X(x)\big)\,; \quad X(x) \equiv \frac{X_g}{x}$$

- $\mathcal{K}(0)$: LO emission kernel for a soft gluon (including $\perp$ non-locality)

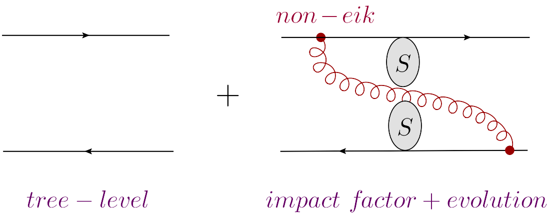



$\mathcal{S}$ *(solution to LO BK equation)*

- The target is not explicit anymore

- The presence of a running coupling complicates the Fourier transform

- Compute (only) the first gluon emission with the exact kinematics




$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\, \mathcal{K}(x)\, \mathcal{S}\big(\boldsymbol{k}, X(x)\big)\,; \quad X(x) \simeq \frac{X_g}{x}$$

- $\mathcal{K}(x)$ : kernel for emitting a gluon with exact kinematics ($x \le 1$)

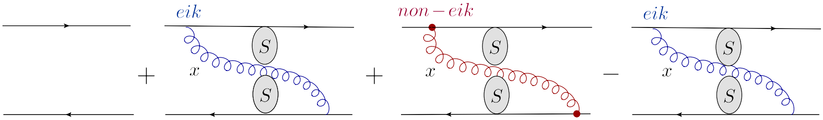
  *(see e.g. Dominguez, Marquet, Xiao, and Yuan, arXiv:1101.0715)*

- This cross-section is almost obviously positive definite

- LO evolution + NLO impact factor ... still mixed with each other

# Recovering $k_\perp$-factorization

- Add and subtract the LO BK equation




$\mathcal{S}$ (solution to $\mathcal{BK}$ equation) $\qquad$ $\mathcal{NLO}$ correction to impact factor

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}\big(\boldsymbol{k}, X(x)\big)$$

- To NLO accuracy, one can replace $\mathcal{S}\big(X(x)\big) \simeq \mathcal{S}(X_g)$ ...
  - the integral over $x$ is now dominated by $x \sim 1$
  - ... and then set $X_g \to 0$ in the lower limit ('plus prescription')
- Local in rapidity : $k_\perp$-factorization in the form presented by CXY

*(Chirilli, Xiao, and Yuan, arXiv:1203.6139 [hep-ph])*

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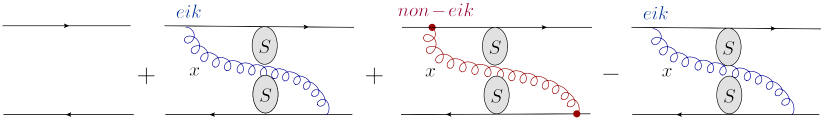


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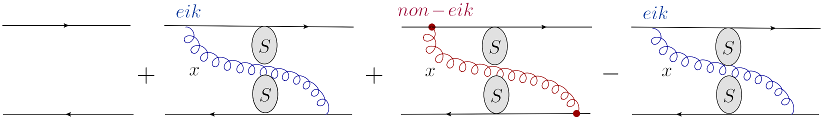
# The fine-tuning problem



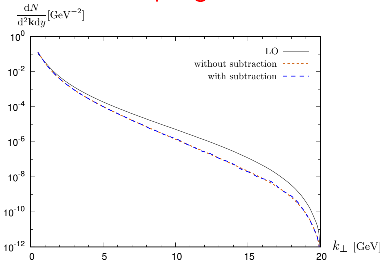

$\mathcal{S}$ (*solution to* $\mathcal{BK}$ *equation*) *NLO correction to impact factor*

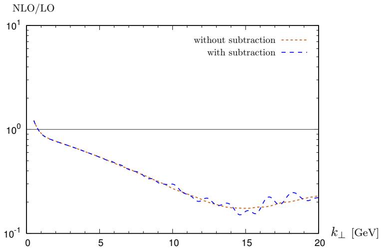
$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}\big(\boldsymbol{k}, X(x)\big)$$

- A mathematical identity ... which may be dangerous in practice
  - one adds and subtracts a large, LO, contribution !
- Any approximation/numerical error in the BK solution or in the subtraction procedure $\Longrightarrow$ mismatch between the 'added' and 'subtracted' pieces
- Potential sources of mismatch (of formally higher order) :
  - the 'plus' prescription $\big[$N.B. $\mathcal{S}(X_g) > \mathcal{S}\big(X(x)\big)$ for any $x < 1\big]$
  - the treatment of the running coupling, e.g. in the Fourier transform

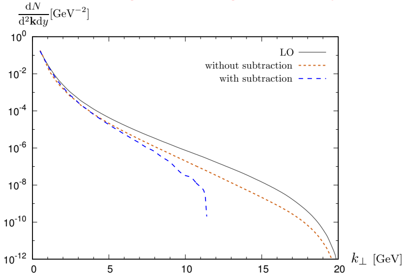
# Numerical results *(Ducloué, Lappi, and Zhu, to appear)*

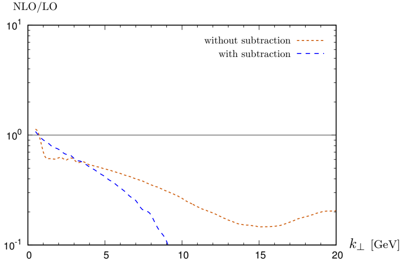
- Fixed coupling: both methods work




- Running coupling: rapidity subtraction leads to negativity

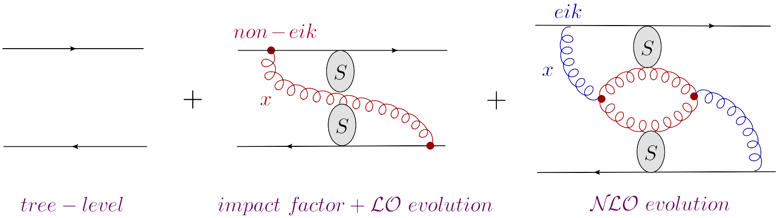


# Completing the NLO evolution

*(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)*

- Recall: the NLO BK evolution also involves 2-loop graphs




$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0 + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\, \mathcal{K}(x)\, \mathcal{S}\big(X(x)\big) + \bar{\alpha}_s^2 \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\, \mathcal{K}_2(0)\, \mathcal{S}\big(X(x)\big)$$

- $\mathcal{K}_2(0)$ : NLO correction to the BK kernel with collinear improvement

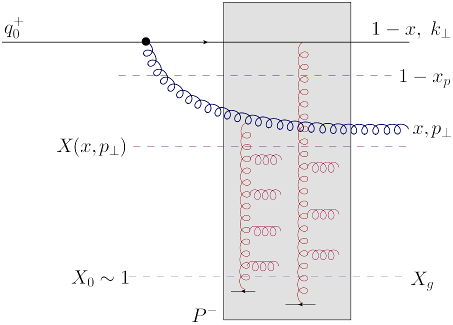
  *(Balitsky and Chirilli, 2008; Iancu et al, 2015)*

# THANK YOU

# Back-up Slides

# Exact kinematics for target evolution

- 'Real amplitude' : the gluon is produced in the final state




- LC energy conservation:

$$\frac{k_\perp^2}{2(1-x)q_0^+} + \frac{p_\perp^2}{2xq_0^+} = XP^-$$

- $\Longrightarrow \ X = X(x, p_\perp)$
- simplifies when $k_\perp \simeq p_\perp \gg Q_s$

$$X(x) \simeq \frac{k_\perp^2}{xs} = \frac{X_g}{x}$$

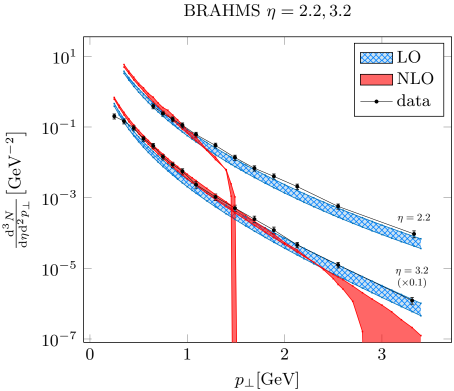
- $X \leq 1 \Longrightarrow x \geq X_g$

- Equivalently: gluon lifetime should be larger than the target width
- The same condition holds for the 'virtual' corrections
  - non-trivial cancellations required by probability conservation

# The negativity problem

*(Stasto, Xiao, and Zaslavsky, arXiv:1307.4057)*

- Sudden drop in the numerical estimate at momenta $p_\perp$ of order $Q_s$




- "NLO evolution is notoriously unstable"
- Sure, but in this calculation $\mathcal{S} \approx \mathcal{S}_{\rm rcBK}$
  - rcBK evolution is well behaved
  - the actual "LO approx" in practice

$$\frac{dN}{dy\, d^2\boldsymbol{k}}\Big|_{\rm LO} = \mathcal{S}_{\rm rcBK}(\boldsymbol{k}, X_g)$$

- The NLO correction to the impact factor is negative (not a real surprise) ... and dominates over the LO result at sufficiently large $k_\perp$

# Some proposals to solve the problem

- General idea: the 'subtracted' term performs an ... over-subtraction
- Strategy: reduce the longitudinal ($x$) phase-space for the 'hard' gluon
  - factorization scale $x_0$ separating 'evolution' from 'impact factor'
    *(Kang, Vitev, and Xing, arXiv:1403.5221)*

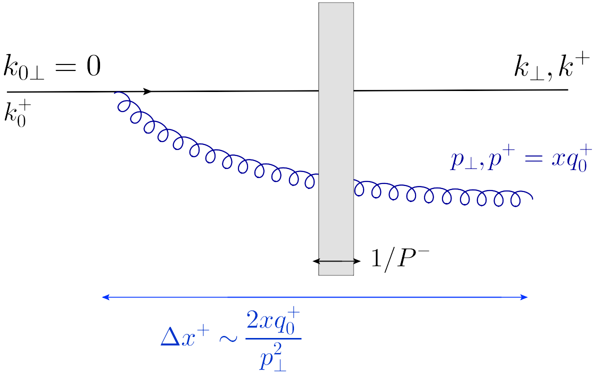
$$\int_0^1 \frac{\mathrm{d}x}{x}\left[\mathcal{K}(x) - \mathcal{K}(0)\right] \Longrightarrow \int_0^{x_0} \frac{\mathrm{d}x}{x}\left[\mathcal{K}(x) - \mathcal{K}(0)\right]$$

  - $x_0$ can depend upon $k_\perp$, say to account for 'time-ordering'
    *(Ducloué, Lappi, and Zhu, arXiv:1604.00225)*
- In principle, it shouldn't matter that much
  - the $x_0$–dependence must cancel in a complete calculation
- In practice, it only pushes the problem up to somewhat higher $k_\perp$
  - also, strongly dependent upon the precise implementation of $x_0$

# Energy conservation *("Ioffe's time")*

*(Altinoluk, Armesto, Beuf, Kovner, and Lublinsky, arXiv:1411.2869)*

- $x$ cannot be arbitrarily small since constrained by energy conservation



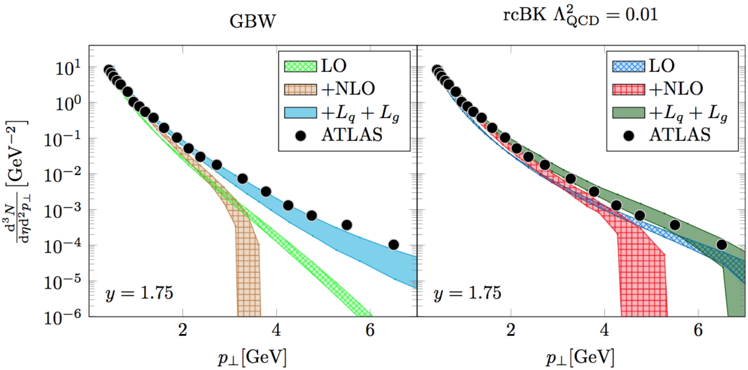

- Gluon lifetime should be larger than the target width

$$\frac{2xq_0^+}{p_\perp^2} > \frac{1}{P^-} \implies x > \frac{p_\perp^2}{s}$$

# Implementing the constraint

*(Watanabe, Xiao, Yuan, and Zaslavsky, arXiv:1505:05183)*

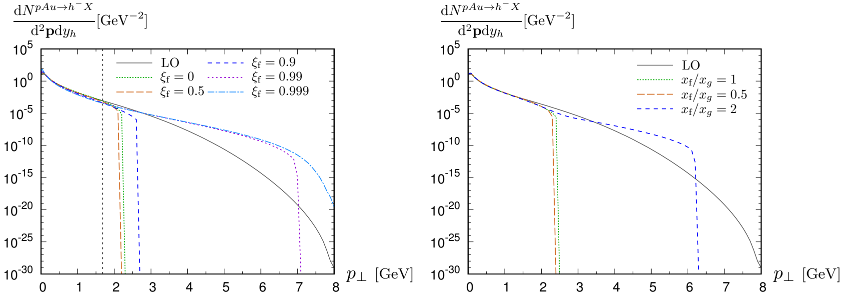
- It matters for the subtraction scheme only if $k_\perp \gg p_\perp$




- Once again, it pushes the problem to higher $k_\perp$
  - ... and strongly dependent upon the model/evolution chosen for $\mathcal{S}$

# Why is this a problem ?

- An extreme example: GBW saturation model $\mathcal{S}_{\rm GBW}(\boldsymbol{k}, X) \propto \mathrm{e}^{-\frac{k_\perp^2}{Q_s^2}}$
  - the 'added' piece is exponentially suppressed at $k_\perp \gg Q_s$
  - the 'subtracted' piece develops a power-law tail $\propto 1/k_\perp^4$
  - the overall result becomes negative at sufficiently large $k_\perp$




*(Ducloué, Lappi, and Zhu, arXiv:1604.00225)*