The problem of Overlapping Formation Times

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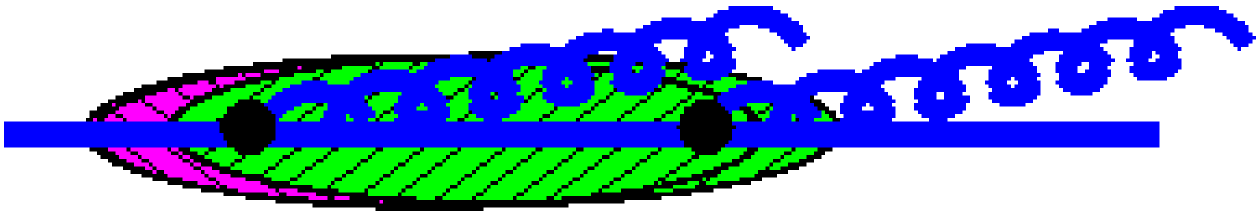
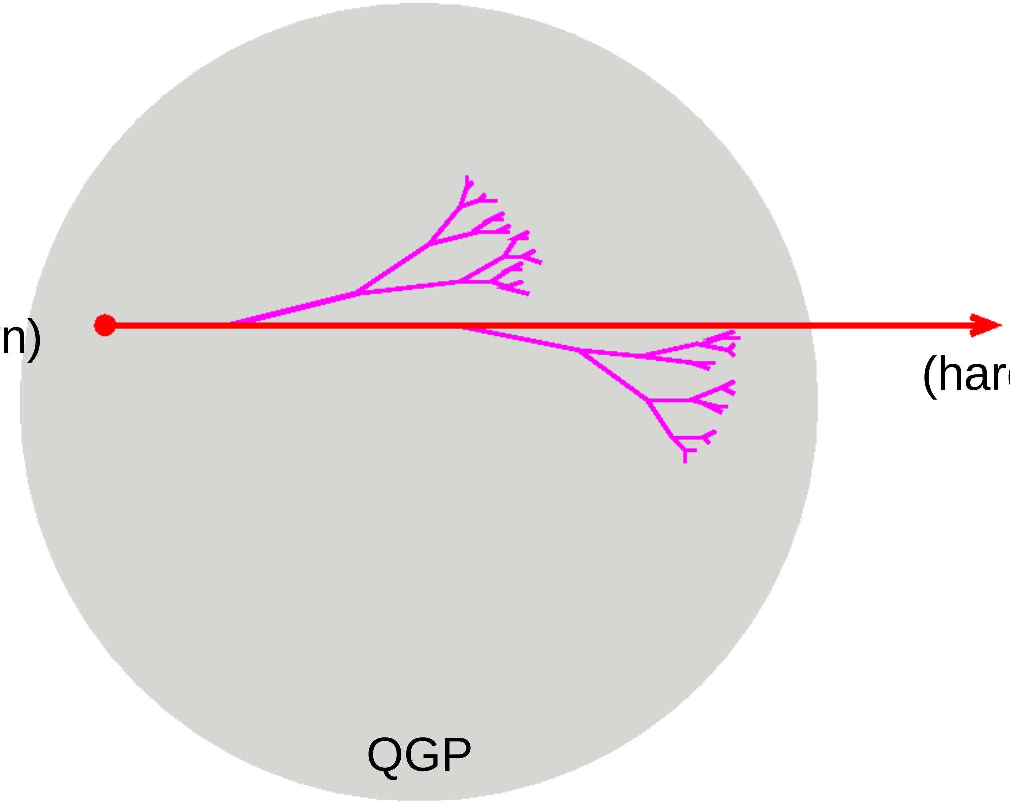


Reporting on work with Shahin Iqbal and Han-Chih Chang

arXiv:1501.04964, 1605.07624, 1606.08853, 1608.05718

Consider cartoon of

In-medium evolution of a jet

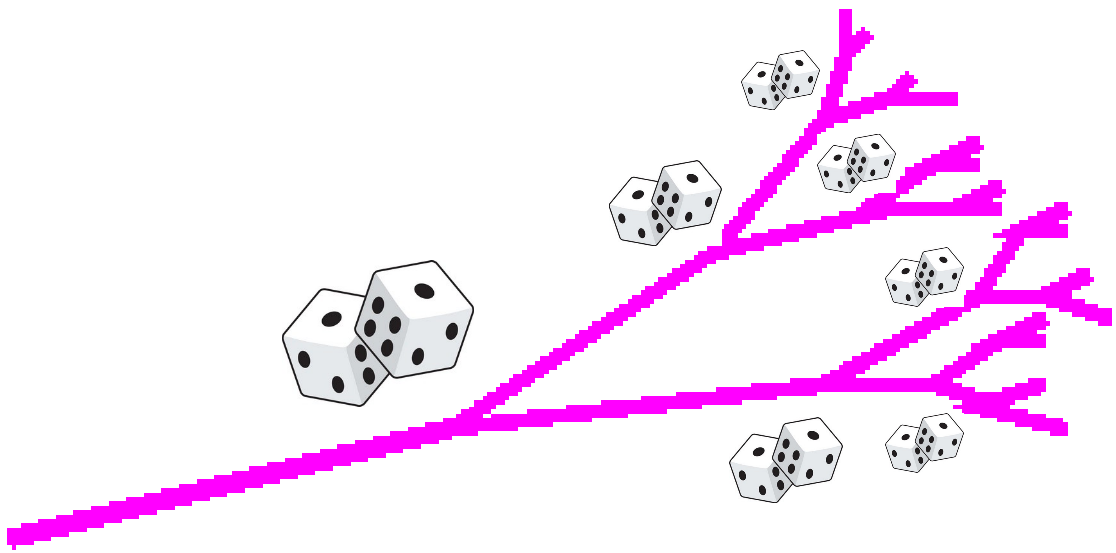


For this talk, simplify discussion by focusing on ...

Cascades that stop in-medium

- Qualitative points we'll discuss generalize.
- Formalism generalizeable as well.

An idealized Monte Carlo picture of in-medium evolution



As time passes,

roll classical dice for probability of each splitting

weighted by the quantum calculation of the single splitting rate

$\frac{d\Gamma_{\rm brem}}{dx}$ for each vertex  shown above.

Built-in assumption:

Consecutive splittings are quantum-mechanically independent.

(*Are they* ?)

Heuristic attempts to improve on this in real Monte Carlos:

JEWEL
recent versions of MARTINI

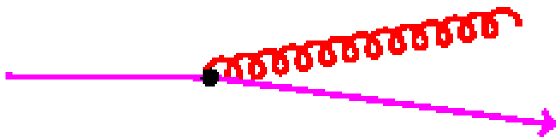
Here, I want to talk about

What's known from first-principles(-ish) QCD calculations?

Review of single splitting

Collisions with the medium

generate chances for bremsstrahlung



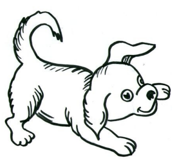
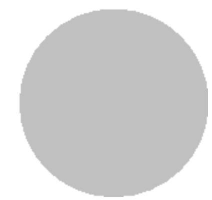
Naively,

prob of emission ~ α per collision

BUT

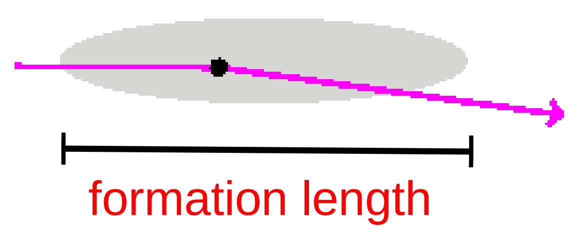
Light can't resolve features on small scales.

Non-relativistic:

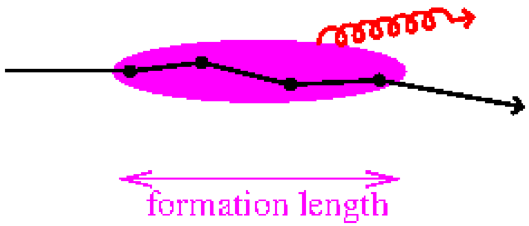
 and  both look like  if $\lambda >> d$.

Extremely relativistic, nearly-collinear motion:

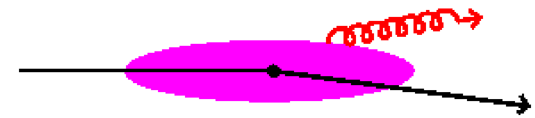
Similar effect, but size of fuzziness stretched out.



$$l_{\rm form} \propto \sqrt{E} \quad \text{(for fixed } x\text{)}$$



indistinguishable from



So

prob of emission ~ α per <u>formation length</u> $l_{\rm form} \propto \sqrt{E}$

Calculated quantitatively by

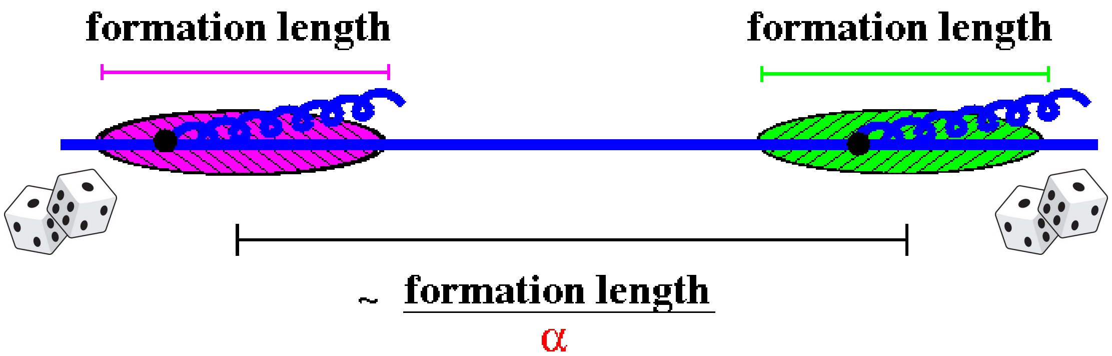
LPM for QED (1950s)
BMDPS-Z for QCD (1990s)

and investigated in numerous ways by many people at this conference.

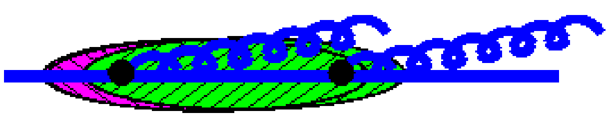
Consecutive emissions

Chance of brem ~ α per formation time

means that two consecutive splittings will typically look like



So chance of overlap (i.e. "rolling dice separately" breaking down) is



$\propto \alpha$

How big is "α" ??

How big is α_s?

Nothing to do with whether medium is

sQGP / perfect liquid [$\alpha_s(T)$ big]	vs.	weakly-coupled QGP [$\alpha_s(T)$ small]

α_s on previous slide associated with emission vertex:

costs roughly $\alpha_s(Q_\perp)$ with $Q_\perp \sim (\hat{q}E)^{1/4} \lesssim$ a few GeV

panic and/or fool around with AdS/CFT energy loss [$\alpha_s(Q_\perp)$ big]	vs.	LPM-based analysis [$\alpha_s(Q_\perp)$ small]

Does the wisdom of the ages tell us if α_s(few GeV) is small?

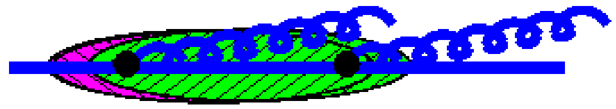
Particle physics in vacuum:

Small for some things, like matching lattice calculations to continuum MS-bar α_s

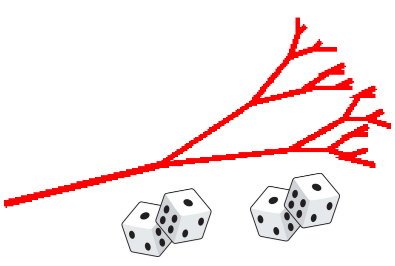
High-temperature physics:

Bad news (except possibly if one does sophisticated resummations of perturbation series)

Overlapping formation times effects on cascade:



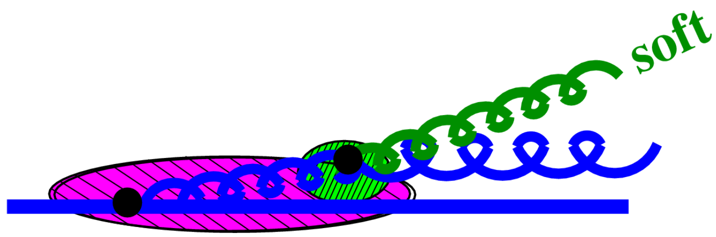
$\propto \alpha$ effect on



We should calculate it and see.

Soft emission

Soft emissions are generally enhanced by logs.
Path-breaking authors found small-*x*-like double logs in this case,



$$\propto \alpha_s \ln^2\left(\frac{E}{\hat{q}\tau_{\rm mfp}}\right)$$

Blaizot & Mehtar-Tani; Iancu; Wu (2014)

This is a BIG effect for large *E*.
But they found soft emission effects could be absorbed into the medium parameter $\hat{q} \to \hat{q}_{\rm eff}(E)$

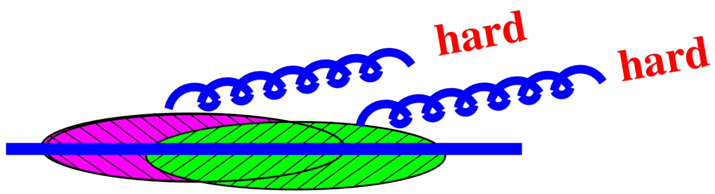
following Liou, Mueller, Wu (2013)

Refined question

What about overlap effects that *can't* be absorbed into $\hat{q}$?

What we've done

Computed the effect of the overlap for hard emissions



$\propto \alpha_s$

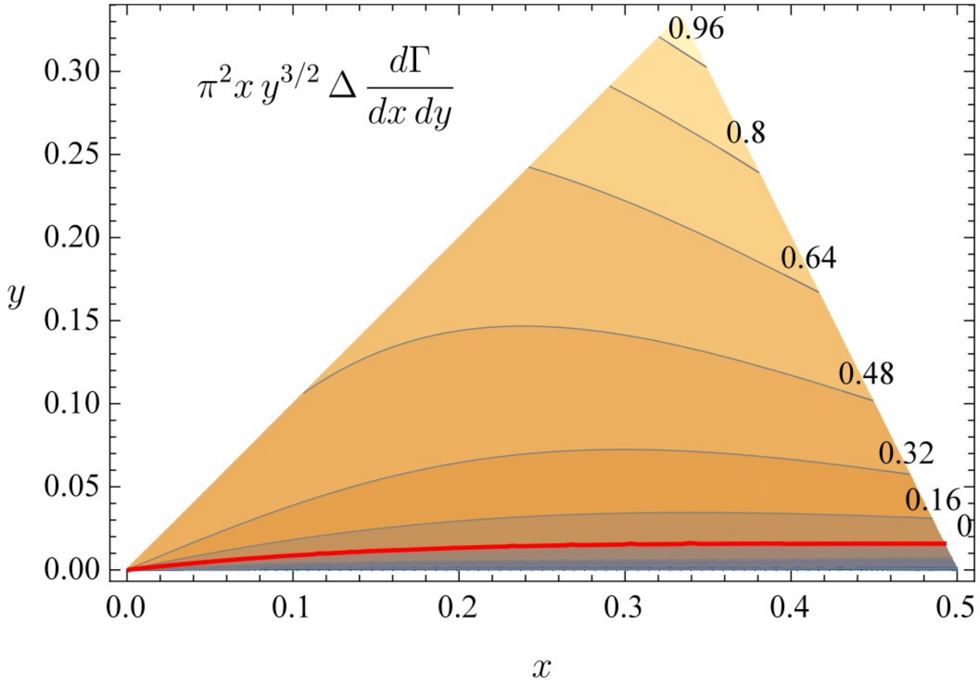
In broad brush: interesting and fun field theory problem.

In calculational detail: a pain in the ass.

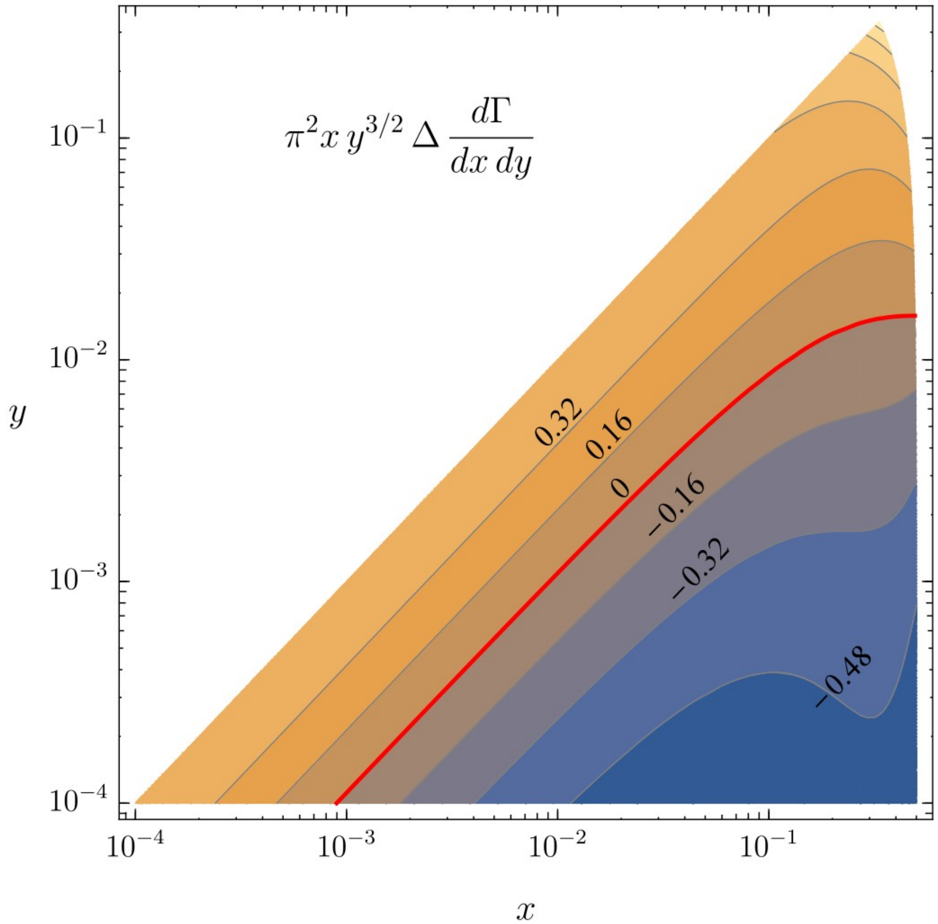
A qualitative lesson so far

Attempts to put overlap effects into various real-world Monte Carlos have assumed (for reasonable-sounding reasons) overlap emissions would be *suppressed*.

We find overlapping emissions are *enhanced* (unless one emission is *very* soft).

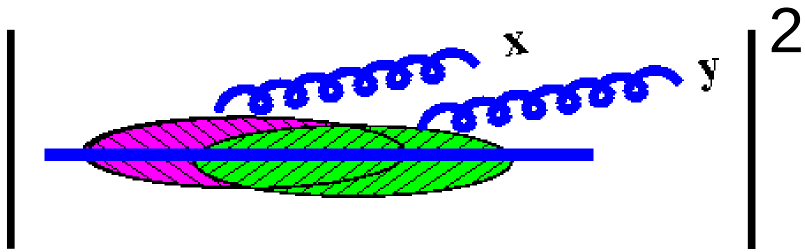


[This is for $g \to ggg$]



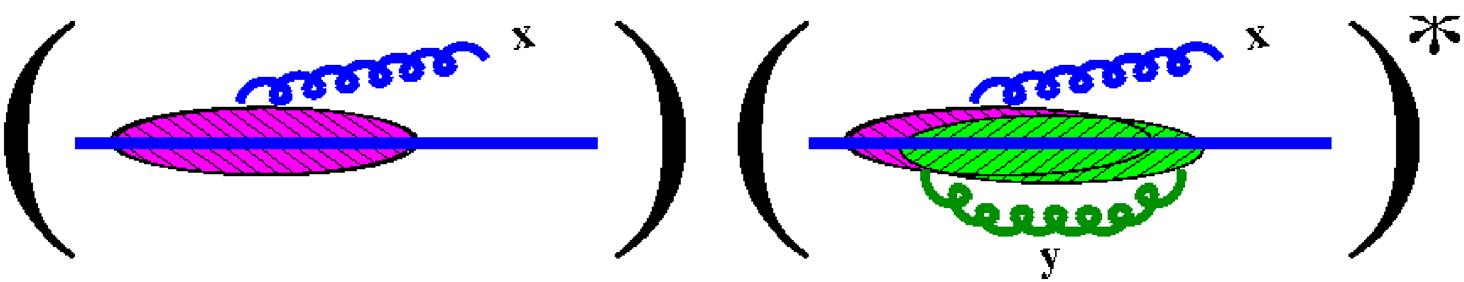
But are (non-absorbable) overlap effects small or big for α_s(few GeV)

$\int dx\, dy$



has soft divergences that require also computing virtual corrections to single emission

$2\,\mathrm{Re}\int dx\, dy$

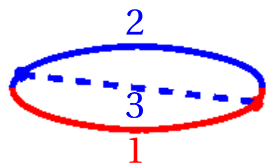


WORK IN PROGRESS...

BACKUP SLIDES

Kinetic terms:

Energy of a high-p_z particle: $$\epsilon_p = \sqrt{p_z^2 + p_\perp^2} \simeq p_z + \frac{p_\perp^2}{2p_z}$$

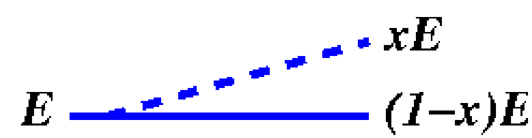
Evolution of 

is $e^{-i\mathcal{H}t}$ with

$$\mathcal{H}_{\rm kin} = -\epsilon_{p_1} + \epsilon_{p_2} + \epsilon_{p_3} \simeq -\frac{p_{\perp 1}^2}{2p_{z1}} + \frac{p_{\perp 2}^2}{2p_{z2}} + \frac{p_{\perp 3}^2}{2p_{z3}}$$

$$\simeq -\frac{p_{\perp 1}^2}{2E} + \frac{p_{\perp 2}^2}{2(1-x)E} + \frac{p_{\perp 3}^2}{2xE}$$

conjugate evolves with e^{+iHt}



This is 2-dimensional non-relativistic QM with

$$(m_1, m_2, m_3) = (-E, (1-x)E, xE)$$

As promised,

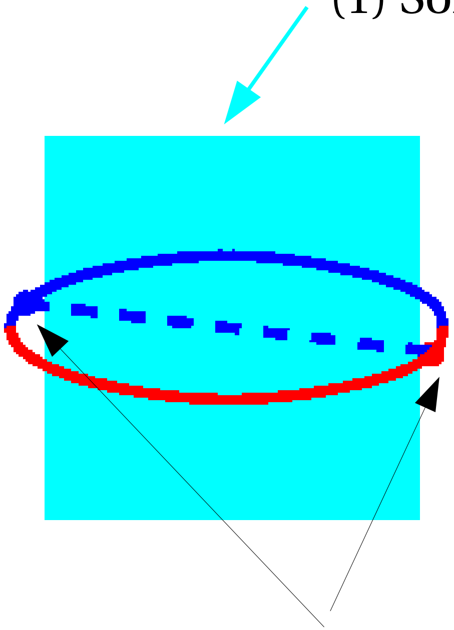
$$m_1 + m_2 + m_3 = 0$$

Potential term:

$V(b_1, b_2, b_3)$ incorporates (statistically averaged) effect of collisions with the medium.

How to put the calculation together:

(1) Solve for propagation in 3-particle QM in shaded region.



(2) Tie together with QFT matrix elements for vertices

$$\propto \sqrt{\textbf{DGLAP splitting functions}}$$

$$\propto \sqrt{P_{i \to j}(x)}$$

Simplifcation: 3-particle QM → 1-particle QM

Can use various symmetries of problem to get rid of 2 d.o.f.

$$\mathcal{H} = \frac{P_B^2}{2M} + V(B)$$

[BDMPS-Z (1990's)]

Method 1. Can solve numerically. [Zakharov (2004+); Caron-Huot & Gale (2010)]

Simplifcation: Harmonic Oscillator (a.k.a. multiple-scattering approx. or $\hat{q}$ approx.)

Method 2. High energies → very collinear → b's small.

So make small B approximation to $V(B)$ → a harmonic oscillator problem

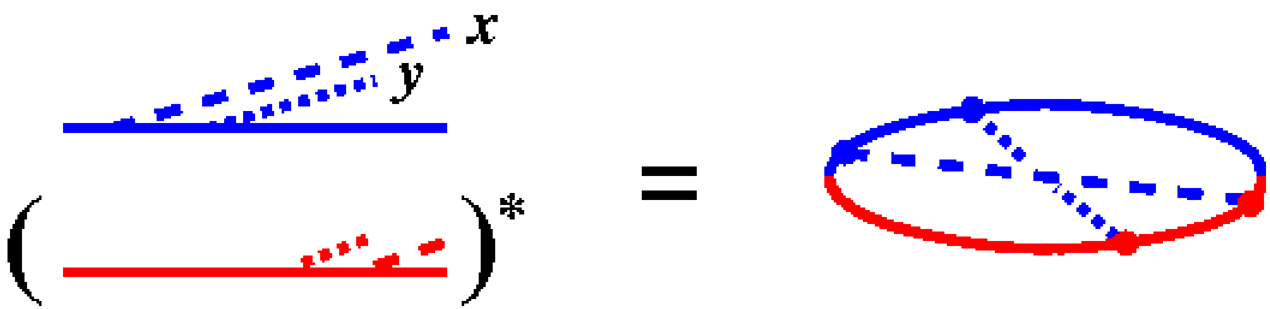
$$\mathcal{H} = \frac{P_B^2}{2M} + \frac{1}{2} M \Omega_0^2 B^2$$

[Baier *et al*. (1998)]

(a non-Hermitian one: $\Omega_0^2 \propto -i$)

What's been done for <u>double</u> brem

Example of an interference contribution:



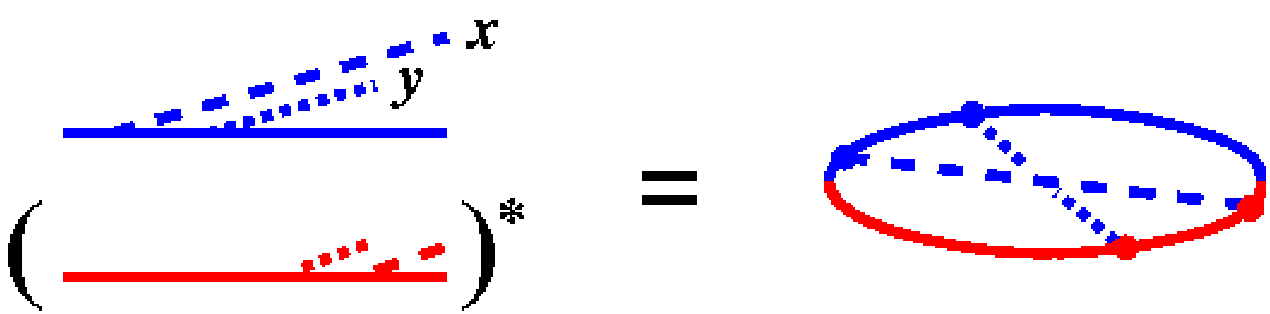
***Previously*:** results in **soft** limit $y << x << 1$ for QCD.

Blaizot & Mehtar-Tani; Iancu; Wu (2014)

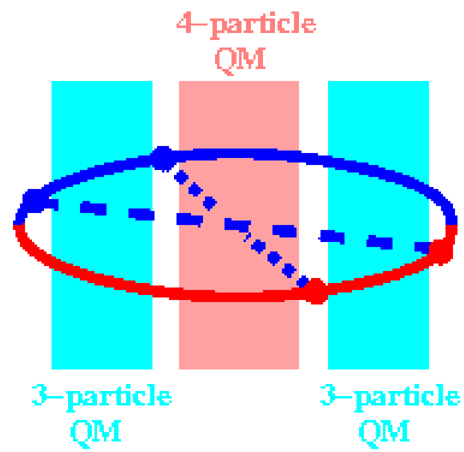
***Our goal*:** Compute effects of overlapping formation times for **any** x and y.

Formalism for LPM: double brem

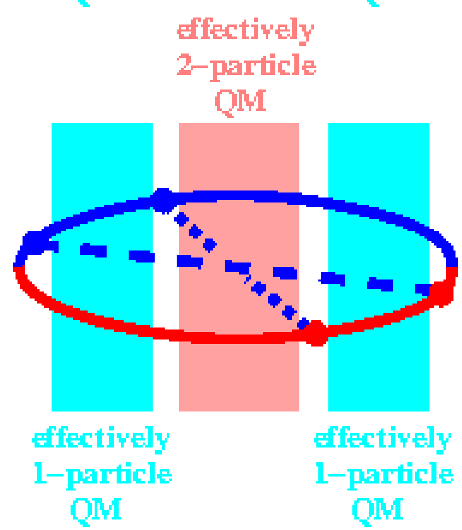
Example of an interference contribution:



To compute: Sew together QFT matrix element for vertices with QM evolution in between.

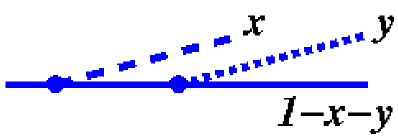


Simplify: Using symmetries, as before.

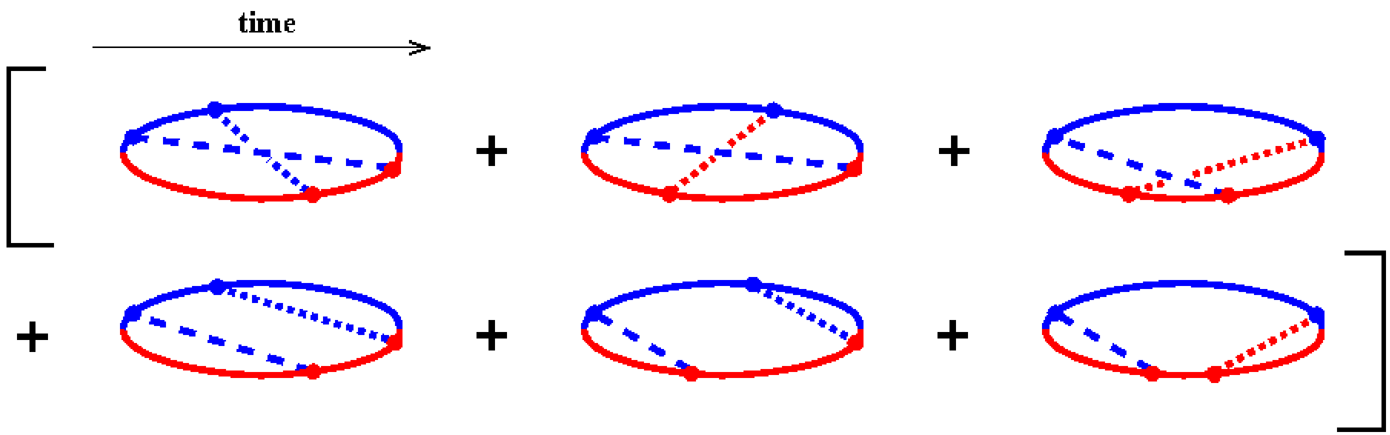


Published Work

[all for $g \to gg \to ggg$]

$\Big|$ 

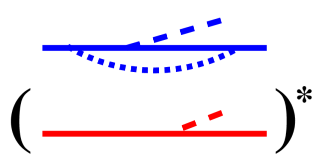
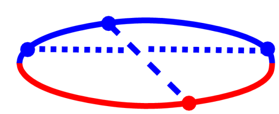
+ permutations $\Big|^2$

= 2 Re 

+ appropriate permutations of (x, y, 1-x-y)

- 4-gluon vertices, e.g. 

Still in progress

- virtual corrections, e.g.  =  correct single brem rate

[parts of which included in $y \ll x \ll 1$ work of earlier refs.]

- Putting it all together to compute physical, infrared-safe characteristics of shower development (including earlier authors' resummation of soft bremsstrahlung).

Some additional ceveats/simplifications

- So far, have assumed QGP is thick compared to formation length
  [not true for the leading parton creating the highest-energy jets in experiments]

- So far, have worked in large N_c limit.
  [One can apply the formalism without this, but much more complicated.]

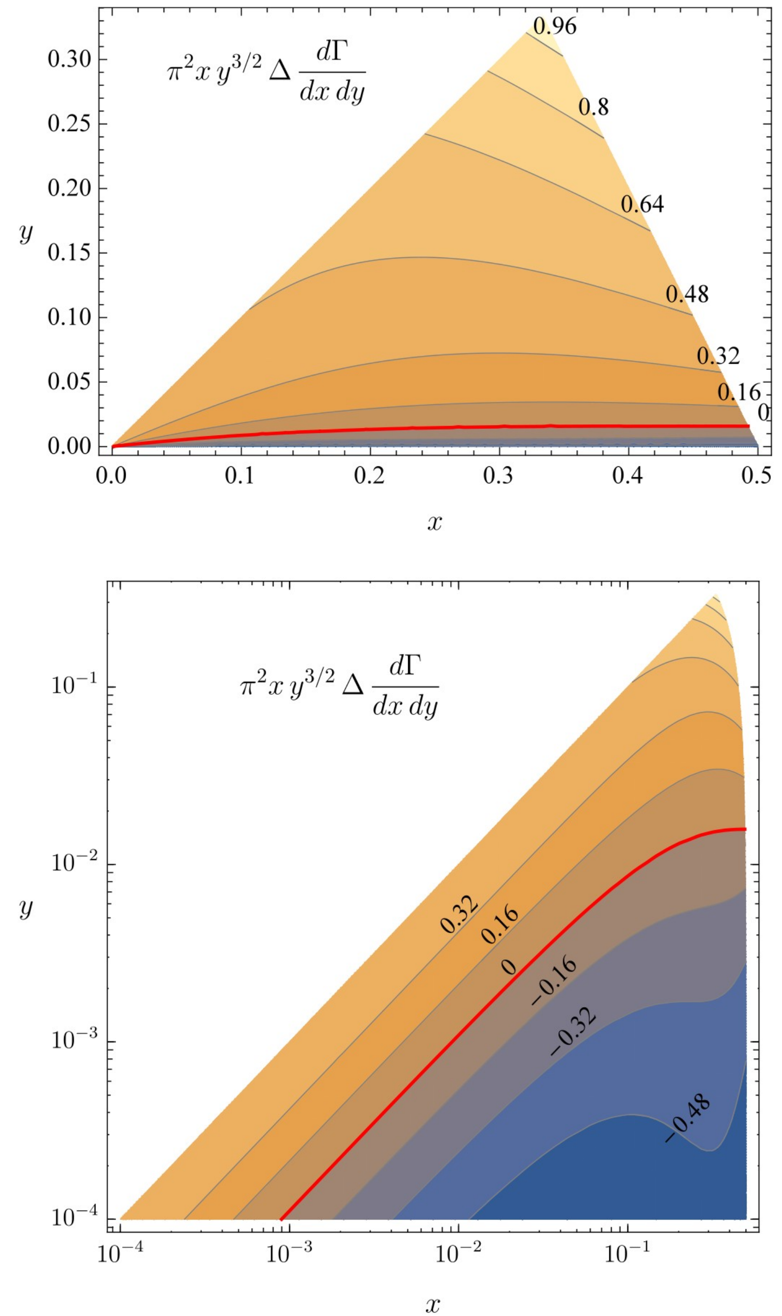
Results

$$\Delta \frac{d\Gamma}{dx\,dy} \equiv$$ correction to double brem due to overlapping formation times

$$= f(x,y)\,\frac{C_A^2 \alpha_s^2}{\pi^2 x y^{3/2}} \sqrt{\frac{\hat{q}_A}{E}}$$

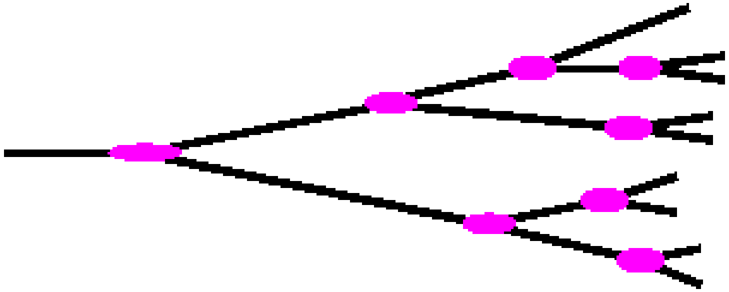
$$(y < x < 1-x-y)$$

where *f*(*x*,*y*) varies from 1.05 to -0.90 and is shown on the right.

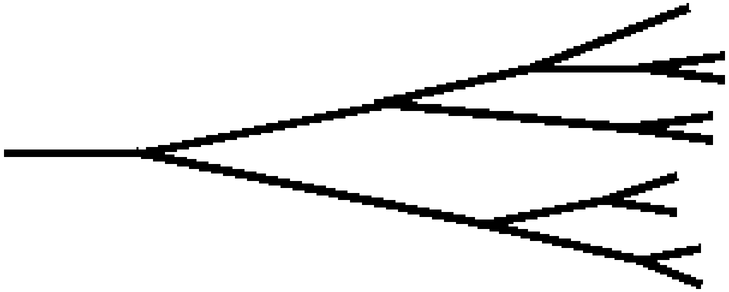


Qualitative Point

Effect of overlapping formation times enhances the rate except when one gluon is very soft.

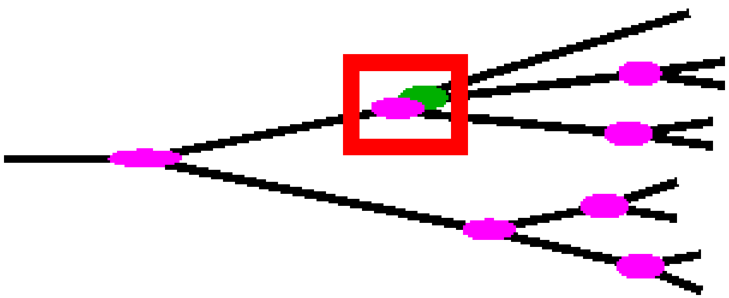


vs



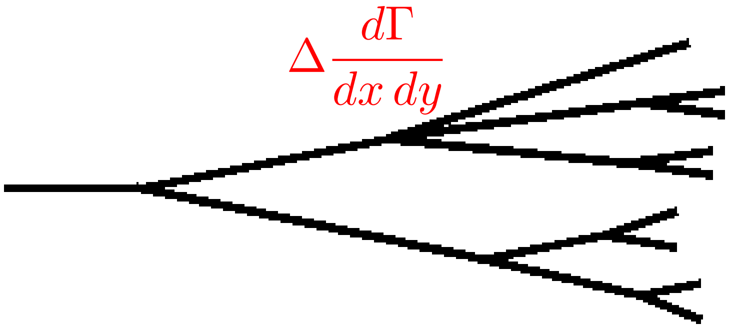
Monte Carlo (MC)

How to account for correction from

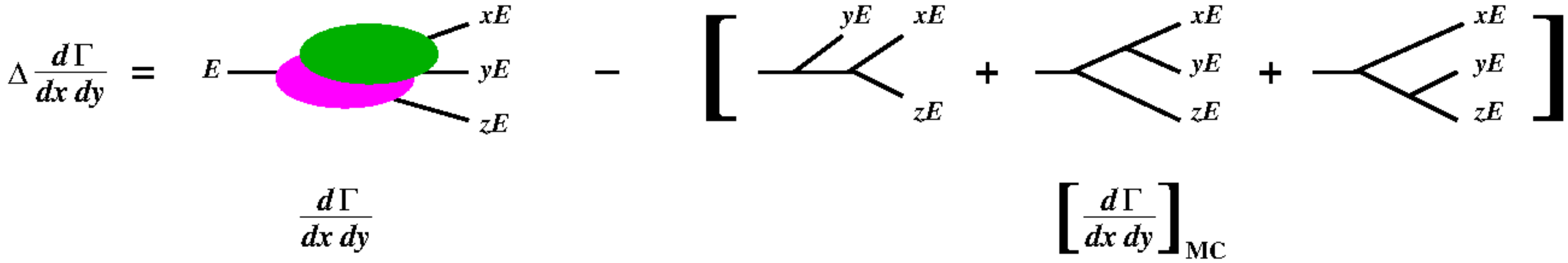


?

Add a $g \to ggg$ Monte Carlo possibility to account for correction:



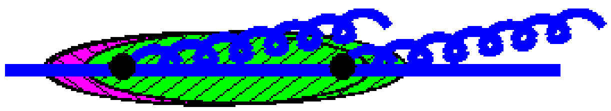
where



$\Delta \frac{d\Gamma}{dx\,dy}$ =
E
xE
yE
zE
−
yE
xE
zE
+
xE
yE
zE
+
xE
yE
zE
$\frac{d\Gamma}{dx\,dy}$
$\left[\frac{d\Gamma}{dx\,dy}\right]_{\mathrm{MC}}$

Summary

Subtle problems in the field theory description of very-high energy showering

e.g. 

can be reduced to problems in

2-dimensional non-relativistic non-Hermitian quantum mechanics

and even

2-dimensional non-relativistic non-Hermitian harmonic oscillators!

(Just when you thought you couldn't learn anything more from the harmonic oscillator...)

Coming in the future

Are the O(α_s) corrections to physical, infrared-safe quantities characterizing shower development small (after accounting for the known running of $\hat{q}(E)$ due to soft brem)?

To wit, is the basic physical assumption behind in-medium Monte Carlo simulations on firm ground?