Jet evolution in a dense medium: event-by-event fluctuations and multi-particle correlations

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JHEP 1605 (2016) 008 and JHEP 1612 (2016) 104. Collaboration with E. Iancu.

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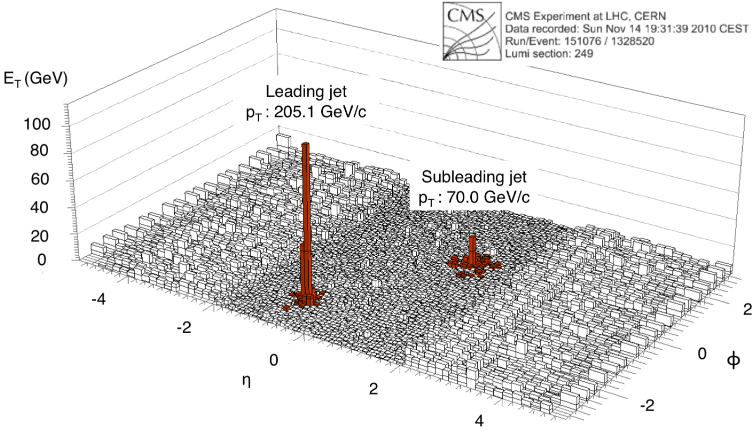
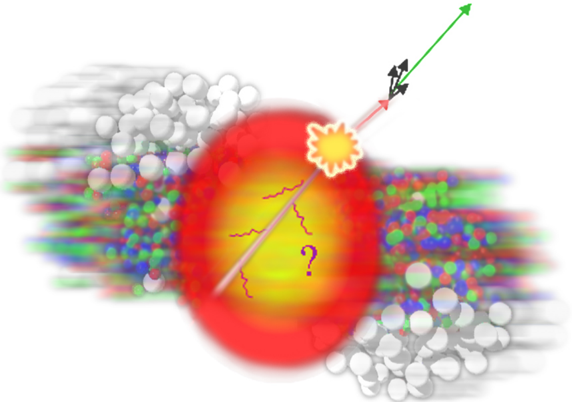
Jet quenching in heavy-ion collisions



Jet quenching, the generally expected picture



Mediums of different sizes are seen by the two jets → asymmetry, but that might not be the full story.

Jet quenching, problems of the naive picture

There is an implicit assumption

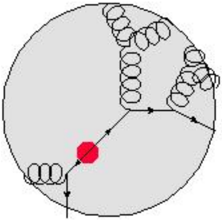
- The energy loss is always the same at fixed medium size. Fluctuations are small.

Is it true?

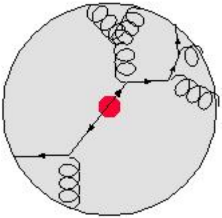
- A recent Monte Carlo computation (Milhano and Zapp, EPJC 76:288 (2016)) shows that fluctuations compete with path length difference.
- In JHEP 1605 (2016) 008 we perform an analytic computation that shows that fluctuations in the energy loss are of the order of the average value.

Fluctuations in size vs fluctuations in energy loss

Asymmetry due to different path length.



Asymmetry due to fluctuations in the branching process.



To estimate the size of the fluctuations in energy is important to better understand the physics behind the dijet asymmetry.

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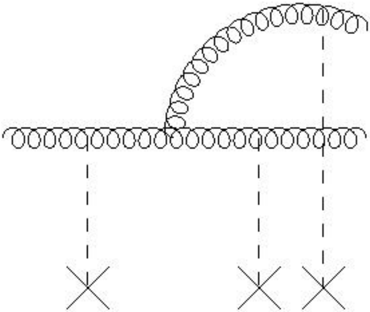
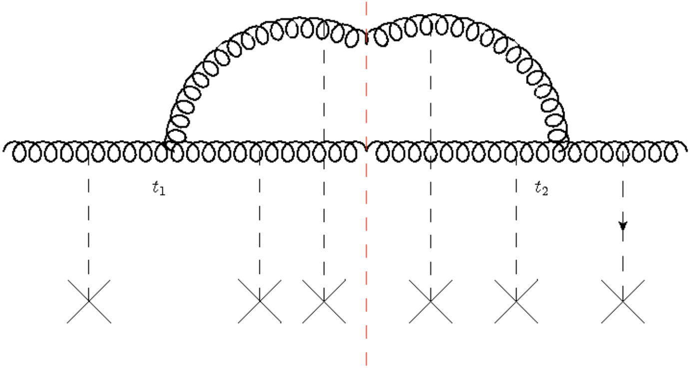
General picture



- All the needed information from the medium is encoded in $\hat{q}$ and the length L.
- Emission probability given by the BDMPS-Z theory.
  Baier, Dokshitzer, Mueller, Peigné, Schiff, Nucl. Phys. B483, 291 and Zakharov, JETP Lett. 63 952.

Medium-induced gluon emission: formation time



- $\tau_f = t_1 - t_2$. By uncertainty relation $\frac{1}{\tau_f} \sim \frac{k_\perp^2}{2\omega}$.
- In a medium the acquired transverse momentum is $k_\perp^2 \sim \hat{q}\tau_f$.
- $\tau_f \sim \sqrt{\frac{2\omega}{\hat{q}}}$

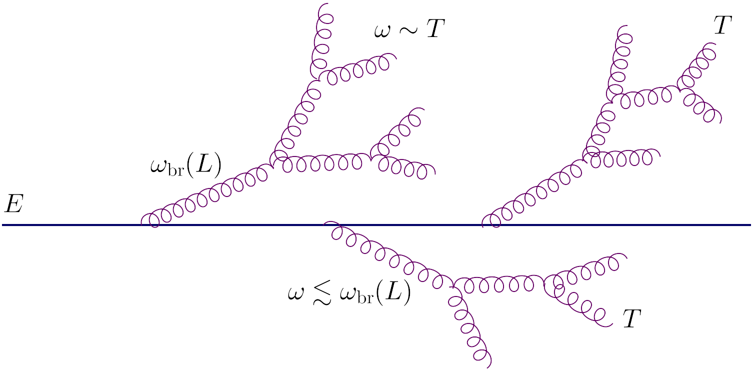
Branching time

The probability of emitting a gluon $P(\omega, \Delta t) \sim \mathcal{O}(1)$ when $\Delta t =$ branching time $\tau_{br}(\omega)$

$$\tau_{br}(\omega) \sim \frac{1}{\alpha_s}\tau_f(\omega) \sim \frac{1}{\alpha_s}\sqrt{\frac{\omega}{\hat{q}}}$$

- $\tau_{br} = L$ when $\omega = \omega_{br} \equiv \alpha_s^2 \hat{q} L$: "branching energy".
- Gluons with $\omega \sim \omega_{br}(L)$ dominate the energy loss of the jet.
- Via their subsequent branching they transfer the energy from the leading particle to the medium.
- Soft gluons with $\omega \ll \omega_{br}(L)$ are abundantly emitted but carry only little energy.

Multiple branching



Successive branching are quasi-independent → a Markovian stochastic process. Blaizot, Dominguez, Iancu and Mehtar-Tani JHEP06(2014)075.

Democratic branching

- BDMPS-Z rate favours democratic branching for gluons with $\omega \leq \omega_{br}(L)$. (When the two daughter gluons carry similar energy fractions :$x \sim 1 - x$).
- Contrast with branching in the vacuum: soft splitting ($x \ll 1$).
- Typical jet evolution: the leading particle emits a number of $\mathcal{O}(1)$ of primary gluons with $\omega \sim \omega_{br}(L)$, which then produce "mini-jets" via democratic branching.

Blaizot, Iancu and Mehtar-Tani *Phys.Rev.Lett* 111, 052001

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The gluon spectrum

Energy density per unit $x \equiv \frac{\omega}{E}$

$$D(x,t) = x\langle \sum_i \delta(x_i - x) \rangle$$

$$\frac{\partial}{\partial \tau} D(x,\tau) = \int \mathrm{d}z \, \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \tau\right) - \frac{z}{\sqrt{x}} D(x,\tau) \right]$$

where $\tau = \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\hat{q}}{E}} t = \frac{t}{\tau_{br}(E)}$.
Two cases:

- $\tau_{br}(E) \gg L$. Interesting for LHC physics.
- $\tau_{br}(E) \sim L$. Interesting to study the case in which the jet is completely absorbed by the medium.

This equation also has an important role in bottom-up thermalization.
Baier, Mueller Schiff and Son *Phys.Lett.*B502(2001) 51-58.

The gluon spectrum

With the initial condition $D(x,0)=\delta(x-1)$ there is an analytic solution with an approximate kernel $\mathcal{K}\to\mathcal{K}_0$.

$$D(x,\tau)=\frac{\tau}{\sqrt{x}(1-x)^{3/2}}\exp\{-\frac{\pi\tau^2}{1-x}\}$$

Average energy inside the jet

$$\langle X(\tau)\rangle=\int_0^1 dxD(x,\tau)=e^{-\pi\tau^2},$$

Blaizot, Iancu and Mehtar-Tani *Phys.Rev.Lett* 111, 052001

Average energy loss

Where does the energy go?

- The energy is transferred to the medium by branching products that thermalize ($\omega \sim T$). Iancu and Wu, JHEP 1510 (2015) 155.
- It emerges at large angles with respect to the jet axis.

$$\mathcal{E} = E(1 - e^{-\pi\tau^2}) = E(1 - e^{-\frac{\pi\omega_{br}(L)}{E}})$$

- Typical kinematics at the LHC: $E \sim 100\ Gev \gg \omega_{br}(L) \sim 5\ GeV$.

$$\mathcal{E} \sim \pi\omega_{br}(L) \sim \alpha_s^2 \hat{q} L^2$$

- Energy loss at large angles is controlled by $\omega_{br}(L)$.

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Fluctuations in the energy loss

$$\langle \mathcal{E}^2(t) \rangle - \langle \mathcal{E}(t) \rangle^2 = E^2(\langle X^2(t) \rangle - \langle X(t) \rangle^2) \,,$$

We need a new ingredient, the average energy squared carried by a pair of gluons with energy fractions equal to x and x'.

$$D^{(2)}(x, x', t) = xx' \langle \sum_{i \neq j} \delta(x_i - x)\delta(x_j - x') \rangle \,,$$

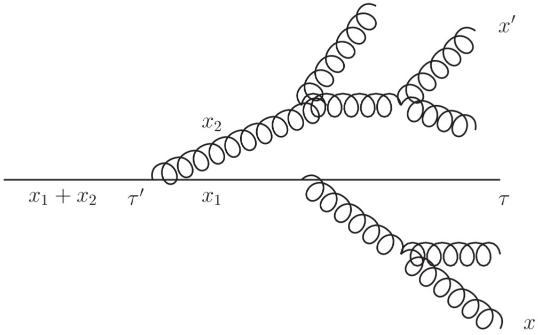
with this we can compute $\langle X^2(t) \rangle$

$$\langle X^2(t) \rangle = \int_0^1 dx x D(x,t) + \int_0^1 dx \int_0^1 dx' D^{(2)}(x, x', t) \,.$$

Escobedo and Iancu, JHEP 1605 (2016) 008

The gluon pair density $D^{(2)}$

How does one create a pair of gluons with energy fraction x and x'?



Intuitively

- Compute all the branchings that happen at any τ' between 0 and τ.
- Evolve independently the results of these branching.
- Integrate for all τ'.

Analytic solution of $D^{(2)}$

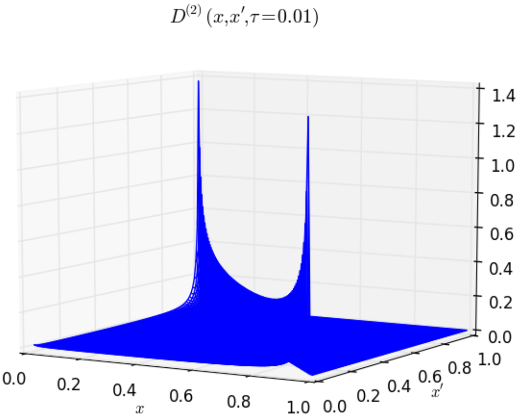
$$D^{(2)}(x,x',\tau) = \int_0^\tau \mathrm{d}\tau'(2\tau-\tau') \frac{\mathrm{e}^{-\frac{\pi(2\tau-\tau')^2}{1-x-x'}}}{\sqrt{xx'}(1-x-x')^{3/2}}\,,$$

The integrand is the contribution to $D^{(2)}$ of all the branchings that happened at τ'.

$$D^{(2)}(x,x',\tau) = \frac{1}{2\pi}\frac{1}{\sqrt{xx'(1-x-x')}}\left[\mathrm{e}^{-\frac{\pi\tau^2}{1-x-x'}} - \mathrm{e}^{-\frac{4\pi\tau^2}{1-x-x'}}\right].$$

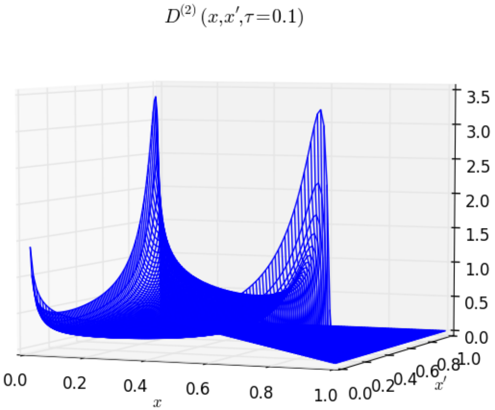
- Correlation due to common ancestors.
- The first term correspond to correlations created by late time branching.
- The second to early time branching.

Plot of $D^{(2)}$



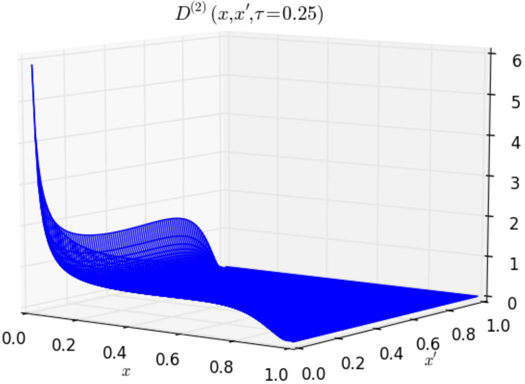
$D^{(2)}(x,x',\tau=0.01)$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
0.0
0.2
0.4
0.6
0.8
1.0
x'

Plot of $D^{(2)}$

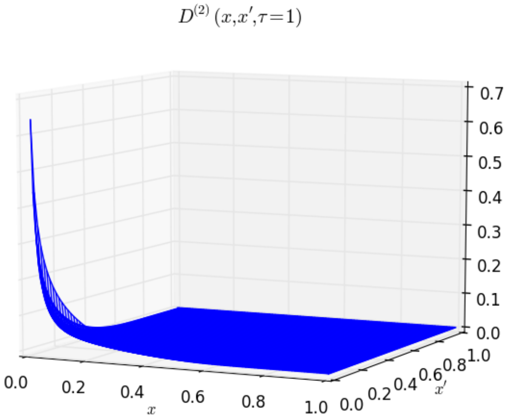


$D^{(2)}(x,x',\tau=0.1)$
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
0.0
0.2
0.4
0.6
0.8
1.0
x'

Plot of $D^{(2)}$



Plot of $D^{(2)}$



$D^{(2)}(x,x',\tau=1)$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
0.0
0.2
0.4
0.6
0.8
1.0
x'

Fluctuations in energy

$$\sigma_{\mathcal{E}}^2 = \langle \mathcal{E}^2(t) \rangle - \langle \mathcal{E}(t) \rangle^2 = E^2 \sigma_{\varepsilon}^2(t)$$

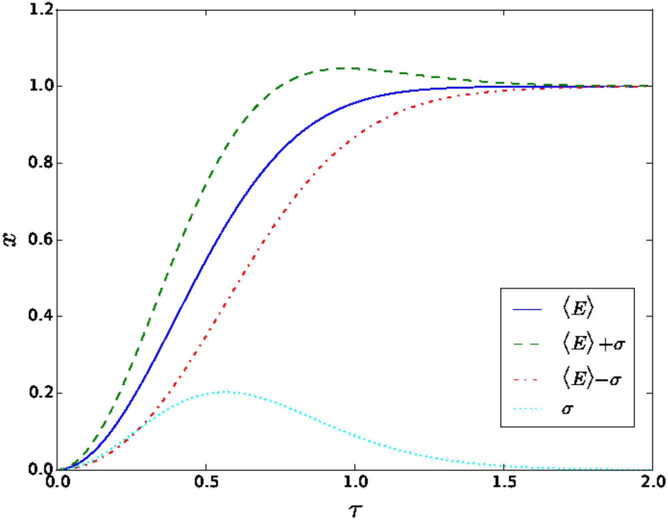
$$\sigma_{\varepsilon}^2(\tau) = 2\pi\tau \left[\text{erf}(\sqrt{\pi}\tau) - \text{erf}(2\sqrt{\pi}\tau)\right] + 2\mathrm{e}^{-\pi\tau^2} - \mathrm{e}^{-4\pi\tau^2} - \mathrm{e}^{-2\pi\tau^2}$$
$$= \frac{1}{3}\pi^2\tau^4 - \frac{11}{15}\pi^3\tau^6 + \mathcal{O}(\tau^8)\,.$$

There are terms that go like τ and τ^2 in the intermediate steps but they cancel out when computing the variance.

$$\sigma_{\mathcal{E}}^2 = \frac{\pi^2}{3} w_{br}^2(L) \sim \langle \mathcal{E} \rangle^2$$

Dispersion in energy loss at large angles is comparable with its mean value.

Fluctuations in energy



Fluctuations in energy and fluctuations in size

Average of all back-to-back jets created in a heavy-ion collision with initial energy E taking into account both fluctuations in energy and in size

$$\langle E_1 - E_2 \rangle^2 = (N_c \alpha_s \hat{q})^2 (\langle L_1^2 \rangle - \langle L_2^2 \rangle)^2$$

$$\sigma^2_{E_1-E_2} = \langle (E_1 - E_2)^2 \rangle - \langle E_1 - E_2 \rangle^2 = (N_c \alpha_s \hat{q})^2 \left[\frac{1}{3}(\langle L_1^4 \rangle + \langle L_2^4 \rangle) + \sigma^2_{L_1^2} + \sigma^2_{L_2^2} \right]$$

The dijet asymmetry is produced

- Asymmetry between the path lengths of the 2 jets in the medium.
- Fluctuations of the energy loss that are present even if the size is fixed.
- Fluctuations in the size of the medium seen by the jet.
- Fluctuations dominate over average whenever $L_1 \sim L_2$.

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Computation of $D^{(n)}$

$\langle E^n \rangle$ and $\langle N^n \rangle$ can be computed if you know $D^{(n)}$.
Recently we have found a exact expression for these quantities

$$D^{(n)}(x_1, \cdots, x_n|\tau) = \frac{(n!)^2}{2^{n-1}n} \frac{\left(1 - \sum_{i=1}^n x_i\right)^{\frac{n-3}{2}}}{\sqrt{x_1 \cdots x_n}} h_n\left(\frac{\tau}{\sqrt{1 - \sum_{j=1}^n x_j}}\right),$$

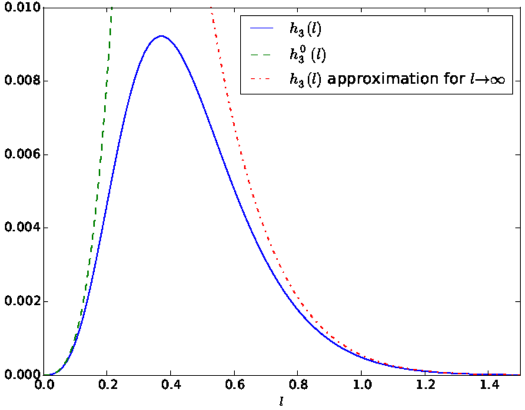
where

$$h_n(l) = \int_0^l dl_{n-1} \cdots \int_0^{l_2} dl_1 (nl - \sum_{i=1}^{n-1} l_i) \mathrm{e}^{-\pi(nl - \sum_{j=1}^{n-1} l_j)^2}.$$

Time dependence enters only through the combination $l = \frac{\tau}{\sqrt{1 - \sum_{j=1}^n x_j}}$.

Example, $D^{(3)}$

$$D^{(3)}(x_1, x_2, x_3|\tau) = \frac{3}{\sqrt{x_1 x_2 x_3}} h_3\left(\frac{\tau}{\sqrt{1 - x_1 - x_2 - x_3}}\right)$$



Multiplicities and KNO scaling

The number of gluons with $\omega > \omega_0$ assuming $\omega_0 \ll \omega_{br}(L)$ is

$$\langle N \rangle(\omega_0) = \int_{\omega_0}^{E} d\omega \frac{dN}{d\omega} \sim 2\sqrt{\frac{\omega_{br}(L)}{\omega_o}}$$

One can similarly compute the higher moments

$$\langle N^p \rangle(\omega_0) \sim (p+1)! \left(\frac{\omega_{br}(L)}{\omega_0} \right)^{p/2}$$

$$C_p = \frac{\langle N^p \rangle}{\langle N \rangle^p} = \frac{(p+1)!}{2^p}$$

C_p is a pure number independent of $\hat{q}$, α_s, E and L. This property is called KNO scaling.

KNO scaling, the negative binomial distribution and jet physics

- We recognize in the previous values of C_p the negative binomial distribution with parameter $k = 2$.
- Probability of having n successful attempts in a Bernoulli trial before having k failures.
- A jet in the vacuum also fulfils KNO scaling and can be approximately described by a negative binomial with $k = 3$. (Dokshitzer, Khoze, Mueller and Troian *Basics of perturbative QCD*).
- The smaller the value of the parameter k, the larger the fluctuations.
- The gluon distribution generated by medium induced radiation stronger statistical fluctuations than that of a jet in the vacuum.

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Conclusions

- Fluctuations are large.
- Fluctuations compete with path length difference in determining the di-jet asymmetry.
- The standard deviation of the energy loss is of the order of its average.
- The gluons multiplicities produced by the jet can be described by a negative binomial distribution and fulfill KNO scaling.
- Correlations and fluctuations in the number of particles are large and bigger that what is found in a jet vacuum cascade.