Charge redistribution from novel magneto-vorticity coupling in anomalous hydrodynamics

KH and Yi Yin, **PRL 117 (2016) 15. [arXiv:1607.01513 [hep-th]]**

KH Y. Hirono, H-U. Yee, and Yi Yin, In preparation.

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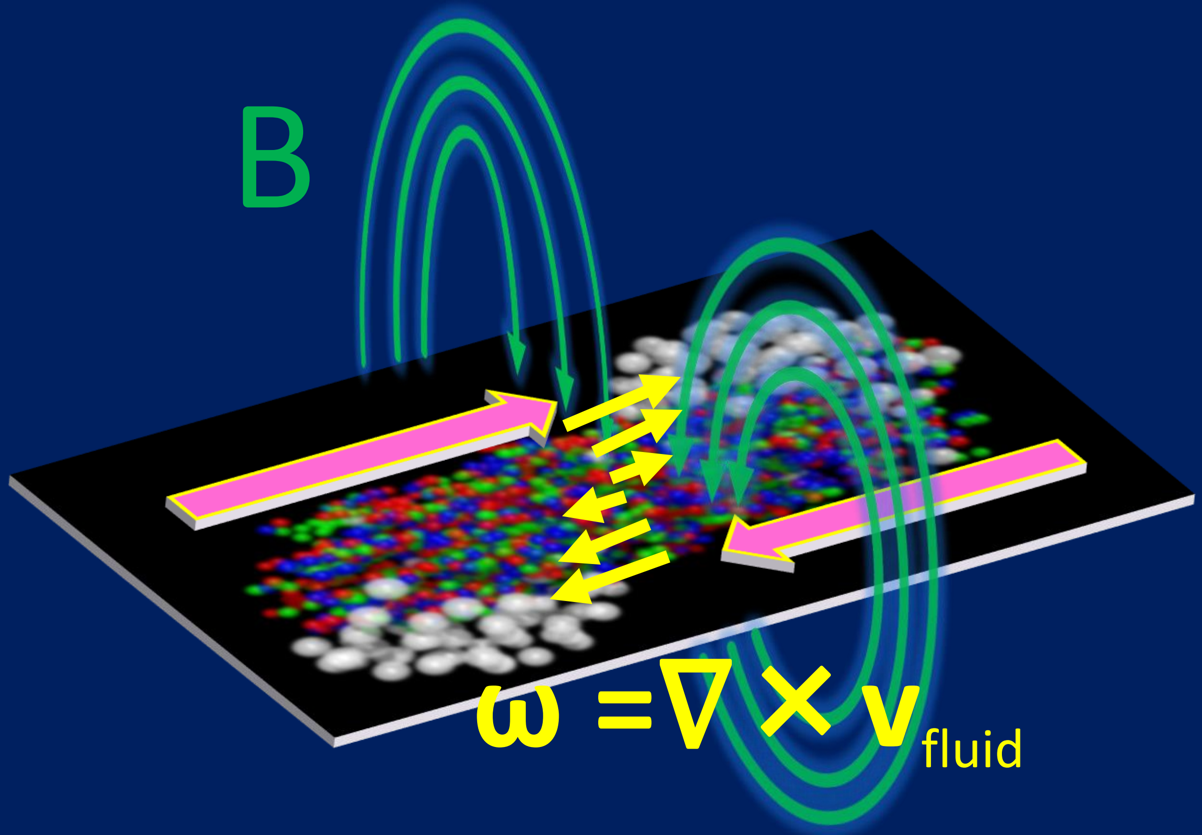
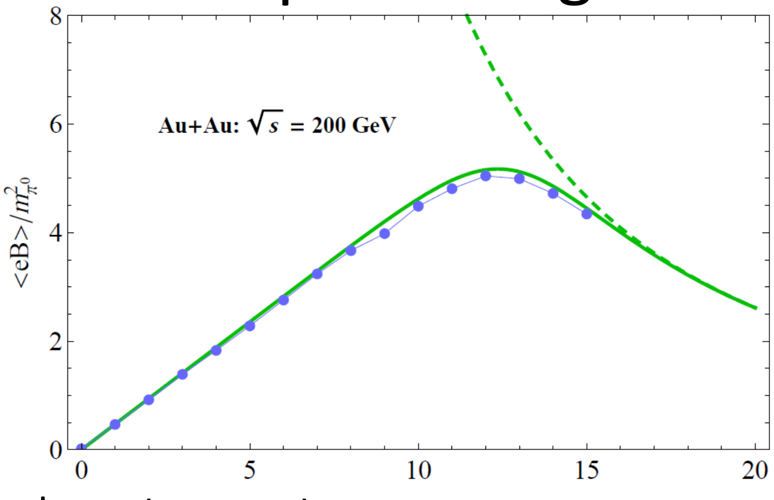
Quark Matter 2017 @ Chicago

Strong magnetic field & vorticity/angular momentum induced by heavy-ion collisions



Theoretical estimates

Super-strong B

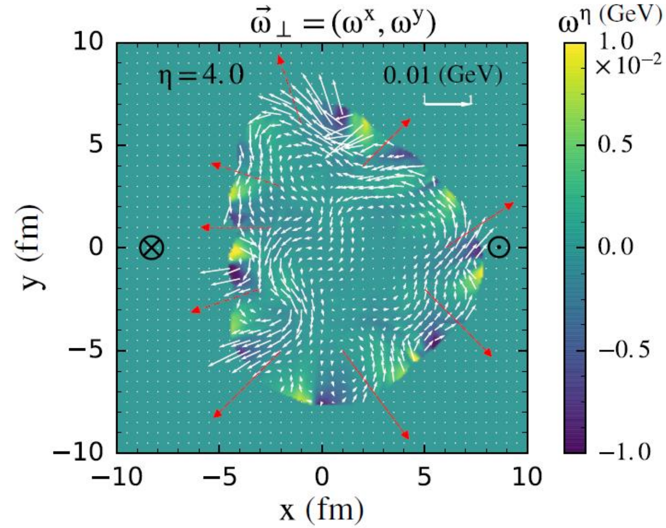


Impact parameter

Deng & Huang (2012), KH & Huang (2016)

Skokov et al. (2009), Voronyuk et al. (2011), Bzdak, Skokov (2012) McLerran, Skokov (2014)

Vorticity in HIC



Pang, Petersen, Wang, Wang (2016)

Becattini et al., Csernai et al., Huang, Huovinen, Wang

Jiang, Lin, Liao(2016) Deng, Huang (2016)

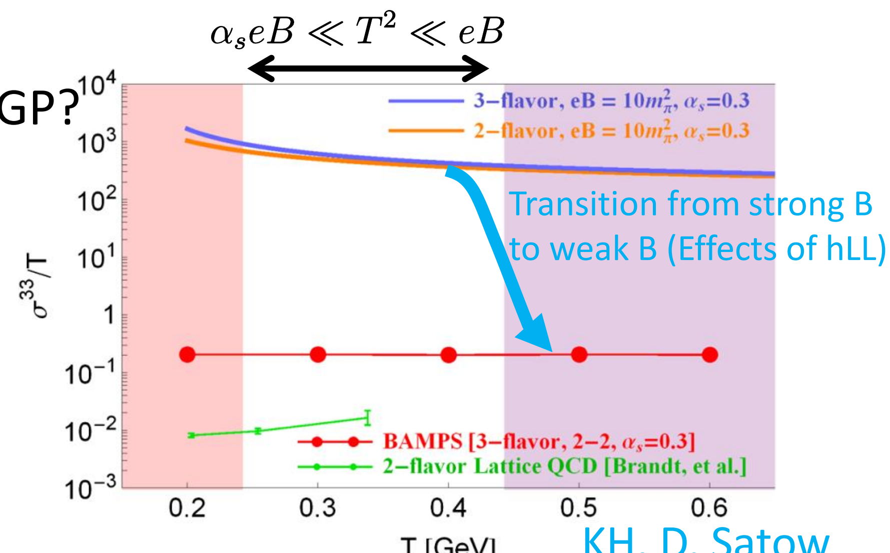
Cf., Long-lived B thanks to back reaction from QGP?

→ Need to know Electrical conductivity in B

KH, S. Li, D. Satow, H.-U. Yee

Extension of AMY' framework in a strong B.

-- Novel damping mechanism emerging only in B.



KH, D. Satow

How do their effects manifest?

Anomaly-induced transports in a magnetic OR vortex field

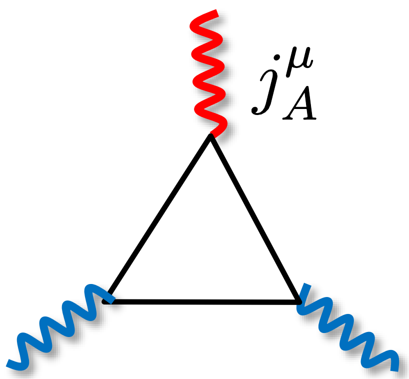
$$\begin{pmatrix} j_V^\mu \\ j_A^\mu \end{pmatrix} = C_A \begin{pmatrix} q_f \mu_A & q_f \mu_V \\ \mu_V \mu_A & (\mu_V^2 + \mu_A^2)/2 + C_A^{-1} T^2/12 \end{pmatrix} \begin{pmatrix} B^\mu \\ \omega^\mu \end{pmatrix}$$

$$B^\mu = \tilde{F}^{\mu\nu} u_\nu \qquad \omega^\mu = \frac{1}{2}\epsilon^{\mu\alpha\beta\gamma} u_\alpha \partial_\beta u_\gamma$$

Coefficients are time-reversal even.
Non-dissipative transport phenomena is
due to topology in quantum anomaly and is nonrenormalizable.

Anomaly relation: $\partial_\mu j_A^\mu = q_f^2 C_A \boldsymbol{E} \cdot \boldsymbol{B}$

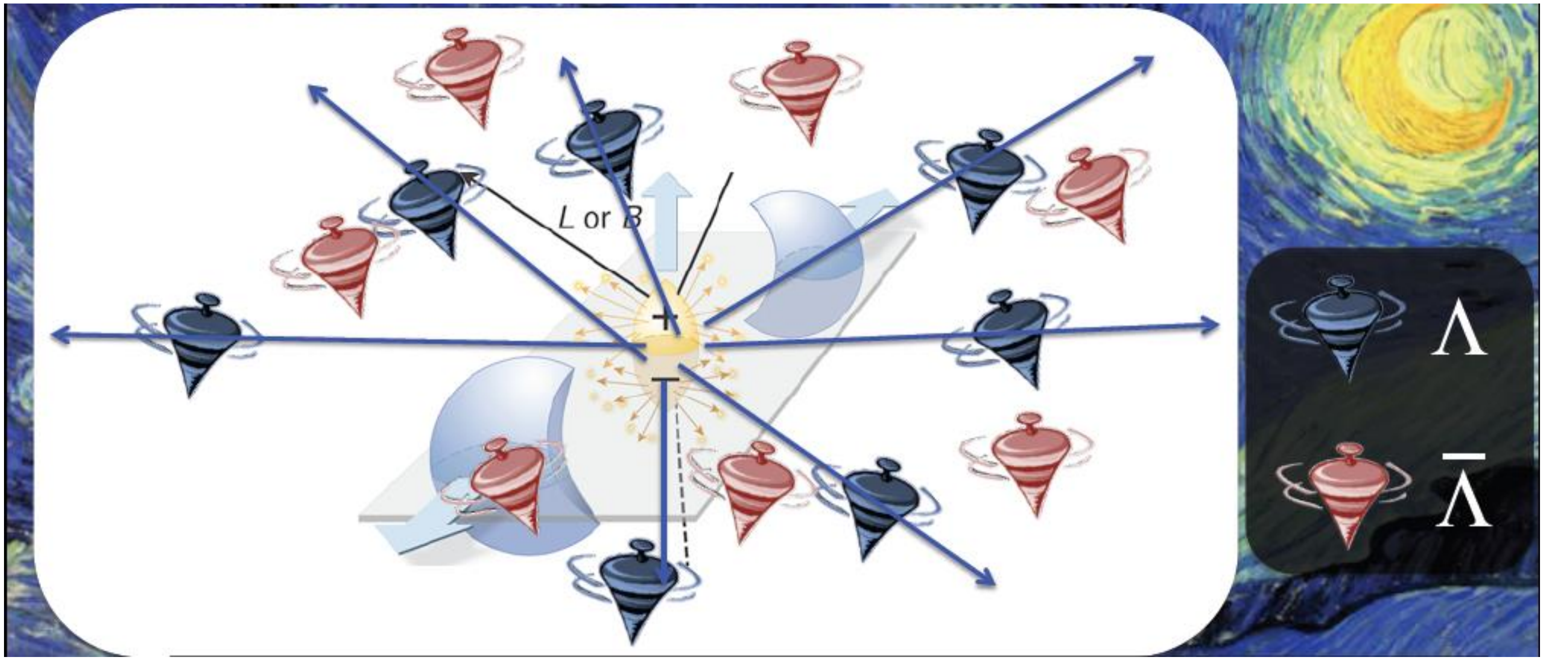
$$C_A = \frac{1}{2\pi^2}$$



Spin polarizations from spin-rotation coupling

$$f^{\pm}(\epsilon,\omega) = f_0^{\pm}(\epsilon - \boldsymbol{S}\cdot\boldsymbol{\omega}) \qquad f_0^{\pm}(\epsilon) = \frac{1}{e^{\pm\beta(\epsilon-\mu)}+1}$$

Λ polarization



Slide by M. Lisa

See talks by Upsal, Karpenko, Pang, Q.Wang

Becattini et al., Glastad & Csernai, Gyulassy & Torrieri, Xie,,,,

What would be missing?

An interplay B ⊗ ω

For dimensional reason, one would get

$$j \sim \# \; \boldsymbol{B} \cdot \boldsymbol{\omega}$$

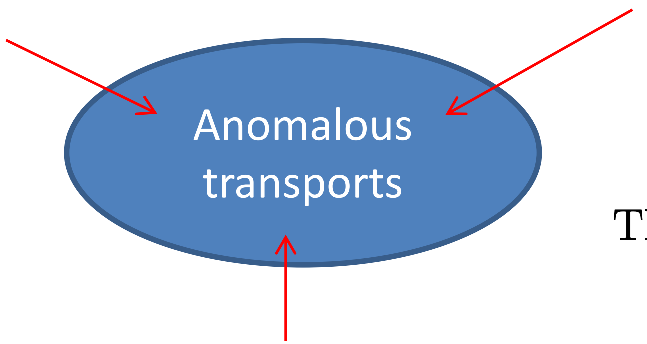
Cf., In CVE, it was

$$j^{\mu}_{R/L} = C_A \mu^2_{R/L} \omega^{\mu}$$

Could the magneto-vorticity coupling be important ??

QFT with Kubo formula

Anomalous hydrodynamics due to Son and Surowka



Chiral kinetic theory

$$j^{\mu} = \sigma_{\rm Ohm} E^{\mu} + \xi_B B^{\mu} + \xi_{\omega} \omega^{\mu}$$

The first-order derivative expansion $[A^{\mu} \sim \mathcal{O}(\partial^0),\ v^{\mu} \sim \mathcal{O}(\partial^0)]$

$$E^{\mu} \sim B^{\mu} \sim \omega^{\mu} \sim \mathcal{O}(\partial^1)$$

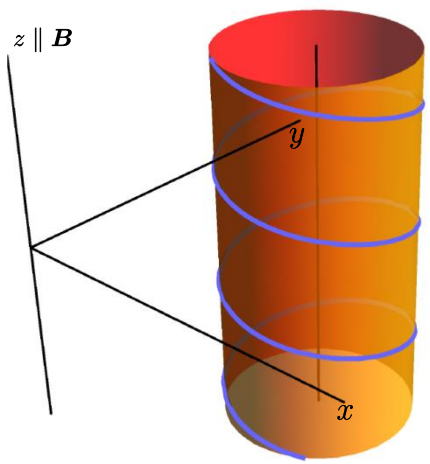
→ Yes, it is important when B is so strong that B >> $O(\partial^1)$.

Q1. Is the coefficient related to any quantum anomaly?
Q2. How is T and/or μ dependence?

Dynamics in the lowest Landau levels
-- A quick reminder

Energy levels in B – Chirality eigenstates in the lowest Landau levels

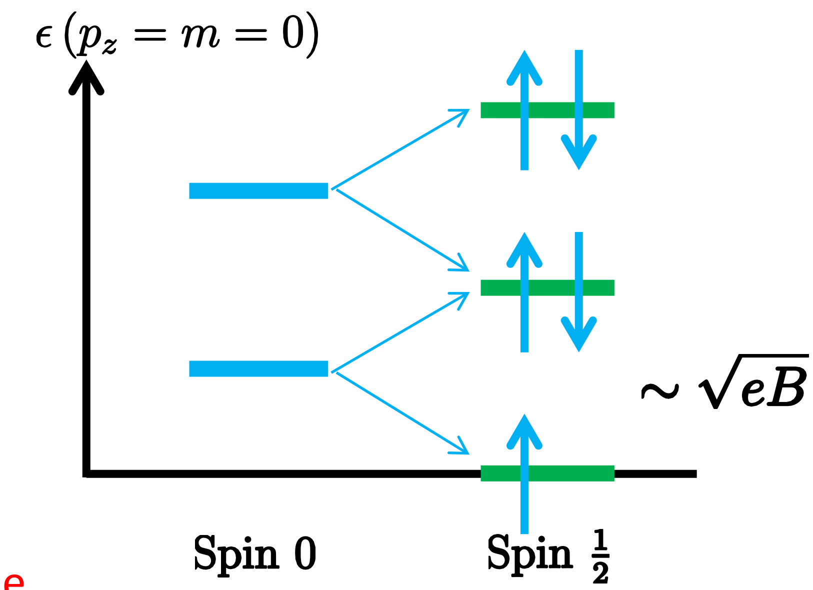
Periodic cyclotron motion
→ Landau level discretization



"Harmonic oscillator" in the transverse plane

$$\epsilon_n^2 = p_z^2 + (2n+1)eB$$

Zeeman splitting

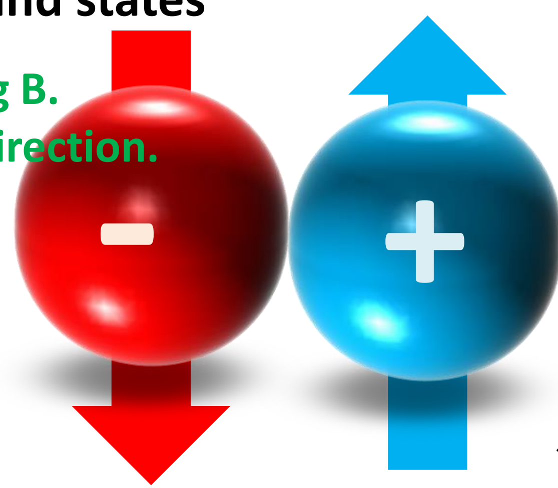


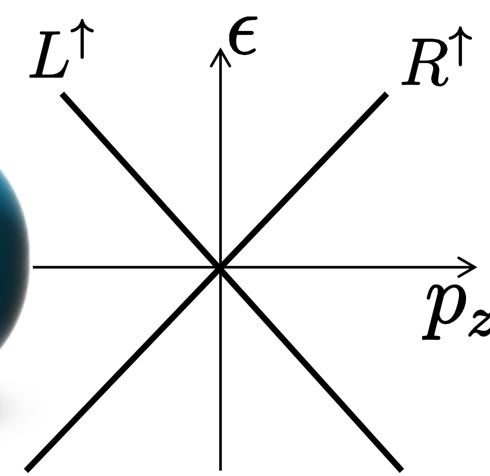
$$\Delta\epsilon_{\rm Zeeman} = \pm eB$$ (If g = 2.)

The lowest Landau levels – Unique ground states

1. **Spin directions are "frozen" due to strong B.**
2. **Chirality is locked with the momentum direction.**
3. **Massless LLL states realize chiral fermions in the (1+1) dim.**

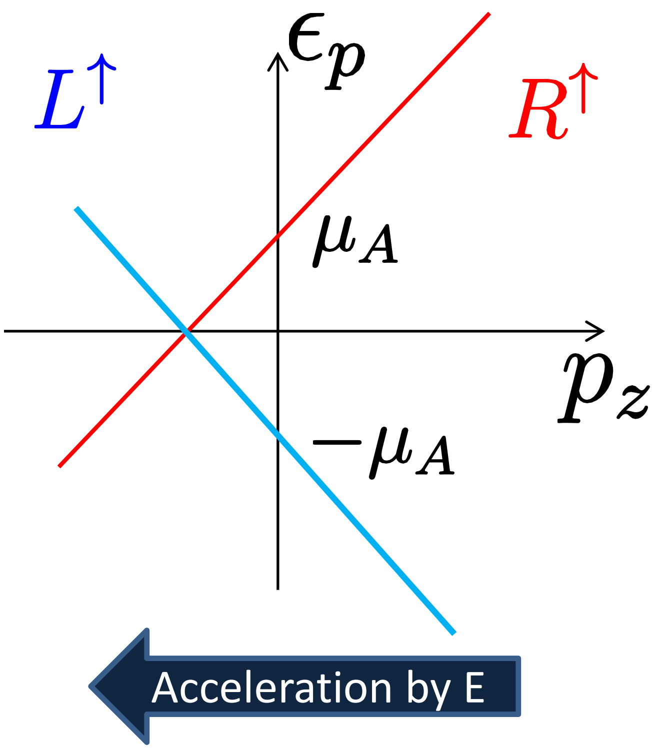
$$\epsilon_{n=0} = \pm p_z$$

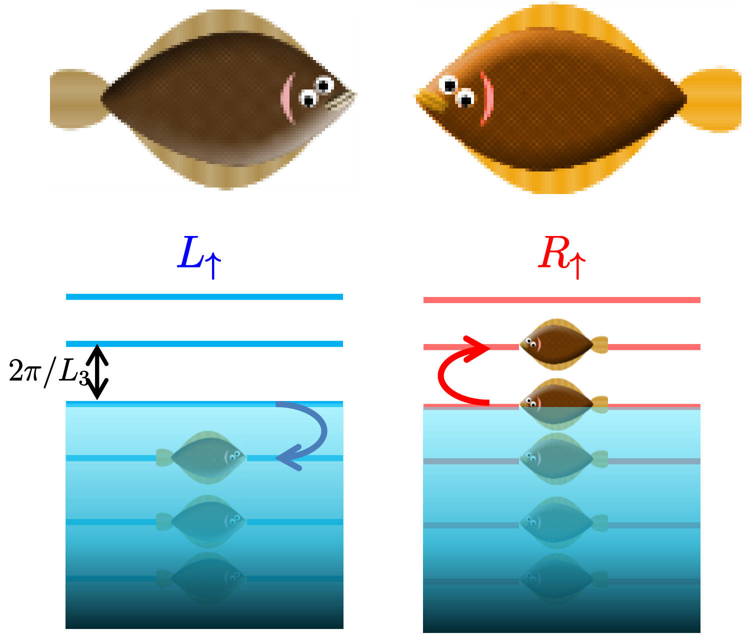




Chiral anomaly in (1+1) dimension

When a parallel electric field (E//B) is applied,





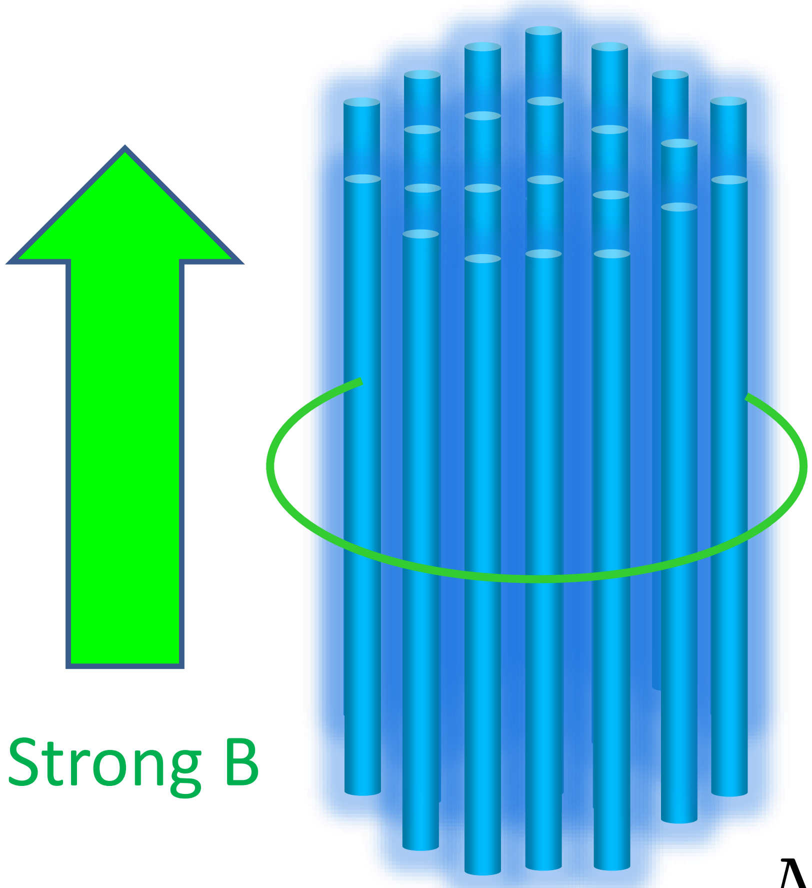
Creation/annihilation near the surface of the bottomless Dirac sea.

Nielsen-Ninomiya, Ambjorn-Greensite-Peterson

"Density" of cyclotron motions

In a homogeneous B, there is a translational invariance.

There are no preferred positions for the center of the cyclotron motion.



Degeneracy in the transverse plane: $\rho = \dfrac{N_{\text{state}}}{S} = \dfrac{eB}{2\pi}$

Consequences of a magneto-vorticity coupling

Shift of thermal distribution functions by the spin-vorticity coupling

Spin-vorticity coupling

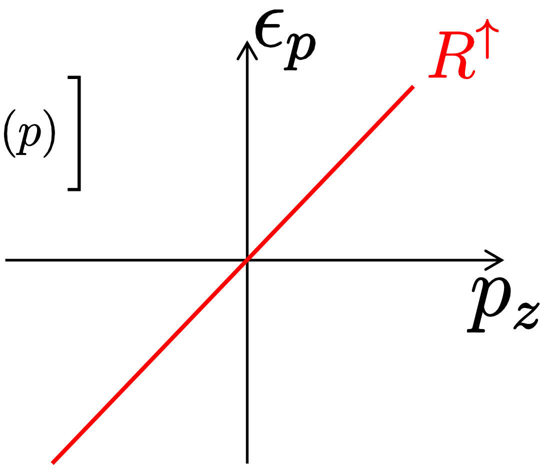
$$f^{\pm}(\epsilon, \omega) = f_0^{\pm}(\epsilon - \boldsymbol{S} \cdot \boldsymbol{\omega}) \qquad f_0^{\pm}(\epsilon) = \frac{1}{e^{\pm\beta(\epsilon-\mu)} + 1}$$

Landau & Lifshitz, Becattini et al.

In the LLL, the spin direction is aligned along a magnetic field .

$$\Delta\epsilon^{\pm} \equiv -\boldsymbol{S} \cdot \boldsymbol{\omega} = \mp \mathrm{sgn}(q_f)\frac{1}{2}\hat{\boldsymbol{B}} \cdot \boldsymbol{\omega}$$

- for particle
+ for antiparticle



Number density

$$n_R = \frac{|q_f B|}{2\pi}\left[\int_0^{\infty}\frac{dp_z}{2\pi}f^{+}(p) + \int_{-\infty}^{0}\frac{dp_z}{2\pi}f^{-}(p)\right]$$

At the LO in the energy shift Δε

$$\Delta n_R = \frac{|q_f B|}{2\pi}\left[\Delta\epsilon^{+}\int_0^{\infty}\frac{dp_z}{2\pi}\frac{\partial f_0^{+}(p_z)}{\partial p_z} + \Delta\epsilon^{-}\int_{-\infty}^{0}\frac{dp_z}{2\pi}\frac{\partial f_0^{-}(p_z)}{\partial p_z}\right]$$

$$\Delta n_R = q_f \frac{C_A}{4} \boldsymbol{B} \cdot \boldsymbol{\omega} [f_0^+(0) + f_0^-(0)]$$
$$= q_f \frac{C_A}{4} \boldsymbol{B} \cdot \boldsymbol{\omega}$$

$f_0^+(0) + f_0^-(0) = 1$ identically for any T and μ.

The shift is independent of the chirality, and depends only on the spin direction.

$$\Delta n_L = \Delta n_R$$

In the V-A basis,

$$\Delta n_V = \Delta n_R + \Delta n_L = q_f \frac{C_A}{2} \boldsymbol{B} \cdot \boldsymbol{\omega}$$
$$\Delta n_A = \Delta n_R - \Delta n_L = 0$$

½ from the size of the spin

Spatial components of the current

$$\Delta j_R^3 = v_R \Delta n_R \qquad j^1 = j^2 = 0 \text{ for the LLL}$$

Velocity: $v_{R/L} = \pm \mathrm{sgn}(q_f B)$

The shift depends on the chirality through the velocity.

$$\Delta j_R^3 = -\Delta j_L^3$$

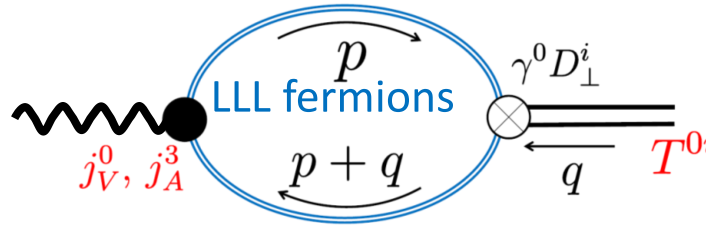
In the V-A basis,

$$\Delta j_V^3 = 0$$
$$\Delta j_A^3 = |q_f| \mathrm{sgn}(B) \frac{C_A}{2} \boldsymbol{B} \cdot \omega$$

Field-theoretical computation by Kubo formula

Perturbative ω in a strong B

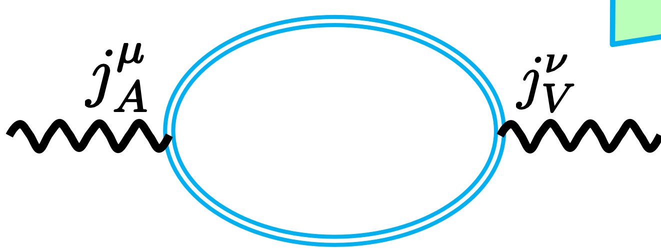
$$\lambda = -2i \lim_{q_x \to 0} \frac{\partial}{\partial q_x} \langle n_V(x) T^{02}(x') \rangle \theta(t-t')$$



Similar to the Kubo formula used to get the T^2 term in CVE (Landsteiner, Megias, Pena-Benitez)

We confirm

1. the previous results obtained from the shift of distributions.
2. a relation of $\langle n_V T^{02} \rangle$ to the chiral anomaly diagram in the (1+1) dim.

$\Pi_{AV}^{\mu\nu} =$ 

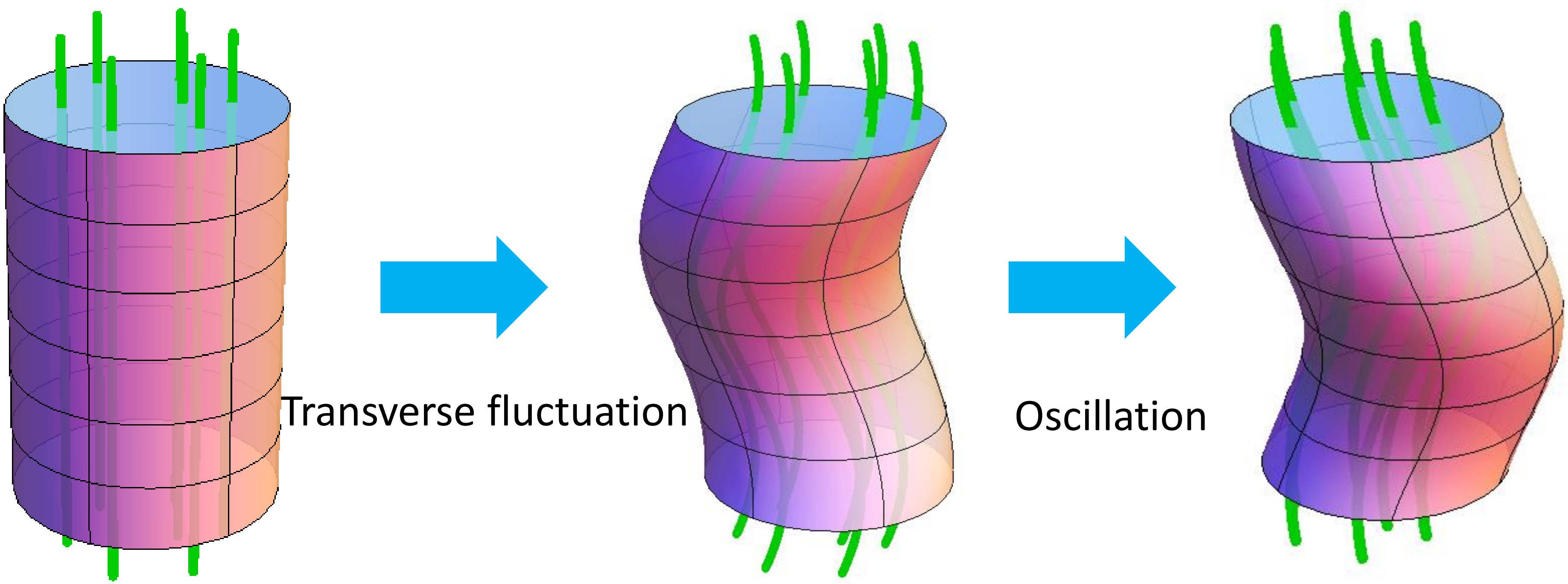
$$q_\mu \Pi_{AV}^{\mu\nu} \neq 0 \ !!!$$

There is no T or μ correction in the massless limit, since it is related to the chiral anomaly!

Collective excitations and instabilities of chiral fluid with CME

Magneto-hydrodynamics (MHD) in conducting plasmas

Alfven's theorem = "Frozen-in" condition:
Magnetic flux is frozen in a fluid volume
and moves together with the fluid.



Tension of B-field = Restoring force
Fluid mass density = Inertia

Transverse Alfven wave

Alfven wave from a linear analysis

$$\boldsymbol{B}_0 \neq 0\,,\ T > 0\,,\ \mu_V = 0$$

Navier-Stokes + Maxwell eqs.

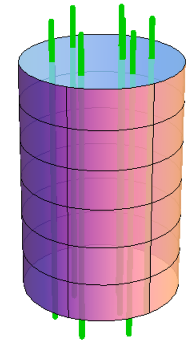
$$\partial_\mu T^{\mu\nu} = F^{\nu\alpha} j_\alpha$$

$$\partial_\mu F^{\mu\nu} = j^\nu,\ \partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$j^\mu = \sigma_{\mathrm{Ohm}} E^\mu$$

Stationary solutions

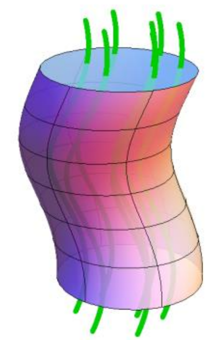
$$u^\mu = (1, \boldsymbol{0}),\ B^\mu = (0, \boldsymbol{B}_0),\ j^\mu = (0, \boldsymbol{0})$$



Transverse perturbations

$$\boldsymbol{B}_0 \perp \delta\boldsymbol{v}, \delta\boldsymbol{B}, \delta\boldsymbol{j}(z)$$

as functions of z.



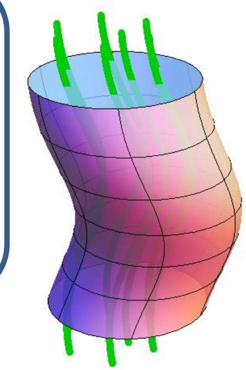
Wave equation

$$\partial_t^2 \delta\boldsymbol{B}(t,z) = \frac{B_0^2}{\epsilon + p} \partial_z^2 \delta\boldsymbol{B}(t,z)$$

Alfven wave velocity

Transverse wave propagating along background B_0

$$\boldsymbol{B}_0 \parallel \boldsymbol{k}$$



Same wave equation for δv

→ Fluctuations of B and v propagate together.

Anomalous MHD in conducting plasmas

$$\mu_A \neq 0\,,\ \boldsymbol{B}_0 \neq 0\,,\ T > 0\,,\ \mu_V = 0$$

A finite CME current without CVE: $j^\mu = \sigma_{\rm Ohm} E^\mu + \sigma_{\rm CME} B^\mu$

Wave equation

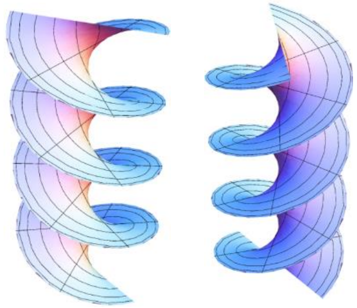
$$\partial_t^2 \delta \boldsymbol{B}(t,z) = v_{\rm Alf}^2\, \partial_z^2 \delta \boldsymbol{B}(t,z) + \sigma_{\rm CME} \nabla \times \partial_t \delta \boldsymbol{B}$$

$$v_{\rm Alf}^2 = \frac{B_0^2}{\epsilon + p}$$

and a similar wave equation for δv.

Helicity decomposition (Circular R/L polarizations)

$$\nabla \times \boldsymbol{e}_{R/L} = \pm \boldsymbol{e}_{R/L}$$



Two modes for each helicity propagating in parallel to B AND in antiparallel to B.

In total, there are 4 modes.

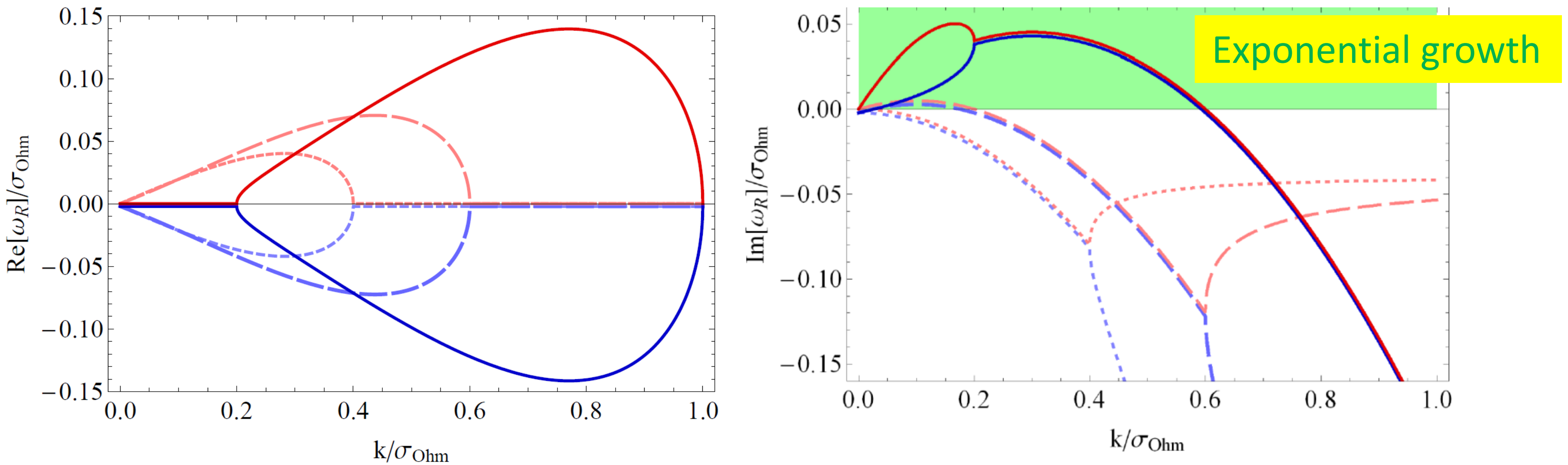
$\sigma_{\rm CME} = 0$

$\sigma_{\rm CME} = 0.2\sigma_{\rm Ohm}$

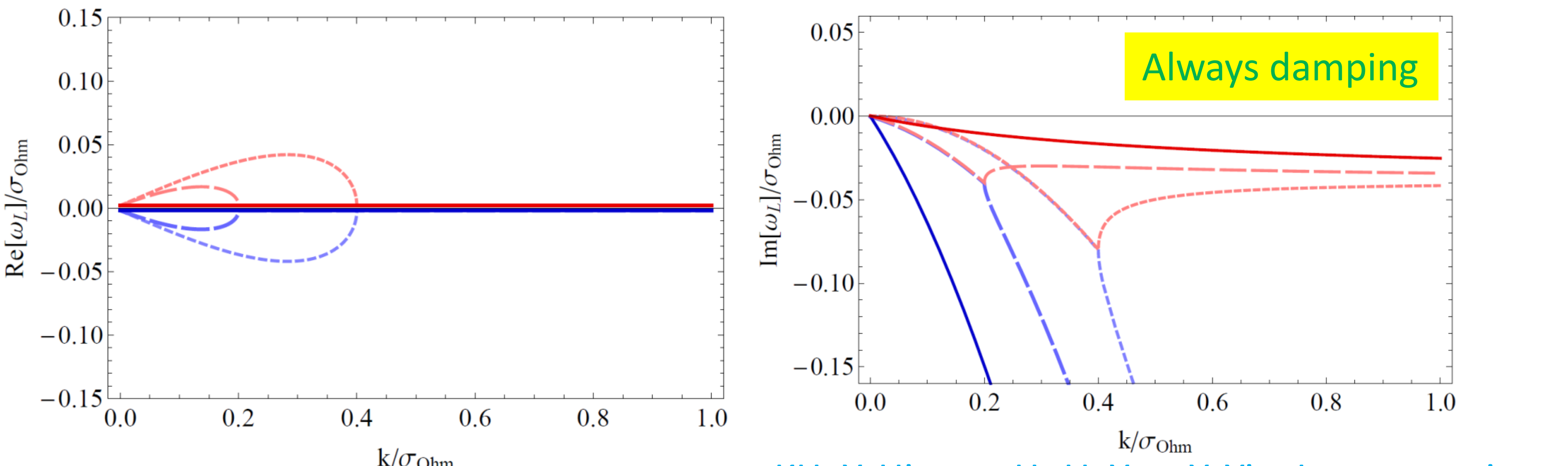
$\sigma_{\rm CME} = 0.6\sigma_{\rm Ohm}$

$v_{\rm Alf} = 0.2$

R-helicity



L-helicity

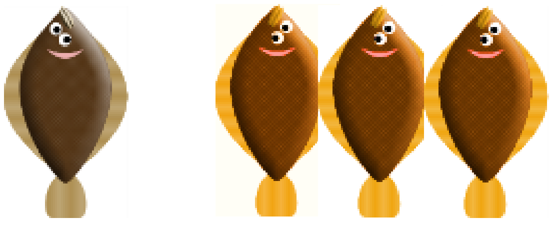


KH, Y. Hirono, H.-U. Yee, Y. Yin, In preparation.

Helicity conversions

Akamatsu, Yamamoto
Hirono, Kharzeev, Yin
Xia, Qin, Wang

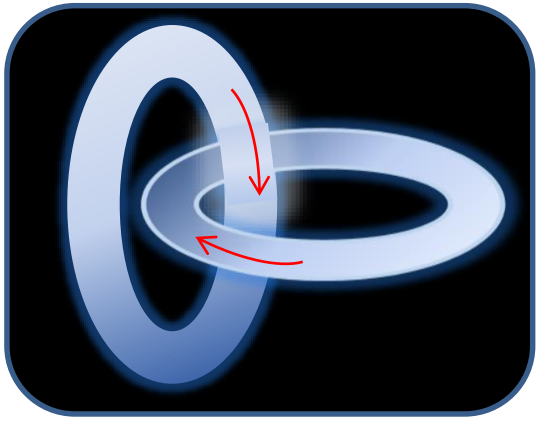
Chiral imbalance
btw R and L fermions



Chiral Plasma Instability (CPI)

Magnetic helicity

Fluid helicity (structures of vortex strings)



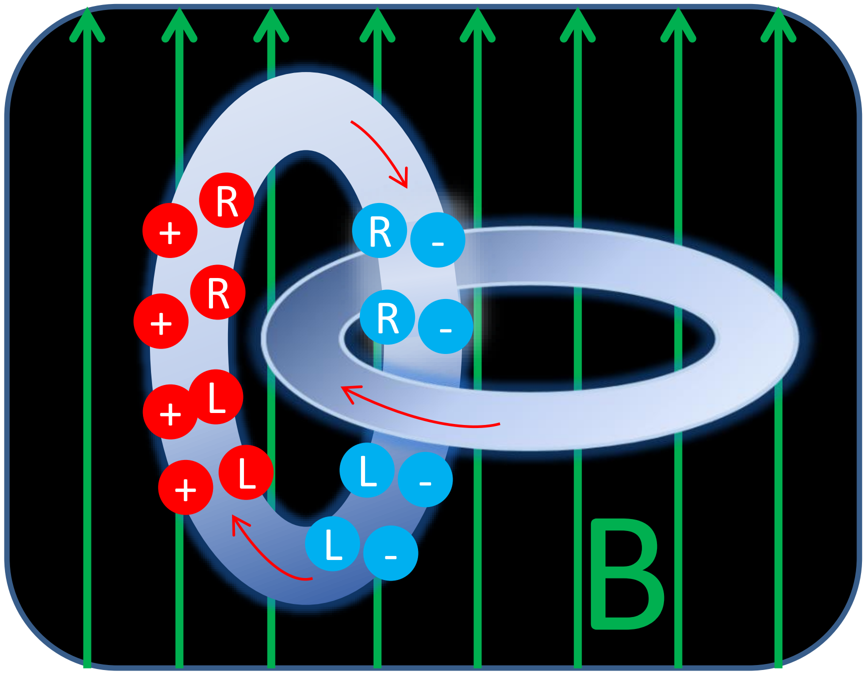
What would be a fate of the instability
in the chiral fluid?

See also Y. Hirono's talk on Feb. 10.

Summary 1

A magneto-vorticity coupling B ⊗ ω induces charge redistributions without μ_A.

- Related to the chiral anomaly in (1+1) dimensions.
- No T or μ correction.



When $\boldsymbol{B} \cdot \boldsymbol{\omega} \neq 0$,

$$j^0_{EM,V} = q_f^2 \frac{C_A}{2} (\boldsymbol{B} \cdot \boldsymbol{\omega})$$

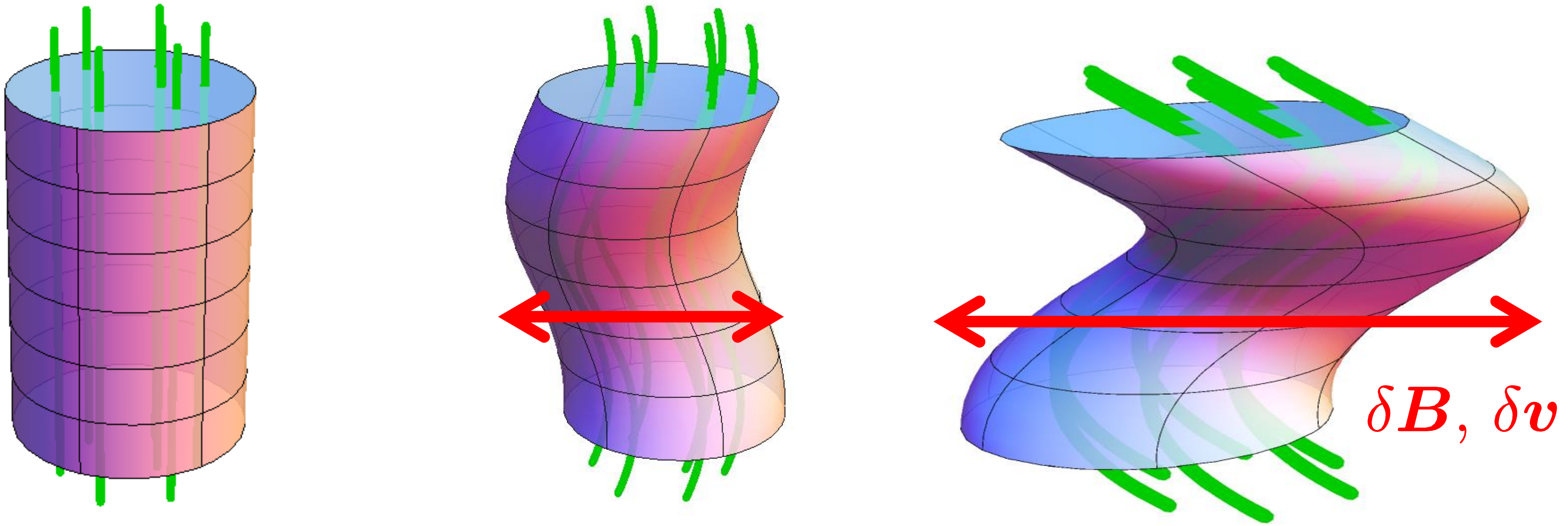
$$j^3_{EM,A} = \mathrm{sgn}(q_f) q_f^2 \frac{C_A}{2} (\boldsymbol{B} \cdot \omega) \hat{\boldsymbol{B}}$$

Emerges even without μ_A.

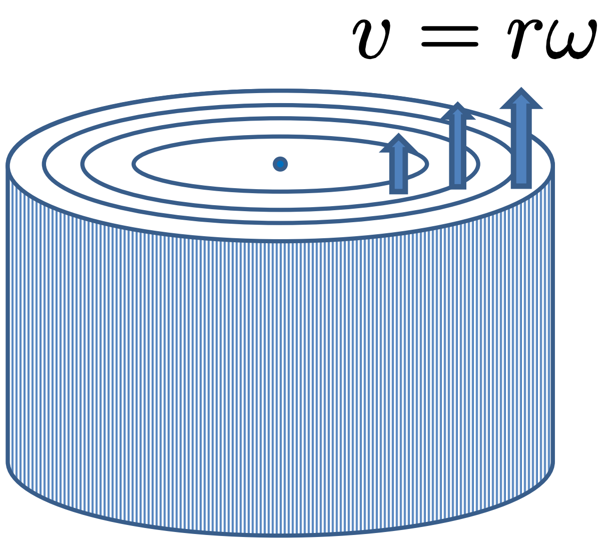
Summary 2

We observed an onset of instabilities in both B and fluid velocity v in anomalous hydrodynamics, when a chiral imbalance induces a CME current.

Amplitudes of both B and v grow exponentially in time.

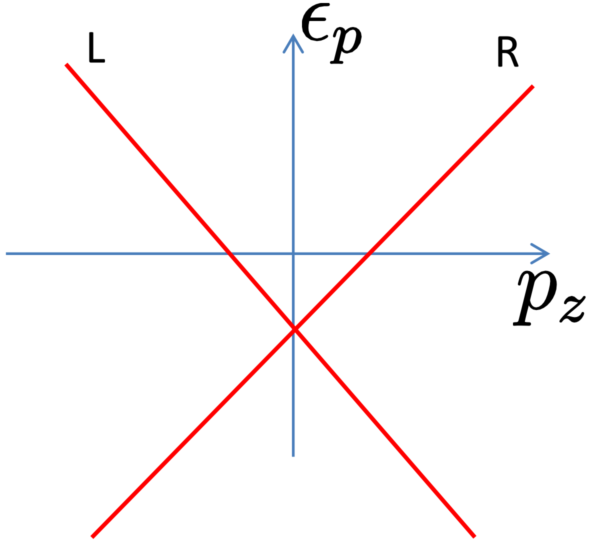


Effects of the viscosity act to suppress the instability
and change the wave velocity. [KH, Y. Hirono, H.-U. Yee, Y. Yin, In preparation.]



Causality problem:
A rigid rotation of an infinite-size system breaks causality in the peripheral region.

One must have a finite-size system with an exterior boundary or a local vortex field.

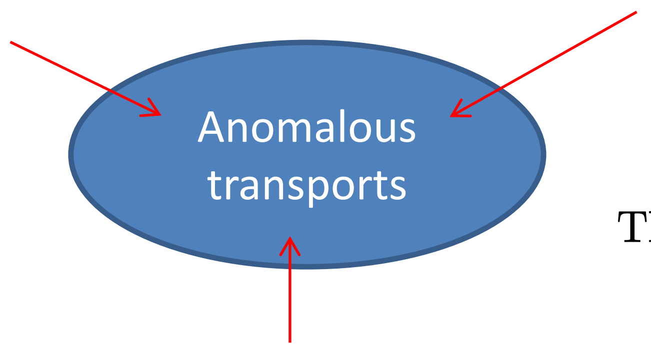


Local shift of the charge density
→ Redistribution of charge in the system

E.g., when $v \to 0$ sufficiently fast,

$$\Delta n_{\mathrm{net}} \propto \int d^3x \boldsymbol{B} \cdot \boldsymbol{\omega} = \frac{1}{2} \int d^3x \nabla \cdot (\boldsymbol{B} \times \boldsymbol{v}) = \frac{1}{2} \int_{\partial S} dS \cdot (\boldsymbol{B} \times \boldsymbol{v}) = 0$$

Consistent predictions from fundamental and effective theories



$$j^\mu = \sigma_{\rm Ohm} E^\mu + \xi_B B^\mu + \xi_\omega \omega^\mu$$

The first-order derivative expansion $[A^\mu \sim \mathcal{O}(\partial^0),\ v^\mu \sim \mathcal{O}(\partial^0)]$

$$E^\mu \sim B^\mu \sim \omega^\mu \sim \mathcal{O}(\partial^1)$$

B^μ and ω^μ terms would violate the second law of thermodyanmics if σ_{CME} and σ_{CVE} were arbitrary.

$$\partial^\mu s_\mu = X(\xi_B, \xi_\omega, C_A) B^\mu + Y(\xi_B, \xi_\omega, C_A)\omega^\mu + (\text{dissipative parts})$$

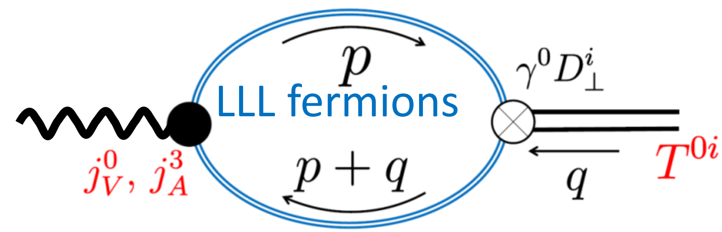
Cured by requiring the coefficients vanish identically.
→ Leads to the CME and CVE conductivities.

$$\xi_B = \sigma_{\rm CME},\ \xi_\omega = \sigma_{\rm CVE} \text{ up to the frame choice}$$

Field-theoretical computation by Kubo formula confirms the simplest calculation

Perturbative ω in a strong B

$$\lambda = -2i \lim_{q_x \to 0} \frac{\partial}{\partial q_x} \langle n_V(x) T^{02}(x') \rangle \theta(t-t')$$



Similar to the Kubo formula for CVE by Landsteiner, Megias, Pena-Benitez

$$n_V(x) = \bar{\psi}_{LLL}(x)\gamma^0\psi_{LLL}(x)$$

$$T^{0i}(x) = \tfrac{i}{2}\bar{\psi}_{LLL}(x)(\gamma^0 D^i + \gamma^i D^0)\psi_{LLL}(x) \qquad D^\mu = \partial^\mu + iq_f A^\mu_{\mathrm{ext}}$$

Retarded correlator in the coordinate space

$$\langle n_V(x) T^{02}(x') \rangle \theta(t-t') = \frac{1}{2i} \mathrm{tr}[\,\gamma^0 \mathcal{P}_+ S_{LLL}(x,x')\gamma^0 D^{i=2}_{x'} \mathcal{P}_+ S_{LLL}(x',x)\,]$$

The correlator has a manifest gauge invariance wrt the gauge of B_{ext}

Cf., $S_{LLL}(x',x) \to e^{-i\alpha(x')} S_{LLL}(x',x) e^{i\alpha(x)}$ $\qquad \mathcal{P}_+ = (1 + i\,\mathrm{sgn}(q_f B)\gamma^1\gamma^2)/2$

$$S_{LLL}(p) = 2e^{-\frac{|\boldsymbol{p}_\perp|^2}{q_f B}} \frac{i}{\not{p}_\parallel + m_f} \mathcal{P}_+$$

$$\langle n_V T^{02} \rangle = i\frac{q_f B}{8\pi} q_x \, \Pi^{00}_{1+1} \qquad \Pi^{\mu\nu}_{1+1} = \frac{1}{\pi}\frac{1}{q_\parallel^2}(q_\parallel^2 g_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu)$$

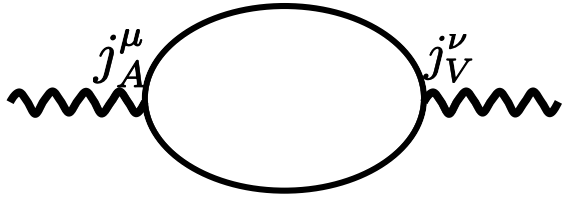
We confirm $n_V = q_f \frac{C_A}{2} \boldsymbol{B}\cdot\boldsymbol{\omega}$ and $\boldsymbol{j}_A = |q_f|\frac{C_A}{2}(\boldsymbol{B}\cdot\omega)\hat{\boldsymbol{B}}$.

Consistent with the previous observation from the shift of distributions.

Easy to establish a relation to the anomaly diagram

$$j_A^\mu = -\epsilon_\parallel^{\mu\nu} j_{V\nu} \qquad \epsilon_\parallel^{03} = -\epsilon_\parallel^{30} = 1$$

$$\Pi^{\mu\nu}_{AV} =$$



$$\Pi^{\mu\nu}_{AV} = -\epsilon_\parallel^{\mu\alpha}(\Pi_{1+1})_\alpha{}^\nu \quad \Rightarrow \quad q_\mu \Pi^{\mu\nu}_{AV} \neq 0 \text{ !!!}$$

$$q_\parallel^\mu \langle j_A \rangle_\mu = q_\parallel^\mu \epsilon_\parallel^{\mu\alpha}(\Pi_{1+1})_{\alpha\beta} A^\beta = \frac{1}{2\pi}\epsilon_\parallel^{\mu\nu} F_{\mu\nu} \qquad \langle j_V^\mu \rangle = -\Pi^{\mu\nu}_{1+1} A_\nu$$

for $q_f B > 0$. An overall minus sign appears when $q_f B < 0$.

There is no T or μ correction in the massless limit!

Alfven wave from a lienar analysis

$$\boldsymbol{B}_0 > 0\,,\ T > 0\,,\ \mu_V = 0$$

EoM

$$\partial_\mu T^{\mu\nu} = F^{\nu\alpha} j_\alpha$$
$$\partial_\mu F^{\mu\nu} = j^\nu,\ \partial_\mu \tilde{F}^{\mu\nu} = 0$$
$$j^\mu = \sigma_{\rm Ohm} E^\mu$$

Stationary solutions

$$u^\mu = (1, \boldsymbol{0}),\ B^\mu = (0, \boldsymbol{B}_0),\ j^\mu = (0, \boldsymbol{0})$$

Transverse perturbations

$$\boldsymbol{B}_0 \perp \delta\boldsymbol{v}, \delta\boldsymbol{B}, \delta\boldsymbol{j}(z)$$

as functions of z.

Linearlized eqs in an ideal hydro.

$$\begin{cases} \partial_t \delta\boldsymbol{B} = B_0\, \partial_z \delta\boldsymbol{v} \\ (\epsilon + p)\, \partial_t \delta\boldsymbol{v} = B_0\, \partial_z \delta\boldsymbol{B} \end{cases}$$

Transverse mode propagating along B0

$$\boldsymbol{B}_0 \parallel \boldsymbol{k}$$

Wave equation for the Alfven wave

$$\partial_t^2 \delta\boldsymbol{B}(t, z) = \frac{B_0^2}{\epsilon + p} \partial_z^2 \delta\boldsymbol{B}(t, z)$$

Alfven wave velocity

Anomalous MHD in conducting plasmas

$$\mu_A \neq 0\,,\ \ \boldsymbol{B}_0 \neq 0\,,\ \ T > 0\,,\ \ \mu_V = 0$$

A finite CME current without CVE: $\quad j^\mu = \sigma_{\rm Ohm} E^\mu + \sigma_{\rm CME} B^\mu$

Linearlized eqs in an ideal hydro.

$$\begin{cases} \partial_t \delta \boldsymbol{B} = B_0\, \partial_z \delta \boldsymbol{v} + \sigma_{\rm CME} \nabla \times \delta \boldsymbol{B} \\ (\epsilon + p)\, \partial_t \delta \boldsymbol{v} = B_0\, \partial_z \delta \boldsymbol{B} \end{cases}$$

$$\partial_t \begin{pmatrix} \delta \boldsymbol{B}_R \\ \delta \boldsymbol{v}_R \end{pmatrix} = \begin{pmatrix} \sigma_{\rm Ohm}^{-1} (\sigma_{\rm CME} k - k^2) & ikv_{\rm Alf}^2 \\ ik & 0 \end{pmatrix} \begin{pmatrix} \delta \boldsymbol{B}_R \\ \delta \boldsymbol{v}_R \end{pmatrix} \qquad v_{\rm Alf}^2 = \frac{B_0^2}{\epsilon + p}$$

$$\omega = -\frac{i}{2} \Xi\, k \pm k \sqrt{v_{\rm Alf}^2 - \frac{1}{4} \Xi^2} \qquad \Xi = \frac{\sigma_{\rm CME} - k}{\sigma_{\rm Ohm}}$$

When $\Xi > 0$, instabilities arise both in $\delta \boldsymbol{B}$ and $\delta \boldsymbol{v}$!!

Effects of the viscosity act to suppress the instability and change the wave velocity.

KH, Y. Hirono, H.-U. Yee, Y. Yin, In preparation.