Chiral magnetic effect and anomalous transport from real-time lattice simulations

Niklas Mueller

Heidelberg University

based on work together with: J. Berges, M. Mace, S. Schlichting, S. Sharma, N. Tanji

PRL 117 (2016) 142301, PRD 93 (2016) 074507, arXiv:1612.02477

Quark Matter 2017 - Hyatt Regency Chicago

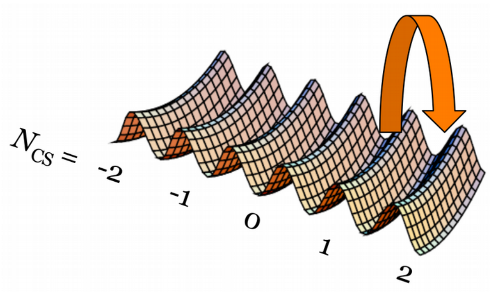
Chicago - *2017 / 02 / 07*

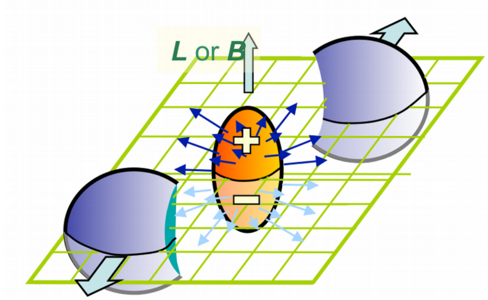
Outline

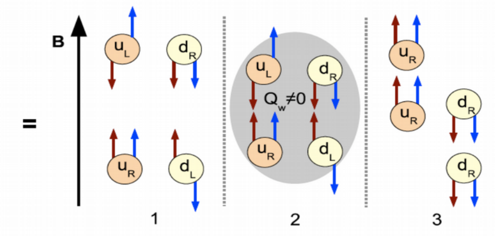
1. Anomalous phenomena in heavy ion collisions

2. Classical-statistical simulations

3. Conclusions



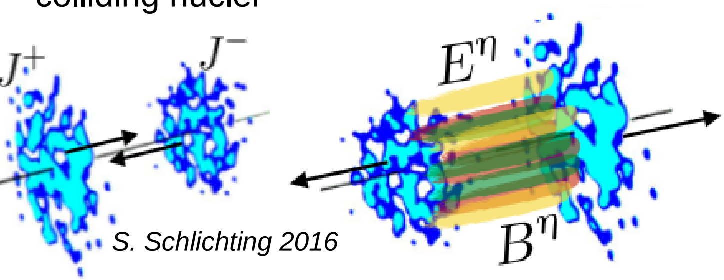




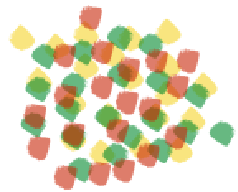
1. Anomalous Phenomena in Heavy Ion Collisions

CGC colliding nuclei

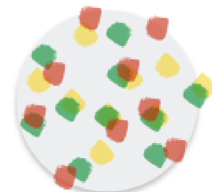
flux tubes



over-occupied plasma



kinetic regime



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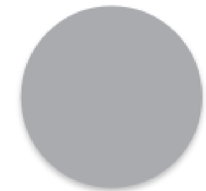

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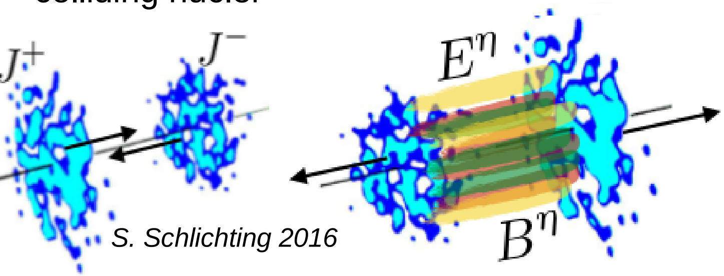

S. Schlichting 2016

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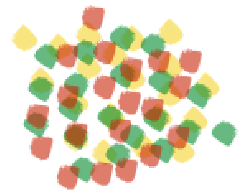

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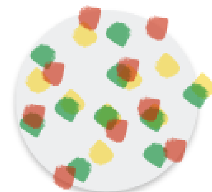

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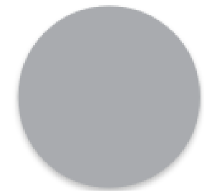

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from coherent fields (Tanji et al. 2016)
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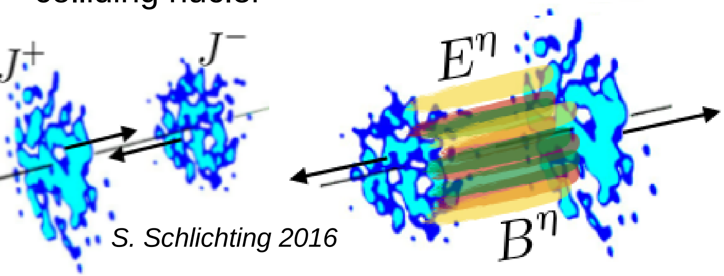

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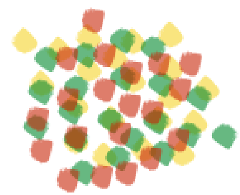

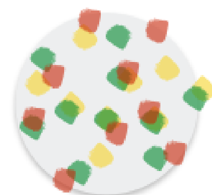

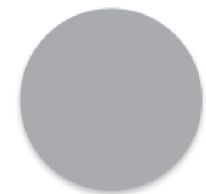

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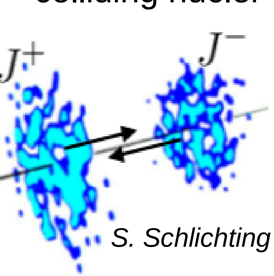

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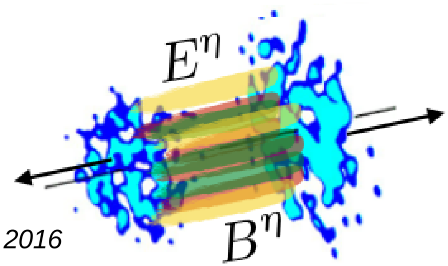

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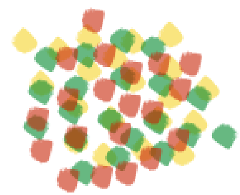

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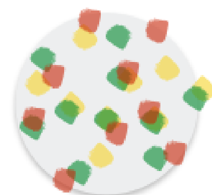

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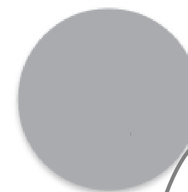

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Subsequent interactions in the fire ball,
axial transport and relaxation
weak to strong coupling

Thermalization / Freezout

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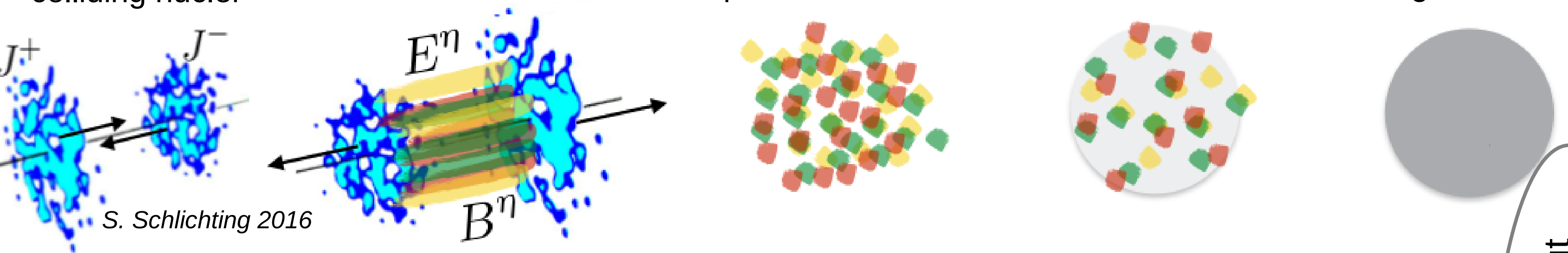

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Theory:

classical statistical simulations + fermions | chiral kinetic theory | anomalous hydrodynamics

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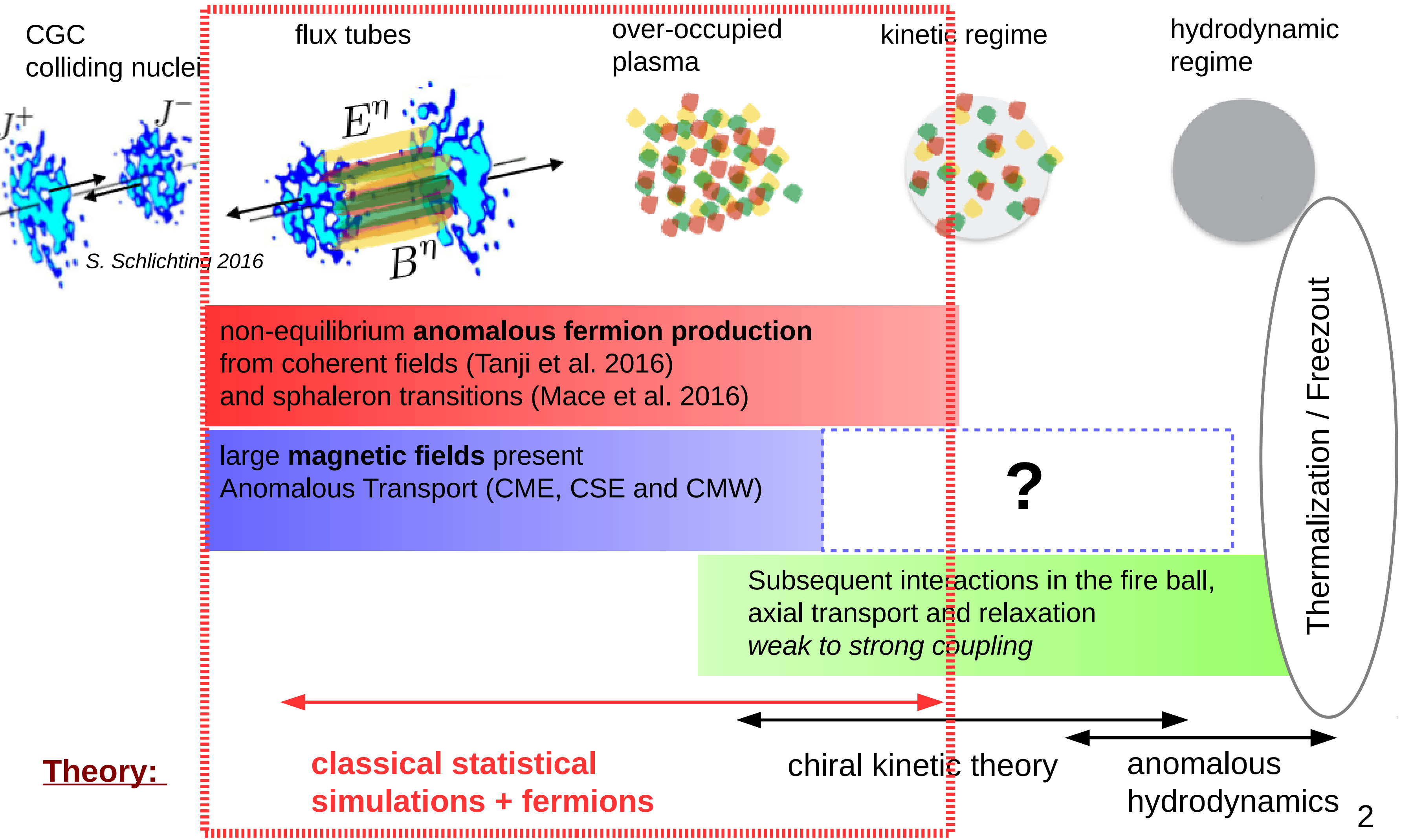


Anomalous fermion dynamics induced by a topologicial transition

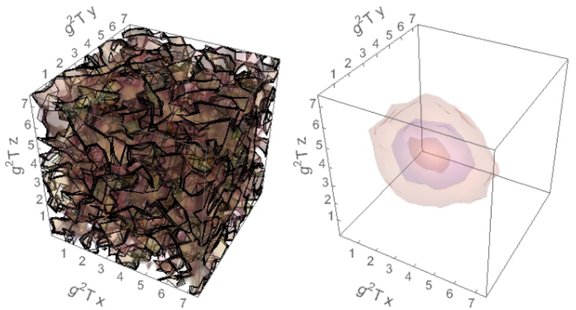
→ **Classical statistical simulations**

2. Real-time simulations

Anomalous fermion dynamics induced by a topologicial transition

→ Classical statistical simulations

simplified situation: setting up an isolated sphaleron transition in background abelian magnetic fields



Mace et al 2016 (see poster)

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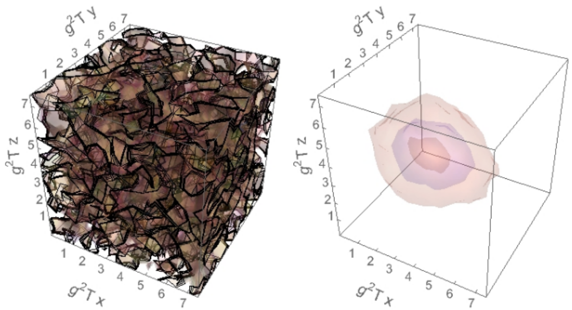

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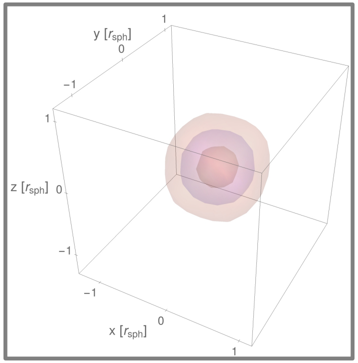

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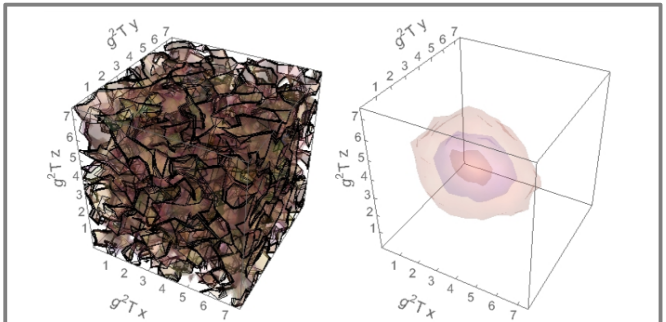

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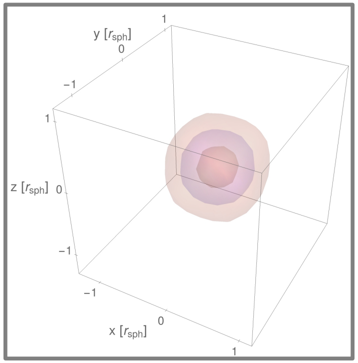

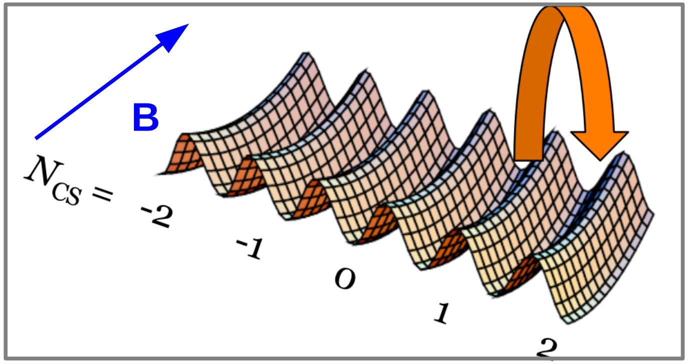

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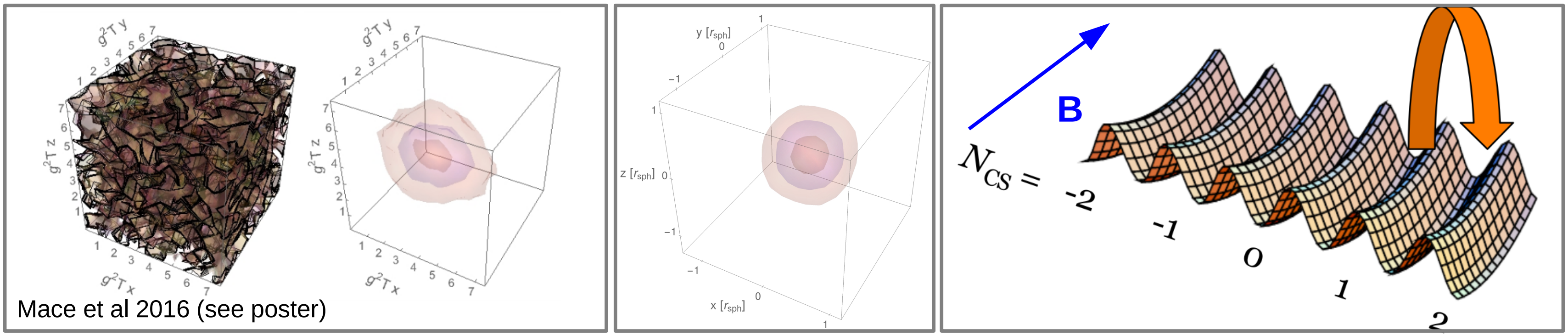

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- consistent treatment of axial charge production, non-abelian gauge fields as **dynamical degrees** of freedom.

Fermions: Challenging!

Solving Dirac operator equation in mode-function expansion

$$i\gamma^0\partial_t\hat{\psi} = (-i\not{D}^s_W + m)\hat{\psi}$$

$$\hat{\psi}_{\mathbf{x}}(t) = \frac{1}{\sqrt{V}}\sum_\lambda \left(\hat{b}_\lambda(0)\phi^u_\lambda(t,\mathbf{x}) + \hat{d}^\dagger_\lambda(0)\phi^v_\lambda(t,\mathbf{x})\right)$$

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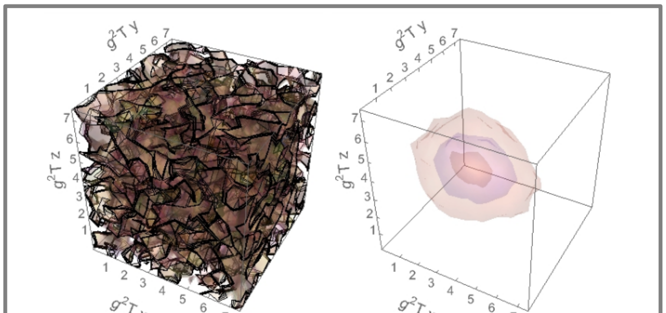

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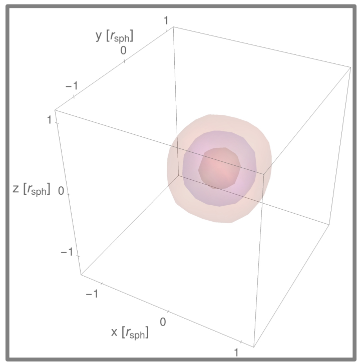

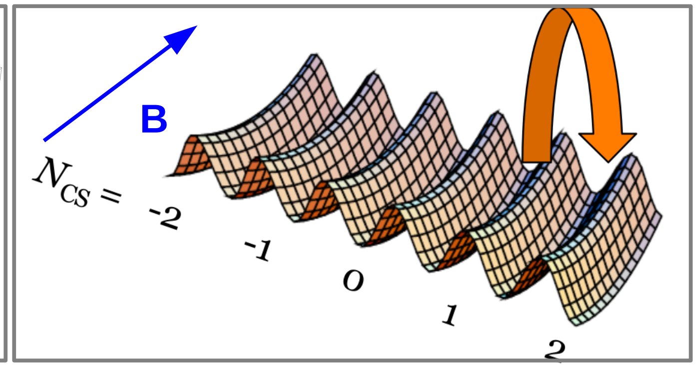

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→ extremely costly (~N^6)

→ big obstacle so far and many attempts at reducing price (e.g. 'low-cost' techniques, Borsányi and Hindmarsh 2009)

Technical breakthroughs – real time fermions

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Evolution of the fermion operators **extremely costly due to mode-function expansion**
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(Borsányi and Hindmarsh 2009, Berges, Gelfand)

→ converge in a limited number of cases, hopeless in many others, *especially for anomalous dynamics!*

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→ **works extremely well, convergence already on small lattices 16x16x16 for smooth gauge fields**

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Check *arXiv:1612.02477* for the current state-of-art for real-time fermion simulations!

Chiral Magnetic and Chiral Separation Effect

NM, Schlichting, Sharma, PRL 117 (2016) 142301; Mace, NM, Schlichting, Sharma, arXiv:1612.02477

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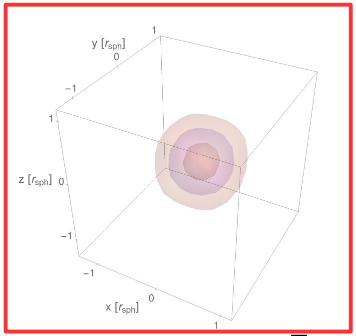

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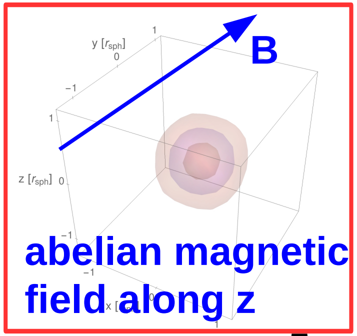

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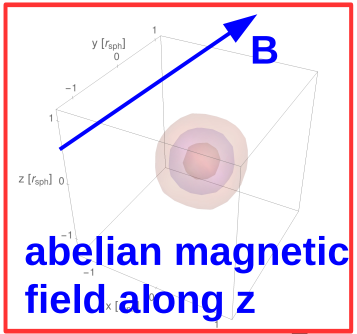

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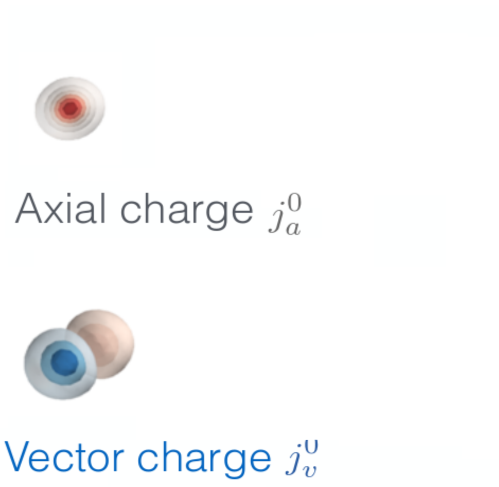

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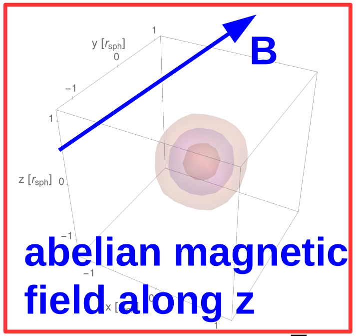

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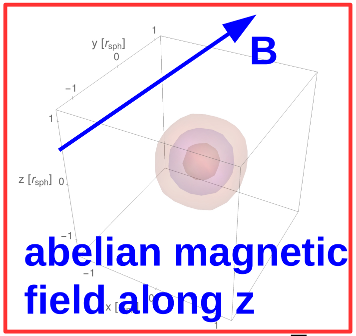

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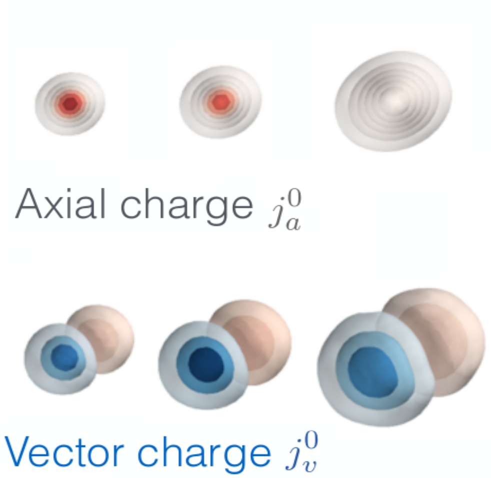

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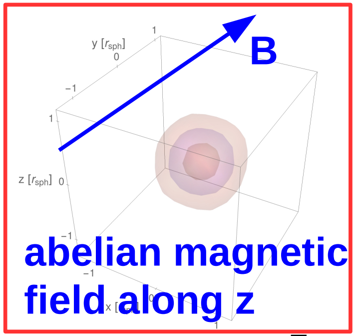

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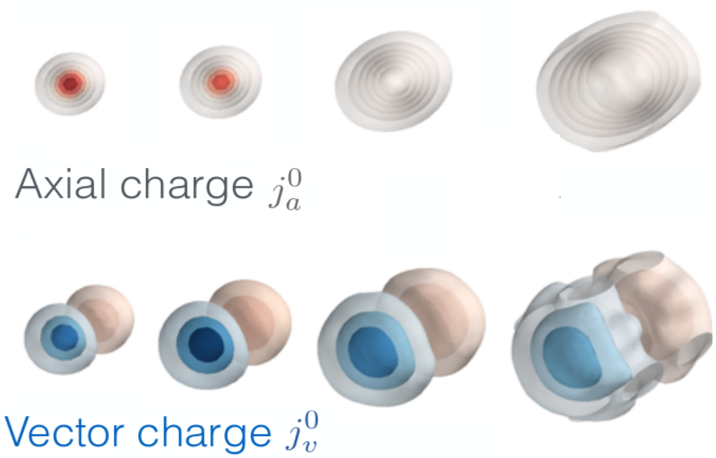

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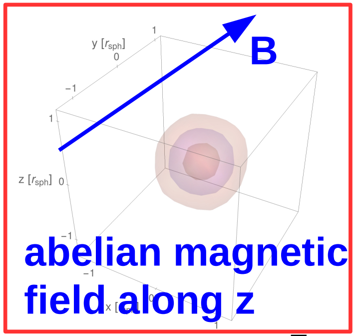

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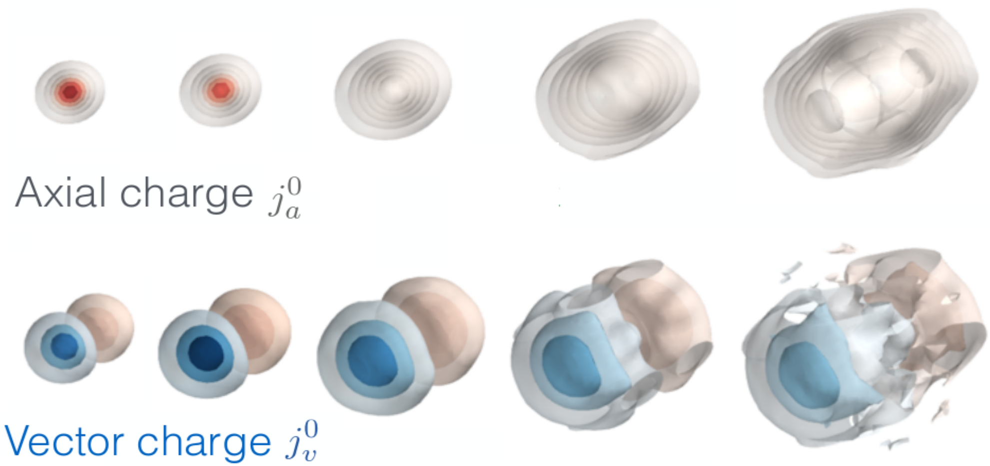

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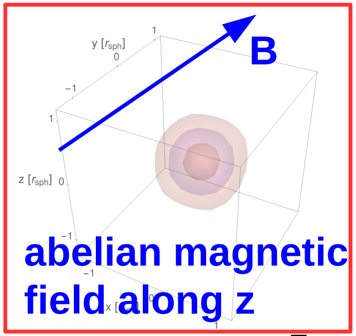

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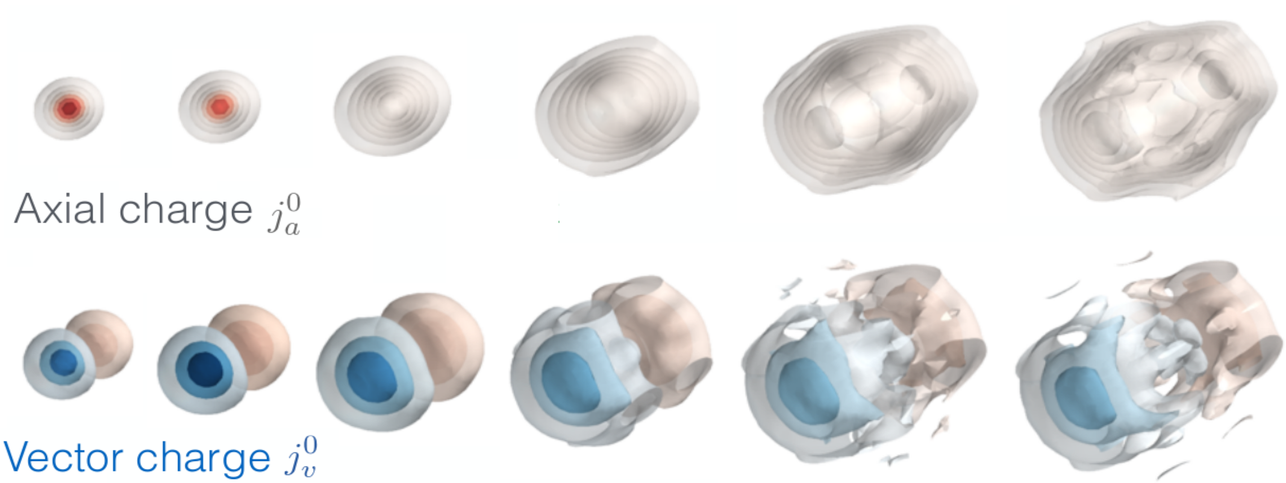

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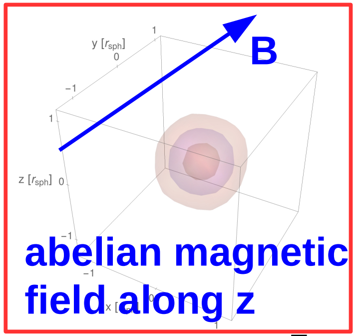
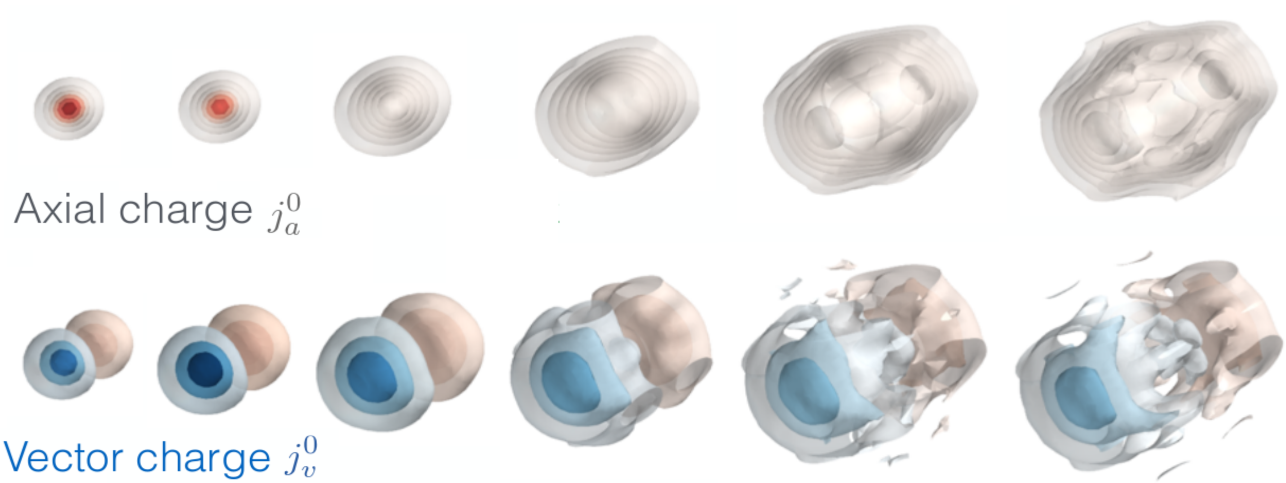


Chiral Magnetic and Chiral Separation Effect

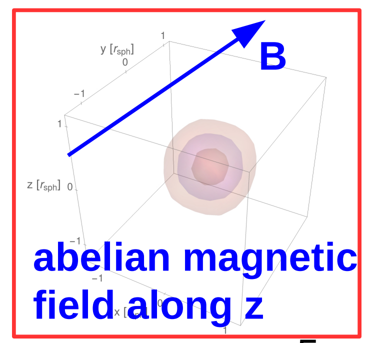
NM, Schlichting, Sharma, PRL 117 (2016) 142301; Mace, NM, Schlichting, Sharma, arXiv:1612.02477



Initially: Vacuum (no fermions, no axial charge)

Chiral Magnetic Effect: Electric current generated due to axial charge produced

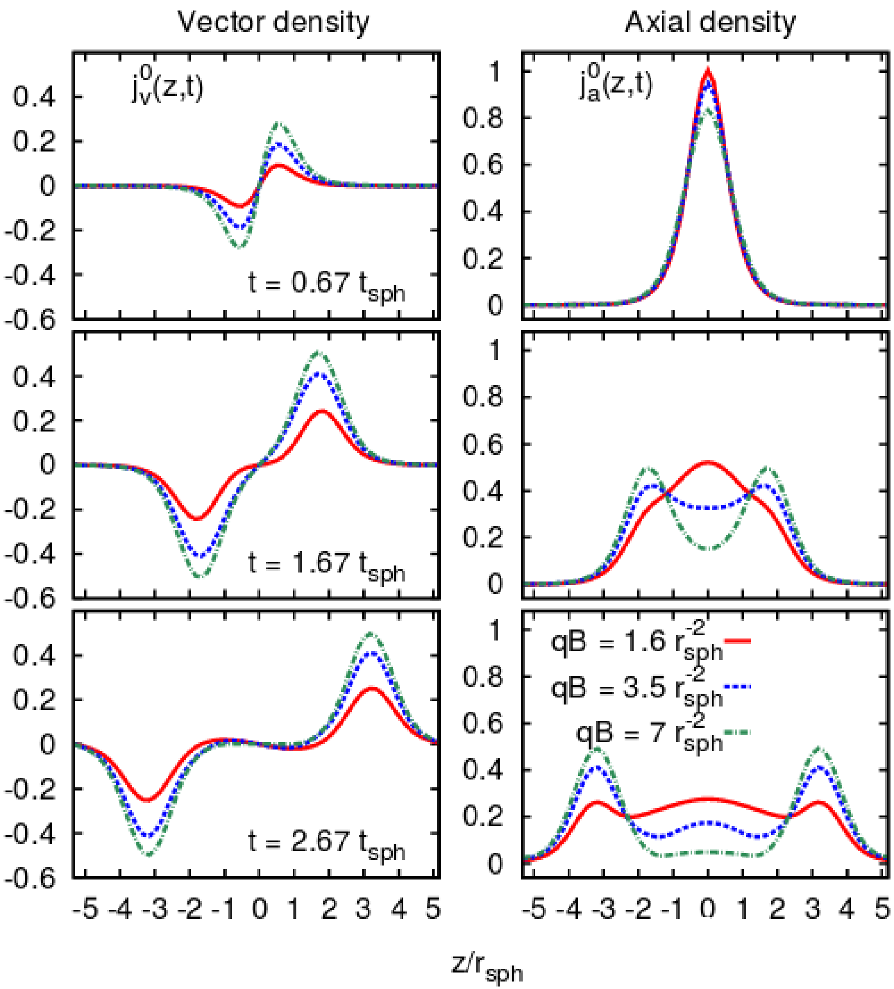
Chiral Separation Effect: Axial current generated due to electric charge



→ Emergence of the **Chiral Magnetic Wave**

Magnetic Field Dependence

Magnetic Field Dependence

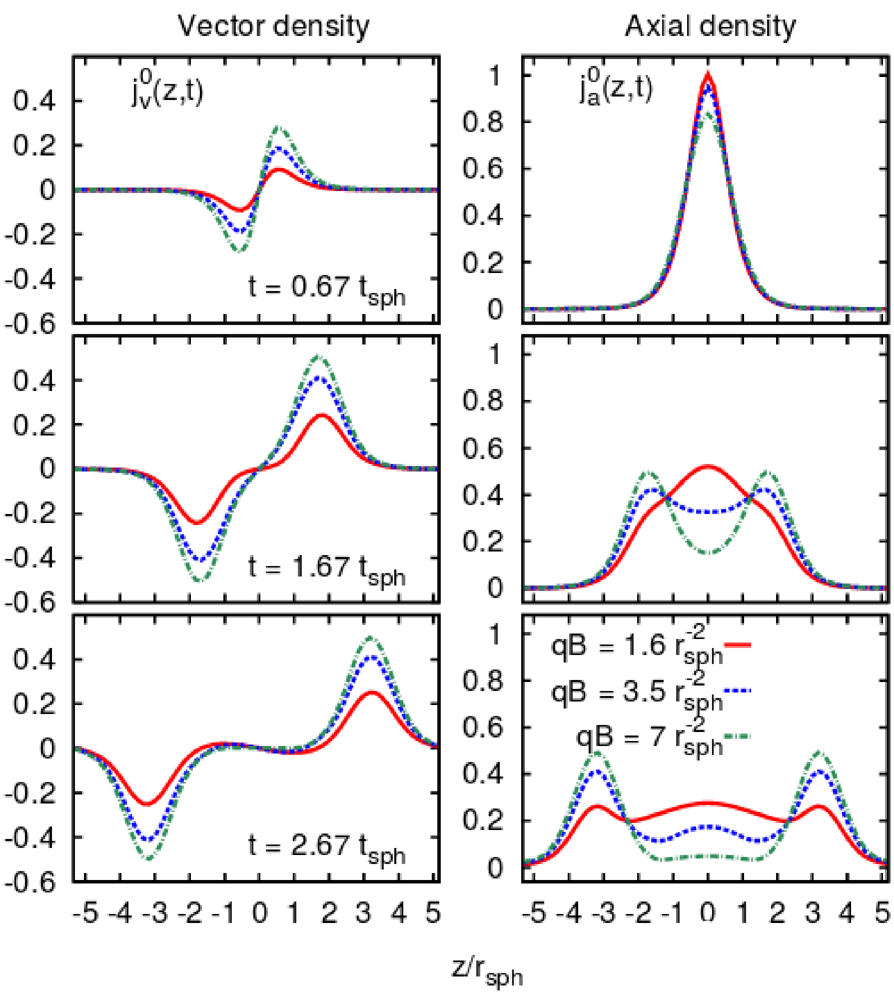


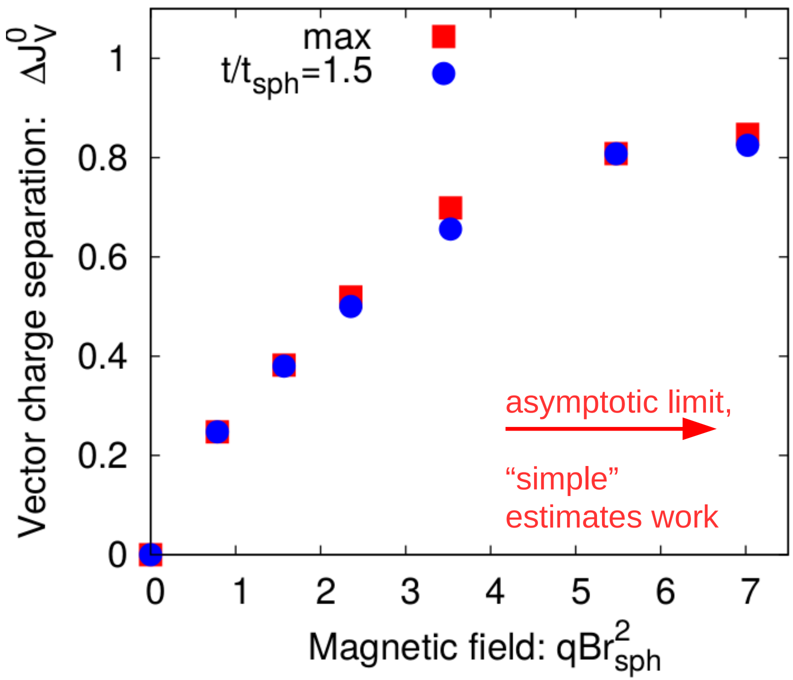
finite magnetic field:

→ important deviations from 'ideal' picture of CME

2. Real-time simulations

Magnetic Field Dependence





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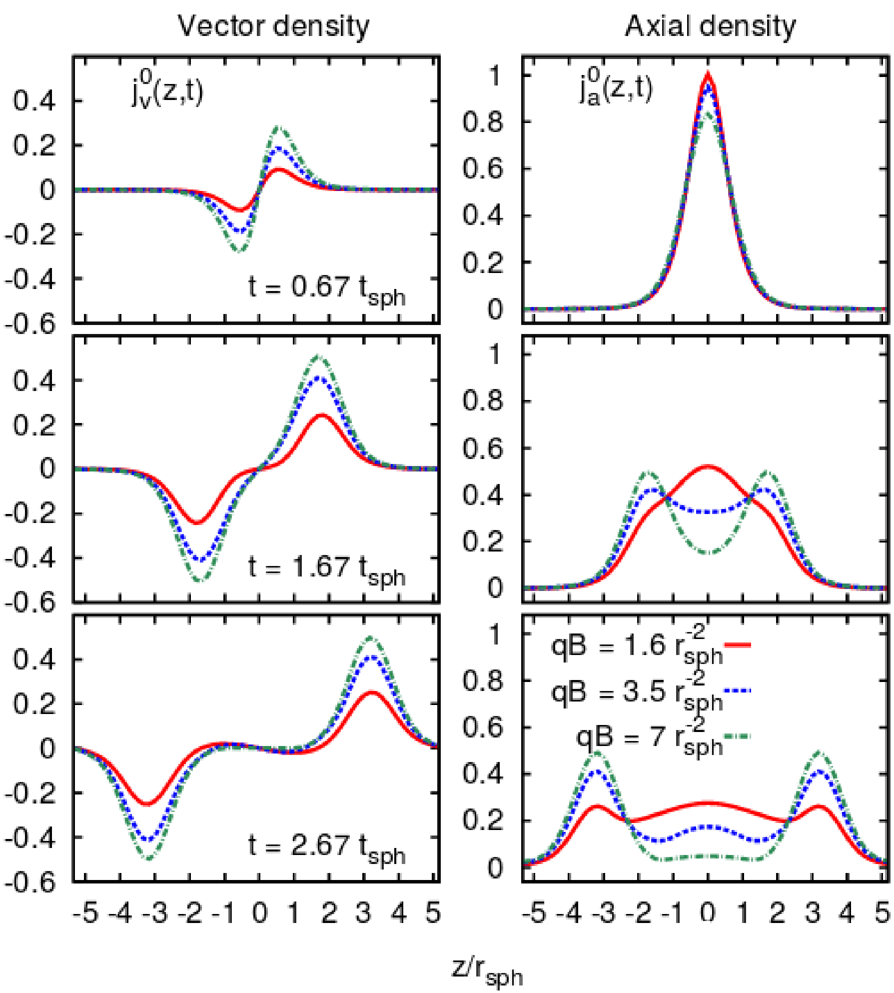
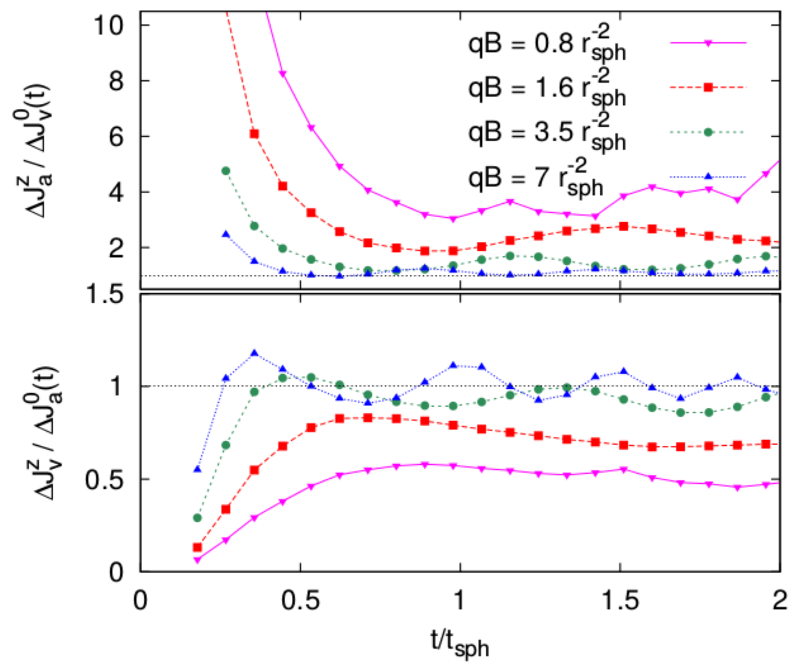
Magnetic Field Dependence





finite magnetic field:

→ important deviations from 'ideal' picture of CME

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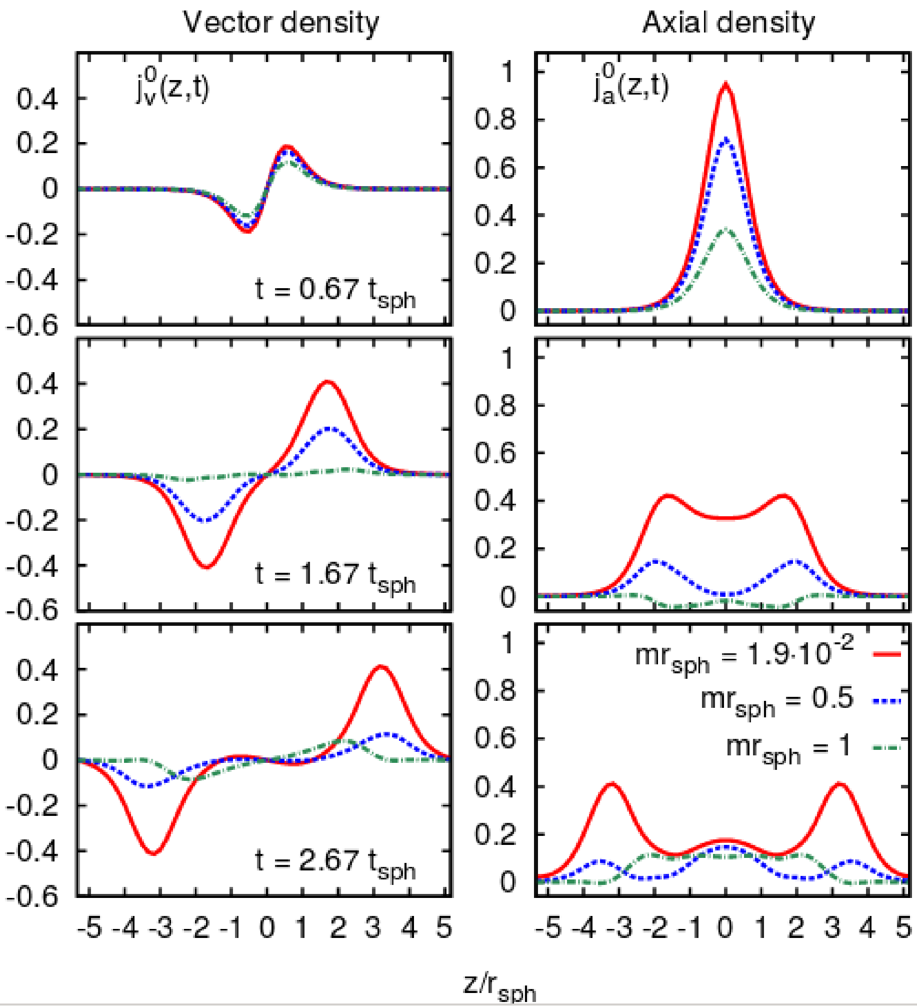
Finite quark mass

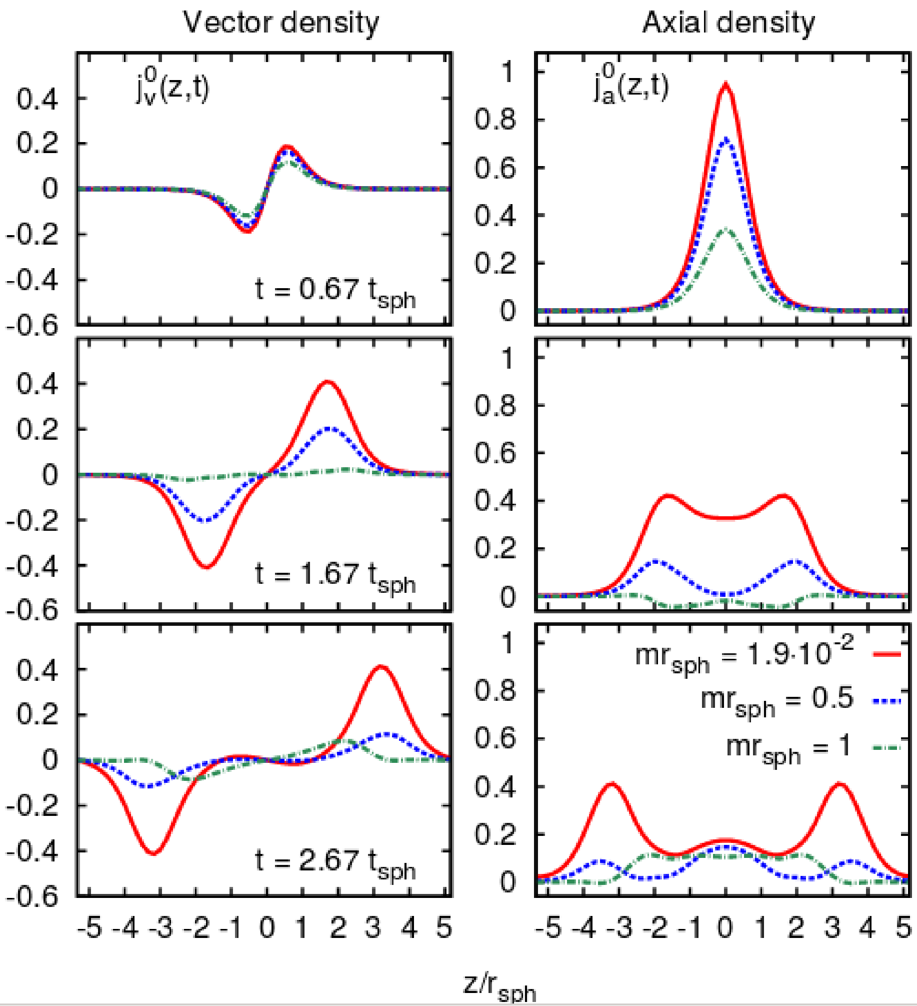
Finite quark mass

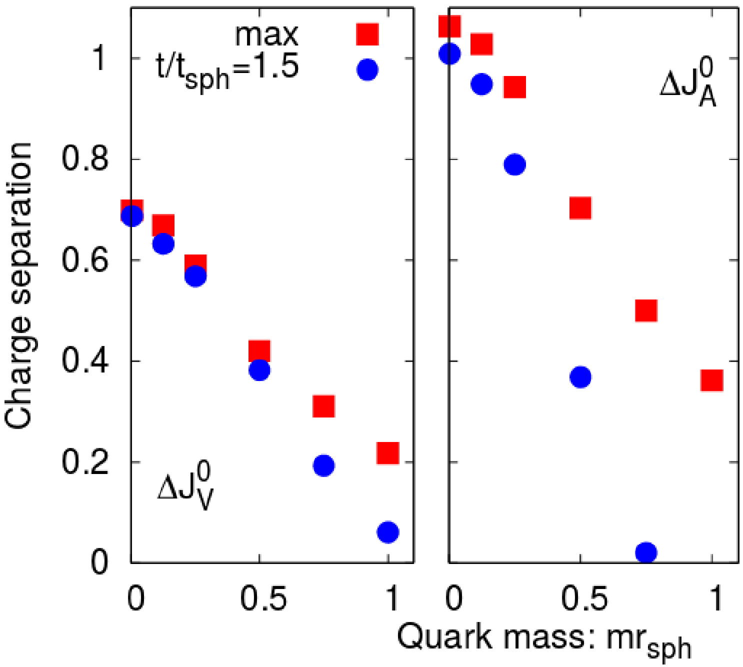


finite quark mass effects
→ anomalous transport
suppressed for heavy quarks

2. Real-time simulations

Finite quark mass





finite quark mass effects
→ anomalous transport suppressed for heavy quarks

Simulating chiral fermions in real-time: *Overlap fermions*

arXiv:1612.02477, arXiv:1701.03331

Simulating chiral fermions in real-time: *Overlap fermions*

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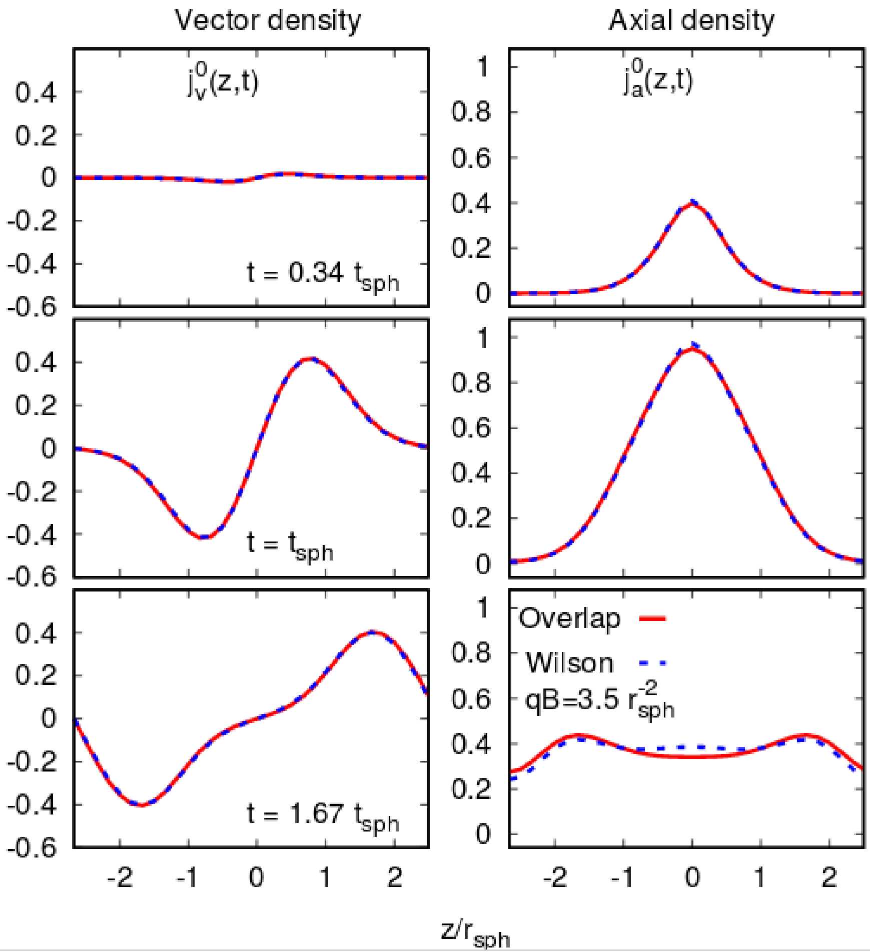
Benchmark:

Wilson-fermions vs. Overlap fermions



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Simulating chiral fermions in real-time: *Overlap fermions*

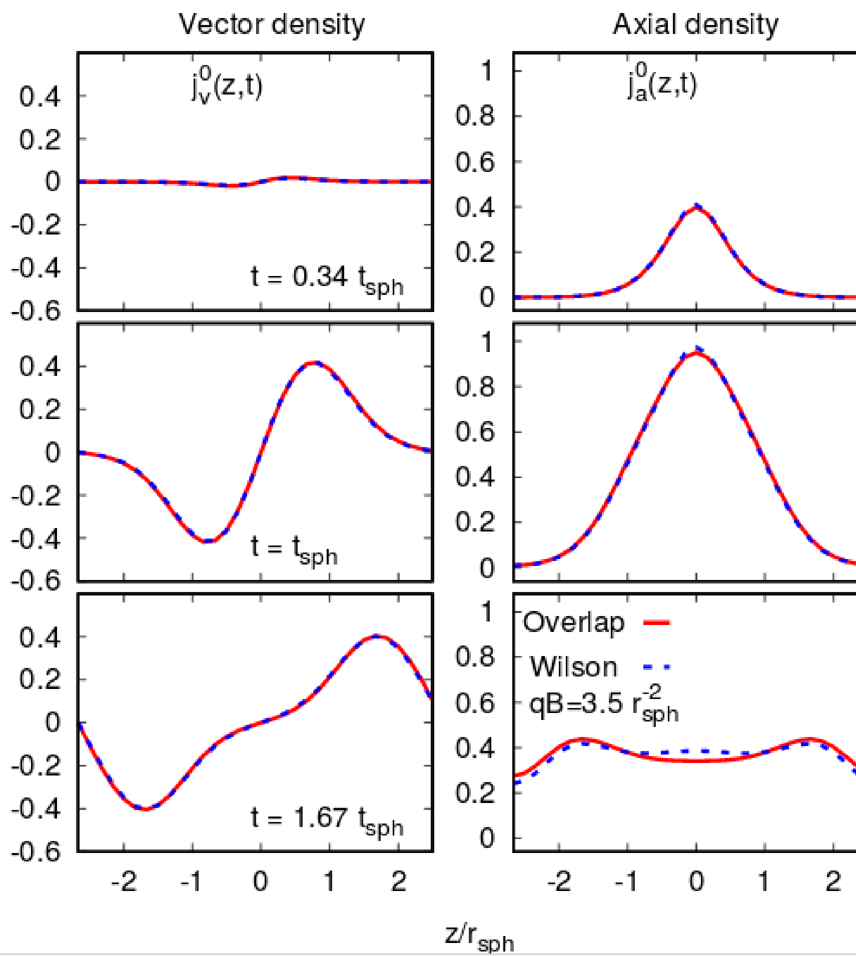
arXiv:1612.02477, arXiv:1701.03331

Benchmark:

Wilson-fermions vs. Overlap fermions

- mass matters! **chiral instabilities**
  helicity transport: heavy ions and astrophysics!

(Yamamoto, Akamatsu, Kaplan, Reddy, Sen, Dvornikov...)



3. Real-time simulations

Simulating <u>chiral</u> fermions in real-time: *Overlap fermions*

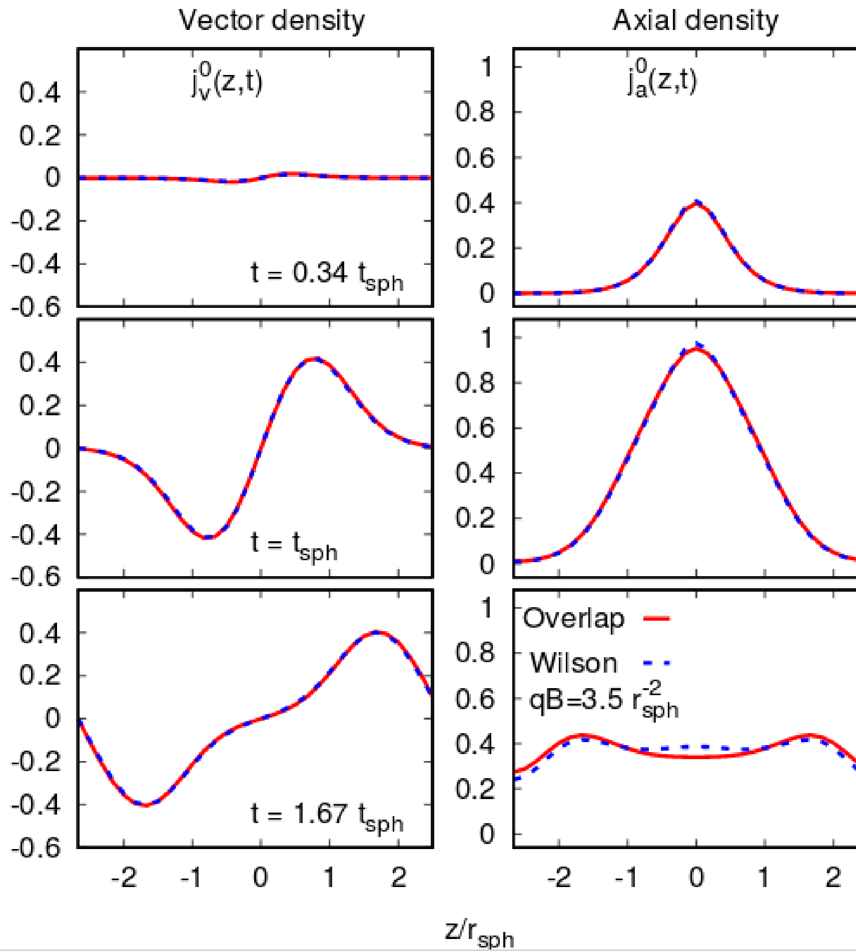
arXiv:1612.02477, arXiv:1701.03331

Benchmark:

Wilson-fermions vs. Overlap fermions

- mass matters! **chiral instabilities**
  helicity transport: heavy ions and astrophysics!
  (Yamamoto, Akamatsu, Kaplan, Reddy, Sen, Dvornikov...)

- real-time evolution beyond early time



3. Conclusions

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- **I have shown you real-time classical statistical simulations of fermion production during sphaleron transitions in background magnetic fields**

- **Axial anomaly realized in lattice simulations using Wilson fermions**

- **Chiral Magnetic and Chiral Separation Effect emerge dynamically**

- **Observation of the Chiral Magnetic Wave**

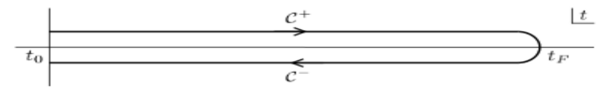
- **Have investigated finite mass and magnetic field dependence.**
  Finite quark mass plays an important role:
  dissipation of anomalous currents

- **Simulated chiral lattice fermions in real-time – overlap fermions!**

- relativistic chiral kinetic theory from world-lines (with R. Venugopalan)
arXiv:1701.03331 & arXiv:1702.01233

Backup
A. Classical Statistical Simulations

(see for example Kasper et al. Phys.Rev. D90 (2014) 2, 025016)



typical situation: large **coherent or highly occupied** gauge fields
→ correspondence principle

$$A^{+}_{\mu,n} = \bar{A}_{\mu,n} + \frac{1}{2}\tilde{A}_{\mu,n}\,,$$

$$A^{-}_{\mu,n} = \bar{A}_{\mu,\bar{n}} - \frac{1}{2}\tilde{A}_{\mu,\bar{n}}\,,$$

- initial stages of a heavy ion collision: A ~ 1/g
- colliding laser beams → large and coherent fields

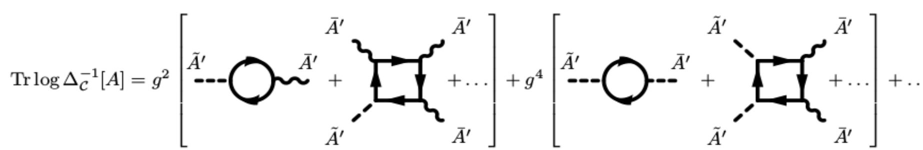
The classical-statistical approximation is a systematic expansion of the 'quantum' fields around the 'classical' fields

'classical' 'quantum'

Fermions are never "classical"

$$Z_{\mathcal{C}} = \int [dA]\,\rho_G(A)\,\exp\left(\mathrm{Tr}\log\Delta_{\mathcal{C}}[A]^{-1} + iS_G\right)$$

$$i\gamma^0\partial_t\hat{\psi} = (-i\not{D}^s_W + m)\hat{\psi}$$



Exact description via **modefunctions** up to **24x24x64** lattices

B. Backup
Algorithmic Improvements

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Fermions: Exact description via **modefunctions** up to 24x24x64 lattices

Improved operators (NM, S.Schlichting, S.Sharma, **arXiv:1606.00342, arXiv:1612.02477**)

We use a tree-level improved version of the lattice Hamiltonian, which takes the form

$$H = \sum_x \psi_x^\dagger m \gamma^0 \psi \quad + \quad \frac{1}{2} \sum_{n,x,i} C_n \psi_x^\dagger \gamma^0 \Big[\Big(-i\gamma^i - n r_w \Big) U_{x,+ni} \psi_{x+ni} + 2 n r_w \psi_x - \Big(-i\gamma^i + n r_w \Big) U_{x,-ni} \psi_{x-ni} \Big]$$

where r_w denotes the Wilson coefficient, the coefficients C_n are chosen to optimize the convergence, and we introduce the following short hand notation for the connecting gauge links

$$U_{x,+ni} = \prod_{k=0}^{n-1} U_{x+ki,i} \; , \qquad U_{x,-ni} = \prod_{k=1}^{n} U^\dagger_{x-ki,i} \tag{1}$$

Wilson-averaging (M.Mace, NM, S.Schlichting, S.Sharma, *arXiv:1612.02477*)

→ improvement of chiral properties
→ extremely important for larger fermion masses
→ average fermionic observables over Wilson parameters with opposite sign
→ leading order errors in the anomaly equation cancel

Backup
C. Magnetic Fields on the lattice

Magnetic fields break translation invariance → magnetic translation group

- Magnetic fields on a torus very non-trivial
(see Al-Hashimi & Wiese “Accidental Symmetries”, also Bali et al.)

$$U_{y,n} = \mathrm{e}^{\mathrm{i}a^2 qBn_x} \; ; \; U_{x,N_x-1,n_y,n_z} = \mathrm{e}^{-\mathrm{i}a^2 qBN_x n_y}$$
$$U_{x,n} = \mathbf{1}, n_x \neq N_x - 1; U_{z,n} = \mathbf{1}$$

- Intriguing lattice artefacts!

→ spoil the low-cost method
--- while there probably are field configurations where low-cost works,
this is certainly not the case in magnetic fields

D. Backup
Anomaly Realization on the Lattice

Chiral Symmetry + Fermion doubling + Chiral Anomaly
= “one of the prettiest connections I have ever seen”

Lattice Fermions: Species Doubling, Chiral Invariance, and the Triangle Anomaly

Luuk H. Karsten (Stanford U., ITP) , Jan Smit (Amsterdam U.)

Sep 1980 - 38 pages

Nucl.Phys. B183 (1981) 103

In *Rebbi, C. (Ed.): Lattice Gauge Theories and Monte Carlo Simulations*, 495-532. (Nucl. Phys. B183 (1981) 103-140) and Stanford Univ. - ITP-677 (80,REC.NOV.) 71p (1981)

DOI: 10.1016/0550-3213(81)90549-6

Conference: C81-06-01, p.495-532

Proceedings

ITP-677-STANFORD

Contributions

- The axial anomaly and the fermion doubling problem are intimately related

- Lattice theory regularized on the basis of the action already

- Anomaly comes from the non-trivial continuum limit of any regulator you put in to remove doublers