Chiral magnetic effect and anomalous transport from real-time lattice simulations

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based on work together with: J. Berges, M. Mace, S. Schlichting, S. Sharma, N. Tanji

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Outline

1. Anomalous phenomena in heavy ion collisions
2. Classical-statistical simulations
3. Conclusions
1. Anomalous Phenomena in Heavy Ion Collisions

CGC colliding nuclei

flux tubes

over-occupied plasma

kinetic regime

hydrodynamic regime

S. Schlichting 2016
1. Anomalous Phenomena in Heavy Ion Collisions

- CGC: colliding nuclei
- Flux tubes
- Over-occupied plasma
- Kinetic regime
- Hydrodynamic regime

Non-equilibrium anomalous fermion production from coherent fields (Tanji et al. 2016) and sphaleron transitions (Mace et al. 2016)
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large magnetic fields present

Anomalous Transport (CME, CSE and CMW)

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Subsequent interactions in the fire ball, axial transport and relaxation

*weak to strong coupling*
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Theory:

classical statistical simulations + fermions

chiral kinetic theory

anomalous hydrodynamics

Thermalization / Freezout
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Anomalous fermion dynamics induced by a topological transition
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simplified situation: setting up an isolated sphaleron transition in background abelian magnetic fields

Mace et al 2016 (see poster)
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- consistent treatment of axial charge production, non-abelian gauge fields as dynamical degrees of freedom.

Fermions: Challenging!
Solving Dirac operator equation in mode-function expansion

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i\gamma^0 \partial_t \hat{\psi} = (-i \slashed{D} + m) \hat{\psi}
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\hat{\psi}(t) = \frac{1}{\sqrt{V}} \sum_{\lambda} \left( \hat{b}_\lambda(0) \phi^u_\lambda(t, x) + \hat{d}^\dagger_\lambda(0) \phi^u_\lambda(t, x) \right)
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Solving Dirac operator equation in mode-function expansion

→ extremely costly (∼N^6)
→ big obstacle so far and many attempts at reducing price (e.g. 'low-cost' techniques, Borsányi and Hindmarsh 2009)
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Technical breakthroughs – real time fermions
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Technical breakthroughs – real time fermions

Evolution of the fermion operators **extremely costly due to mode-function expansion** (cost with lattice size: $N^6$ for fermions vs. $N^3$ for 'YM only')

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→ stochastic descriptions have been put forward early on: 'low-cost fermions'

(Borsányi and Hindmarsh 2009, Berges, Gelfand)

→ converge in a limited number of cases, hopeless in many others,
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Our approach:
- tree-level operator improvements (Eguchi and N. Kawamoto 1984)
- Wilson-averaging

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**Computational Resources:**

[NERSC] [SCC]
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Computational Resources:

Check arXiv:1612.02477 for the current state-of-art for real-time fermion simulations!
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Chiral Magnetic and Chiral Separation Effect
NM, Schlichting, Sharma, PRL 117 (2016) 142301; Mace, NM, Schlichting, Sharma, arXiv:1612.02477
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Initially: Vacuum (no fermions, no axial charge)
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Chiral Magnetic Effect: Electric current generated due to axial charge produced
Chiral Separation Effect: Axial current generated due to electric charge

→ Emergence of the Chiral Magnetic Wave
2. Real-time simulations

Magnetic Field Dependence
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Magnetic Field Dependence

finite magnetic field:
→ important deviations from 'ideal' picture of CME
2. Real-time simulations

Magnetic Field Dependence

Finite magnetic field: important deviations from 'ideal' picture of CME

→ asymptotic limit, "simple" estimates work

max $t/t_{sph} = 1.5$

finite magnetic field: important deviations from 'ideal' picture of CME
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Magnetic Field Dependence

finite magnetic field:
→ important deviations from 'ideal'
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Finite quark mass
2. **Real-time simulations**

**Finite quark mass**

Finite quark mass effects → anomalous transport suppressed for heavy quarks
2. Real-time simulations

Finite quark mass

Finite quark mass effects → anomalous transport suppressed for heavy quarks
3. Real-time simulations

Simulating chiral fermions in real-time: Overlap fermions

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**Benchmark:**

Wilson-fermions vs. Overlap fermions
3. Real-time simulations

Simulating chiral fermions in real-time: *Overlap fermions*


**Benchmark:**

Wilson-fermions vs. Overlap fermions

- mass matters! chiral instabilities
- helicity transport: heavy ions and astrophysics!

(Yamamoto, Akamatsu, Kaplan, Reddy, Sen, Dvornikov...)

![Graphs showing vector and axial density plots](image)
3. Real-time simulations

Simulating chiral fermions in real-time: *Overlap fermions*


**Benchmark:**

**Wilson-fermions vs. Overlap fermions**

- mass matters! **chiral instabilities**
- helicity transport: heavy ions and astrophysics!
- (Yamamoto, Akamatsu, Kaplan, Reddy, Sen, Dvornikov...)

- real-time evolution beyond early time
3. Conclusions

- I have shown you real-time classical statistical simulations of fermion production during sphaleron transitions in background magnetic fields.

- Axial anomaly realized in lattice simulations using Wilson fermions.

- Chiral Magnetic and Chiral Separation Effect emerge dynamically.

- Observation of the Chiral Magnetic Wave.

- Have investigated finite mass and magnetic field dependence. Finite quark mass plays an important role: dissipation of anomalous currents.

- Simulated chiral lattice fermions in real-time – overlap fermions!

- relativistic chiral kinetic theory from world-lines (with R. Venugopalan)
A. Classical Statistical Simulations

(see for example Kasper et al. Phys.Rev. D90 (2014) 2, 025016)

typical situation: large coherent or highly occupied gauge fields → correspondence principle

- initial stages of a heavy ion collision: \( A \sim 1/g \)
- colliding laser beams → large and coherent fields

The classical-statistical approximation is a systematic expansion of the 'quantum' fields around the 'classical' fields

Fermions are never “classical”

\[
Z_c = \int [dA] \rho_G(A) \exp \left( \text{Tr} \log \Delta_c [A]^{-1} + iS_G \right)
\]

\[
i\gamma^0 \partial_t \hat{\psi} = (-iD^s_W + m)\hat{\psi}
\]

Exact description via \textbf{modefunctions} up to 24x24x64 lattices
B. Algorithmic Improvements

Fermions: Exact description via \textit{modefunctions} up to 24x24x64 lattices


We use a tree-level improved version of the lattice Hamiltonian, which takes the form

\[
H = \sum_x \psi_x^\dagger m^0 \gamma^0 \psi + \frac{1}{2} \sum_{n,x,i} C_n \psi_x^\dagger \gamma^0 \left[ \left( -i \gamma^i - nr_w \right) U_{x,+ni} \psi_{x+ni} + 2nr_w \psi_x - \left( -i \gamma^i + nr_w \right) U_{x,-ni} \psi_{x-ni} \right]
\]

where \(r_w\) denotes the Wilson coefficient, the coefficients \(C_n\) are chosen to optimize the convergence, and we introduce the following short hand notation for the connecting gauge links

\[
U_{x,+ni} = \prod_{k=0}^{n-1} U_{x+ki,i}, \quad U_{x,-ni} = \prod_{k=1}^{n} U_{x-ki,i}^\dagger
\]


→ improvement of chiral properties
→ extremely important for larger fermion masses
→ average fermionic observables over Wilson parameters with opposite sign
→ leading order errors in the anomaly equation cancel
C. Magnetic Fields on the lattice

Magnetic fields break translation invariance → magnetic translation group

- Magnetic fields on a torus very non-trivial
  (see Al-Hashimi & Wiese “Accidental Symmetries”, also Bali et al.)

\[ U_{y,n} = e^{ia^2qBn_x} ; \quad U_{x,N_x-1,n_y,n_z} = e^{-ia^2qBN_xn_y} \]
\[ U_{x,n} = 1, \quad n_x \neq N_x - 1; \quad U_{z,n} = 1 \]

- Intriguing lattice artefacts!

→ spoil the low-cost method
  --- while there probably are field configurations where low-cost works,
  this is certainly not the case in magnetic fields
D. Anomaly Realization on the Lattice

Chiral Symmetry + Fermion doubling + Chiral Anomaly = “one of the prettiest connections I have ever seen”

- The axial anomaly and the fermion doubling problem are intimately related

- Lattice theory regularized on the basis of the action already

- Anomaly comes from the non-trivial continuum limit of any regulator you put in to remove doublers