Cumulants and correlation functions vs the QCD phase diagram at low energies

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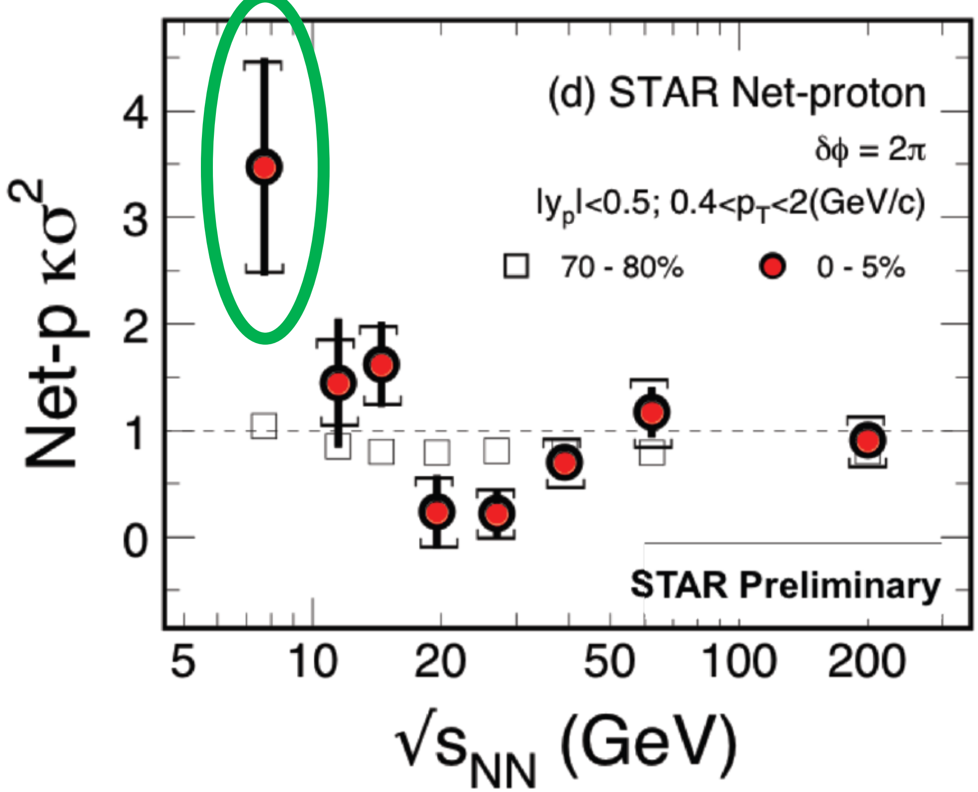
STAR results
multi-particle correlations
minimal model
possible clusters
exclusion plots
conclusions

AB, V. Koch, N. Strodthoff , 1607.07375
AB, V. Koch, V. Skokov, 1612.05128

Preliminary STAR data

X.Luo, N.Xu, 1701.02105



my notation

K_4/K_2

Is the signal at 7.7 GeV large?

I will demonstrate that **the signal is surprisingly large**

Cumulants are not the best choice

$$K_2 = \langle (N - \langle N \rangle)^2 \rangle$$

$$K_3 = \langle (N - \langle N \rangle)^3 \rangle$$

$$K_4 = \langle (N - \langle N \rangle)^4 \rangle - 3\langle (N - \langle N \rangle)^2 \rangle^2$$

N – number of protons

we neglect anti-protons,
good at low energies

$$K_n = \langle N \rangle + physics[2, \dots, n]$$

$physics$ = two-, three-, n-particle correlation functions

for Poisson $K_n = \langle N \rangle$, $(physics = 0)$

We have

$$K_2 = \langle N \rangle + \boldsymbol{C_2}$$

$$K_3 = \langle N \rangle + 3\boldsymbol{C_2} + \boldsymbol{C_3}$$

$$K_4 = \langle N \rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

cumulants mix correlation functions of different orders

For example:

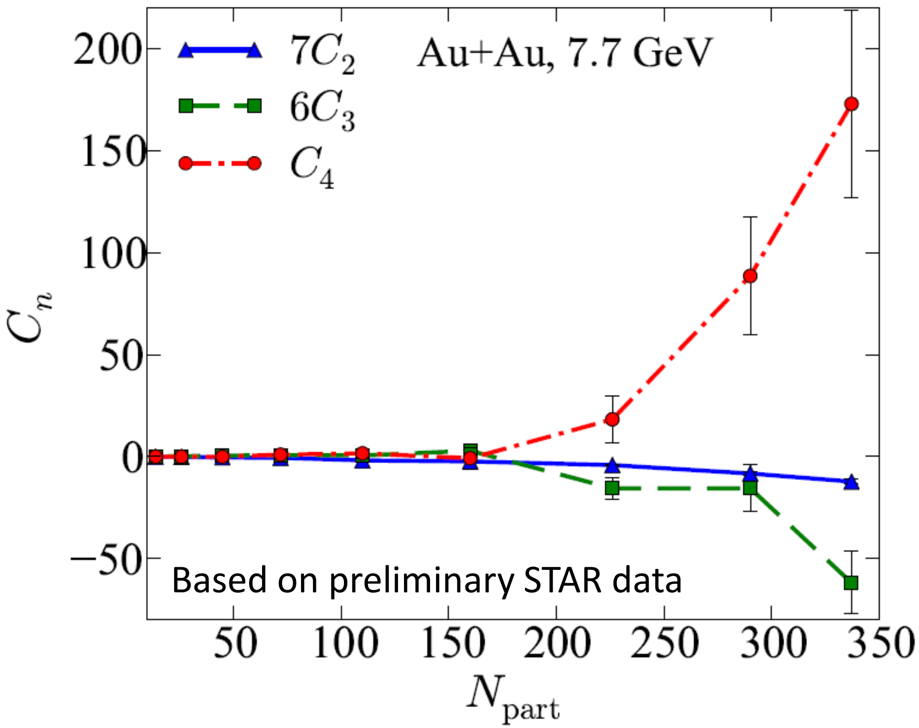
$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2) dy_1 dy_2 \qquad \boldsymbol{C_2} = \langle N \rangle^2 \boldsymbol{c_2}$$

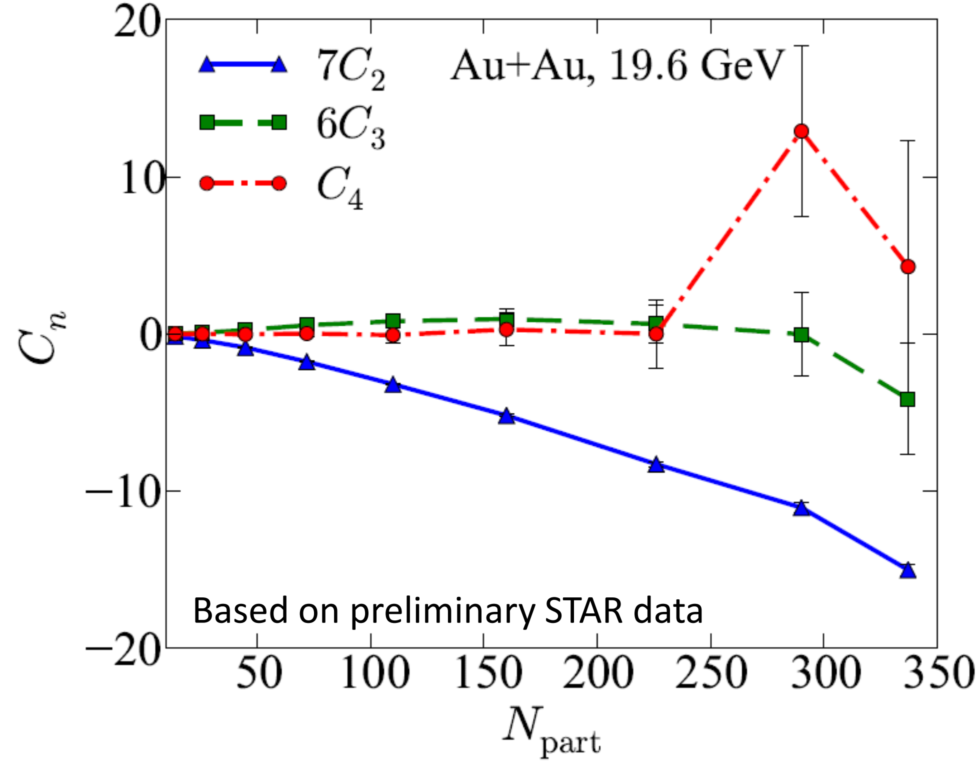
See, e.g.,
B. Ling, M. Stephanov, PRC 93 (2016) no.3, 034915
AB, V. Koch, N. Strodthoff , 1607.07375

Using preliminary STAR data we can obtain C_n

central signal at 7.7 GeV is driven by large 4-particle correlations



central signal at 19.6 GeV is driven by 2-particle correlations



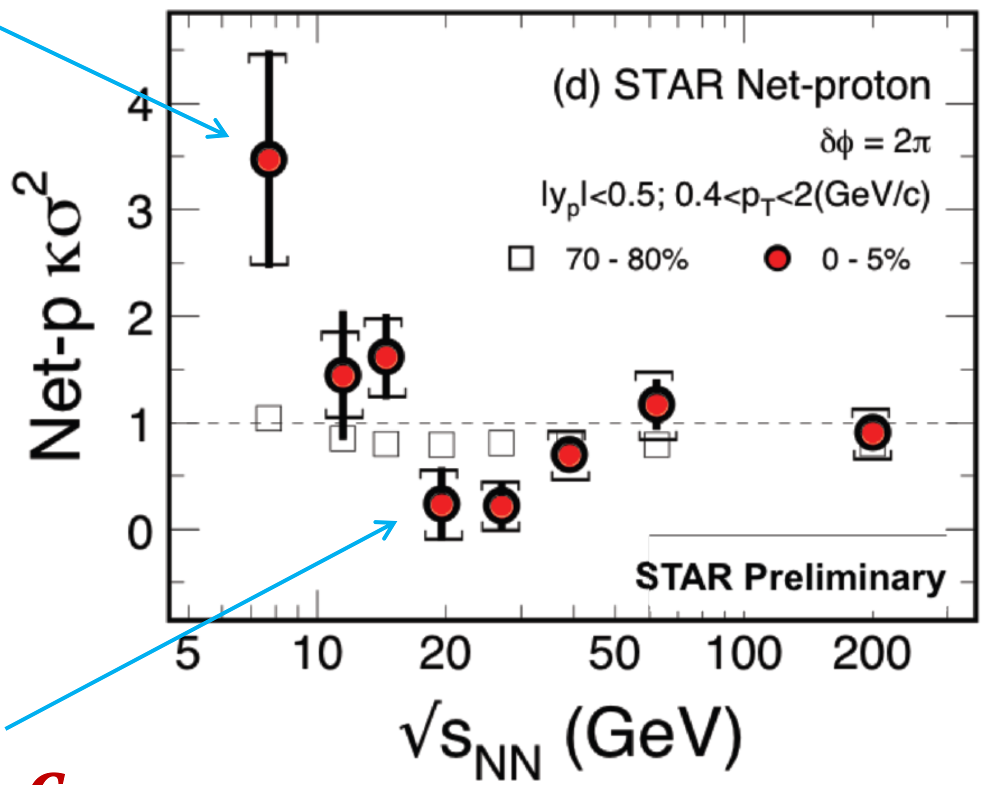
$C_4(7.7) \sim \mathbf{170}$

C_4 and $6C_3$ cancelation in most central coll.

AB, V. Koch, N. Strodthoff, 1607.07375

here we see C_4

X.Luo, N.Xu, 1701.02105



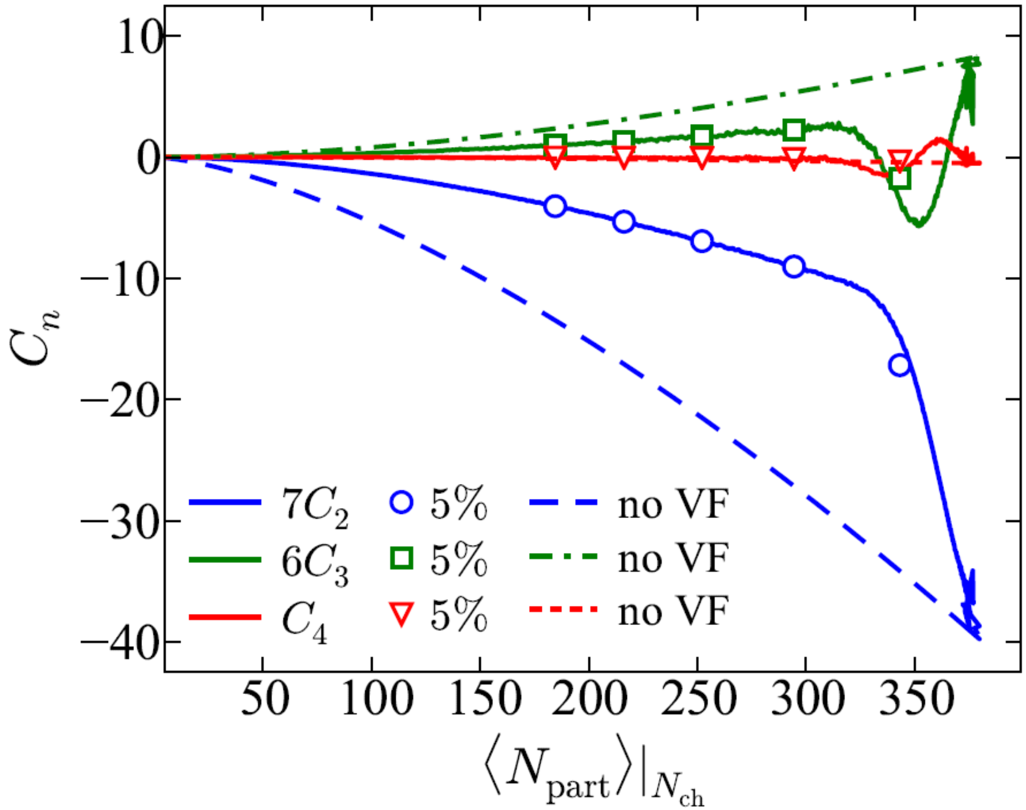
and here C_2

e.g., baryon conservation

Minimal model (MM) at low energies

- independent baryon stopping (baryon conservation by construction)
- N_{part} fluctuations (volume fluctuation - VF)

Au+Au, $\sqrt{s} = 7.7$ GeV



STAR

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$

we follow the STAR way (centrality etc.) as closely as possible

AB, V. Koch, V. Skokov, 1612.05128

Let's put the STAR numbers in perspective.

Suppose that we have clusters (distributed according to Poisson) decaying always to 4 protons

$$\boldsymbol{C_k} = \langle N_{\mathrm{cl}} \rangle \cdot 4!/(4-k)!$$

↑ mean number of clusters

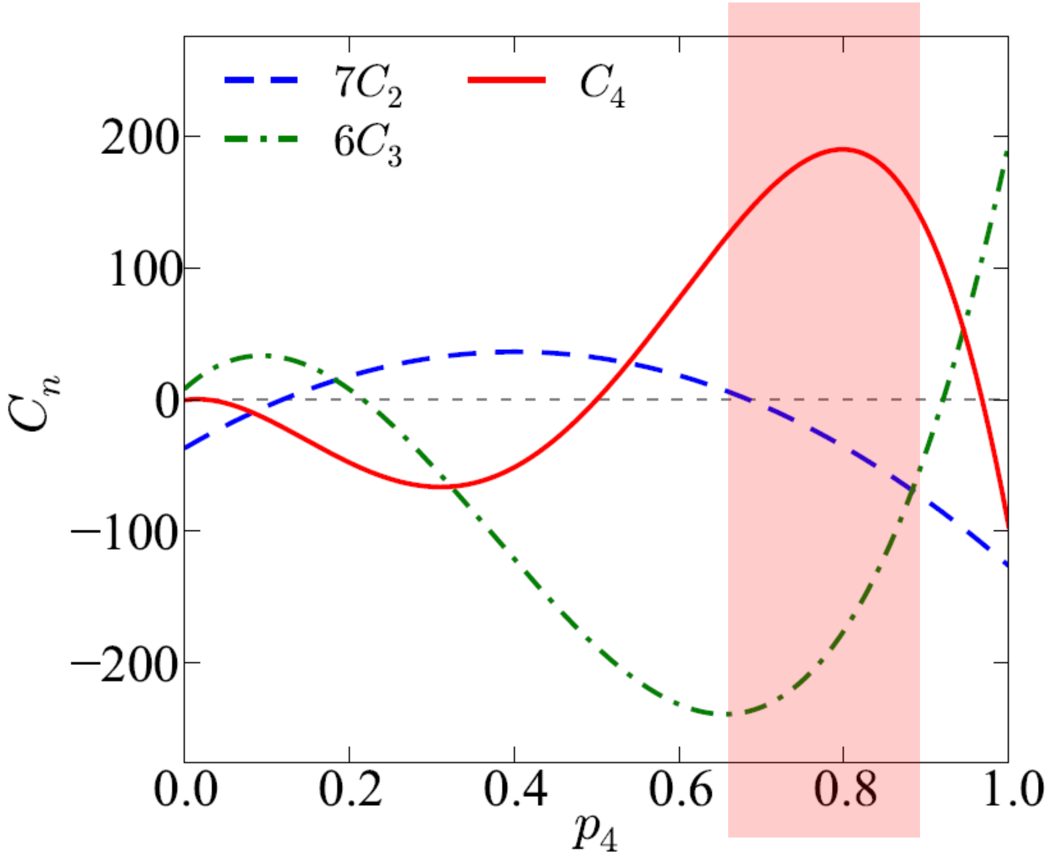
$$\boldsymbol{C_4} = \langle N_{\mathrm{cl}} \rangle \cdot 24$$

To obtain $C_4 \approx 170$ we need $\langle N_{\mathrm{cl}} \rangle \sim 7$, it means 28 protons.
STAR sees on average 40 protons in central collisions.

In this model $C_2 > 0$ and $C_3 > 0$ contrary to the STAR data

Toy model:

- 16 protons stop in quartets with probability p_4
- remaining protons stop independently with some small probability $p_1 \sim 0.1$



qualitatively consistent with STAR

STAR

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$

AB, V. Koch, V. Skokov, 1612.05128

Take-home message:

C_4 (four-proton correlation function) observed by STAR is larger by almost **three orders of magnitude** than the minimal model.

To explain C_4 we need a strong source of multi-proton correlations. **Proton clusters**?

Similar story with 3-proton correlation function.

Centrality dependence

$\boldsymbol{C}_2(y_1,y_2)=\rho(y_1)\rho(y_2)\boldsymbol{c}_2(y_1,y_2)$

$$K_4=\langle N\rangle+7\boldsymbol{C}_{\boldsymbol{2}}+6\boldsymbol{C}_{\boldsymbol{3}}+\boldsymbol{C}_{\boldsymbol{4}}$$

$$\boldsymbol{C}_{\boldsymbol{k}}=\langle N\rangle^k\boldsymbol{c}_{\boldsymbol{k}}$$

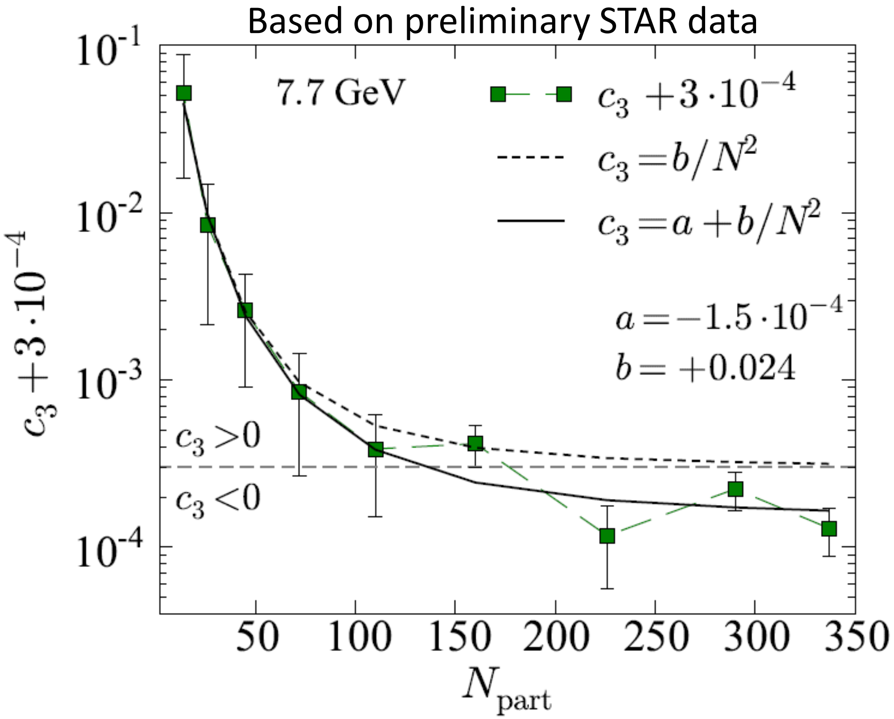
$$K_4=\langle N\rangle+7\langle N\rangle^2\boldsymbol{c}_{\boldsymbol{2}}+6\langle N\rangle^3\boldsymbol{c}_{\boldsymbol{3}}+\langle N\rangle^4\boldsymbol{c}_{\boldsymbol{4}}$$

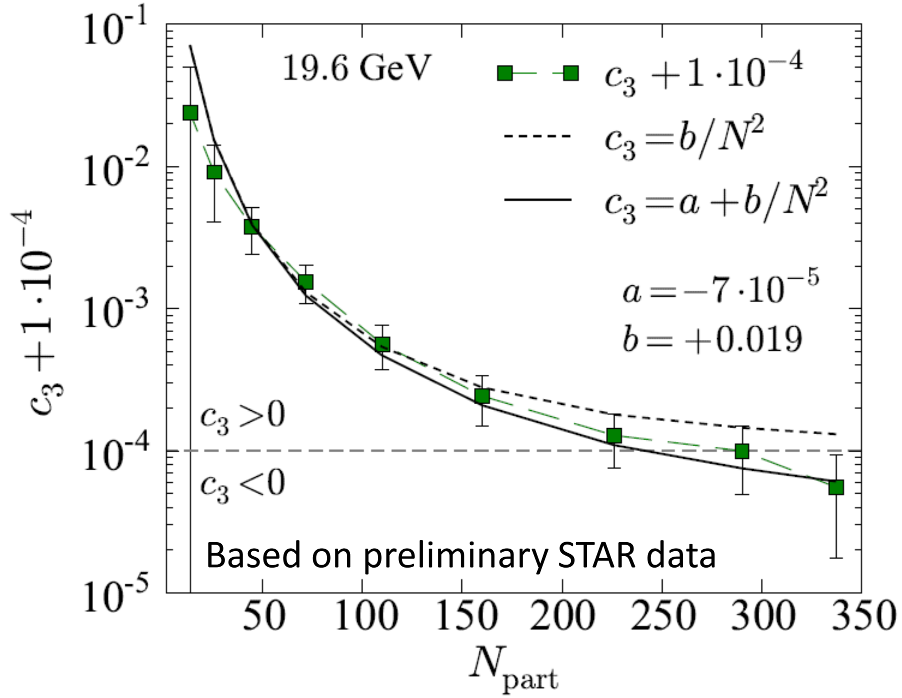
Suppose we have N_s **independent sources** of correlations (resonances, superposition of p+p etc.)

$$\boldsymbol{c}_{\boldsymbol{k}}\sim\frac{N_s}{N^k}\sim\frac{1}{N^{k-1}}$$

Using preliminary STAR data we obtain c_3

AB, V. Koch, N. Strodthoff, 1607.07375



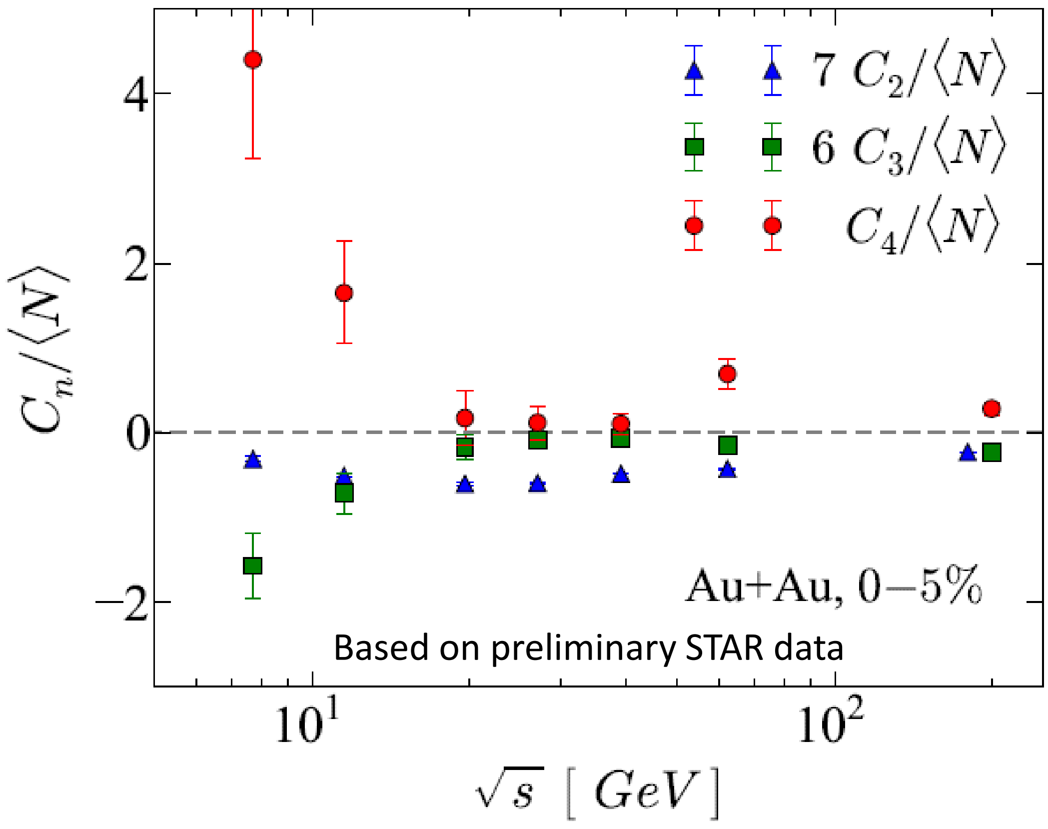


At 7.7 GeV we see $1/N^2$ for small N_{part} then c_3 **changes sign** and stays roughly constant…

Similar story for c_4 (see backup)

Exclusions plots

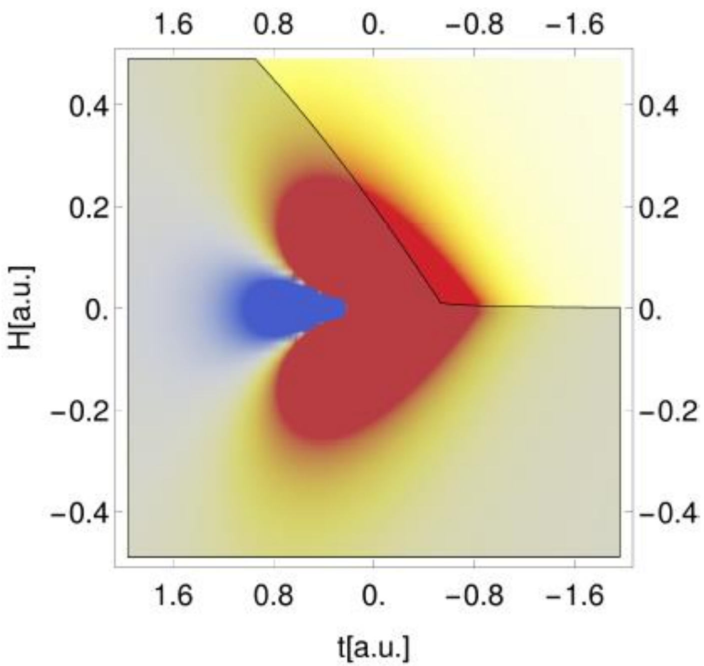
We propose to make the phase-diagram exclusion plots based on the **signs** of the correlation functions.



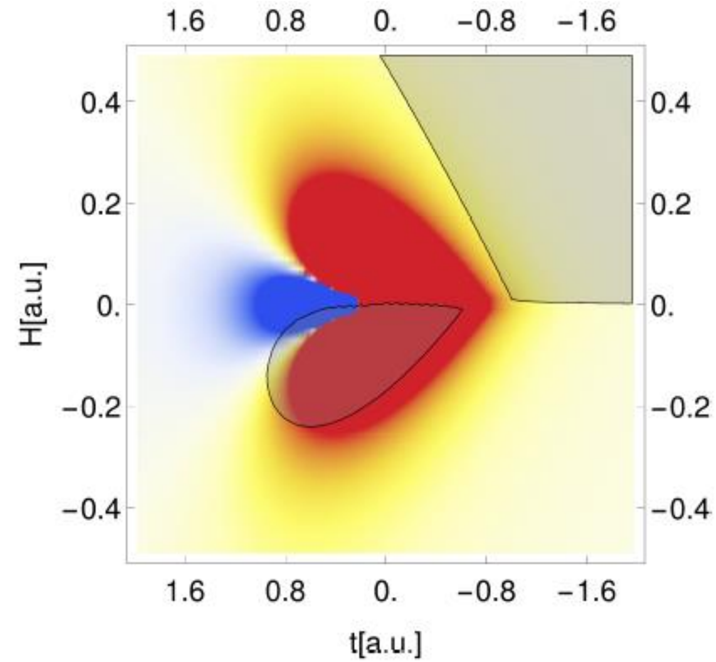
C_4 at 62 GeV !

$$C_4 > 0,\ C_3 < 0 \qquad C_2 < 0$$

Exclusions plots based on the Ising model

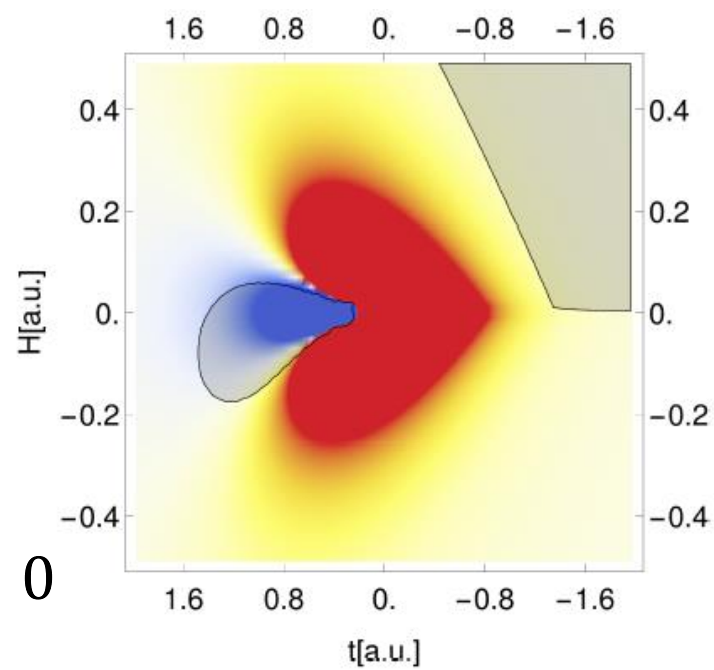


$\boldsymbol{C_2} < 0$



$\boldsymbol{C_3} < 0$

Clearly we need to use more realistic model with various effects included



$\boldsymbol{C_4} > 0$

Conclusions:

Four-proton correlation function observed by STAR is larger by almost **three orders of magnitude** than the minimal model.

To explain C_4 we need a strong source of multi-proton correlations. **Proton clusters**?

Correlation sign change for $N_{\mathrm{part}} \sim 200$ and the transition from independent sources to “collective” source(s)

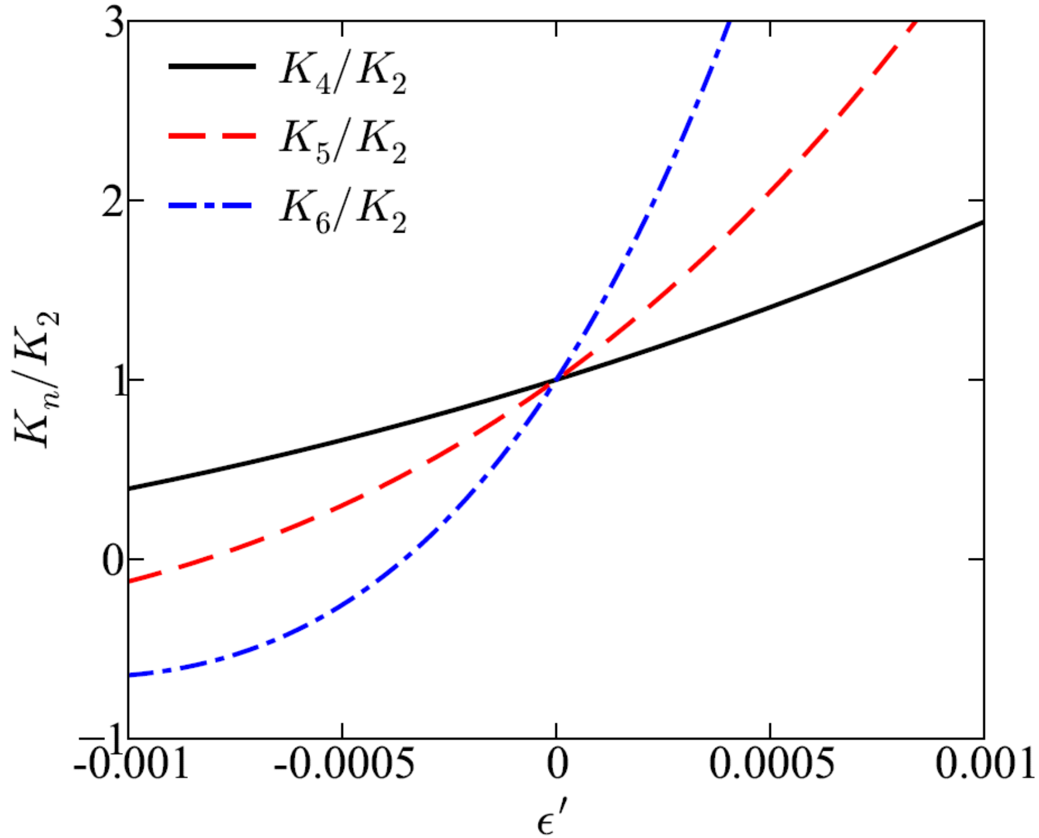
We propose to make the phase-diagram exclusion plots based on the **signs** of the correlation functions

See backup: The observed correlation functions are long-range in rapidity (in comparison to $|y| < 0.5$)

Backup

Multiplicity dependent efficiency

AB, R.Holzmann, V.Koch
1603.09057



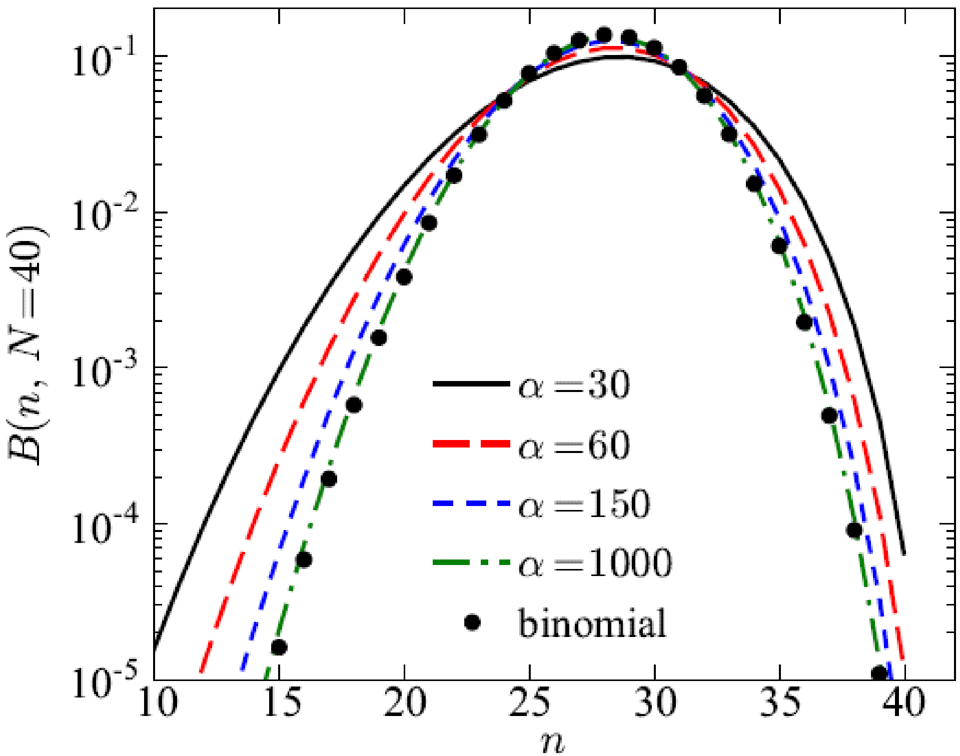
$$P(N) = \frac{\langle N \rangle^N}{N!} e^{-\langle N \rangle},$$

$$\epsilon(N) = \epsilon_0 + \epsilon'(N - \langle N \rangle)$$

Large corrections for small ϵ'

Non-binomial efficiency, e.g., beta-binomial distribution

AB, R.Holzmann, V.Koch
1603.09057



Beta-binomial	$\alpha = 30$	$\alpha = 60$	$\alpha = 150$	$\alpha = 1000$
K_3/K_2	1.28	1.24	1.13	1.02
K_4/K_2	0.82	1.45	1.35	1.07
K_5/K_2	-1.11	1.15	1.63	1.16
K_6/K_2	5.71	-0.44	1.80	1.32

Observations

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2}$$

$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

Rapidity dependence:

short-range correlation

$$c_2(y_1, y_2) = c_2^0 \delta(y_1 - y_2)$$

$$c_2 \sim 1/(\Delta y)$$

$$K_n \sim \Delta y$$

long-range correlation

$$c_n(y_1, \ldots, y_n) = c_n^0$$

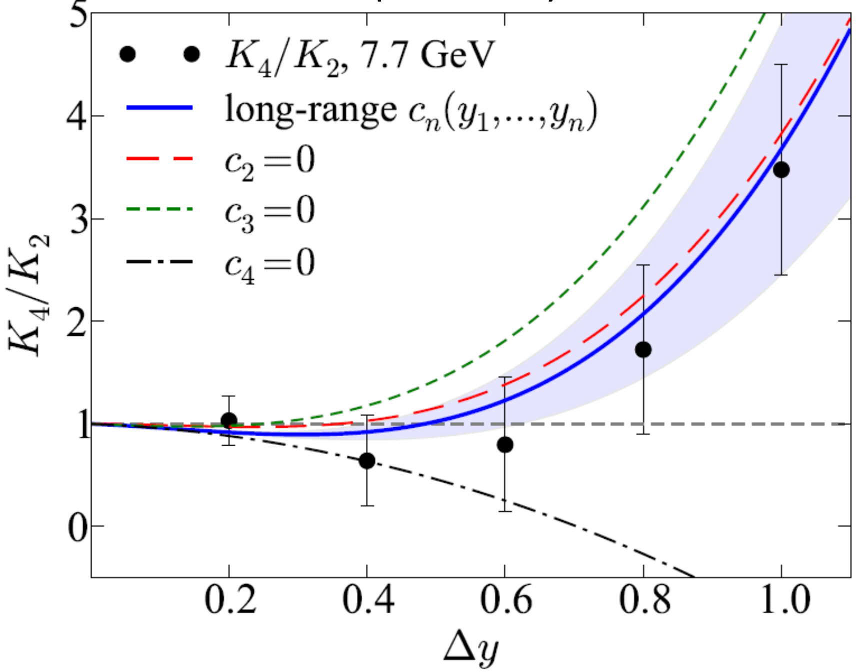
$$c_n = c_n^0$$

$$K_2 = \langle N \rangle + c_2^0 \langle N \rangle^2, \qquad \langle N \rangle \sim \Delta y$$

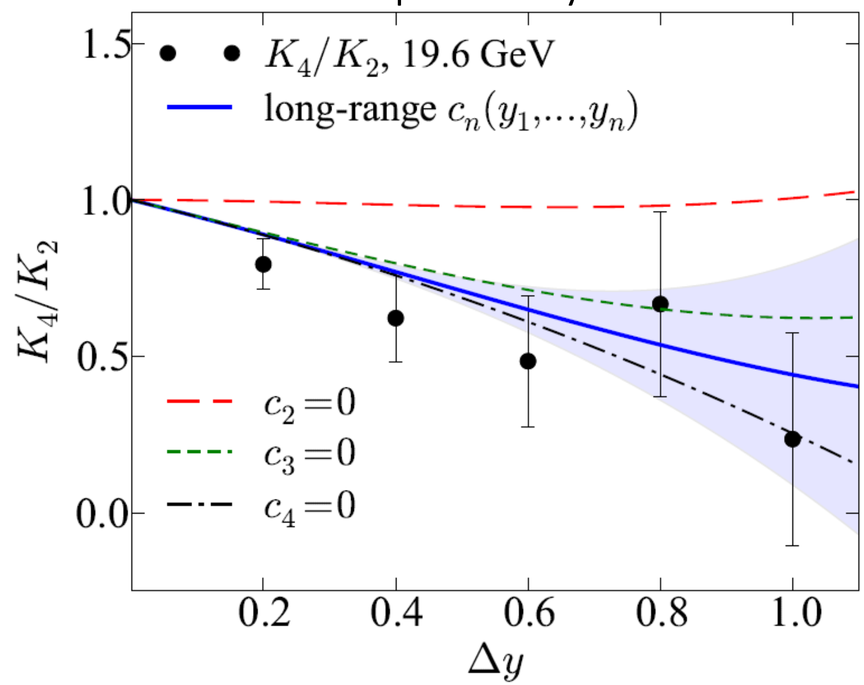
$$K_4 = \langle N \rangle + 7c_2^0\langle N \rangle^2 + 6c_3^0\langle N \rangle^3 + c_4^0\langle N \rangle^4$$

Rapidity dependence consistent with long-range correlations

Based on preliminary STAR data

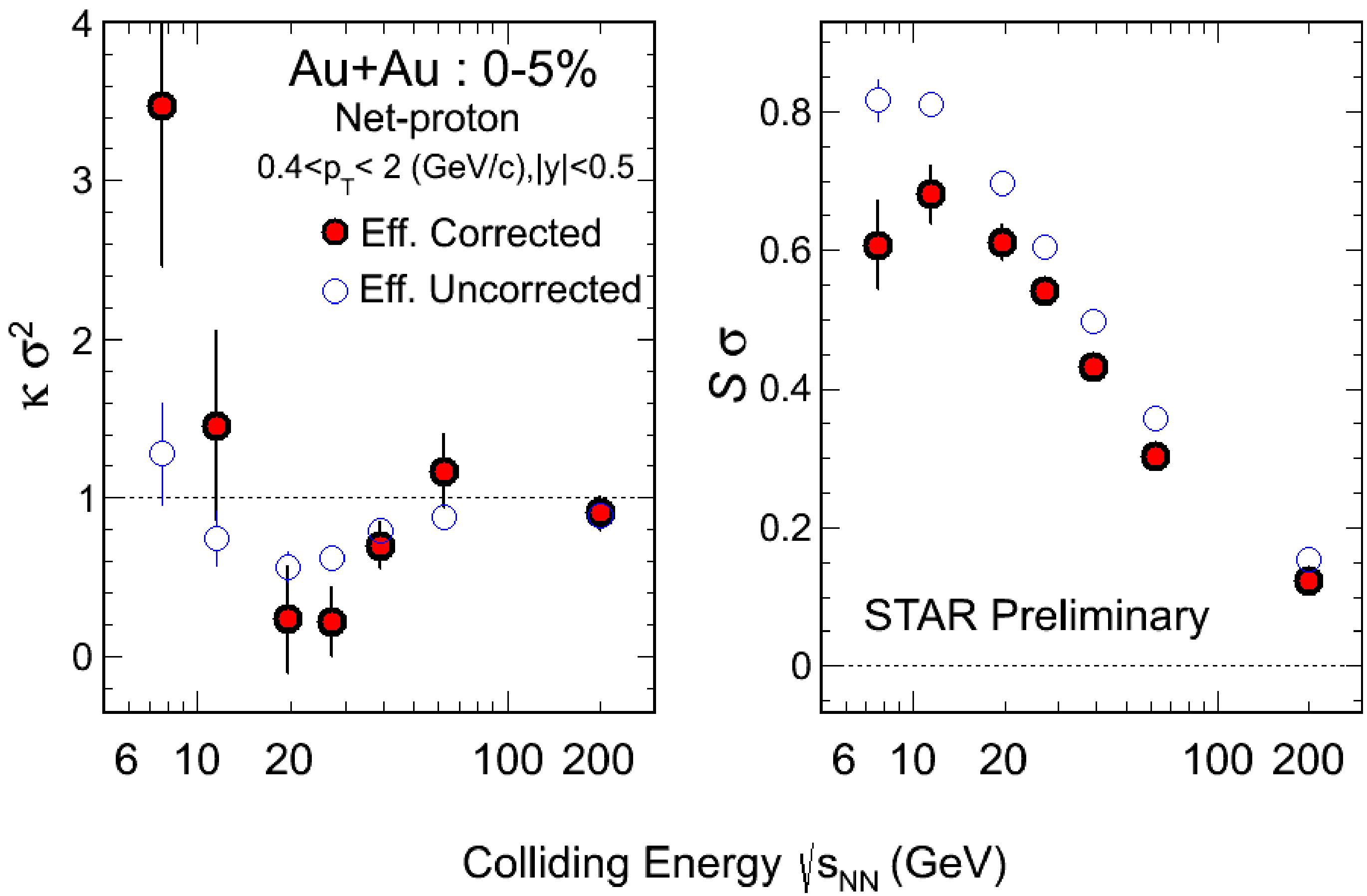


Based on preliminary STAR data



$|y| < 0.5$ is not particularly large

Preliminary STAR data



K_4/K_2 my notation K_3/K_2

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)[1 + \boldsymbol{c_2}(y_1, y_2)]$$

reduced correlation function

$$\langle N(N-1)\rangle = \langle N\rangle^2 + \langle N\rangle^2 \boldsymbol{c_2}$$

$$\boldsymbol{c_2} = \frac{\int \rho(y_1)\rho(y_2)\boldsymbol{c_2}(y_1, y_2)\,dy_1 dy_2}{\int \rho(y_1)\rho(y_2)\,dy_1 dy_2}$$

and the second order cumulant

$$K_2 = \langle N\rangle + \underbrace{\langle N\rangle^2 \boldsymbol{c_2}}_{\boldsymbol{C_2}}$$

In the same way

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3)[1 + \boldsymbol{c_2}(y_1, y_2\ ) + \cdots + \boldsymbol{c_3}(y_1, y_2, y_3)]$$

$$F_3 = \langle N(N-1)(N-2)\rangle = \langle N\rangle^3 + 3\langle N\rangle^3 \boldsymbol{c_2} + \langle N\rangle^3 \boldsymbol{c_3}$$

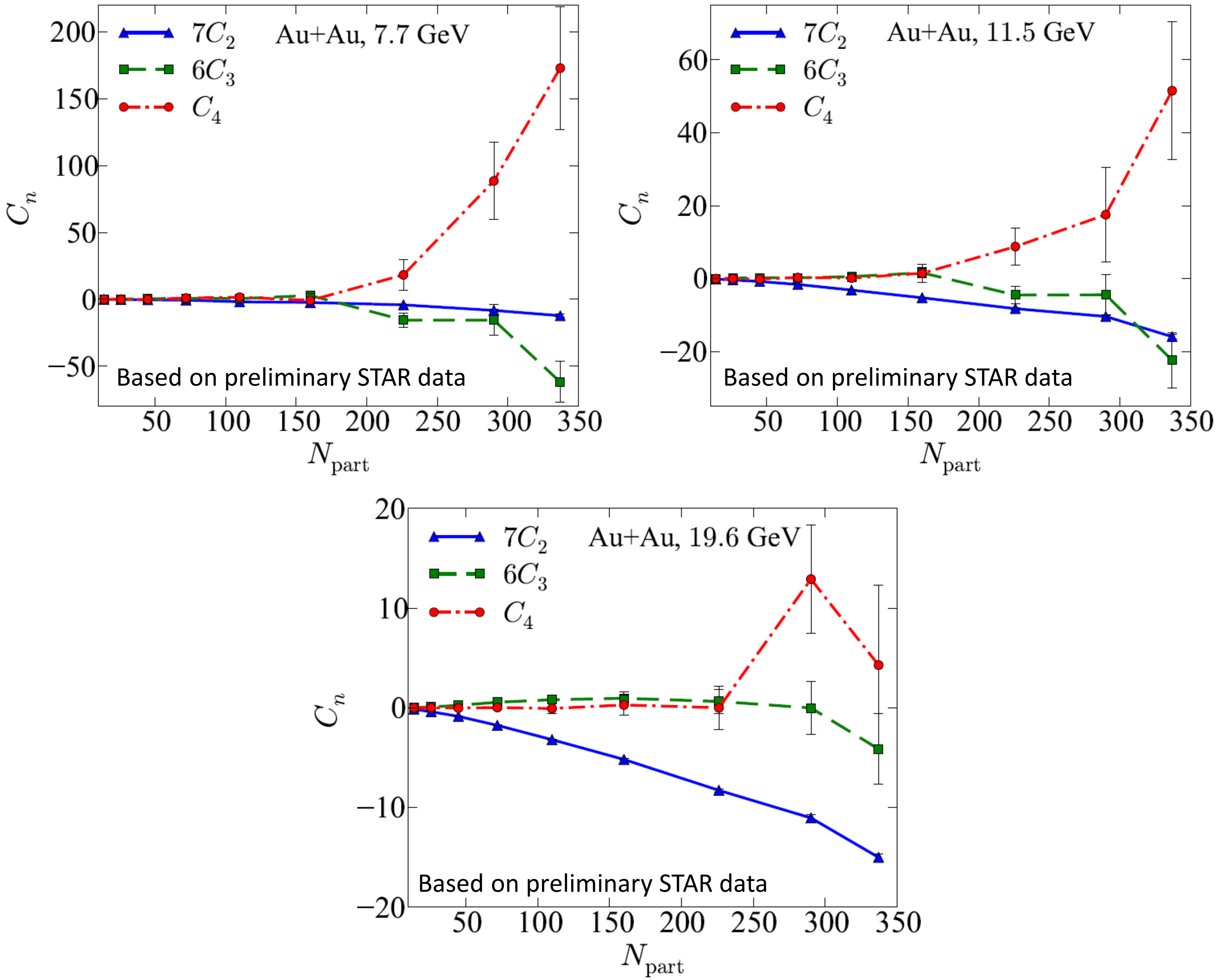
$$\boldsymbol{c_3} = \frac{\int \rho(y_1)\rho(y_2)\rho(y_3)\boldsymbol{c_3}(y_1, y_2, y_3)dy_1 dy_2 dy_3}{\int \rho(y_1)\rho(y_2)\rho(y_3)dy_1 dy_2 dy_3}$$

and the third order cumulant

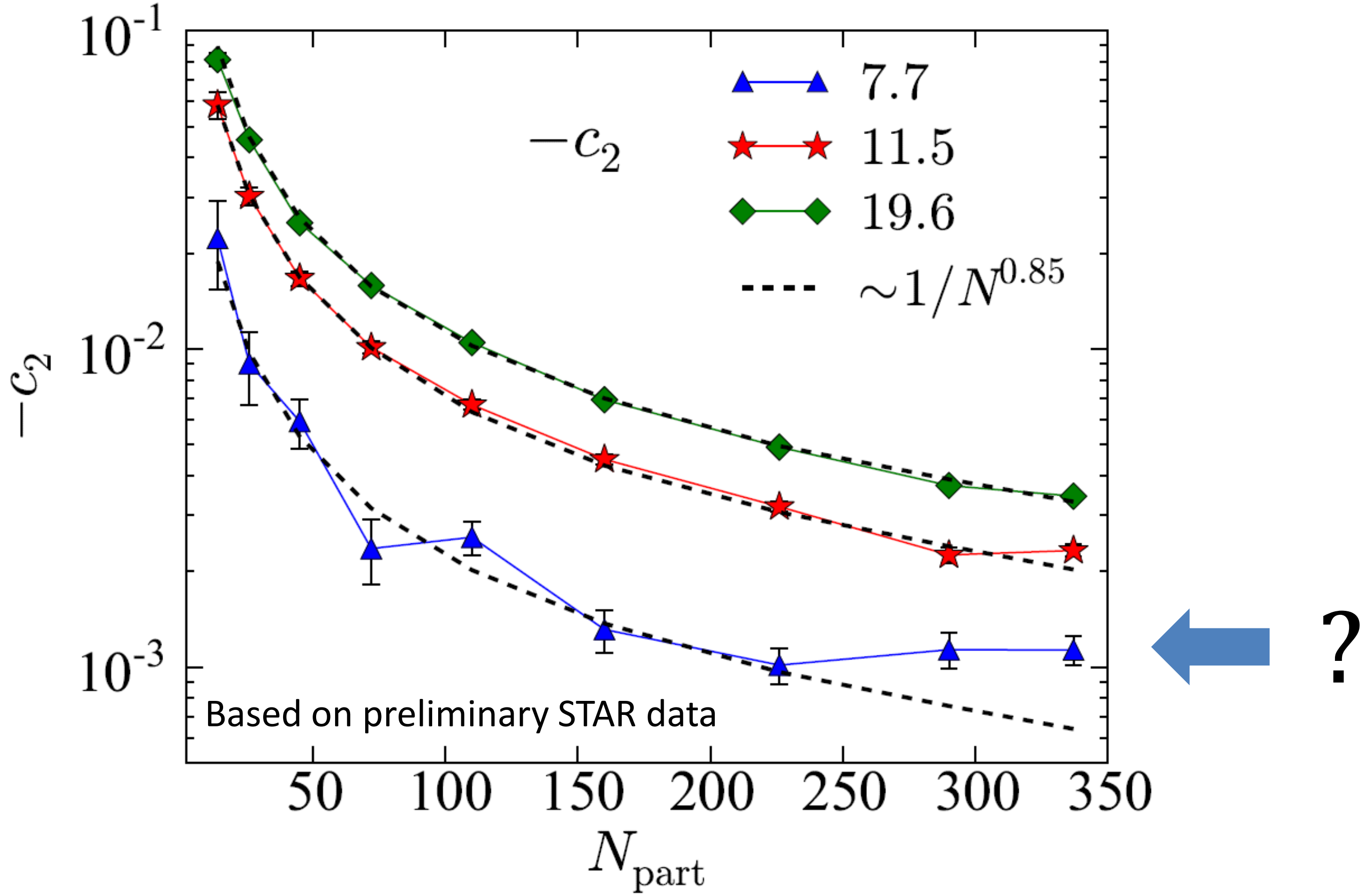
$$K_3 = \langle N\rangle + 3\langle N\rangle^2 \boldsymbol{c_2} + \langle N\rangle^3 \boldsymbol{c_3}$$

$3\boldsymbol{C_2}$ $\boldsymbol{C_3}$

Comparison of 7.7, 11.5 and 19.6 GeV



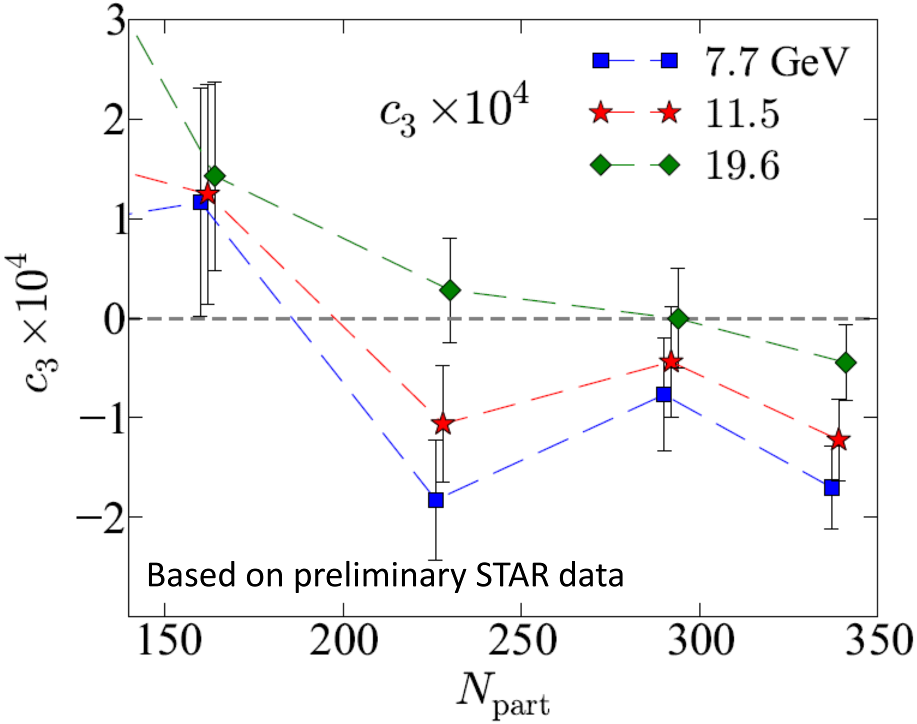
results for c_2

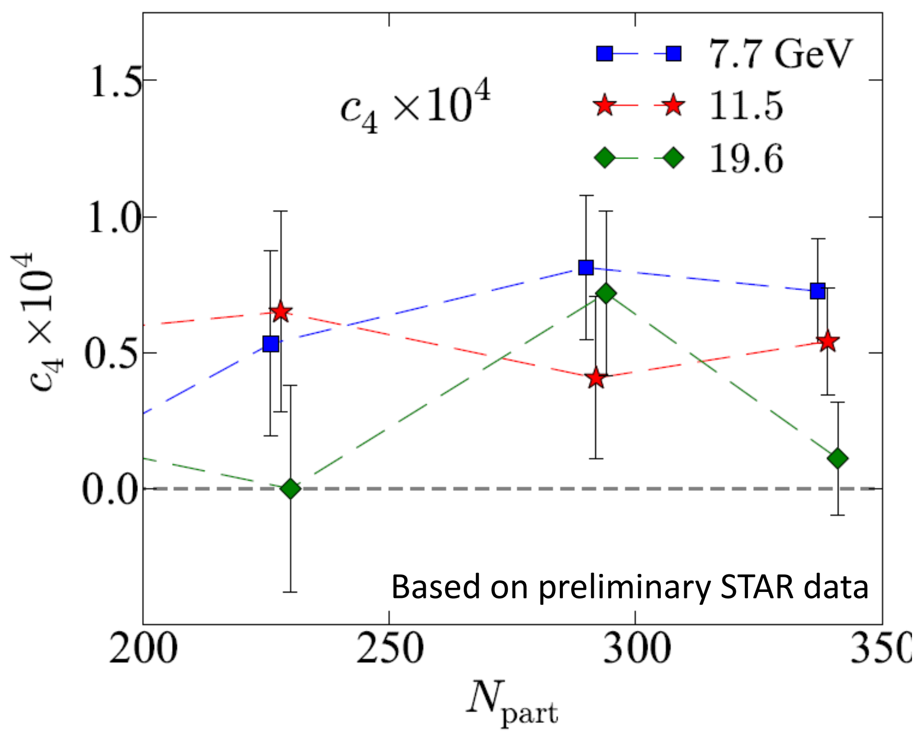


central 7 GeV points are somehow special

AB, V. Koch, N. Strodthoff , 1607.07375

results for central c_3 and c_4





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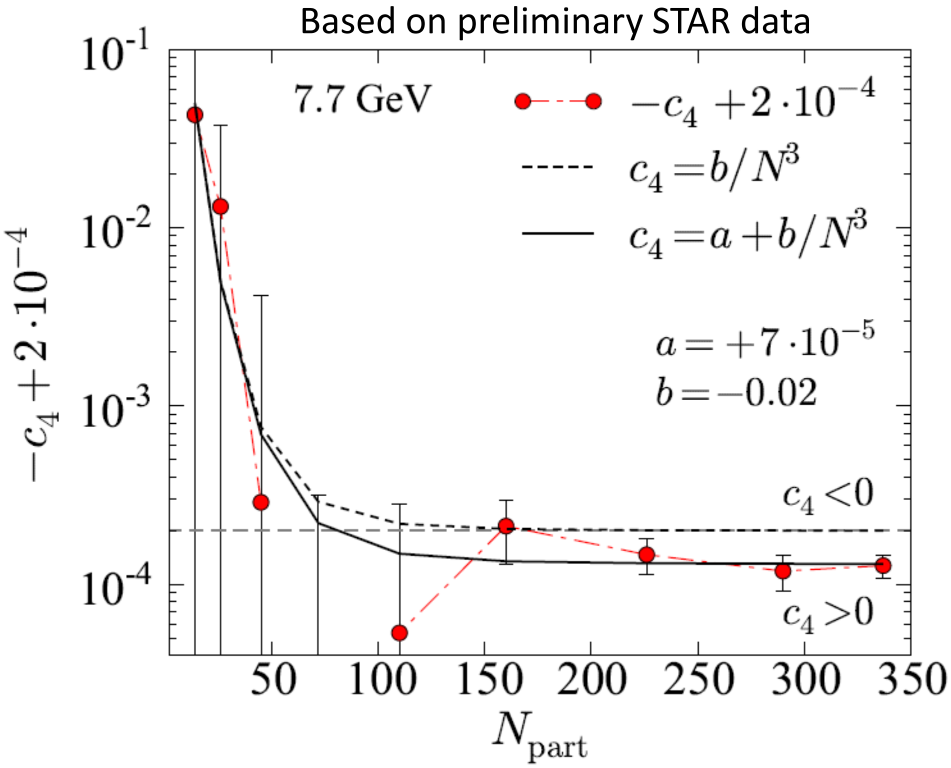
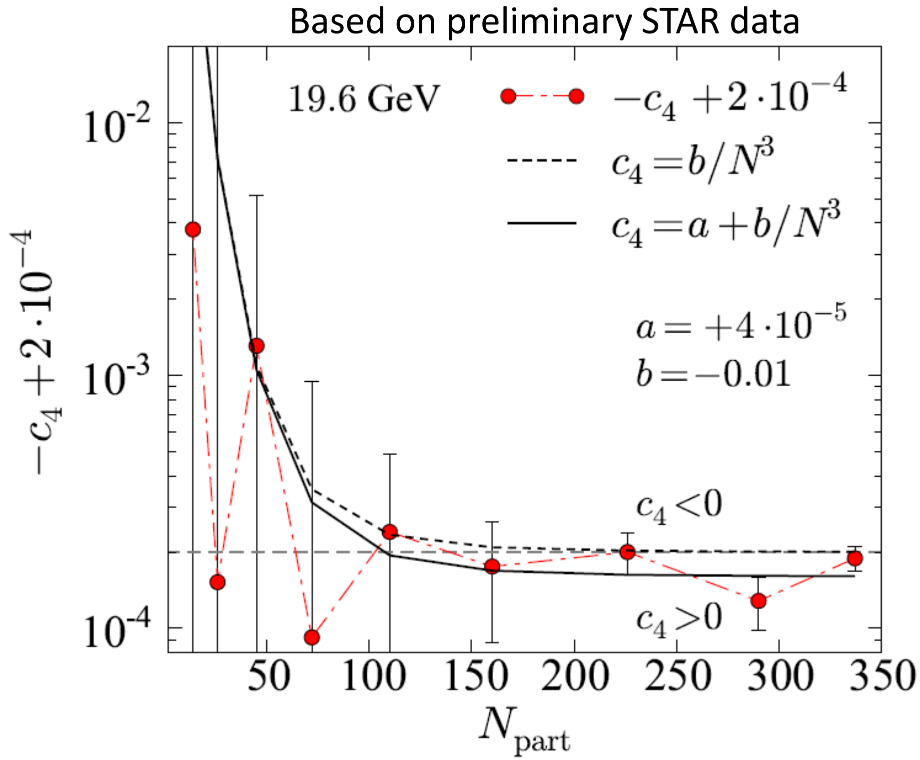
results for c_4





AB, V. Koch, N. Strodthoff , 1607.07375

At low energy protons are not produced. They are transferred from incoming nucleus.

There is no infinite deceleration. It take some time and length to slow down or stop a proton.

$$E_z = E_i - \sigma(z - z_c)$$

E_i − initial energy

z_c − collision point

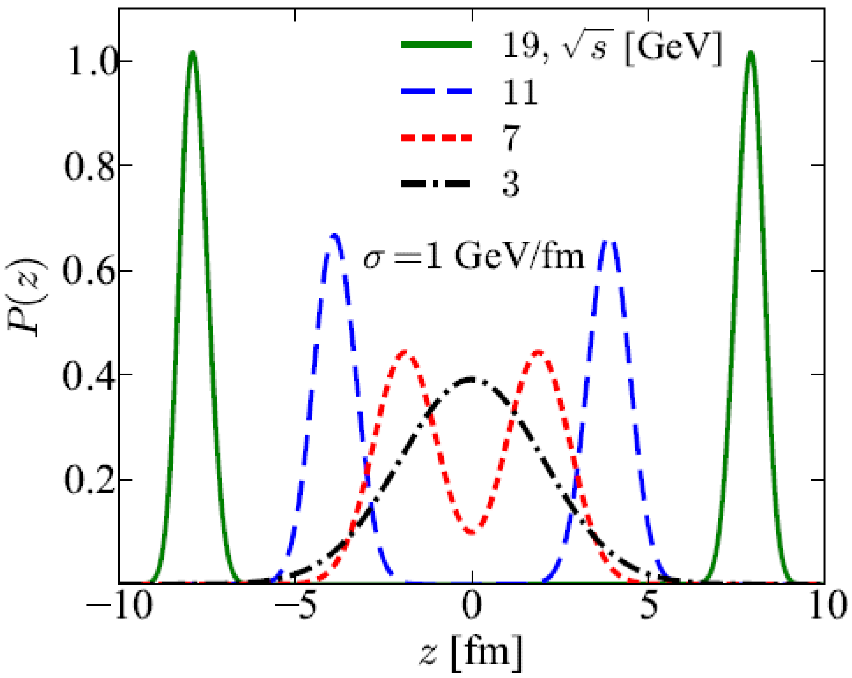
E_z − energy at a point z

$$E_z \rightarrow M_t \cosh(y)$$

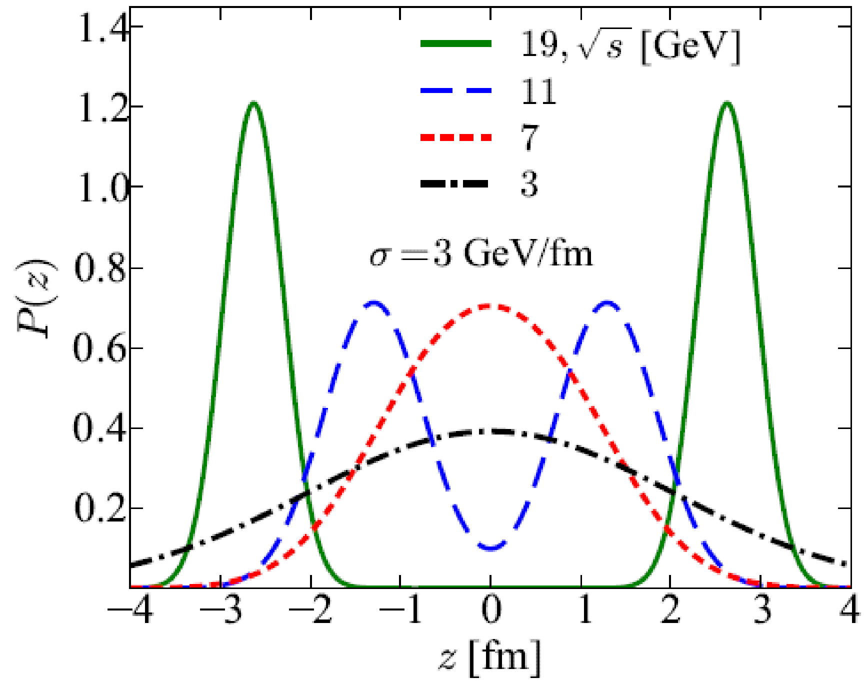
σ − energy loss per unit length

Distribution of protons in z

A.Bialas, AB, V.Koch, 1608.07041



wounded nucleon model



wounded quark model

Are protons stopped in pairs, triplets etc.?

Correlation between pions and protons from stopping?