Effective Field Theories for thermal QCD

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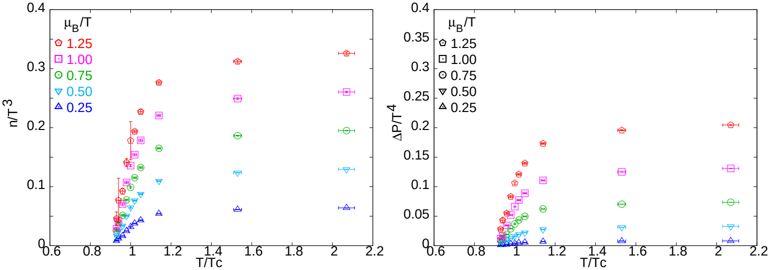
State of the art



Datta, Gavai, SG, 1612.06673

Lattice making strong progress in predicting EOS etc. What about long wavelength physics, slow dynamics? Try EFT

What is an effective field theory?

Intent: describe the long distance, slow dynamics of a system.

Technique:

1. Identify the global symmetries of the system: G
2. Select the fields on which the symmetries act: ψ
3. Choose a cutoff momentum below which one wants a description: $\Lambda \gg p$
4. Write down all possible terms in the action using ψ, derivatives, etc, which are invariant under G. High dimensional terms down by powers of Λ. Keep terms of dimension n; order of accuracy: $(p/\Lambda)^n$.
5. Many couplings in the theory. Sufficient data needed to fix these. Everything else predictions.

Weinberg, Wilson

Global symmetries of QCD at finite temperature

Approximate chiral symmetry $SU_L(N_f) \times SU_R(N_f)$ of QCD acts globally on quark fields. Softly broken even in the limit when the symmetry is exact. Explicitly broken by quark masses. What N_f is appropriate for the EFT?

Scale anomaly of QCD generates an intrinsic scale: Λ_{QCD}. When quark masses $m \gg \Lambda_{QCD}$, inappropriate for EFT. As a result, $N_f = 2$ or 3. We choose $N_f = 2$ as a first exercise.

Lorentz invariance is broken at finite temperature: spatial invariances (rotation, P) remain, T remains, CP symmetry remains. Temporal and spatial components of vectors distinguished. Theory is still relativistic ($p \gg m$), *i.e.*, Lorentz covariant. The counting of mass dimensions is unchanged.

Grossman+, Yaffe+

The effective Lagrangian up to dimension 6

$$L = d^0 + d^3 T_0 \overline{\psi}\psi + \overline{\psi}\partial\!\!\!/_4 \psi + d^4 \overline{\psi}\partial\!\!\!/_i \psi + \frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi} i\gamma_5 \tau^a \psi)^2\right]$$

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&\quad + \frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 + (\overline{\psi} i\gamma_5\psi)^2\right] + \frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2}(\overline{\psi} i\gamma_i\psi)^2
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&+ \frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 + (\overline{\psi} i\gamma_5 \psi)^2\right] + \frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2}(\overline{\psi} i\gamma_i\psi)^2 \\
&+ \frac{d^{65}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 + \frac{d^{66}}{T_0^2}(\overline{\psi} i\gamma_5\gamma_i\psi)^2 \\
&+ \frac{d^{67}}{T_0^2}\left[(\overline{\psi}\gamma_4\tau^a\psi)^2 + (\overline{\psi}\gamma_5\gamma_4\tau^a\psi)^2\right] \\
&+ \frac{d^{68}}{T_0^2}\left[(\overline{\psi} i\gamma_i\tau^a\psi)^2 + (\overline{\psi} i\gamma_5\gamma_i\tau^a\psi)^2\right]
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& + \frac{d^{69}}{T_0^2}\left[(\overline{\psi} i S_{i4}\psi)^2 + (\overline{\psi} S_{ij}\tau^a\psi)^2\right] + \frac{d^{60}}{T_0^2}\left[(\overline{\psi} i S_{i4}\tau^a\psi)^2 + (\overline{\psi} S_{ij}\psi)^2\right]
\end{aligned}
$$

The mean field approximation

Mean field approximations: a field operator is replaced by a number: condensate. Here we can write

$$\psi_\alpha \overline{\psi}_\beta = \delta_{\alpha\beta} \langle \overline{\psi} \psi \rangle.$$

Then doing the Fierz transformations, all terms reduce to a single combination of couplings

$$\lambda = (\mathcal{N} + 2) d^{61} - 2d^{62} - d^{63} + d^{64} + d^{65} - d^{66} + d^{69} - d^{60}.$$

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Using this, MFT same as ordinary NJL:

$$L_6 = -\mathcal{N} \left(\frac{T_0^2}{4\lambda} \right) \Sigma^2 + \Sigma \, \overline{\psi} \psi,$$

where $\Sigma = 2\lambda \langle \overline{\psi} \psi \rangle / T_0^2$. Quadratic Lagrangian, easy to write free energy density:

$$\Omega = -\frac{\mathcal{N} T_0^2 \Sigma^2}{4\lambda} - \mathcal{N} I_0, \qquad I_0 = \int \frac{d^4p}{(2\pi)^4} \log \left(\frac{(p^4)^2 + (d^4)^2 \mathbf{p}^2 + m^2}{T^2} \right).$$

Regularization

After doing the Matsubara sum

$$\Omega = \int \frac{d^3p}{(2\pi)^3} \left[TS(p) + E(p) + E(p, T) \right].$$

$TS(p)$ is a vacuum entropy and is UV divergent. $E(p)$ is a vacuum energy and is UV divergent. $E(p, T)$ is the thermal part and not UV divergent.

Cutoff regularization renders everything finite. But we want more: entropy of the vacuum must vanish. Pauli-Villars regularization and dimensional regularization remove vacuum entropy altogether. Adjust d_0 to get the correct high temperature behaviour. Finite contributions from $E(p)$ remain and are regularization dependent. We work in DR with a "renormalization scale" $M^2 = 4\pi\mu^2 \exp(-2\gamma)$

The gap equation

The pressure is dominated by the UV modes: not captured in any EFT. A good EFT gets: the IR-dominated part of the free energy, critical points, singular parts near it. The usual gap equation is obtained, whose solution is

$$\Sigma = -\frac{2\lambda}{T_0^2}\,\frac{\partial I_0}{\partial m}.$$

Well known chiral critical point at $T_c(\lambda, d^4)$ when $d^3 = 0$. Extend to finite μ. Then we find

$$\frac{T_c^2(\mu)}{T_c^2(0)} = 1 - K\frac{\mu^2}{T_c^2(0)},$$

where $K = 3/\pi^2$. Parameter free prediction of MFT, to be compared with lattice $K \simeq 0.01$–0.05. Lattice may need extrapolation to $d^3 = 0$ and MFT may need correction.

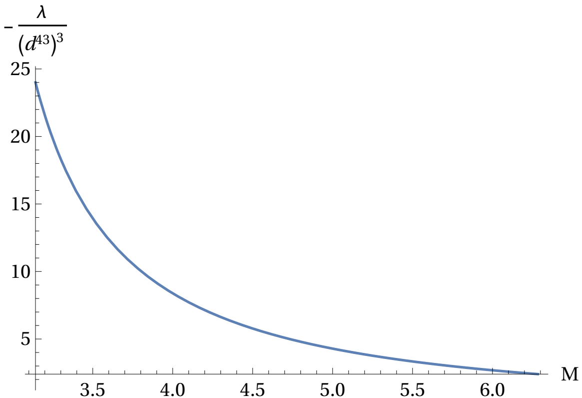
Fitting couplings

With this EFT we want to describe low-momentum physics in a small range of T near T_c: choose $T_0 = T_c$. Far away, the EFT could be different. For example, at $T \gg T_c$ a dimensionally reduced theory may be fine. At very low temperature a pionic theory makes a good EFT.

"Renormalization scale" M is unphysical parameter. If M changes, then the couplings can be changed to keep physics fixed. This is the running of couplings in the EFT. (Different from usual NJL)

For the MFT in the chiral limit ($d^3 = 0$) only two couplings: d^4 and λ. First we fix a combination of these in the chiral limit. Then we may fix d^4 and d^3 separately from quark and meson correlation functions.

Fitting couplings



$M \simeq$ 500–700 MeV is not extreme. Then d^4 and $\lambda/\mathcal{N}$ can be chosen to be $O(1)$ *i.e.*, natural.

Fluctuations

Replace MFT ansatz by

$$\overline{\psi}_\alpha \psi_\beta = \langle \overline{\psi}\psi \rangle U_{\alpha\beta} \qquad \text{where} \qquad U = \exp\left(\frac{i\tilde{\pi}\gamma_5}{f}\right).$$

The pion fields, π, are fluctuations around the MFT. Introduce this into the MFT quadratic Lagrangian, integrate over the fermions, and find the EFT for pions. Result in the form

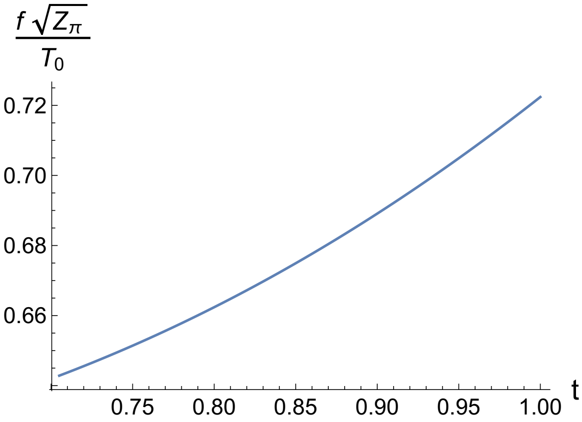
$$L_f = \frac{1}{2}(\partial_0 \pi)^2 + \frac{c_\pi}{2}(\nabla \pi)^2 + m_\pi^2 \pi^2 + \cdots$$

where $\pi = \sqrt{Z_\pi}\widetilde{\pi}$.

Finite T GMOR obtained. **Klevansky (review)**. Similar pion theory used before, using intuition from hydrodynamics. **Son, Stephanov**

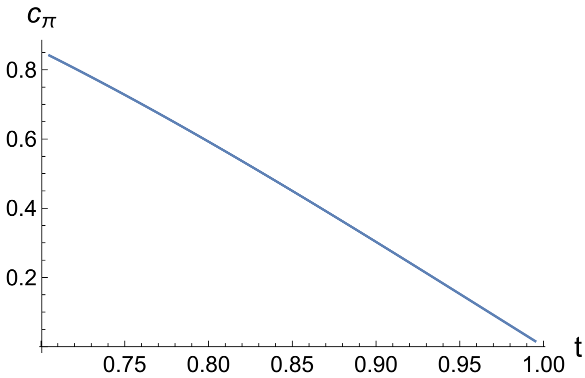
Our contribution: microscopic derivation. Systematic extension to higher order terms in pion EFT possible. Investigating extension to finite μ.

Pion wavefunction renormalization



$t = T/T_c$. Completely smooth behaviour at $t = 1$.

Pion velocity



$c_\pi \simeq (1-t)$ near $t=1$. Also $c_\pi = 0$ for $t>1$. When $c_\pi = 0$, equation of motion gives no propagating mode.

Summary

1. Full fermionic EFT (including V, AV, T terms) after Fierz transformations equivalent to pure NJL model in mean field approximation.
2. Parameter free prediction of curvature of the critical line in the chiral limit of MFT: $K = 3/\pi^2$.
3. Microscopic derivation of the pion EFT. One immediate result: no propagating pion for $T > T_c$.
4. Work in progress: extension to finite μ, pion corrections to susceptibilities, curvature of the critical line, etc.