Evolution of critical fluctuations in a heavy-ion collision scenario

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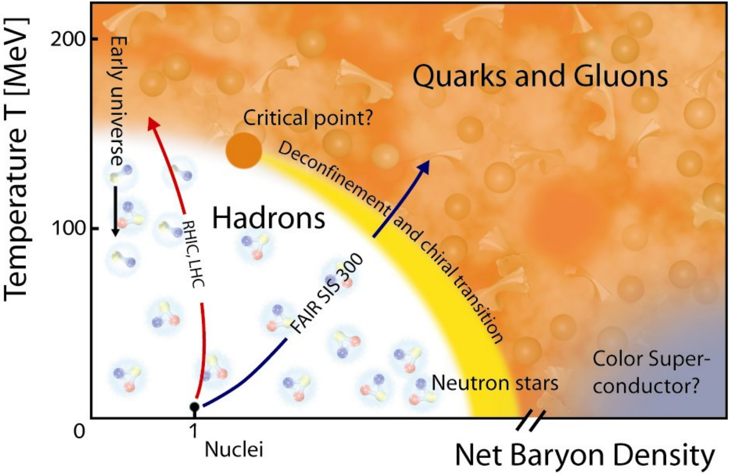
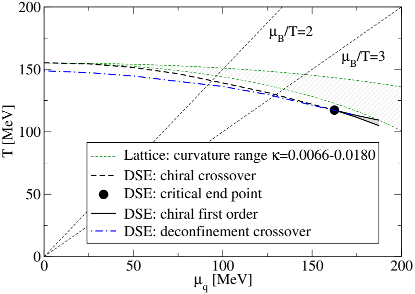
The QCD phase diagram

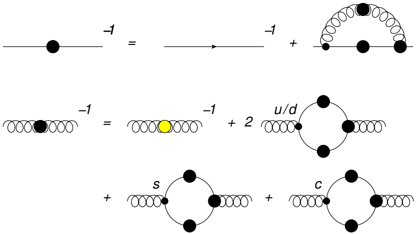


Finding the critical point - I

1. From the QCD Lagrangian

- Solve partition function $\mathcal{Z}$ on a lattice (sign problem for finite μ)
- Solve Dyson-Schwinger equations



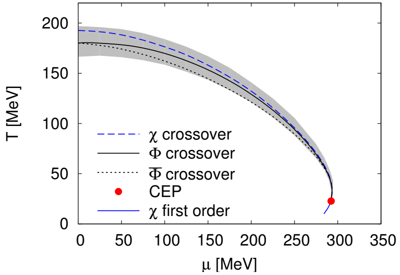


(Fischer, Luecker, Welzbacher, Phys. Rev. D **90** (2014))

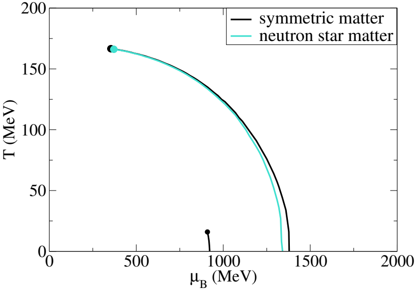
Finding the critical point - II

2. From effective models

- Respect chiral symmetry (Sigma model, NJL model, ...)
- Existence/location of CP not universal!



(Herbst, Pawlowski, Schaefer, Phys. Lett. B **696** (2011) 58-67)



(Dexheimer, Schramm, Phys. Rev. C **81** (2010) 045201)

Finding the critical point - III

3. From experiment

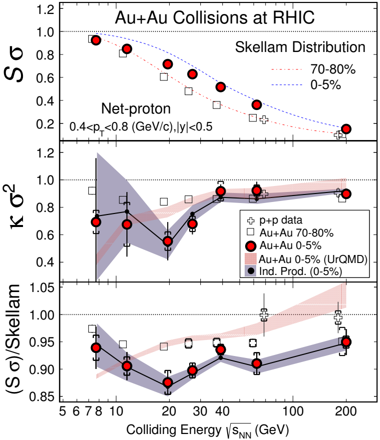
- Fluctuations sensitive to critical region

$$\sigma^2 = \langle \delta N^2 \rangle \sim \xi^2$$

$$S\sigma = \frac{\langle \delta N^3 \rangle}{\langle \delta N^2 \rangle} \sim \xi^{2.5}$$

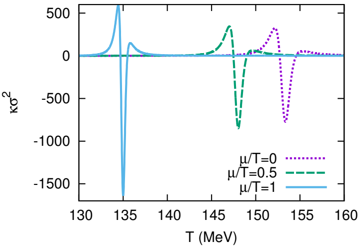
$$\kappa\sigma^2 = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle \sim \xi^5$$

(Stephanov, Phys. Rev. Lett. **102** (2009))



(STAR collaboration, Phys. Rev. Lett. **112** (2014))

Finding the critical point - III



(CH, Nahrgang, in preparation)

In equilibrium:

- Generalized susceptibilities:

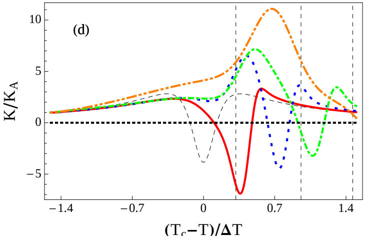
$$c_2 = \left(\frac{\delta^2\Gamma}{\delta\tilde{\sigma}^2}\right)^{-1}$$

$$c_4 = -\frac{\delta^4\Gamma}{\delta\tilde{\sigma}^4}\left(\frac{\delta^2\Gamma}{\delta\tilde{\sigma}^2}\right)^{-1} + 3\left(\frac{\delta^3\Gamma}{\delta\tilde{\sigma}^3}\right)^2\left(\frac{\delta^4\Gamma}{\delta\tilde{\sigma}^4}\right)^{-5}$$

(Effective action Γ, $\tilde{\sigma} = \sigma/T$)

Important in nonequilibrium:

- Critical slowing down
- Memory effects



(Mukherjee, Venugopalan, Yin, Phys. Rev. C **92**, (2015))

The NχFD model

Ingredients for **NχFD** model

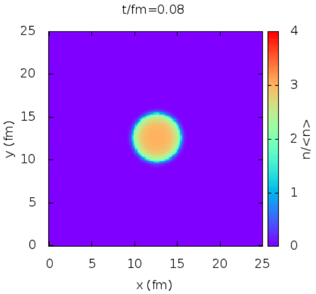
- Fluctuations (chiral fields)
- Fluid (quarks)

$$\frac{\partial^2\sigma}{\partial t^2} - \nabla^2\sigma + \eta\frac{\partial\sigma}{\partial t} + \frac{\delta\Omega}{\delta\sigma} = \xi$$

$$\partial_\mu T_{\mathrm{q}}^{\mu\nu} = S_\sigma^\nu \;, \quad \partial_\mu n^\mu = 0$$

- coarse-grained noise, $\xi_{\mathrm{corr}} = 1/m_\sigma$

(Nahrgang, Leupold, CH, Bleicher, Phys. Rev. C 84 (2011))



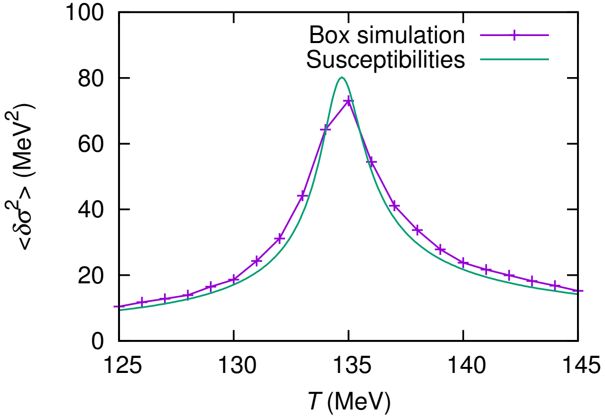
Based on LσM

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu\partial_\mu - g\sigma\right) + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma)\,,$$ possibly extended with ℓ, χ

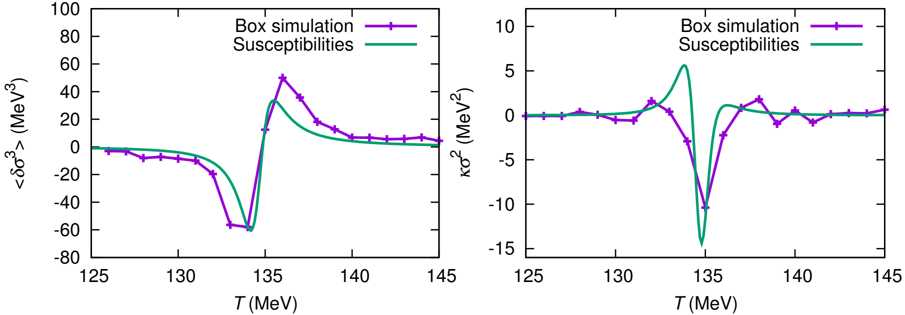
- **Self-consistent** coupling of order parameter fluctuations to fluid dynamical space-time evolution
- Successfully describes: **critical fluctuations in and out of equilibrium**

Cumulants in a thermalized box - sigma I

- Isothermal box with periodic boundary conditions

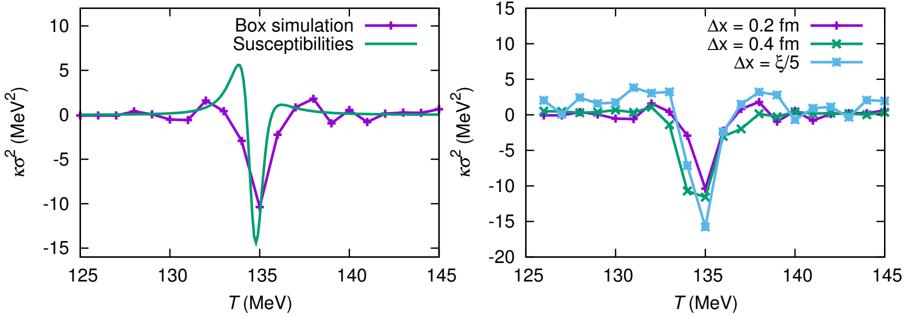


Cumulants in a thermalized box - sigma II



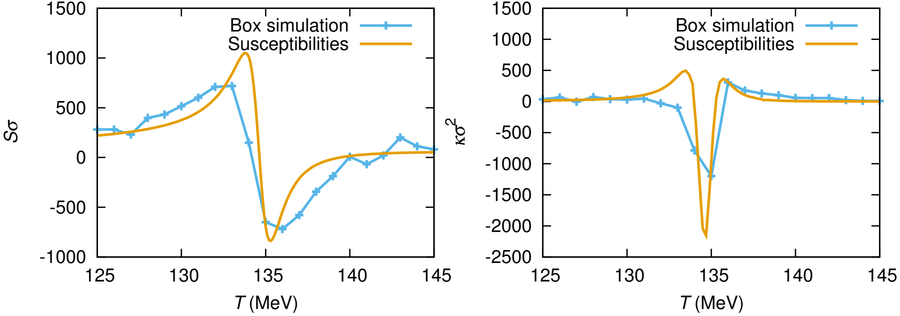
- Moments/Cumulants follow trend of susceptibilities

Cumulants in a thermalized box - sigma III



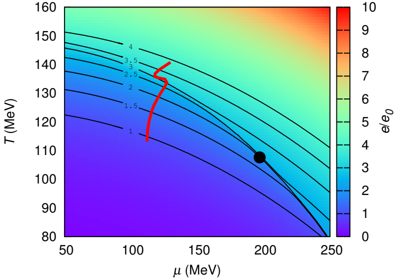
- Moments/Cumulants follow trend of susceptibilities
- Stable for several lattice spacings

Cumulants in a thermalized box - baryons

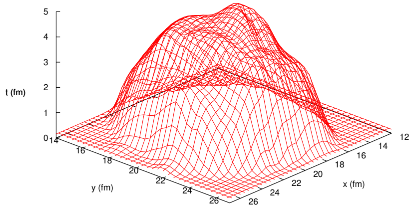


- Calculate $n(x)$ locally from T, μ, $\sigma(x)$
- Moments/Cumulants follow trend of susceptibilities

The kurtosis in an expanding medium - I

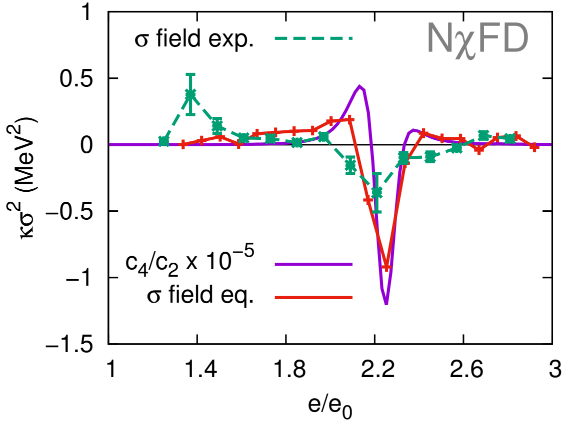


(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))



- Study crossover evolution left of CP
- Determine net-proton kurtosis on energy hypersurfaces
- Smooth hypersurfaces at crossover

The kurtosis in an expanding medium - II



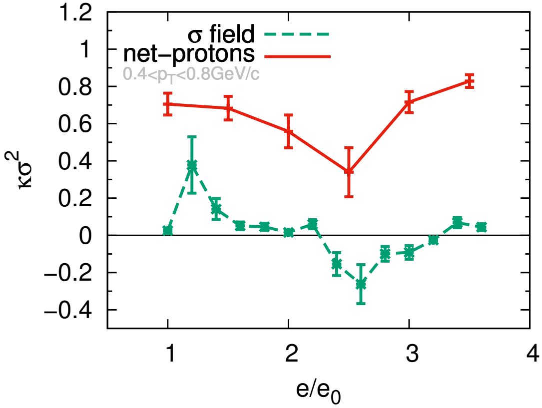
(CH, Nahrgang, in preparation)

- Here, e is determined from T and μ along the trajectory

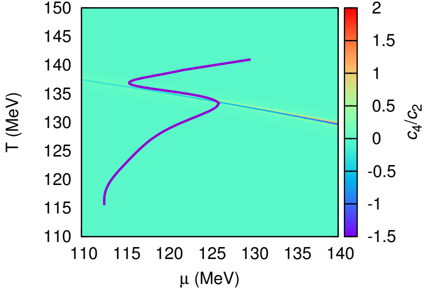
Kurtosis - net-proton vs. sigma

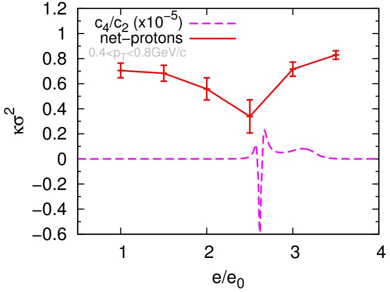
- Perform Cooper-Frye freezeout
- Total net-baryon number exactly conserved in each event
- Net-proton kurtosis follows kurtosis of sigma field

(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))



Kurtosis - net-proton vs. susceptibilities





(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))

- Comparison of net-proton kurtosis to equilibrium fluctuations
- Characteristic dip imprints signal on net-proton kurtosis

The kurtosis - net-proton dynamical vs. mean-field

- Net-proton kurtosis follows kurtosis of sigma field
- In contrast: Mean-field kurtosis remains flat
- In mean-field (hydro/eos): critical fluctuations do not build up

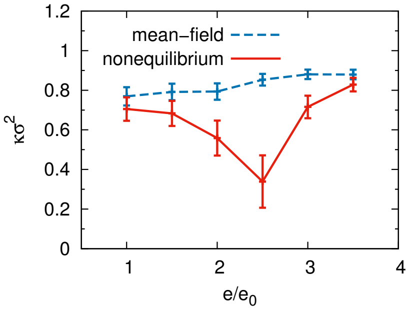
(CH, Nahrgang, Yan, Kobdaj, PRC **93** (2016))

- Mean-field

$$\left.\frac{\partial\Omega}{\partial\sigma}\right|_{\sigma=\langle\sigma\rangle} = 0$$

- Nonequilibrium

$$\frac{\partial^2\sigma}{\partial t^2} - \nabla^2\sigma + \eta\frac{\partial\sigma}{\partial t} + \frac{\delta\Omega}{\delta\sigma} = \xi$$



Summary

- Modeling phase transitions in HICs with **NχFD**
- Formation of dynamical fluctuations of the chiral order parameter with Langevin equation
- Criticality visible in nonmonotonic sigma kurtosis
- Signal imprinted in kurtosis of net-protons

Outlook

- Compare net-proton with net-baryon, acceptance range
- Study beam energy dependence of kurtosis
- Include baryonic degrees of freedom (eos)

Thanks to

BEST COLLABORATION

FIAS Frankfurt Institute for Advanced Studies

The NχFD model

Two possible evolutions:

- Mean-field, local thermal equilibrium without fluctuations

$$\left.\frac{\partial\Omega(T,\mu;\sigma)}{\partial\sigma}\right|_{\sigma=\sigma_{\mathrm{eq}}}=0$$

$$p(T,\mu;\sigma)=-\Omega(T,\mu;\sigma)\ ,\quad \partial_\mu T^{\mu\nu}=0$$

- Full nonequilibrium dynamics with damping and stochastic fluctuations

$$\frac{\partial^2\sigma}{\partial t^2}-\nabla^2\sigma+\eta\frac{\partial\sigma}{\partial t}+\frac{\delta\Omega}{\delta\sigma}=\xi$$

$$p(T,\mu;\sigma)=-\Omega_{\bar{\mathrm{q}}\mathrm{q}}(T,\mu;\sigma)\ ,\quad \partial_\mu T^{\mu\nu}=S^\nu$$

In both cases quark densities are propagated via

$$\partial_\mu n^\mu=0$$