Evolution of critical fluctuations in a heavy-ion collision scenario

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The QCD phase diagram

Temperature $T$ [MeV]

Early universe

Critical point?

Deconfinement and chiral transition

Neutron stars

Color Superconductor?

Hadrons

Net Baryon Density

Nuclei
Finding the critical point - I

1. From the QCD Lagrangian
   - Solve partition function $\mathcal{Z}$ on a lattice (sign problem for finite $\mu$)
   - Solve Dyson-Schwinger equations

(Fischer, Luecker, Welzbacher, Phys. Rev. D 90 (2014))
2. From effective models

- Respect chiral symmetry (Sigma model, NJL model, ...)
- Existence/location of CP not universal!


3. From experiment

- Fluctuations sensitive to critical region

\[ \sigma^2 = \langle \delta N^2 \rangle \sim \xi^2 \]

\[ S\sigma = \frac{\langle \delta N^3 \rangle}{\langle \delta N^2 \rangle} \sim \xi^{2.5} \]

\[ \kappa\sigma^2 = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle \sim \xi^5 \]

(Stephanov, Phys. Rev. Lett. 102 (2009))

(STAR collaboration, Phys. Rev. Lett. 112 (2014))
Finding the critical point - III

In equilibrium:

- Generalized susceptibilities:

\[
c_2 = \left( \frac{\delta^2 \Gamma}{\delta \tilde{\sigma}^2} \right)^{-1}
\]

\[
c_4 = -\frac{\delta^4 \Gamma}{\delta \tilde{\sigma}^4} \left( \frac{\delta^2 \Gamma}{\delta \tilde{\sigma}^2} \right)^{-1} + 3 \left( \frac{\delta^3 \Gamma}{\delta \tilde{\sigma}^3} \right)^2 \left( \frac{\delta^4 \Gamma}{\delta \tilde{\sigma}^4} \right)^{-5}
\]

(Effective action \( \Gamma, \tilde{\sigma} = \sigma / T \))

Important in nonequilibrium:

- Critical slowing down
- Memory effects

(Mukherjee, Venugopalan, Yin, Phys. Rev. C 92, (2015))
The $N\chi$FD model

Ingredients for $N\chi$FD model

- Fluctuations (chiral fields)
- Fluid (quarks)

\[
\frac{\partial^2 \sigma}{\partial t^2} - \nabla^2 \sigma + \eta \frac{\partial \sigma}{\partial t} + \frac{\delta \Omega}{\delta \sigma} = \xi \\
\partial_\mu T_{\mu \nu}^q = S_{\sigma}^\nu, \quad \partial_\mu n_\mu = 0
\]

- Coarse-grained noise, $\xi_{\text{corr}} = 1/m_\sigma$

(Nahrgang, Leupold, CH, Bleicher, Phys. Rev. C 84 (2011))

Based on $L\sigma M$

\[
\mathcal{L} = \bar{q} \left( i \gamma_\mu \partial_\mu - g \sigma \right) + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma), \quad \text{possibly extended with } \ell, \chi
\]

- Self-consistent coupling of order parameter fluctuations to fluid dynamical space-time evolution

- Successfully describes: critical fluctuations in and out of equilibrium
Cumulants in a thermalized box - sigma 1

- Isothermal box with periodic boundary conditions

![Graph showing box simulation and susceptibilities against temperature](image-url)
Moments/Cumulants follow trend of susceptibilities
Cumulants in a thermalized box - sigma III

- Moments/Cumulants follow trend of susceptibilities
- Stable for several lattice spacings
Calculate $n(x)$ locally from $T$, $\mu$, $\sigma(x)$

- Moments/Cumulants follow trend of susceptibilities
The kurtosis in an expanding medium - I

- Study crossover evolution left of CP
- Determine net-proton kurtosis on energy hypersurfaces
- Smooth hypersurfaces at crossover

(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))
Here, \( e \) is determined from \( T \) and \( \mu \) along the trajectory.
Kurtosis - net-proton vs. sigma

- Perform Cooper-Frye freezeout
- Total net-baryon number exactly conserved in each event
- Net-proton kurtosis follows kurtosis of sigma field

(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))
Comparison of net-proton kurtosis to equilibrium fluctuations

Characteristic dip imprints signal on net-proton kurtosis

(Kurtosis - net-proton vs. susceptibilities)

(Ch, Nahrgang, Yan, Kobdaj, PRC 93 (2016))
The kurtosis - net-proton dynamical vs. mean-field

- Net-proton kurtosis follows kurtosis of sigma field
- In contrast: Mean-field kurtosis remains flat
- In mean-field (hydro/eos): critical fluctuations do not build up

(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))

- Mean-field

\[ \left. \frac{\partial \Omega}{\partial \sigma} \right|_{\sigma = \langle \sigma \rangle} = 0 \]

- Nonequilibrium

\[ \frac{\partial^2 \sigma}{\partial t^2} - \nabla^2 \sigma + \eta \frac{\partial \sigma}{\partial t} + \frac{\delta \Omega}{\delta \sigma} = \xi \]
Summary

- Modeling phase transitions in HICs with $N_{\chi}$FD
- Formation of dynamical fluctuations of the chiral order parameter with Langevin equation
- Criticality visible in nonmonotonic sigma kurtosis
- Signal imprinted in kurtosis of net-protons

Outlook

- Compare net-proton with net-baryon, acceptance range
- Study beam energy dependence of kurtosis
- Include baryonic degrees of freedom (eos)

Thanks to

[Best Collaboration logo]

[FIAS logo]
The $N\chi$FD model

Two possible evolutions:

- Mean-field, local thermal equilibrium without fluctuations

\[
\frac{\partial \Omega(T, \mu; \sigma)}{\partial \sigma} \bigg|_{\sigma=\sigma_{eq}} = 0
\]

\[
p(T, \mu; \sigma) = -\Omega(T, \mu; \sigma), \quad \partial_\mu T^{\mu \nu} = 0
\]

- Full nonequilibrium dynamics with damping and stochastic fluctuations

\[
\frac{\partial^2 \sigma}{\partial t^2} - \nabla^2 \sigma + \eta \frac{\partial \sigma}{\partial t} + \frac{\delta \Omega}{\delta \sigma} = \xi
\]

\[
p(T, \mu; \sigma) = -\Omega_{\bar{q}q}(T, \mu; \sigma), \quad \partial_\mu T^{\mu \nu} = S^{\nu}
\]

In both cases quark densities are propagated via

\[
\partial_\mu n^\mu = 0
\]