The QCD equation of state at finite density from analytical continuation

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for the Wuppertal-Budapest-Collaboration

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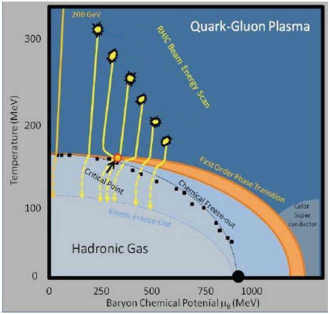
SFB TR55

BERGISCHE UNIVERSITÄT WUPPERTAL

WB collaboration

The (T, μ_B)-phase diagram of QCD



Our observables:

T_c

R. Bellwied et al., Phys. Lett. B751, 559 (2015), arXiv:1507.07510

The Equation of State along trajectories of constant $\frac{S}{N_B}$ and its Taylor coefficients determined by the method of analytical continuation

J. Günther et al., arXiv:1607.02493

The sign problem

The QCD partition function:

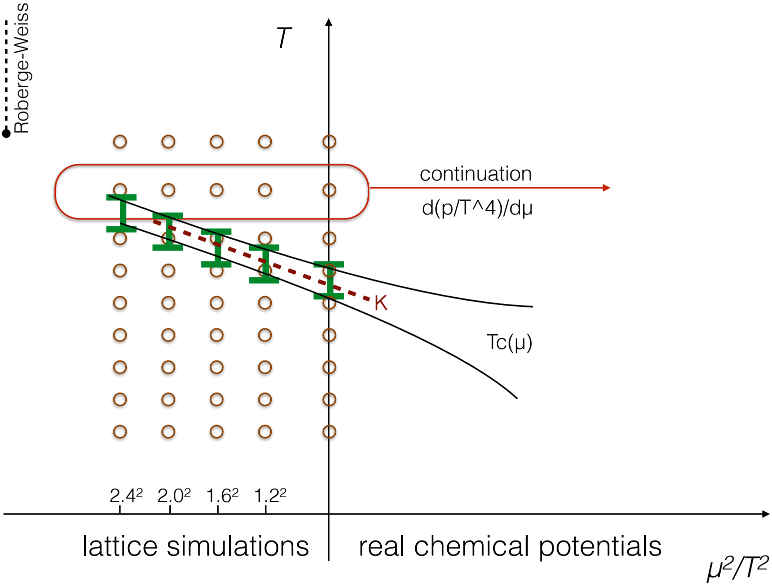
$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi} e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U \det M(U) e^{-\beta S_G(U)}$$

- For Monte Carlo simulations $\det M(U)e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle- antiparticle-symmetry $\det M(U)$ is real
- If $\mu^2 > 0$ $\det M(U)$ is complex

Dealing with the sign problem

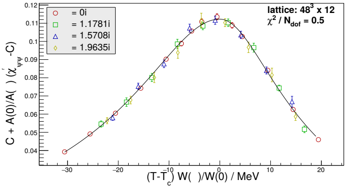
- Reweighting technics
- Canonical ensemble
- Complex Langevin
- Lefshetz Thimble
- Density of state methods
- Dual veriables
- Taylor expansion $\longrightarrow$ Talk at 15:20 by Sayantan Sharma
- Imaginary μ
- ...

Analytic continuation

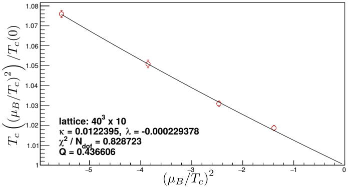


Overview over the Analysis

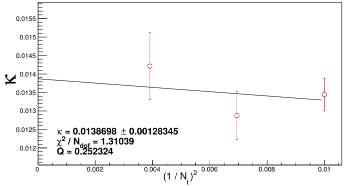
1. Simulationen at $\langle n_S \rangle \approx 0$
2. Extrapolation to $\langle n_S \rangle = 0$
3. Fit in the T direction



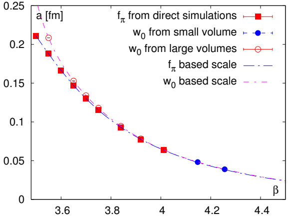
4. Deterine T_c
5. Fit in the μ_B direction
6. Determine κ



7. Fit in the $\frac{1}{N_t^2}$ direction



Simulation details



- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\langle n_S \rangle = 0$ (as for heavy ion collisions, in contrast to simulations with $\mu_s = 0$ or $\mu_S = 0$ where $\mu_S = \frac{1}{3}\mu_B - \mu_s$)
- Lattice sizes: $32^3 \times 8$, $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 3, 4, 5$ and 6
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

Observables

Chiral susceptibility:

$$\chi_{\bar{\psi}\psi} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial (m_q)^2}$$

$$\chi^r_{\bar{\psi}\psi} = \left(\chi_{\bar{\psi}\psi}(T,\beta) - \chi_{\bar{\psi}\psi}(0,\beta)\right) \frac{m_l^2}{m_\pi^4}$$

Chiral condensate:

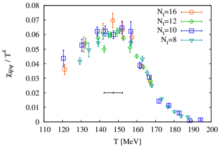
$$\langle\bar{\psi}\psi\rangle = \frac{T}{V} \frac{\partial \ln Z}{\partial m_q}$$

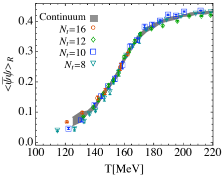
$$\langle\bar{\psi}\psi\rangle^r = -\left(\langle\bar{\psi}\psi\rangle(T,\beta) - \langle\bar{\psi}\psi\rangle(0,\beta)\right) \frac{m_l}{m_\pi^4}$$

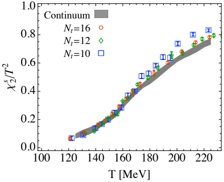
Strangeness susceptibility:

$$\chi_{SS} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial (\mu_S)^2}$$

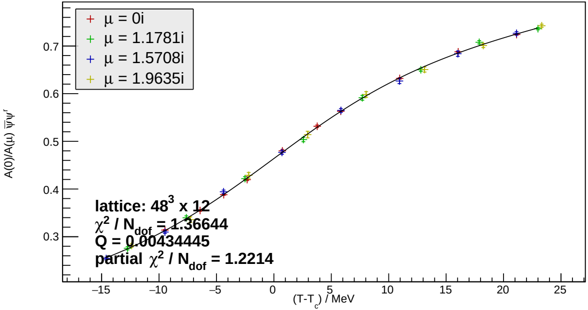
S. Borsányi et al (2010, arXiv:1005.3508)







$\bar{\psi}\psi$

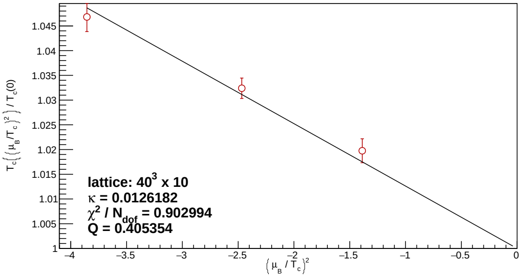


Fit function:

$$\langle\bar{\psi}\psi\rangle^r(\mu, T) = A(\mu)\left(1 + B\tanh\left[C\left(T - T_c(\mu)\right)\right] + D\left(T - T_c(\mu)\right)\right)$$

$$\left(\text{ or } \bar{\psi}\psi^r(\mu, T) = A(\mu)\left(1 + B\arctan\left[C\left(T - T_c(\mu)\right)\right] + D\left(T - T_c(\mu)\right)\right)\right)$$

Curvature



Curvature function:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$

For error analysis we also fit:

$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1 + ax}{1 + bx}$$

$$C_3(x) = \frac{1}{1 + ax + bx^2}$$

Continuum extrapolation

Continuum extrapolation:
$\kappa = \kappa^{\mathrm{c}} + A\left(\frac{1}{N_t}\right)^2$
Combined curvature fit and continuum extrapolation with:
$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \left(\kappa^{\mathrm{c}} + c_1 \frac{1}{N_t^2}\right)\left(\frac{\mu_B}{T_c}\right)^2$

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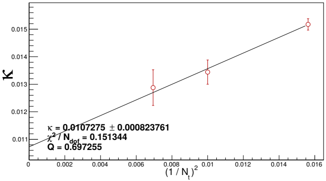
Extrap. with $Nt = 8, 10, 12$



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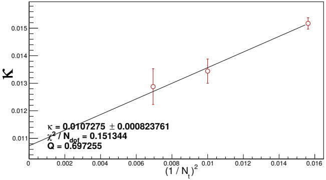

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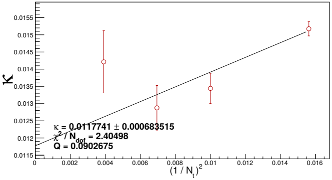

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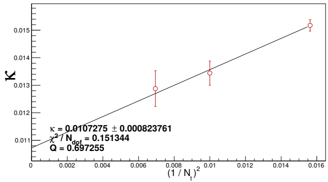

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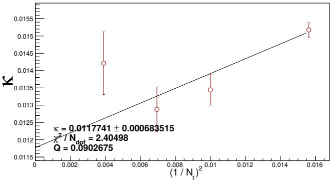

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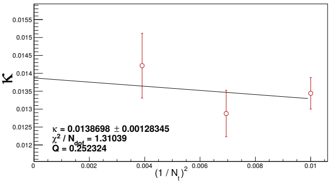
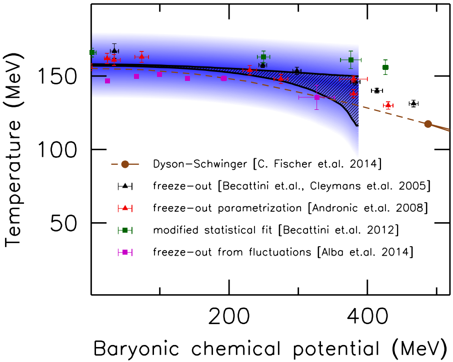


T_c extrapolation

Determining $T_c(\mu_B)$ by solving the equation $\frac{T_c(\mu_B)}{T_c(0)} = C_i\left(-\frac{\mu_B^2}{T_c^2(\mu)}\right)$.



$$C_0(x) = 1 + ax$$

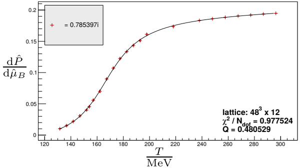
$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1 + ax}{1 + bx}$$

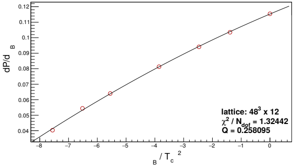
$$C_3(x) = \frac{1}{1 + ax + bx^2}$$

Overview over the Analysis

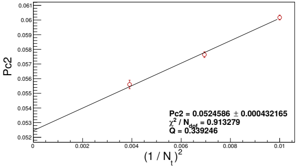
1. Do the simulations at $\langle n_S \rangle \approx 0$
2. Extrapolate to $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$
3. Make a fit in the T direction



4. Determine everything you need for the observables
5. Make a fit in the μ_B direction



6. Make a fit in the $\frac{1}{N_t^2}$ direction
7. Determine the observables



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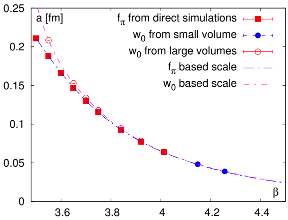

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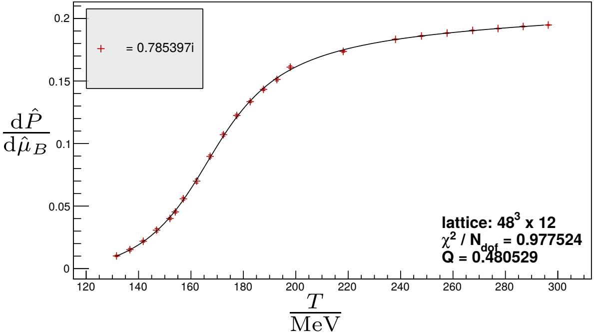
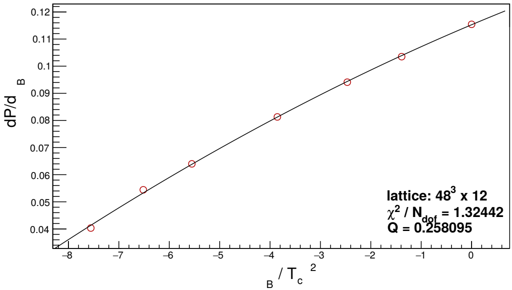
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

Fit in the T direction



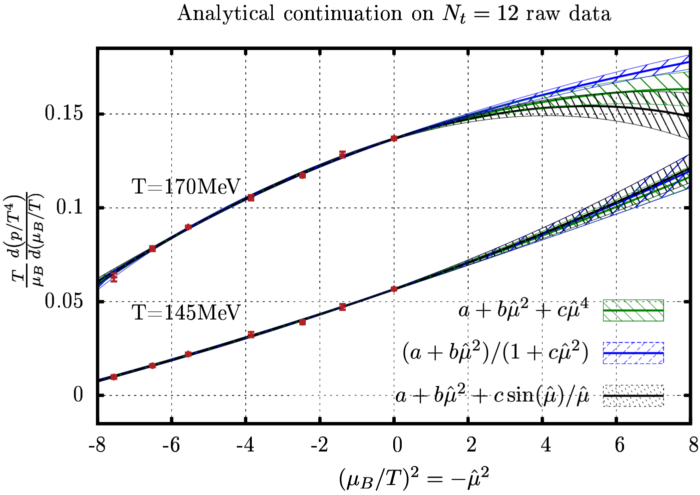
$$\begin{aligned}
A_1(T) &= a + bT + c/T + d \arctan(e(T-f)) \\
A_2(T) &= a + bT + c/T + d/(1 + e(T-f)^g)^{1/g}, \\
A_3(T) &= a + bT + cT^2 + d \arctan(e(T-f)) \\
A_4(T) &= a + bT + cT^2 + d/(1 + e(T-f)^g)^{1/g}.
\end{aligned}$$

Fit in the μ_B direction



$$
\begin{aligned}
B_1(\hat{\mu}) &= a + b\hat{\mu}^2 + c\hat{\mu}^4 \\
B_2(\hat{\mu}) &= (a + b\hat{\mu}^2)/(1 + c\hat{\mu}^2) \\
B_3(\hat{\mu}) &= a + b\hat{\mu}^2 + c\sin(\hat{\mu})/\hat{\mu}
\end{aligned}
$$

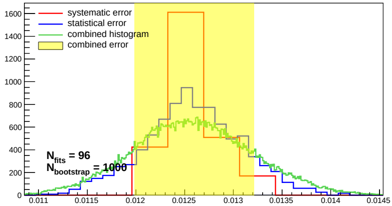
Extrapolation from different fit functions



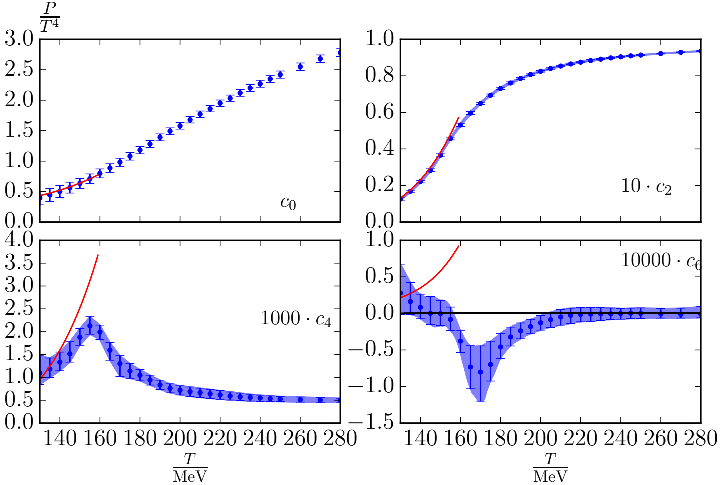
Error estimation

- Statistical error:
  Bootstrap method
- Systematic error:
  Using different way of analysis, combining them in a histogram:
  - 4 fit functions for the T direction
  - 3 fit functions in the μ_B direction
  - Doing continuum extrapolation and μ_B-fit in one or two steps
  - 2 methods of scale setting: f_π and w_0
  - 2 temperatures from where we use the extrapolated data

  This adds up to 96 ways of analysis



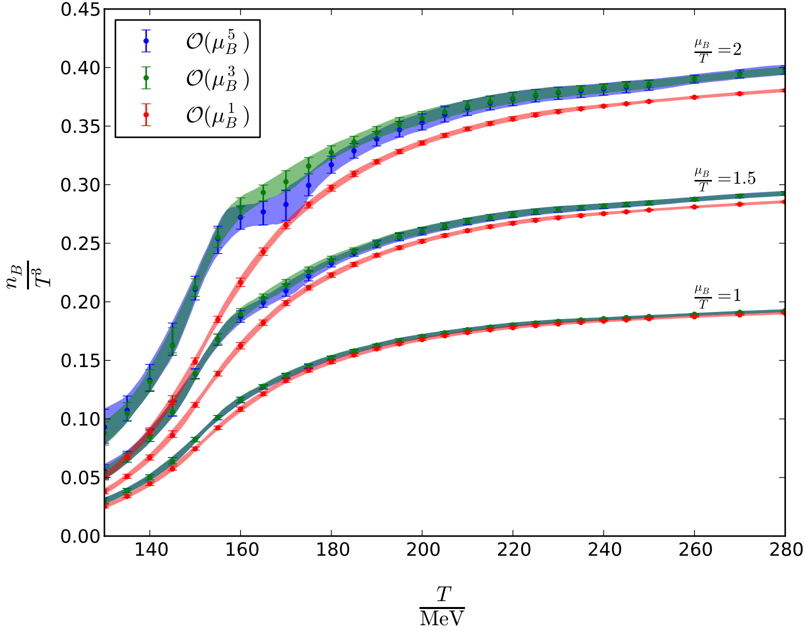
Taylor coefficients



The Taylor coefficients of $\frac{P}{T^4} = c_0 + c_2 \left(\frac{\mu_B}{T}\right)^2 + c_4 \left(\frac{\mu_B}{T}\right)^4 + c_6 \left(\frac{\mu_B}{T}\right)^6$

c_0 is from *S. Borsanyi et al, Phys. Lett. B730, 99 (2014), arXiv:1309.5258*

n_B to different orders



Entropy

- $S = \left.\frac{\partial P}{\partial T}\right|_{\mu_i}$
- We have $\frac{\mathrm{d}\hat{P}}{\mathrm{d}\hat{\mu}_B}$
- And we can only do a total derivative in T

$$\frac{\mathrm{d}\hat{P}}{\mathrm{d}T} = \left.\frac{\partial \hat{P}}{\partial T}\right|_{\mu_i} + \frac{\mathrm{d}\mu_B}{\mathrm{d}T}\frac{\hat{n}_B}{T} + \frac{\mathrm{d}\mu_Q}{\mathrm{d}T}\frac{\hat{n}_Q}{T}$$

How we correct this:

$$\frac{\mathrm{d}\mu_B}{\mathrm{d}T} = \frac{\mathrm{d}(\hat{\mu}_B T)}{\mathrm{d}T} = \hat{\mu}_B$$

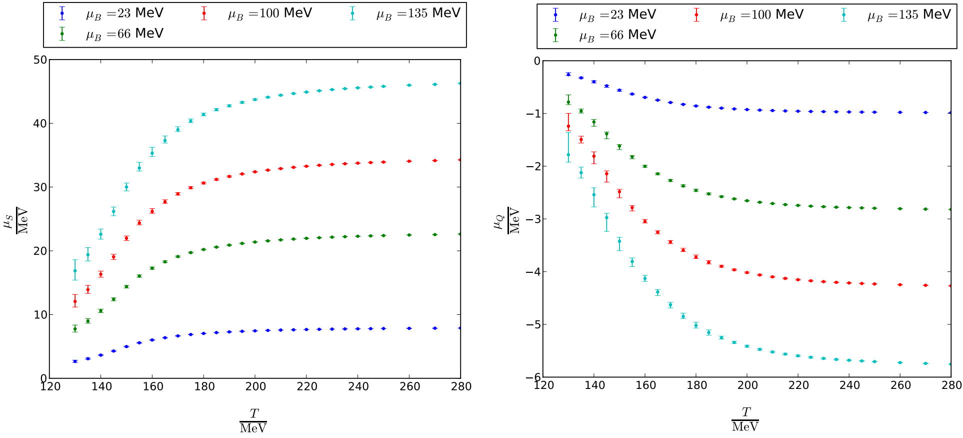
$$\frac{\mathrm{d}\mu_Q}{\mathrm{d}T} = \frac{\mathrm{d}(\hat{\mu}_Q T)}{\mathrm{d}T} = \hat{\mu}_Q + T\frac{\mathrm{d}\hat{\mu}_Q}{\mathrm{d}T}$$

$$\hat{S} = 4\hat{P} + T\frac{\mathrm{d}\hat{P}}{\mathrm{d}T} - \hat{\mu}_B\hat{n}_B - \hat{\mu}_Q\hat{n}_Q - T\frac{\mathrm{d}\hat{\mu}_Q}{\mathrm{d}T}$$

$$= 4\hat{P} + T\frac{\mathrm{d}\hat{P}}{\mathrm{d}T} - \hat{n}_B\left(\hat{\mu}_B + 0.4\hat{\mu}_Q\right) - T\frac{\mathrm{d}\hat{\mu}_Q}{\mathrm{d}T}$$

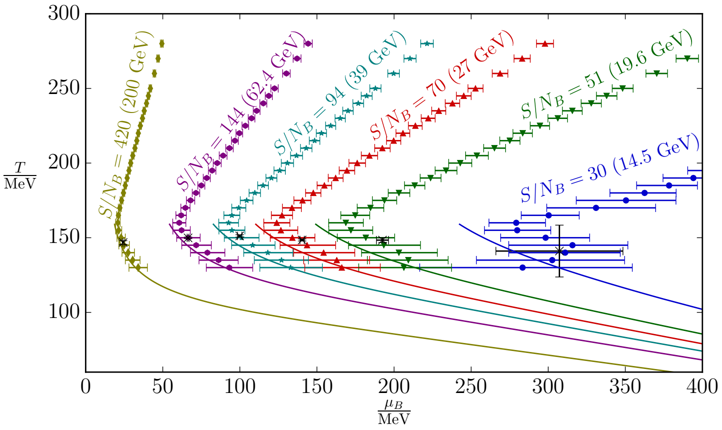
New terms we have to determine

μ_S and μ_Q

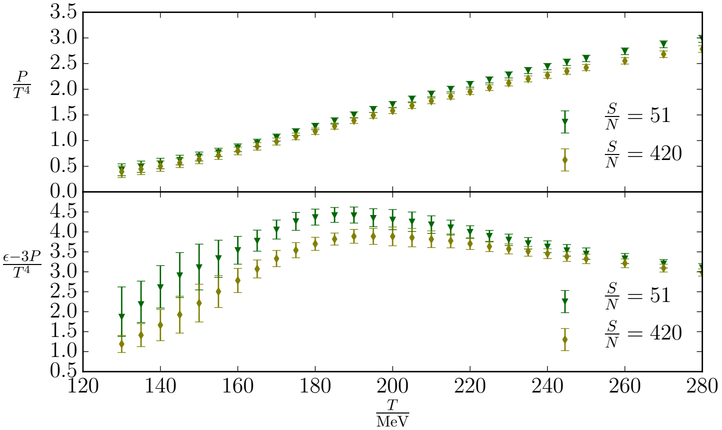


$$\langle n_s \rangle = 0 \text{ and } \langle n_Q \rangle = 0.4 \langle n_B \rangle$$

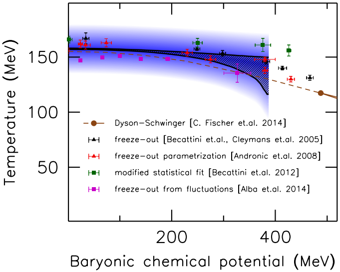
Trajectories



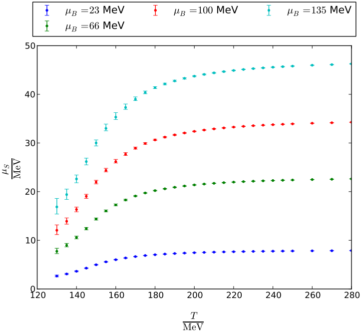
Equation of state



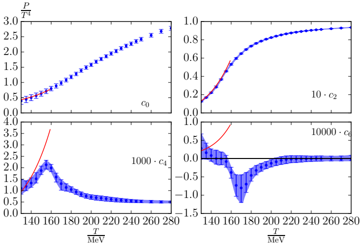
Summary



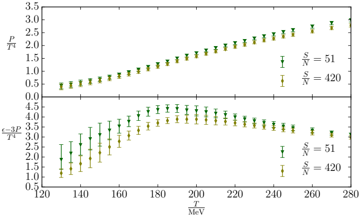
Temperature (MeV)
150
100
50
200
400
Baryonic chemical potential (MeV)
Dyson−Schwinger [C. Fischer et.al. 2014]
freeze−out [Becattini et.al., Cleymans et.al. 2005]
freeze−out parametrization [Andronic et.al. 2008]
modified statistical fit [Becattini et.al. 2012]
freeze−out from fluctuations [Alba et.al. 2014]



μ_B =23 MeV
μ_B =100 MeV
μ_B =135 MeV
μ_B =66 MeV
$\frac{\mu_S}{\text{MeV}}$
$\frac{T}{\text{MeV}}$



$\frac{P}{T^4}$
c_0
$10 \cdot c_2$
$1000 \cdot c_4$
$10000 \cdot c_6$
$\frac{T}{\text{MeV}}$



$\frac{P}{T^4}$
$\frac{S}{N} = 51$
$\frac{S}{N} = 420$
$\frac{\epsilon - 3P}{T^4}$
$\frac{T}{\text{MeV}}$