QCD Equation of state and critical end-point estimates at $\mathcal{O}(\mu_B^6)$

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Based on arXiv: 1701.04325, Bielefeld-BNL-CCNU collaboration

A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa, S. Mukherjee, H. Ohno, P. Petreczky, H. Sandmeyer, C. Schmidt, S. Sharma, W. Soeldner, P. Steinbrecher, M. Wagner

Outline

1. The QCD phase diagram: outstanding issues from lattice

2. Equation of state at finite μ_B

3. Critical-end point search from Lattice

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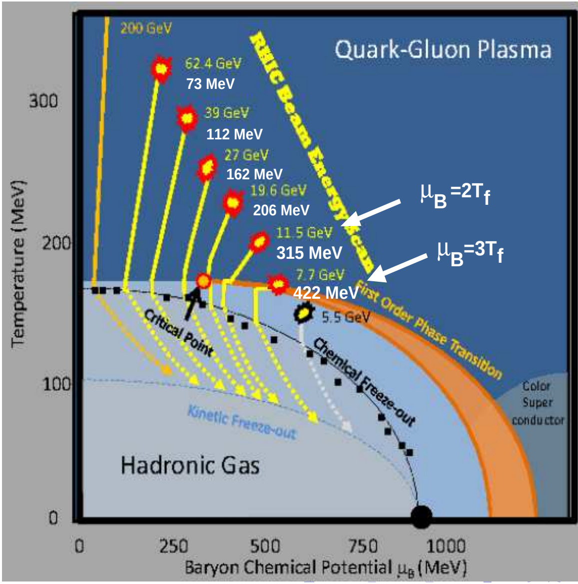
Major Themes from Lattice



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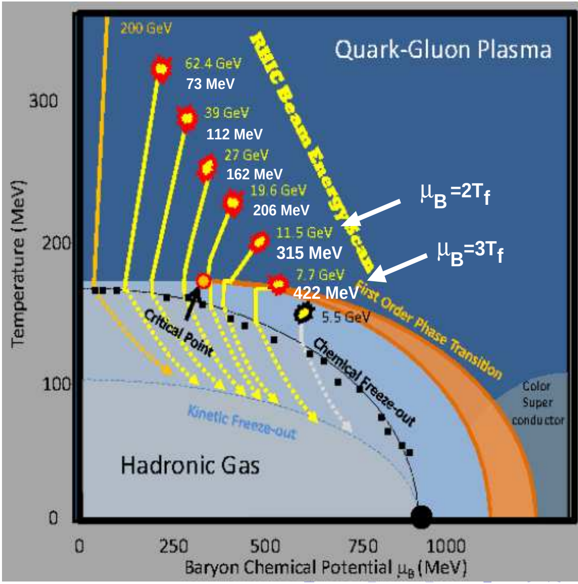
- In view of the RHIC Beam Energy Scan-II in 2019-20 it is important to have control over the Equation of State for $\mu_B/T \leq 3$.



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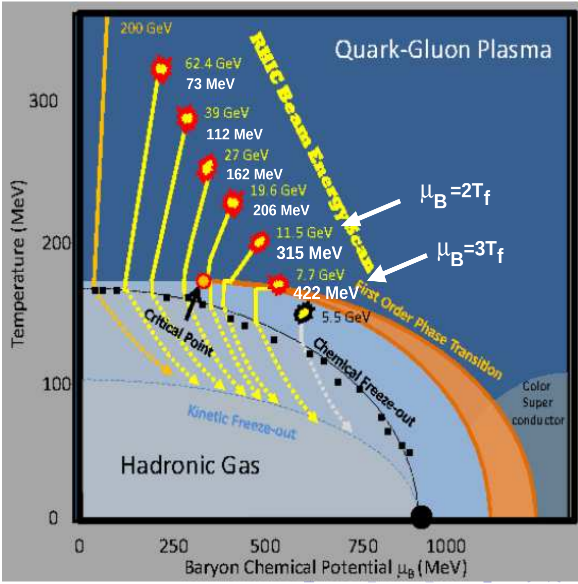

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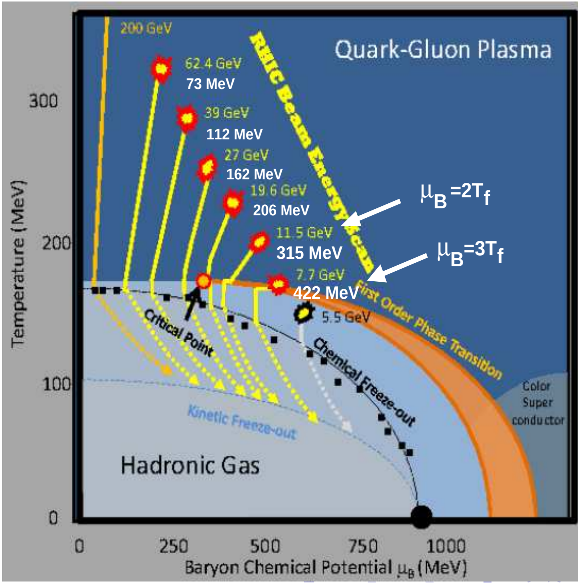
- Measure the curvature of chiral and freezeout curves expected from QCD thermodynamics.
- Look for possible existence and bracket the position of critical end-point in the phase diagram.



Basic observables

- One of the methods to circumvent sign problem at finite μ:
Taylor expansion of physical observables around $\mu = 0$ in powers of μ/T [Bi-Swansea collaboration, 02]

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0,T)}{T^4} + \left(\frac{\mu_B}{T}\right)^2 \underbrace{\frac{\chi_2^B(0,T)}{2T^2}}_{P_2} + \left(\frac{\mu_B}{T}\right)^4 \underbrace{\frac{\chi_4^B(0)}{4!}}_{P_4} + \ldots$$

- The series for $\chi_2^B(\mu_B)$ should diverge at the critical point. On finite lattice χ_2^B peaks, ratios of Taylor coefficients equal, indep. of volume [Gavai& Gupta, 03]

Challenges for Lattice computations

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- Higher derivatives $\rightarrow$ more inversions
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- Why extending to higher orders so difficult?
  - Matrix inversions increasing with the order
  - Delicate cancellation between a large number of terms for higher order QNS.

Recent developments: A new method to introduce μ

- The staggered fermion matrix used at finite μ [Hasenfratz, Karsch, 83]

$$
\begin{aligned}
D(\mu)_{xy} &= \sum_{i=1}^{3} \eta_i(x) \left[U_i^\dagger(y)\delta_{x,y+\hat{i}} - U_i(x)\delta_{x,y-\hat{i}} \right] \\
&+ \eta_4(x) \left[\mathrm{e}^{\mu \mathrm{a}} U_4^\dagger(y)\delta_{x,y+\hat{4}} - \mathrm{e}^{-\mu \mathrm{a}} U_4(x)\delta_{x,y-\hat{4}} \right]
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\end{aligned}
$$

- One can also add μ coupled to the conserved number density as in the continuum.

$$
D(0)_{xy} - \frac{\mu a}{2} \eta_4(x) \left[U_4^\dagger(y)\delta_{x,y+\hat{4}} + U_4(x)\delta_{x,y-\hat{4}} \right] .
$$

Pros and Cons

- Linear method: $D' = \sum_{x,y} N(x,y)$, and
  $D'' = D''' = D''''... = 0$

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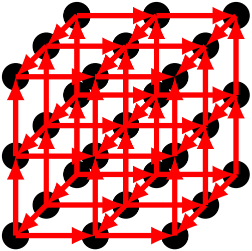
- Linear method: χ_n have additional zero-T artifacts. $\rightarrow$ explicit counter terms needed for $\chi_{2,4}$, discussed in detail [Gavai & Sharma, 15]
- In Exp method: counter terms already at the Lagrangian level. We use this method for χ_n^B, $n = 2, 4$.

Our Set-up



- $V = N^3 a^3$, $T = \frac{1}{N_\tau a}$. We use $N_\tau = 6, 8, 12, 16$ lattices for $\chi_{2,4}$ and $N_\tau = 6, 8$ for higher order fluctuations.
- Box size: $m_\pi \ V^{1/3} > 4$
- Input m_s physical and $m_\pi^G = 160$ MeV for $T > 175$ MeV and $m_\pi^G = 140$ MeV for $T <= 175$ MeV.
- Calculating explicitly the lowest eigenvalues improves performance of the fermion inverter.

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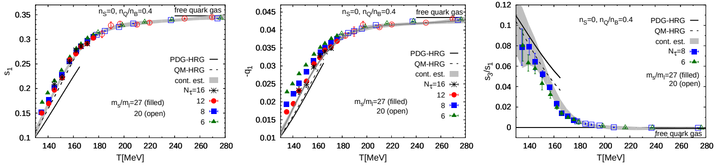
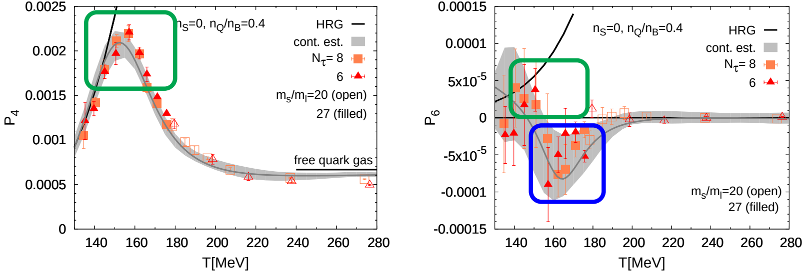
EoS in the constrained case

- In most central heavy-ion experiments typically:
  $n_S = 0$, Strangeness neutrality,
  $\frac{n_Q}{n_B} = \frac{n_P}{n_P + n_N} = 0.4$.
  [Bi-BNL collaboration, 1208.1220]
- For lower $\sqrt{s}$ collisions: Need to understand baryon stopping!
- Imposes non-trivial constraints on the variation of μ_S and μ_Q.
- Possible to vary them by only varying μ_B through

$$\mu_S = s_1 \mu_B + s_3 \mu_B^3 + s_5 \mu_B^5 + \ldots .$$
$$\mu_Q = q_1 \mu_B + q_3 \mu_B^3 + q_5 \mu_B^5 + \ldots .$$

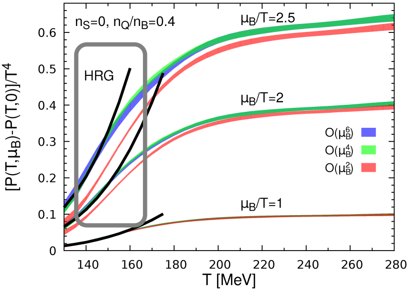


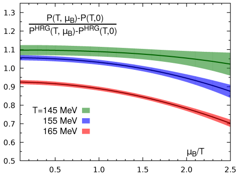
- Central values of P_4, P_6 already deviate from Hadron Resonance gas model at $T > 145$ MeV $\rightarrow$ need to analyze the errors on P_6 better.
- P_6 has characteristic structure at $T > T_c$ $\rightarrow$ remnant of the chiral symmetry due to the light quarks. Effects of $U_A(1)$ anomaly?
- Essentially non-perturbative $\rightarrow$ cannot be predicted within Hard Thermal Loop perturbation theory.



EoS in the constrained case

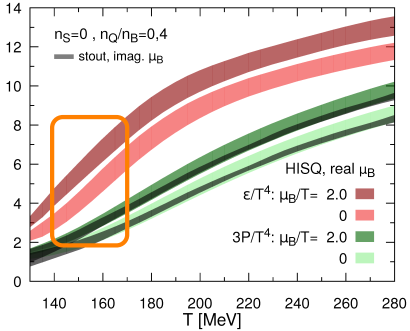
- The EoS for the constrained case is well under control for $\mu_B/T \sim 2.5$ with χ_6.
- Full parametric dependence for N_B on T available in arxiv: 1701.04325.
- Expanding to $\mu_B/T = 3$, need to calculate χ_8!



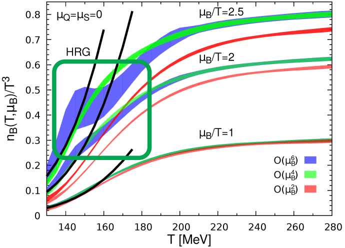


Summary for the EoS

- Continuum estimates from two different fermion discretization agree for $\mu_B/T \leq 2$.
  [Bielefeld-BNL-CCNU collaboration, 1701.04325, Borsanyi et. al, 1606.07494].
- Steeper EoS for RHIC energies compared to LHC energy.



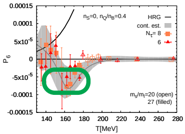
Baryon number density

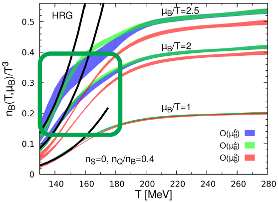


- χ_6 contribution is 30-times larger than in pressure.

$$\frac{N(\mu_B)}{T^3} = \frac{\mu_B}{T}\chi_2^B(0) + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \frac{1}{4!}\left(\frac{\mu_B}{T}\right)^6 \chi_6^B(0) + \ldots$$

- Strongly sensitive to the singular part of χ_6^B.

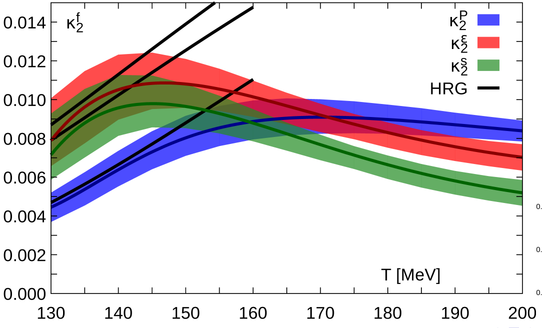


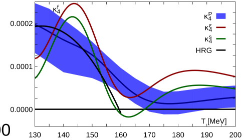


- For strangeness neutral system, effect is milder.

Curvature of freeze-out line

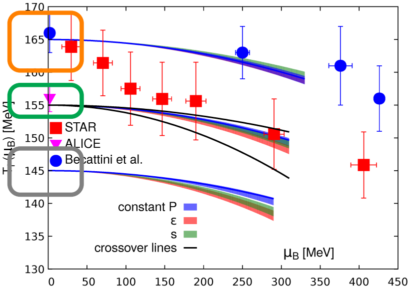
- The lines of constant $f \equiv \epsilon$ or p is characterized as:
  $$T_f(\mu_B) = T_0\left(1 - \kappa_2^f\left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4^f\left(\frac{\mu_B}{T_0}\right)^4\right)$$
- For $145 \leq T \leq 165$ MeV: $0.0064 \leq \kappa_2^P \leq 0.0101$, $0.0087 \leq \kappa_2^\epsilon \leq 0.012$.
- Consistent with the curvature of the chiral 'crossover' transition curve $0.0066(7)$ to $0.013(3)$. [arxiv:1011.3130, 1507.03571, 1507.07510, 1508.07599]
- For $\mu_B/T \leq 2$ the contribution from κ_4 to $T_f(\mu_B)$ within errors of κ_2.





Curvature of freeze-out line: Final summary

- Different LCP's agree within 2 MeV for $\mu_B/T \leq 2$ for 3 initial choices of T_0.
- For lines $P = \text{const}$, the entropy density changes by $15\% \rightarrow$ better description of LCP for viscous medium formed in heavy-ion collisions.
  [Bi-BNL-CCNU collaboration, 1701.04325].



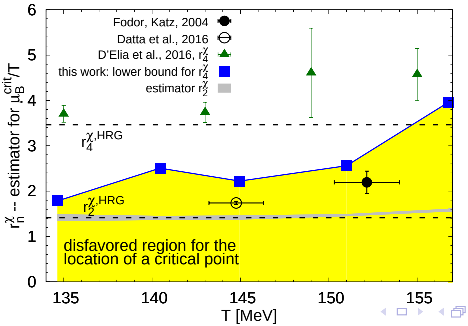
- STAR results give a steeper curvature. arXiv:1412.0499.
- Agreement with the recent ALICE results. arXiv:1408.6403.
- Consistent with phenomenological models. Becattini et. al., 1605.09694.

For more details,
talk by Frithjof Karsch, Wed, 2:40 pm.

Outline

Critical-end point search from Lattice

- The series for χ_2^B should diverge at the critical point. On finite lattice ratios of Taylor coefficients equal, indep. of volume [Gavai& Gupta, 03]
- Radius of convergence from Taylor expansion: $r_{2n} \equiv \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$.
- Definition is true for $n \to \infty$. How large n could be on a finite lattice?
- New studies from Taylor expansions and imaginary μ sets a current bound for CEP to be $\mu_B/T > 2$ [Bielefeld-BNL-CCNU, 1701.04325, D'Elia et. al., 1611.08285] though some studies point to a lower bound. [Datta et. al., 1612.06673, Fodor and Katz, 04]



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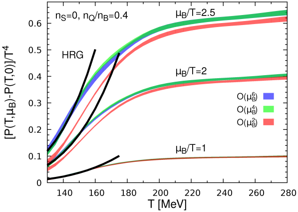

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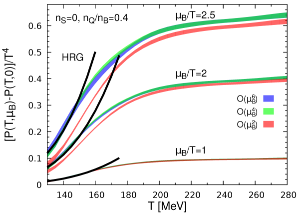

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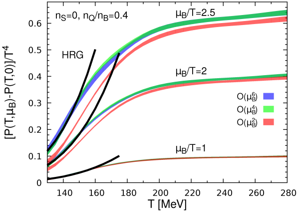

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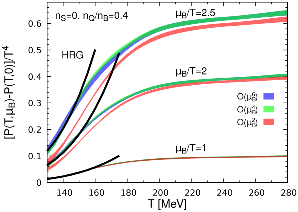

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- The lines of constant ϵ, p consistent with LQCD estimates of curvature of chiral crossover line.
- Higher order cumulants will also help in bracketing the possible CEP. Most LQCD calculations suggest $\mu_B(\text{CEP})/T \geq 2$.

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