Lattice Calculations of Heavy Quark Potential at non-Zero Temperature

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- Motivation : the study and interpretation of static meson correlators is much more easier than the analysis of heavy meson correlators

  static correlators ⟶ potential model ⟶ spectral functions

- Spectral decomposition of static meson correlators and moments
- Wilson loops in HTL perturbation theory
- Extracting the real and the imaginary parts of the potential

in collaboration with Alexei Bazavov and Johannes Weber

EFT, potential models and static energy

Above deconfinement the binding energy is reduced and eventually $E_{bind} \sim mv^2$ is the smallest scale in the problem (zero binding) $mv^2 \ll \Lambda_{QCD}, 2\pi T, m_D$ => most of medium effects can be described by a *T*-dependent potential

Potential = Static Energy

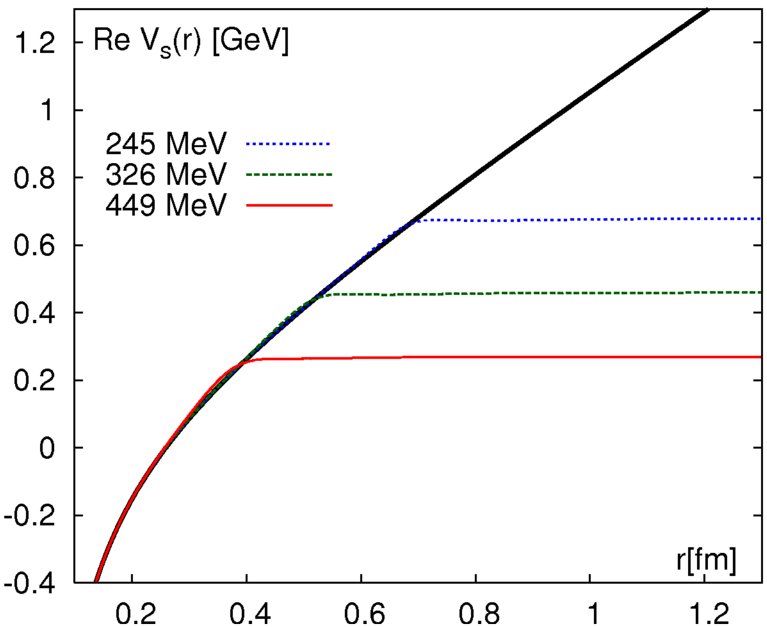
Determine the potential by non-perturbative matching to static quark anti-quark potential calculated on the lattice

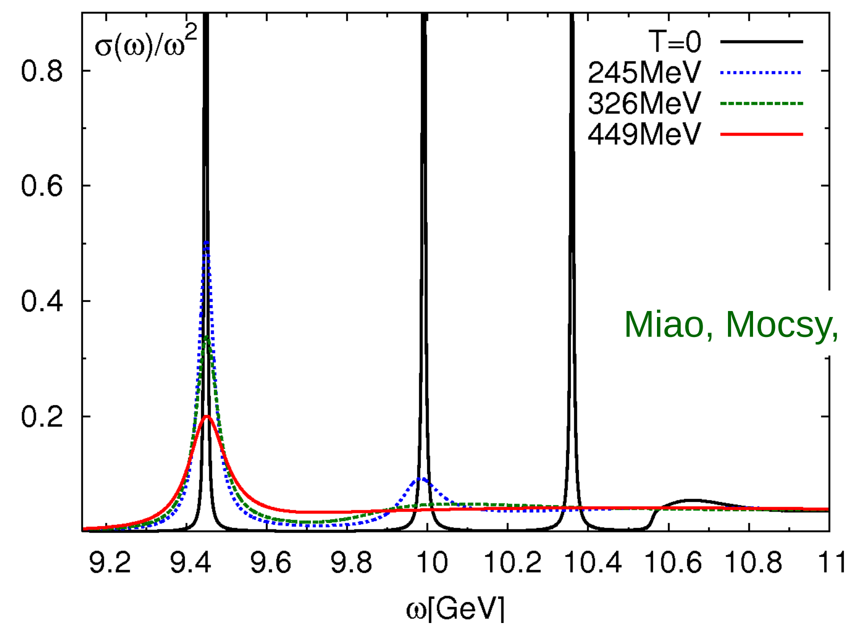
"Maximal" value for the real part

Mócsy, P.P., 2007

Minimal (perturbative) value for imaginary part

Laine et al, 2007, Brambilla et al, 2008





Miao, Mocsy, PP 2010

Wilson loops and potential at T>0

Correlation function of static Q and $\bar{Q}$ at distance r ($\tau \times r$ Wilson loop)

$$W_r(\tau, T) = \int_{-\infty}^{\infty} d\omega \sigma_r(\omega, T) e^{-\omega\tau}$$

not related to the free energy !

Rothkopf 2009, Hatsuda, Rothkopf, Hatsuda 2011

potential at $T > 0$ is related to a peak structure in $\sigma(\omega, T)$:
peak position: $\mathrm{Re}V(r,T)$ peak width: $\mathrm{Im}V(r,T)$

HISQ action, $48^3 \times 12$ (configs from HotQCD and TUMQCD)

$m_\pi \simeq 160$ MeV
$\beta = 10/g^2 = 7.03, 7.28, 7.373, 7.596, 7.825 \leftrightarrow T = 200 - 407$MeV

$m_\pi \simeq 300$ MeV $\beta = 8.0, 8.2, 8.4 \leftrightarrow T = 450 - 660$ MeV

additional calculations on $32^3 \times 8$ and $40^3 \times 10$ to check cutoff effects

Choices of the spatial links:

or use Coulomb gauge and *U(x,y)=1*

Moments of the spectral functions

$$m_1(\tau,r,T)=-\frac{\partial_\tau W_r(\tau,T)}{W_r(\tau,T)},\quad m_n=(-1)^n\partial_\tau m_{n-1}(\tau,r,T),\ n>1$$

moments of the spectral function around the peak, e.g

$$m_1(\tau)=\frac{\int_{-\infty}^{\infty}\omega\sigma(\omega,T)e^{-\omega\tau}}{\int_{-\infty}^{\infty}\sigma(\omega,T)e^{-\omega\tau}}=\langle\omega\rangle$$

On the lattice

$$m_1(\tau-\frac{s}{2a},r,T)=-\frac{1}{s}\ln\left(\frac{W_r(\tau-s/a,T)}{W_r(\tau,T)}\right)=V_{eff}(r,T,\tau)$$

$$m_n(\tau-\frac{s}{2a},r,T)=\frac{1}{s}\left(m_{n-1}(\tau,r,T)-m_{n-1}(\tau-\frac{s}{a},r,T)\right),\ n=2,3\ldots$$

Static potential at $T=0$: $m_1(\tau\to\infty,r,T=0)=V_0(r)$

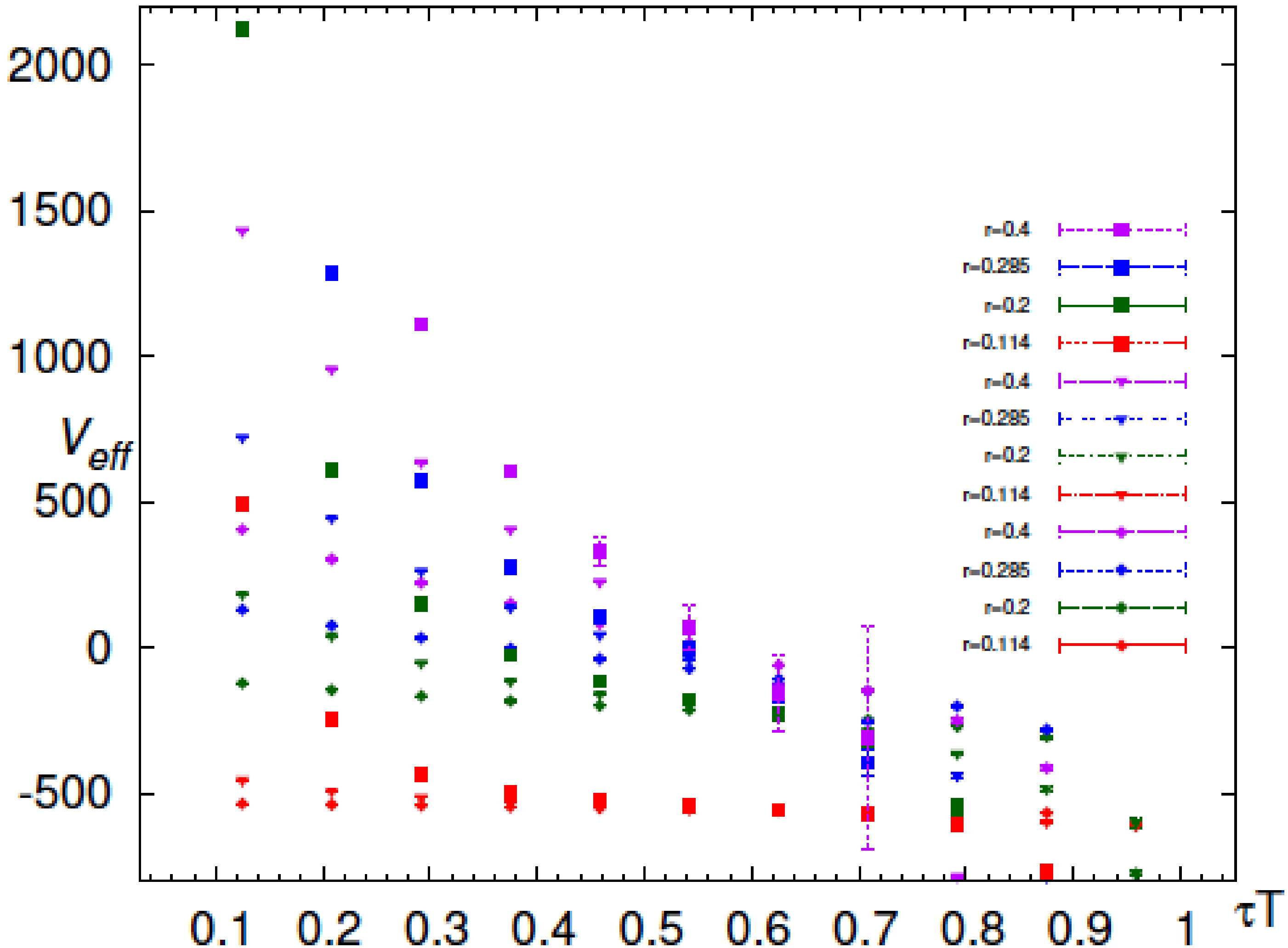
$T>0$: $\tau<1/T\Rightarrow$ use Lorentzian form of the spectral function:

$$\sigma_r(\omega,T)=\frac{1}{\pi}\frac{\Gamma}{(\omega-\mu)^2+\Gamma^2}\Theta(\omega-\eta),\ \mu=\mathrm{Re}V(r,T),\ \Gamma=\mathrm{Im}V(r,T)$$

η -tail regulator; $\Gamma\ll\mu$: $m_1\simeq\mu$.

Lattice results for the 1st moment

$\beta = 7.825, T = 407$ MeV: unsmeared Wilson loops (squares), smeared Wilson loops (triangles), Wilson line correlators in Coulomb gauge (bullets)

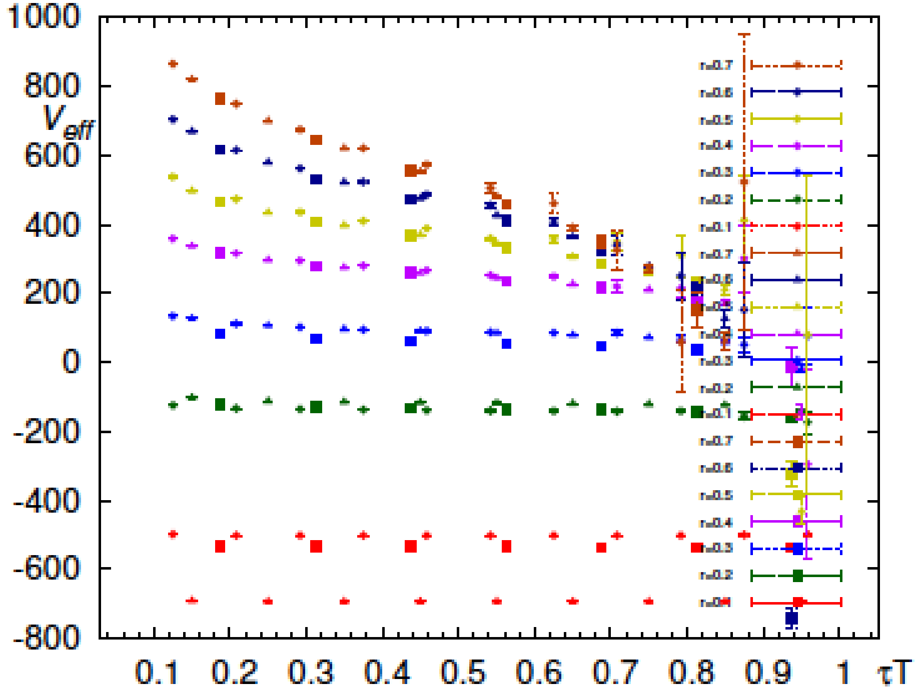


at large enough τ V_{eff} is independent of the choice of the operator

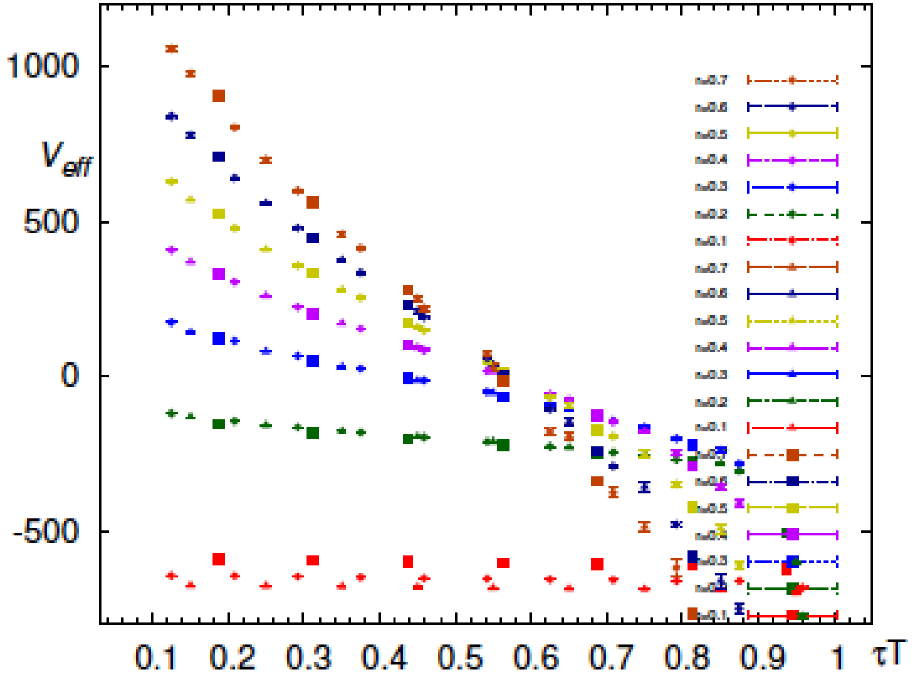
Lattice results for the 1st moment (cont'd)

Correlator of Wilson lines:

$\beta = 7.03, T = 199$ MeV



$\beta = 7.825, T = 407$ MeV

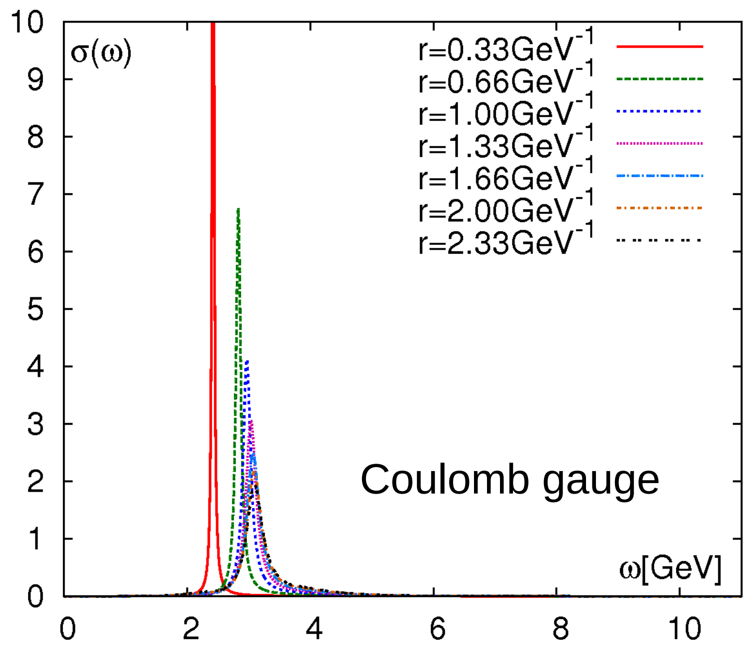


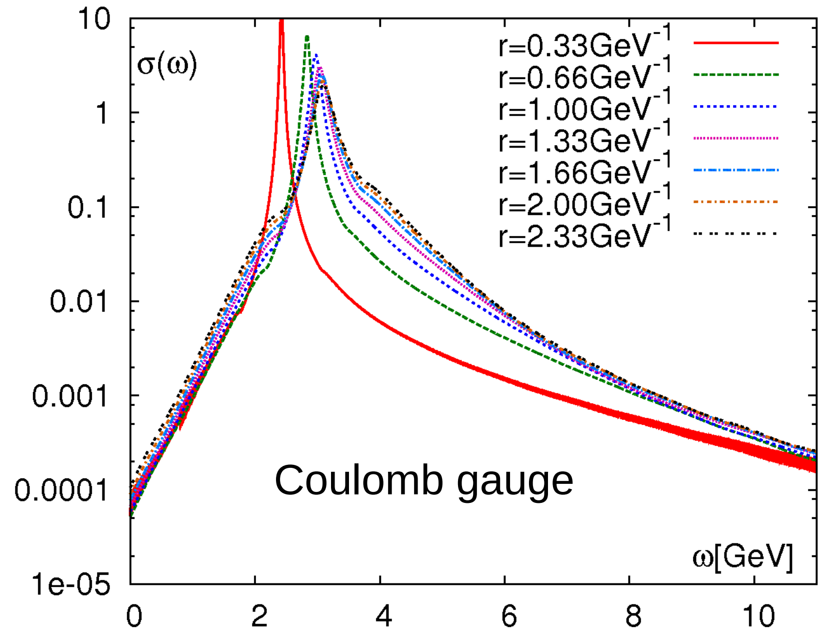
larger τ dependence for large temperature; small cutoff effects

Spectral functions in HTL perturbation theory

Perturbative hard thermal loop (HTL) calculations for $T=2.33\ T_c$, $T_c=270$ MeV $(N_f = 0)$:

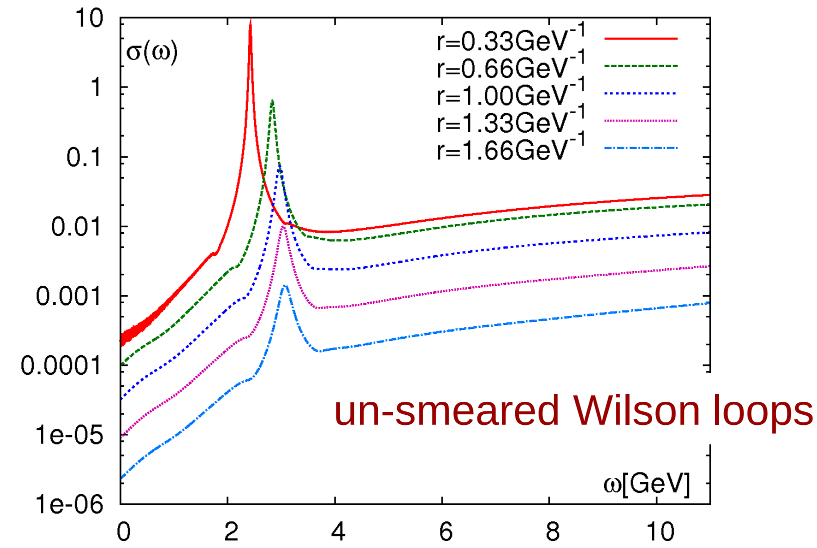
Burnier, Rothkopf, 2013





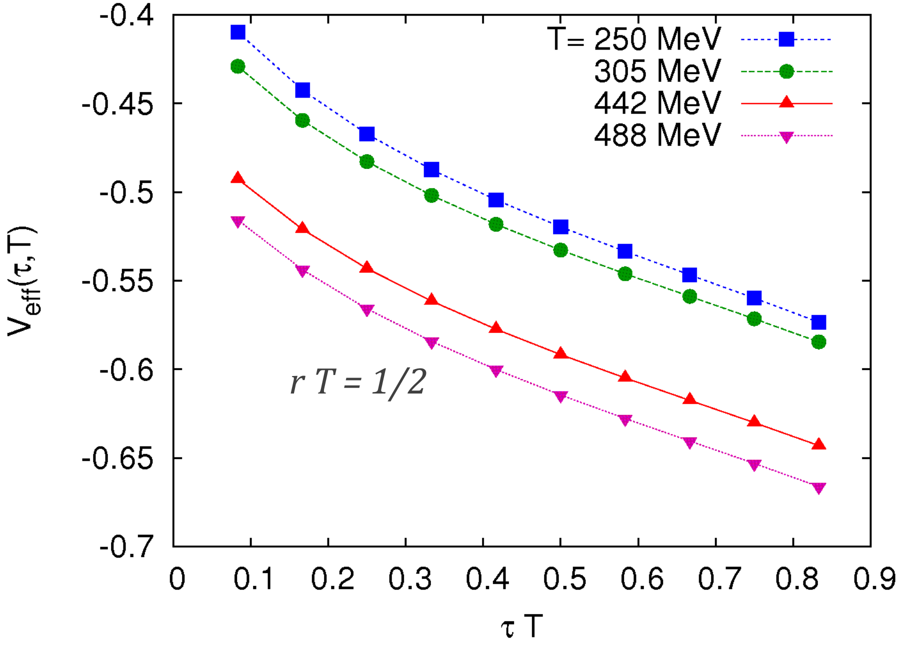
Spectral functions has long tails and non-Lorentzian away from the peak

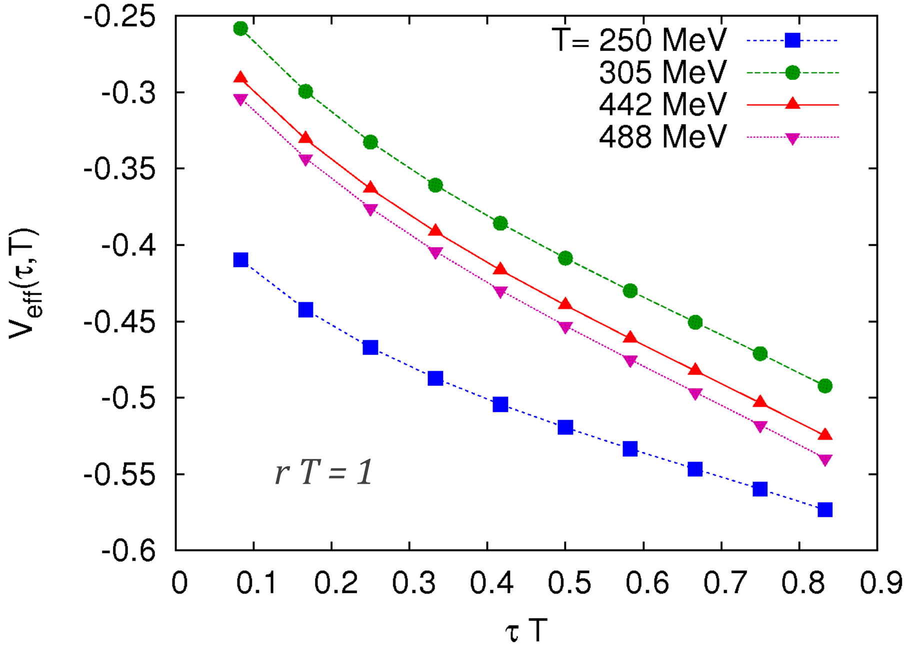
For un-smeared Wilson loops the peak height is much suppressed compared to the Coulomb gauge case



Effective potential at high temperatures

Effective potential V_{eff} in HTL perturbation theory:

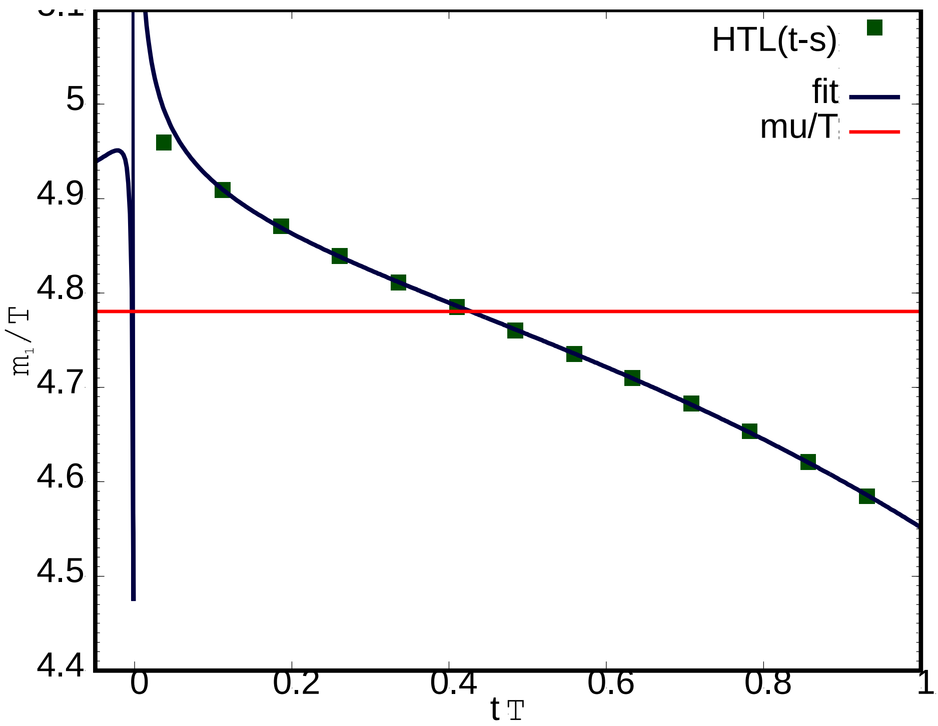


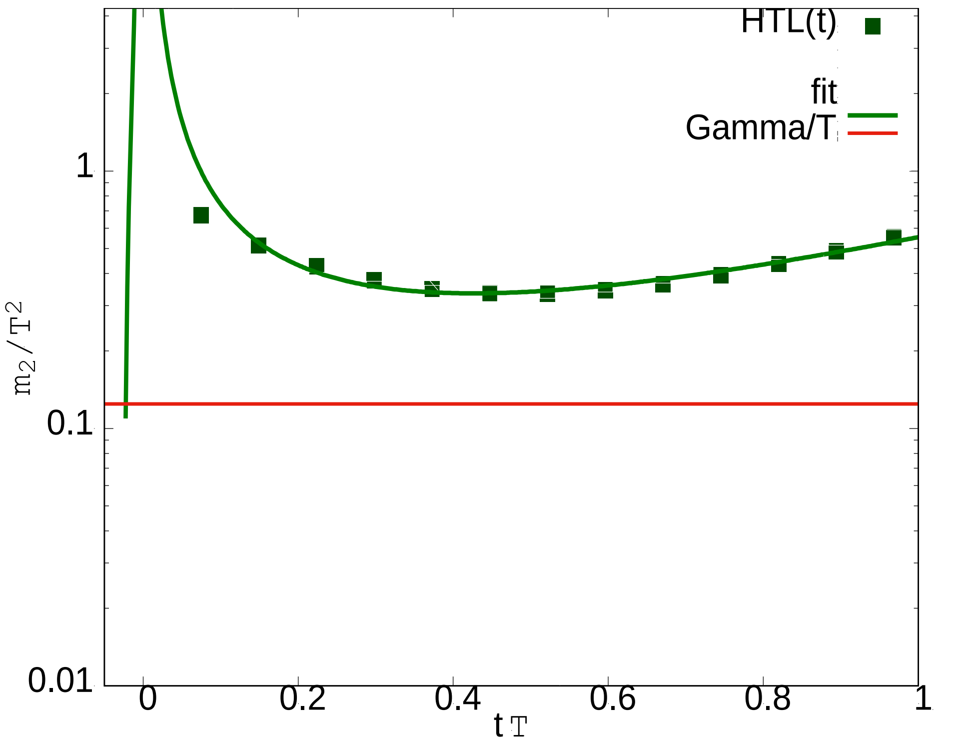


V_{eff} decreases with τ due to the width of the spectral functions, its slope increases with T and the distance r as observed in the lattice calculations

Tests with HTL spectral functions

Construct the moments from HTL correlator and fit with Lorentzian:



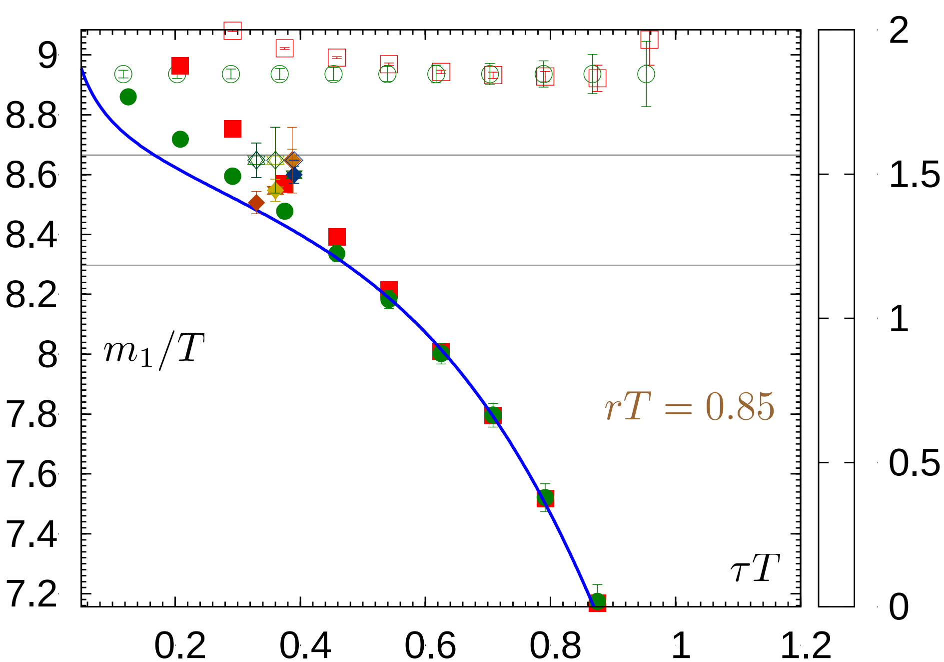


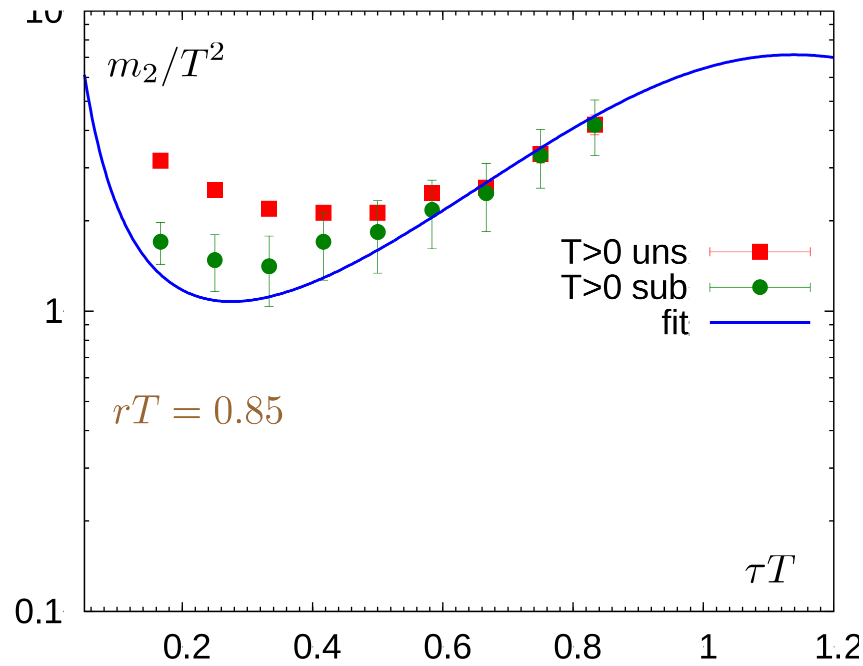
$m_1^{input}/T = 4.72$ $\Gamma^{input}/T = 0.122$

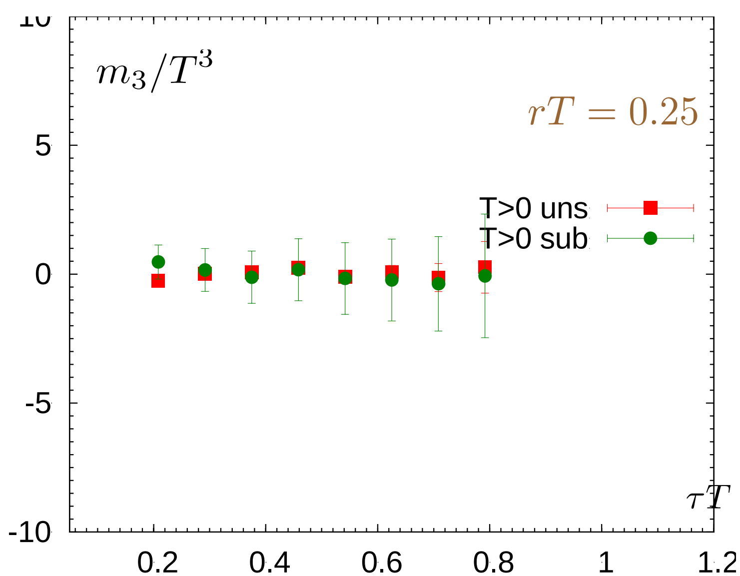
Lorentzian form can describe the moments very well (except at very small τ) even-though it is not the correct away from the peak.

Fit of the moments can reproduce the peak position and width with 1% accuracy

Fits to the lattice moments







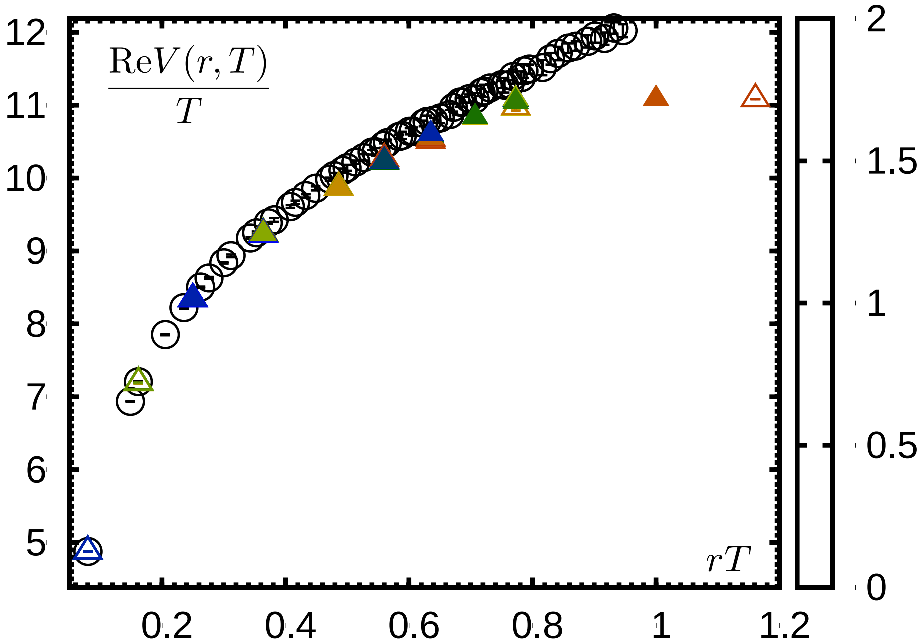
1) Identify the ground state at $T = 0$ and estimate the contribution of higher states to $W_r(\tau, T = 0)$

2) Subtract the contribution due to higher states estimated at $T = 0$ from $W_r(\tau, T)$ and calculate moments

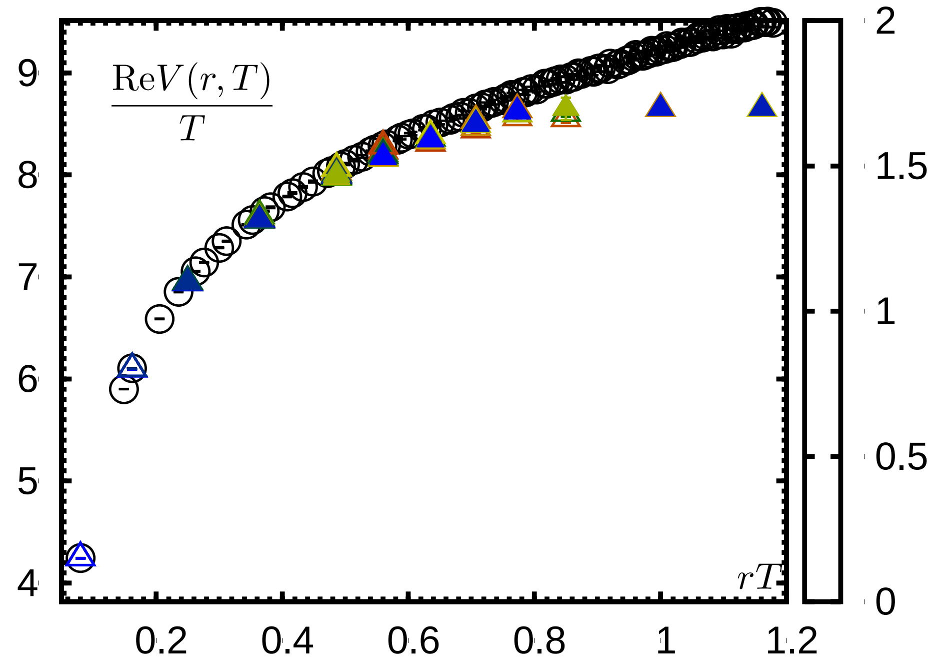
3) Fit the subtracted 1st moment skipping data at small and large τ as well as imposing constraints on the fit parameters to estimate systematic effects

The real part of the potential

$\beta = 7.280, T = 251$ MeV



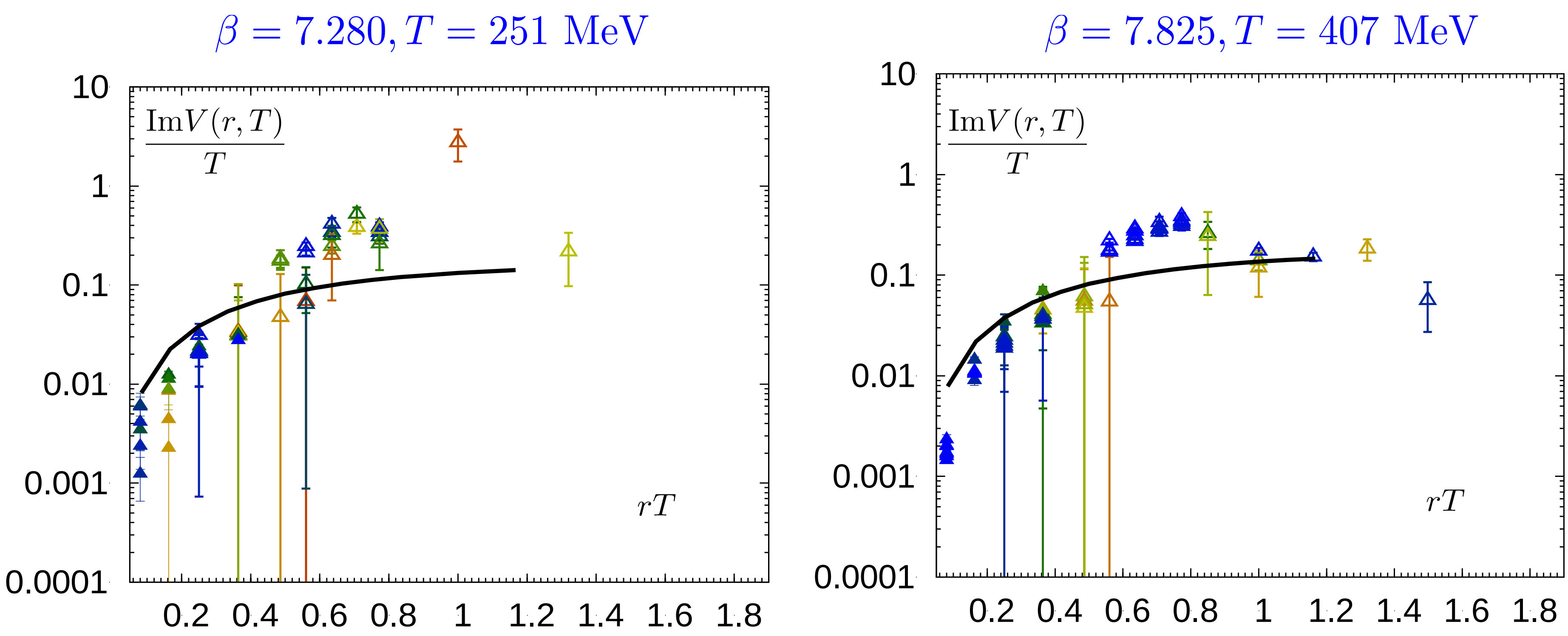
$\beta = 7.825, T = 407$ MeV



Fits without the data at small τ + constraints on μ

$\mathrm{Re}V(r,T)$ agrees with $T=0$ potential up to $rT=0.7$ and is larger than the singlet free energy

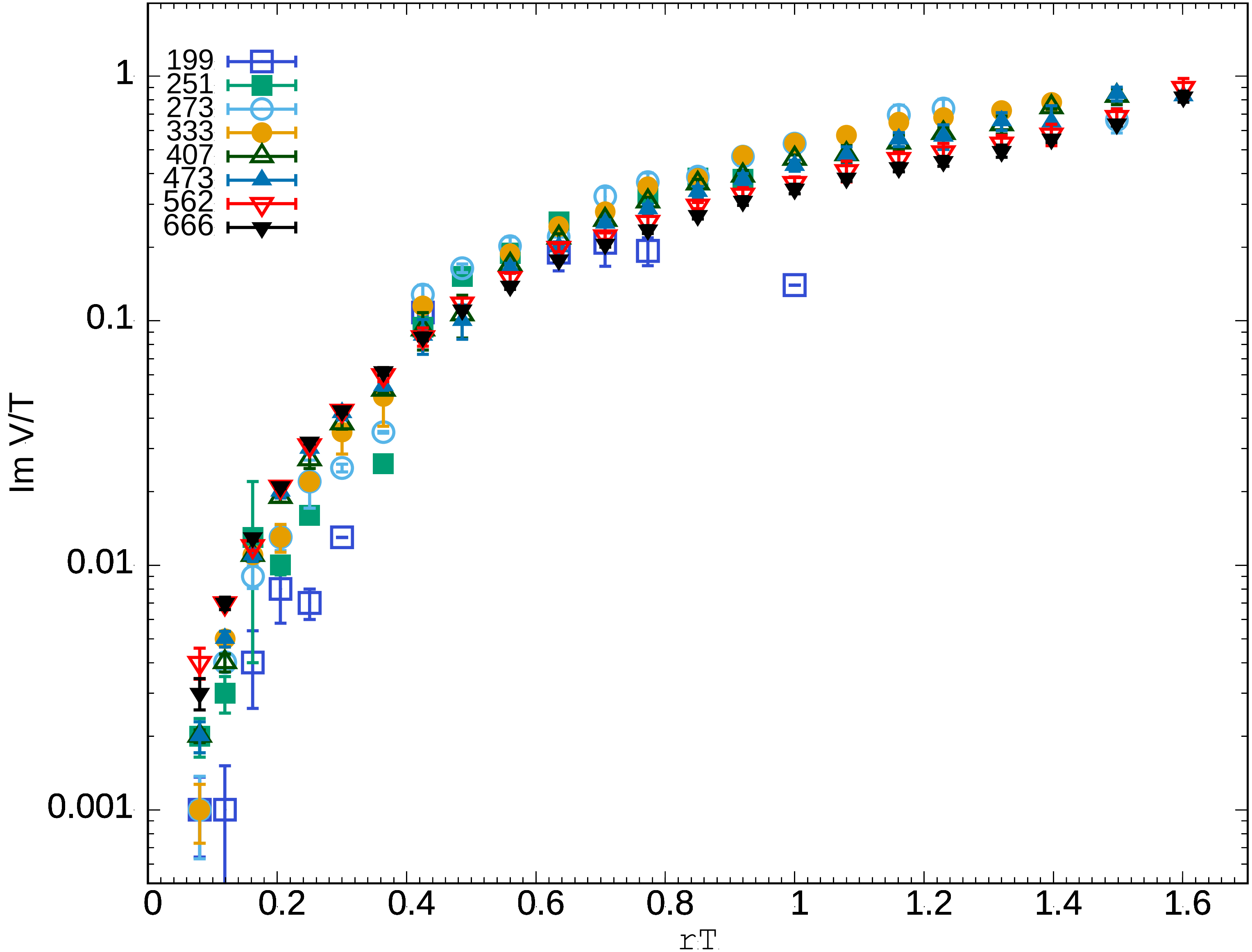
The imaginary part of the potential



Fits with no data at small τ: $\mathrm{Im}V(r,T)/T$ as function of rT is approximately T-independent

There are hints that lattice results from $\mathrm{Im}V$ are larger than the perturbative HTL results (black lines)

Temperature dependence of the imaginary part



Fits with data at small τ: Im $V(r,T)/T$ as function of rT is approximately T-indepedent and larger compared to the previous fits

Summary

- The moments of static meson correlators provide an efficient tool to extract the real and imaginary parts of the potential at $T>0$ as well as insights into the spectral decomposition of static meson correlators

- The behavior of the Wilson line correlators calculated on the lattice is qualitatively the same as in HTL perturbation theory

- Different static meson correlators (Wilson line, smeared and unsmeared Wilson loops) show very similar behavior for large enough τ

- The potential at $T>0$ shows screening effects at for $T>200$ MeV, its real part is larger than the singlet free energy and is compatible with the "maximally binding" potential used in potential model calculations

- The imaginary part of the potential is larger than in the HTL perturbation theory

- A better control of the high energy part of the spectral function will be needed for a more reliable extraction of the real and imaginary part of the potential

- For temperatures $T \approx 200$ MeV the extraction of the potential is limited mostly by statistics rather than N_τ