

Quarkonium spectral functions and heavy quark diffusion of charm and bottom quarks from lattice QCD at finite temperature

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in collaboration with

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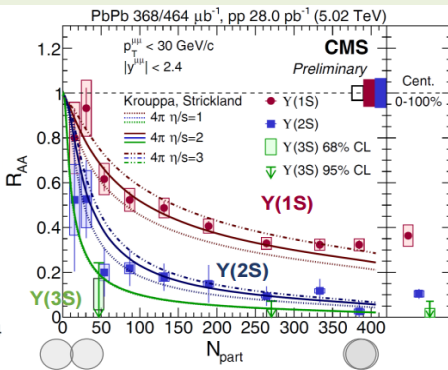
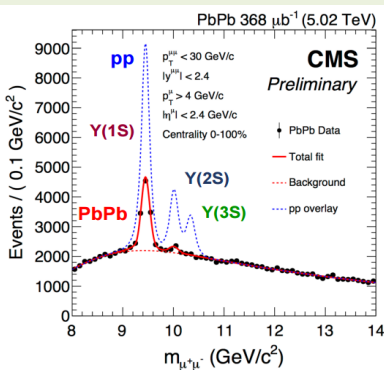
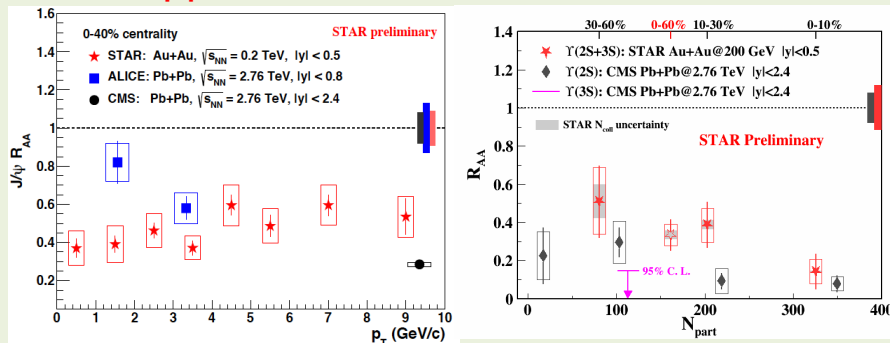
Quark Matter 2017
Hyatt Regency Chicago, Chicago, USA
February 7, 2017

Motivation

- Quarkonium spectral functions (SPFs)
 - have all information about in-medium properties of quarkonia

Quarkonium dissociation temperature

→ Important to understand quarkonium suppression

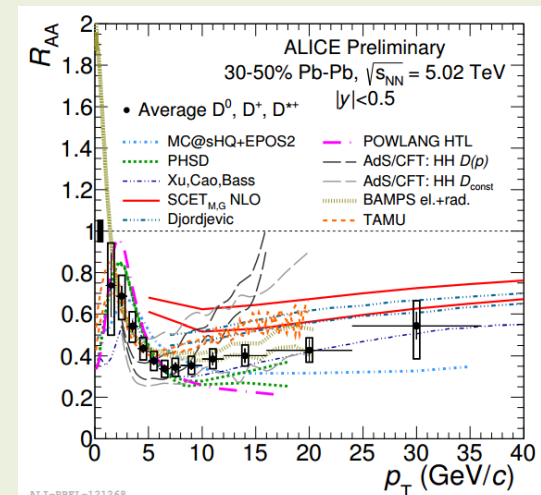


Heavy quark diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \rightarrow 0} \sum_{i=1}^3 \frac{\rho_{ii}^V(\omega, 0)}{\omega}$$

$\rho_{ii}^V(\omega)$: vector SPF

→ Important input for heavy quark transport models



Quarkonium correlation and spectral functions

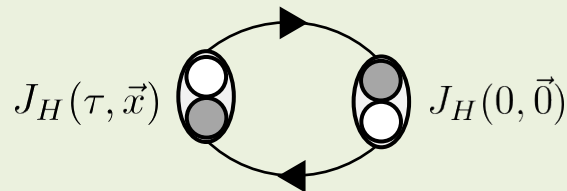
Euclidian (imaginary time) meson correlation function

$$G_H(\tau, \vec{p}) \equiv \int d^3x e^{-i\vec{p}\cdot\vec{x}} \langle J_H(\tau, \vec{x}) J_H(0, \vec{0}) \rangle$$

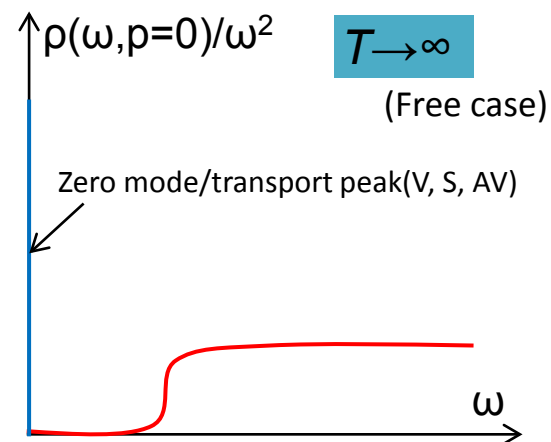
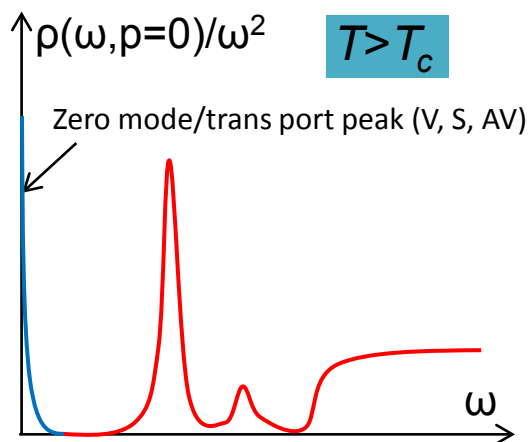
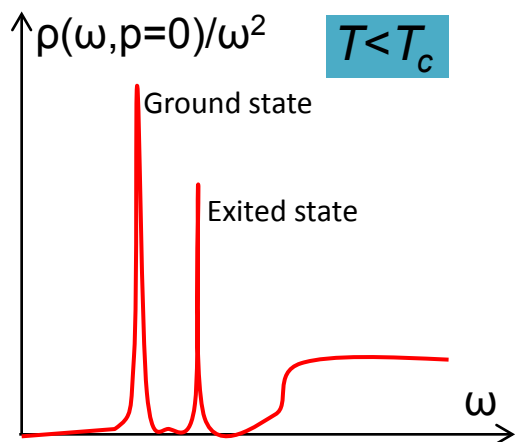
$$= \int_0^\infty \frac{d\omega}{2\pi} \rho_H(\omega, \vec{p}) K(\omega, \tau)$$

Spectral function

$$J_H(\tau, \vec{x}) \equiv \bar{\psi}(\tau, \vec{x}) \Gamma_H \psi(\tau, \vec{x})$$



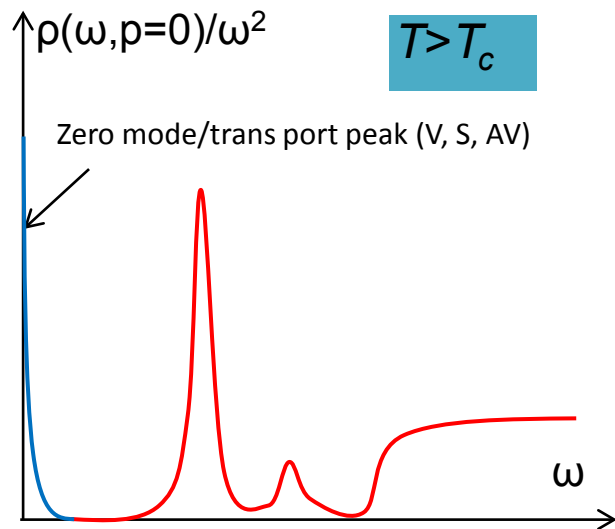
$$K(\omega, \tau) \equiv \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$



Heavy quark diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \rightarrow 0} \sum_{i=1}^3 \frac{\rho_{ii}^V(\omega, \mathbf{0})}{\omega}$$

$\rho_{ii}^V(\omega)$: spatial component of vector spectral function



χ_{00} : Quark number susceptibility

$$\rho_{00}^V(\omega) = 2\pi\chi_{00}\omega\delta(\omega) \quad \rightarrow \quad G_{00}^V(\tau) = T\chi_{00}$$

D is related to the slope of the vector spectral function around zero frequency.

This study

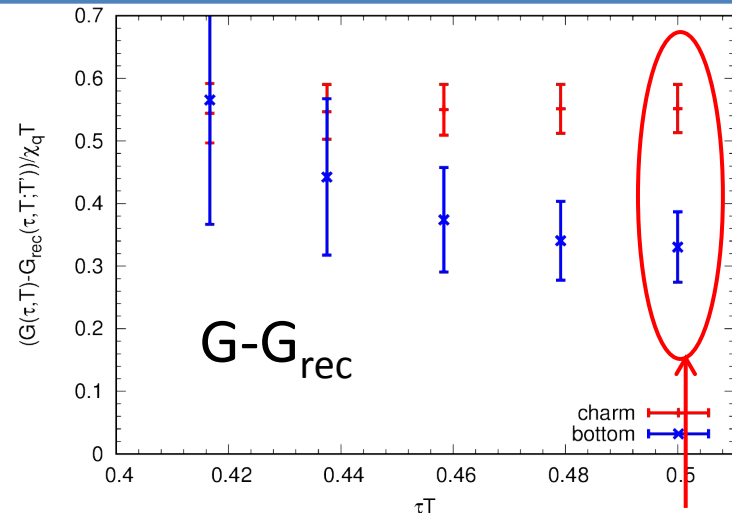
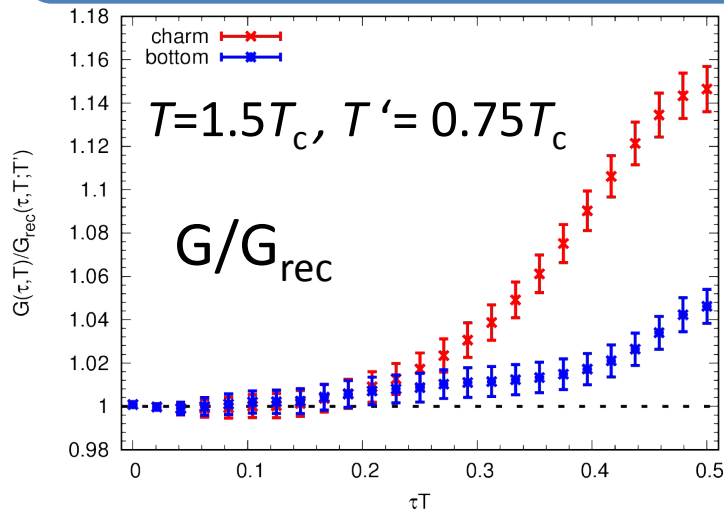
- Finite temperature lattice QCD simulations
 - large and fine isotropic lattices ($N_\sigma = 192$, $a \approx 0.009$ fm)
 - $N_\tau = 96, 48 \rightarrow T = 0.75T_c, 1.5T_c$
 - quenched approximation
(no dynamical quark)
 - both charm and bottom valence quarks treated relativistically
 - vector (V) channel
- Investigating quarkonium SPFs (and heavy quark diffusion)
 - indirectly with reconstructed correlators
 - directly by using both MEM and stochastic methods

M. Asakawa, T. Hatsuda and Y. Nakahara,
Prog.Part.Nucl.Phys. 46 (2001) 459-508

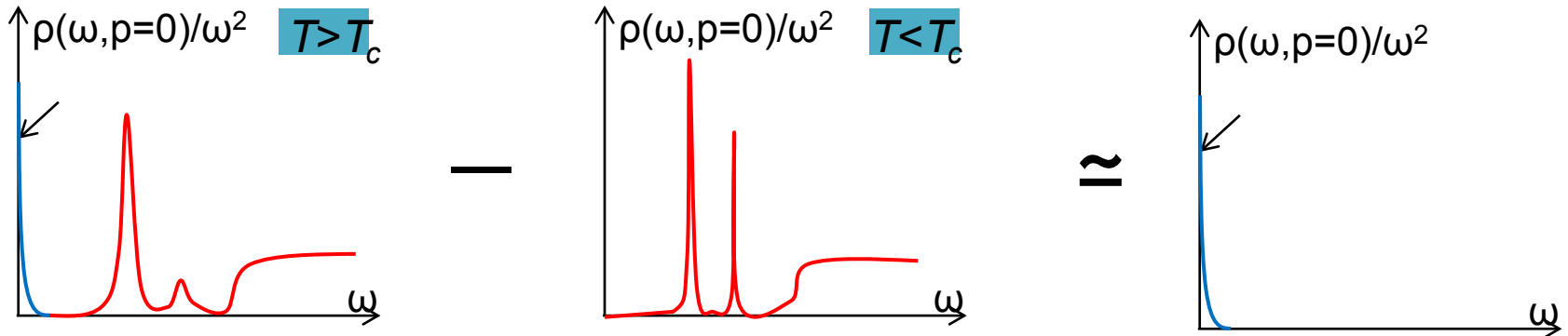
H.-T. Shu *et al*, PoS LATTICE 2015, 180 (2016)
HO, PoS LATTICE 2015, 175 (2016)

Estimation of the heavy quark diffusion coefficient

Reconstructed correlator:
$$G_{\text{rec}}(\tau, T; T') = \int \frac{d\omega}{2\pi} \rho(\omega, T') K(\omega, \tau, T)$$



Contribution from the transport peak is assumed to be dominant for $G - G_{\text{rec}}$ at $\tau T = 1/2$.



Estimation of the heavy quark diffusion coefficient

Heavy quark diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \rightarrow 0} \sum_{i=1}^3 \frac{\rho_{ii}^V(\omega, \mathbf{0})}{\omega}$$



H.-T. Ding *et al.*, PRD 86 (2012) 014509

Ansatz: $\rho_{ii}^V(\omega \ll T) = 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2} \quad \eta \equiv \frac{T}{MD}$

Charm: for $M = 1 - 1.5 \text{ GeV}$

$2\pi TD \simeq 0.5 - 0.7$ at $1.5T_c$

Bottom: for $M = 4 - 5 \text{ GeV}$

No solution at $1.5T_c$

Reconstruction of spectral functions

- Known to be an ill-posed problem
 - Simple χ^2 -fitting does not work.
- Maximum entropy method (MEM)
 - Based on Bayes' theorem
 - Prior knowledge (Default models) → Shannon-Jaynes entropy
 - Analytically minimizing χ^2 term + entropy term → a most likely solution

M. Asakawa, T. Hatsuda and Y. Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459-508
- Stochastic methods
 - Also based on Bayes' theorem
 - Stochastically finding a free-energy minimum = a most likely solution
 - Default models can be introduced
 - Stochastic Analytical Inference (SAI)

K.S.D. Beach, arXiv:cond-mat/0403055
S. Fuchs *et al.*, PRE81, 056701 (2010)
 - There is also a default-model-free method
 - Stochastic Optimization Method (SOM)

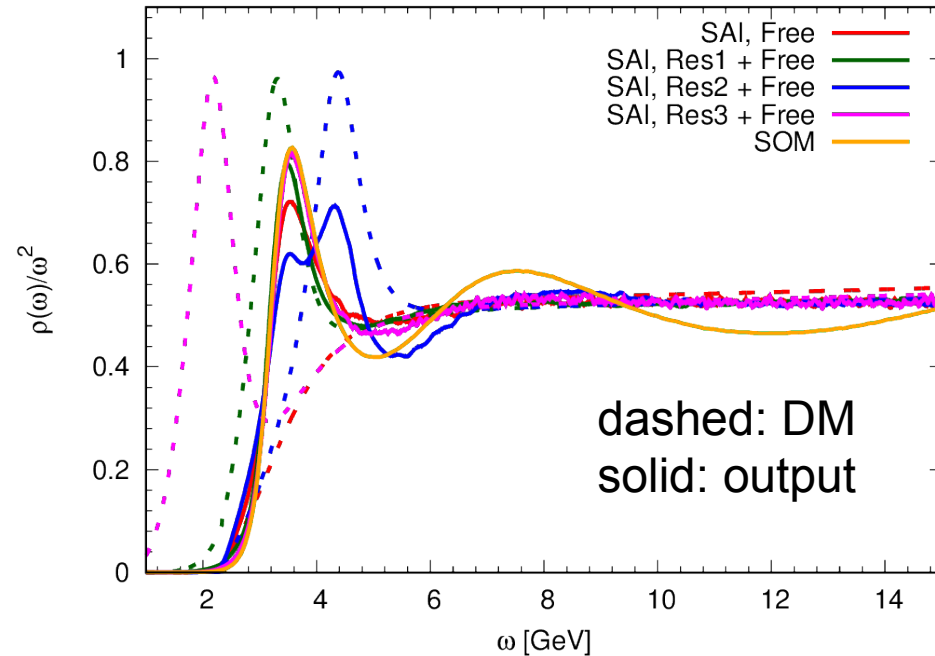
A. S. Mishchenko *et al.*, Phys. Rev. B62, 6317 (2000)

There is also another type of Bayesian methods proposed recently.

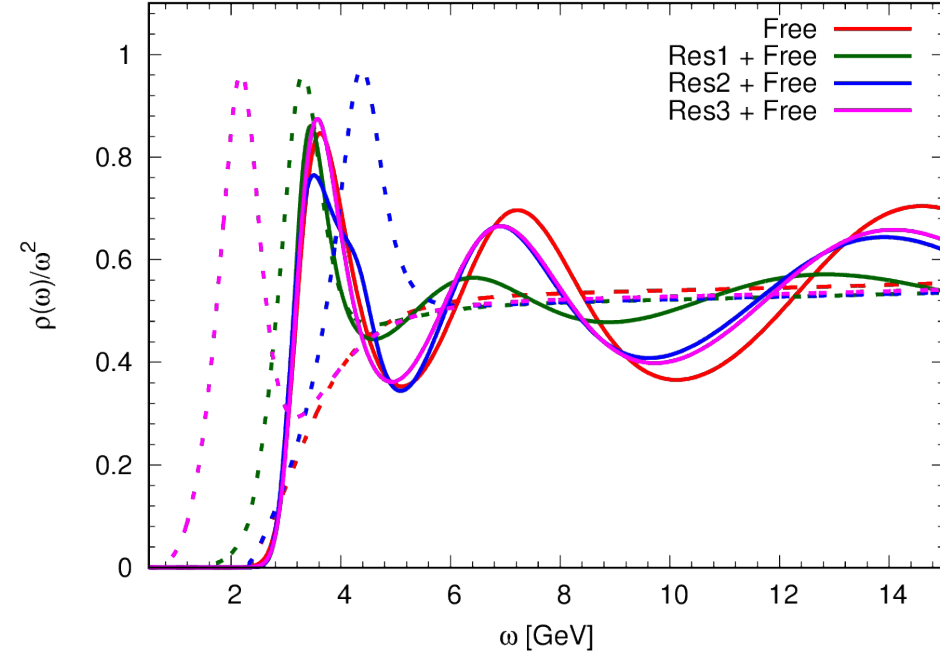
Y. Burnier and A. Rothkopf, PRL 111 (2013) 18, 182003

Default model (DM) dependence of the charmonium SPF at $0.75T_c$

Stochastic methods



MEM



DM = Free, Res(1-3) + Free

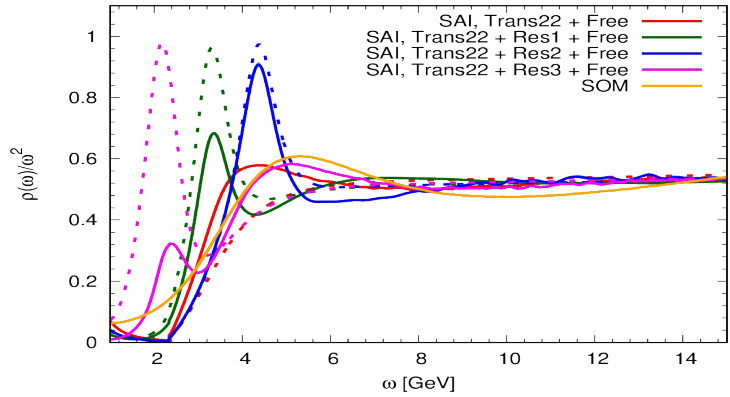
Peak location: Res1 \sim J/ Ψ mass, Res2 $>$ J/ Ψ mass, Res3 $<$ J/ Ψ mass

**Continuum part behaves differently between the stochastic methods and MEM.
Location of the first peak \sim J/ Ψ mass, small DM dependence
 \rightarrow There is a stable bound state corresponding to J/ Ψ .**

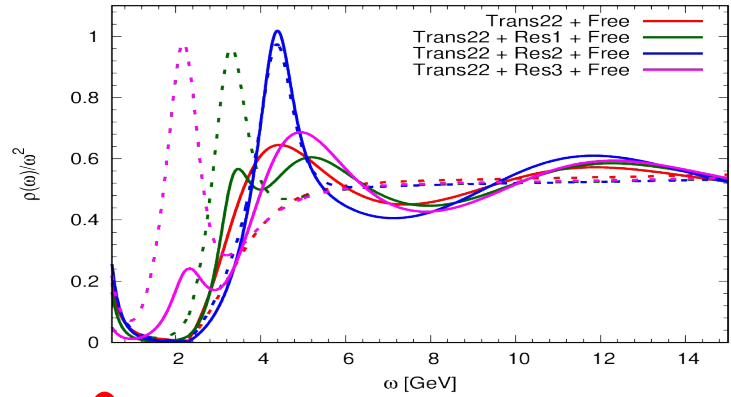
DM dependence of the charmonium SPF at $1.5T_c$ (1)

High ω

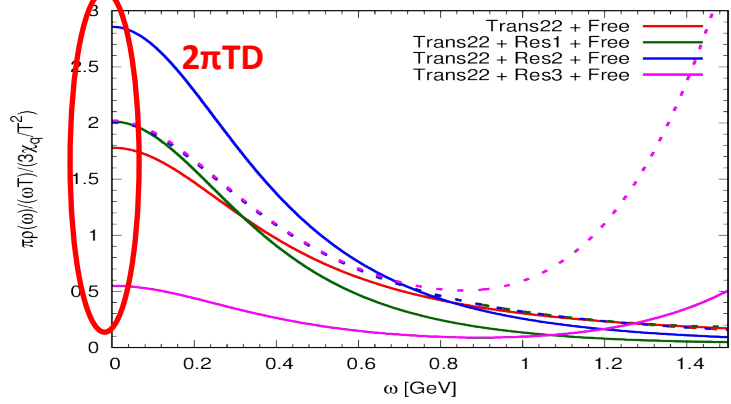
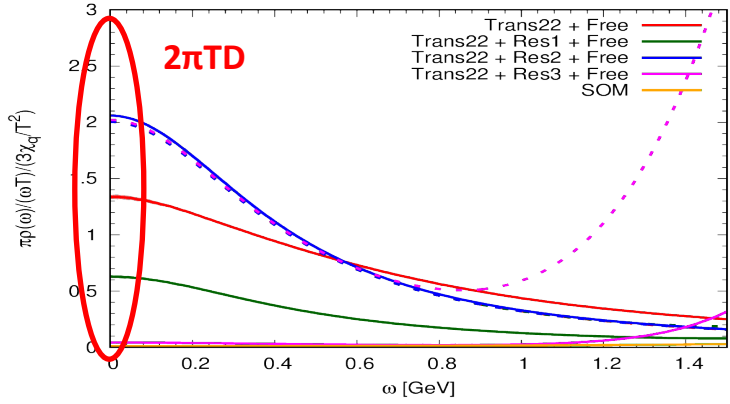
Stochastic methods



MEM



Low ω



DM = Trans + Free, Trans + Res(1-3) + Free (Trans is fixed)

The resonance peak is unstable and highly sensitive to DMs.

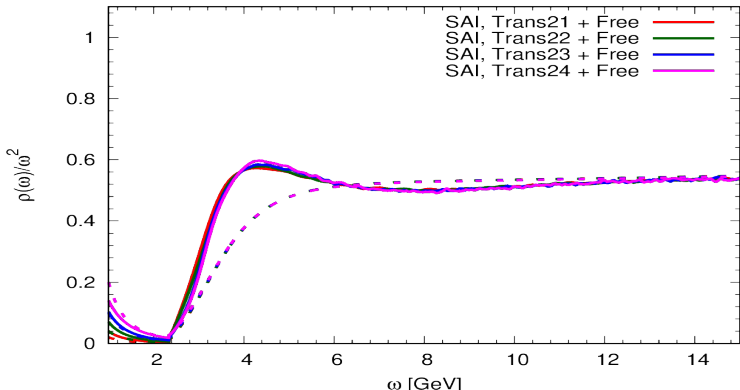
→ J/ψ seems to melt already $T < 1.5T_c$.

The transport peak is also sensitive to DMs.

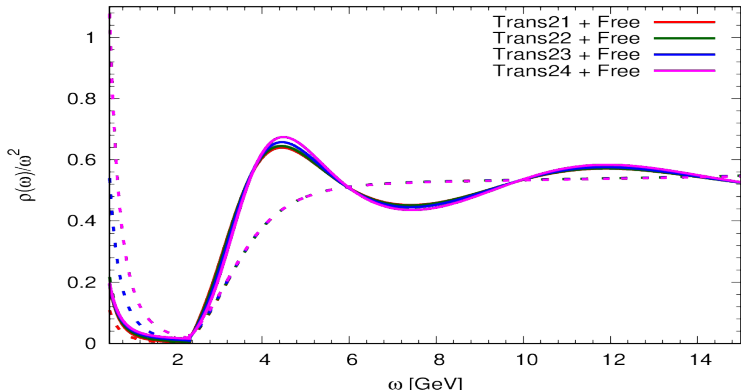
DM dependence of the charmonium SPF at $1.5T_c$ (2)

High ω

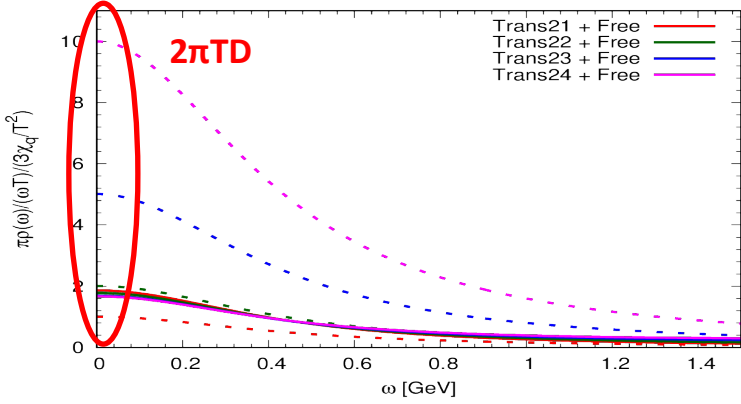
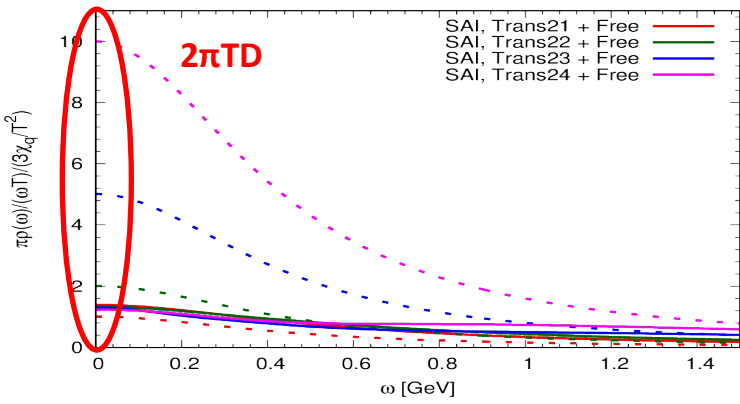
Stochastic methods



MEM



Low ω



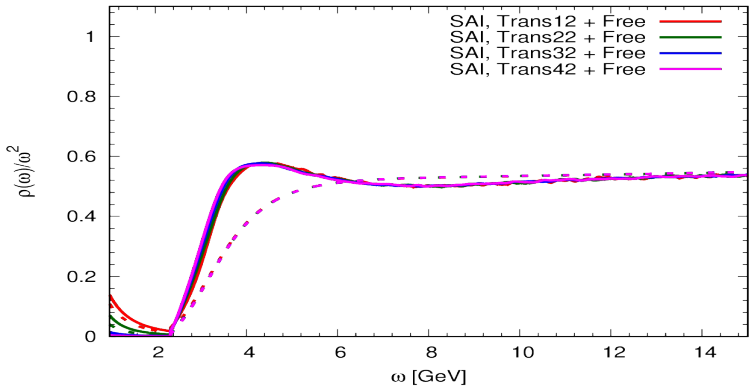
DM = Trans(21-24) + Free (width of the transport peak is fixed)
 Trans21: $2\pi TD \sim 1$, Trans22: $2\pi TD \sim 2$, Trans23: $2\pi TD \sim 5$, Trans24: $2\pi TD \sim 10$

Both high and low frequency parts have small DM dependence.
 $\rightarrow 2\pi TD \sim 1-2$

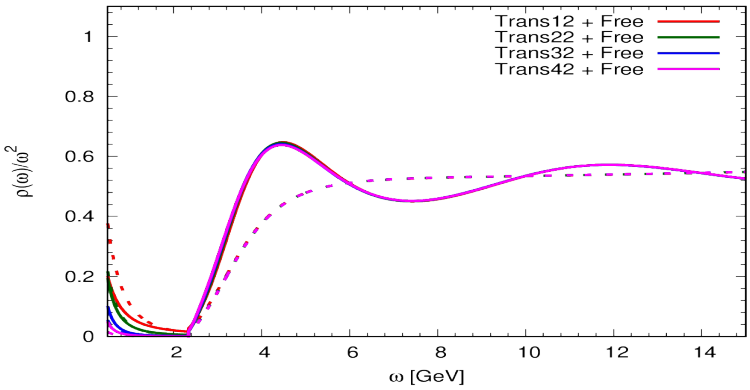
DM dependence of the charmonium SPF at $1.5T_c$ (3)

High ω

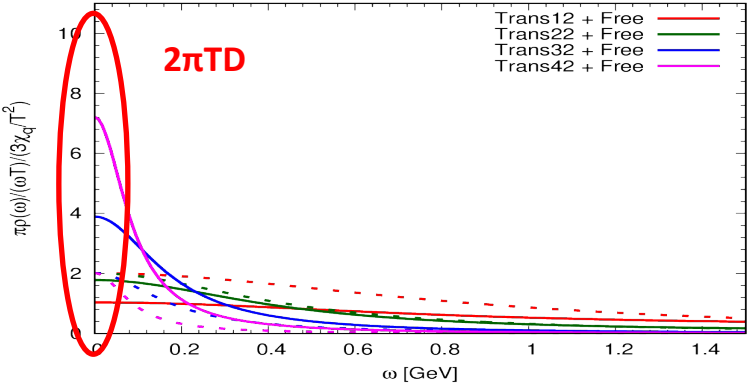
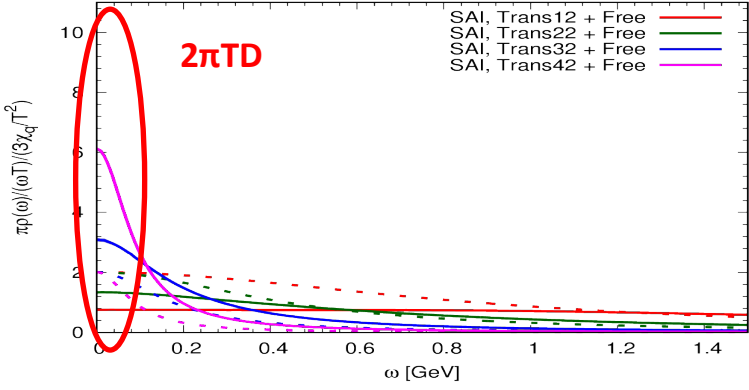
Stochastic methods



MEM



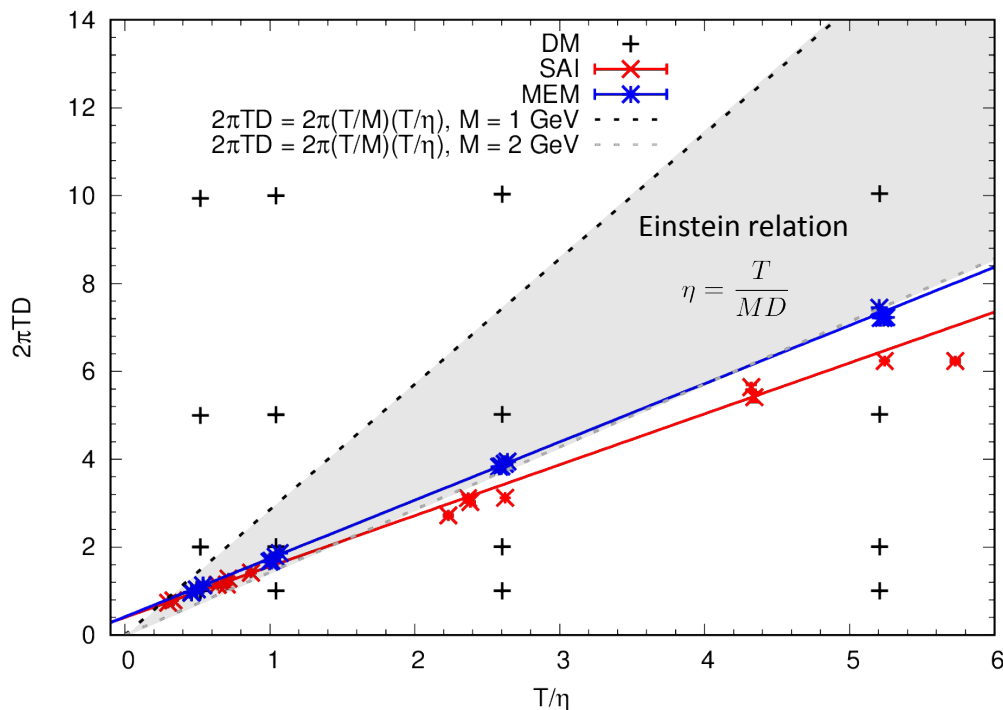
Low ω



DM = Trans(12-42) + Free (height of the transport peak is fixed)
 Trans12: $\eta/T \sim 2$, Trans22: $\eta/T \sim 1$, Trans32: $\eta/T \sim 0.2$, Trans42: $\eta/T \sim 0.1$

High frequency part has small DM dependence.
Low frequency part is sensitive to DMs. $\rightarrow 2\pi TD \sim 1-7$

DM dependence of the charm quark diffusion coefficient at $1.5T_c$



Output $2\pi TD$ varies as DM η/T changes.

Output η/T is almost the same to DM η/T for MEM

It seems that $D \cdot \eta$ is always fixed.

$$G(\tau) = \int \frac{d\omega}{2\pi} c \frac{\omega \eta}{\omega^2 + \eta^2} \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

$$\propto c$$

$$\omega \ll T$$

$$D = \frac{1}{6\chi_q} \lim_{\omega \rightarrow 0} c \frac{\eta}{\omega^2 + \eta^2} \Rightarrow c \propto D\eta$$

Both SAI and MEM can only determine the coefficient c or $D \cdot \eta$

→ The diffusion coefficient cannot be determined unless η/T is fixed.

$$2\pi TD = 1.16(4) T/\eta + 0.40(2) \text{ for SAI}$$

$$2\pi TD = 1.33(4) T/\eta + 0.42(2) \text{ for MEM}$$

for $T/\eta = 1-5$



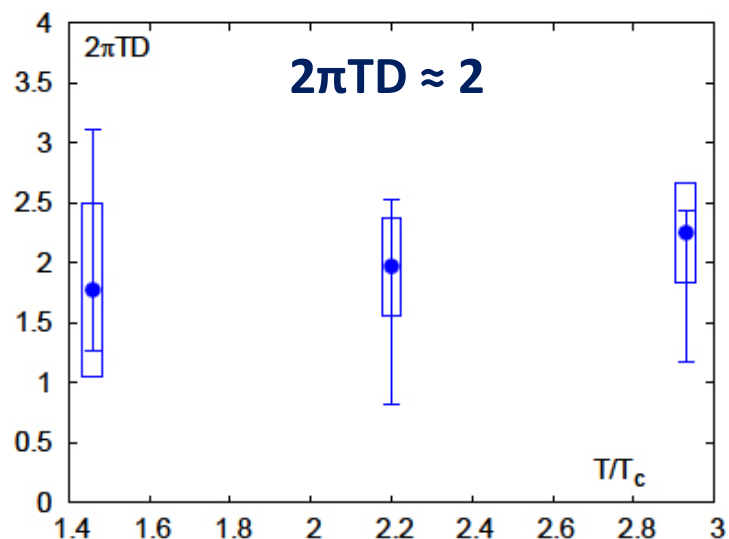
$$2\pi TD \sim 1.6-6.2 \text{ for SAI}$$

$$2\pi TD \sim 1.8-7.0 \text{ for MEM}$$

The Einstein relation suggests $2\pi TD \lesssim 6$.

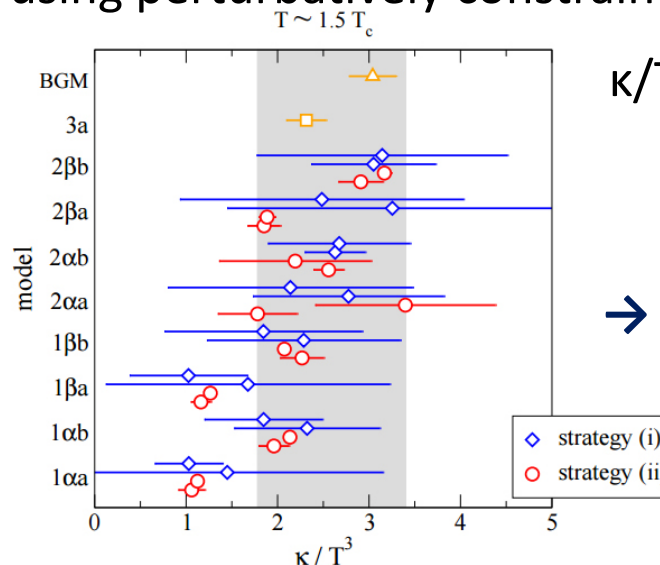
Comparison with recent lattice results

- Quenched QCD, using MEM



H.-T. Ding *et al.*, PRD 86 (2012) 014509

- Heavy quark effective theory, continuum limit, using perturbatively constrained fits



$$\kappa/T^3 = 1.8 - 3.4$$

$$D = \frac{2T^2}{\kappa}$$

$$\rightarrow 2\pi TD \approx 3.7 - 7.0$$

A. Francis, O. Kaczmarek, M. Laine, T. Neuhaus and HO, PRD 92 (2015) no.11, 116003

Our results are consistent with other lattice results.

- Perturbative estimate

$$2\pi DT \approx 71.2 \text{ in LO}$$

Moore and Teaney, PRD 71 (2005) 064904

$$2\pi DT \approx 8.4 \text{ in NLO}$$

Caron-Hout and Moore, PRL 100 (2008) 052301

- Strong coupling limit

$$2\pi DT \approx 1$$

Kovtun, Son and Starinets, JHEP 0310 (2004) 064

Summary & outlook

- We investigated vector charmonium and bottomonium SPFs on very large and fine quenched lattices in both indirect and direct ways.
- Charmonium SPFs for the vector channel at $0.75T_c$ and $1.5T_c$ were reconstructed with both MEM and stochastic methods.
 - Both MEM and the stochastic methods gave almost DM-independent stable SPFs having a clear J/ψ peak at $0.75T_c$.
 - Most of the results suggest that J/ψ might melt already $T < 1.5T_c$
 - So far we observed a relation between $2\pi TD$ and T/η , which gives a range $1 \lesssim 2\pi TD \lesssim 7$ for $1 \lesssim T/\eta \lesssim 5$.
- More studies on SPF reconstruction are needed.
 - further checks of the DM-dependence and other systematic uncertainties
 - analysis of the temperature and quark mass dependence as well as other channels
 - continuum extrapolation

End

Backup slides

Reconstructed correlator

$$G_{\text{rec}}(\tau, T; T') \equiv \int_0^\infty d\omega \rho(\omega, T') K(\omega, \tau, T)$$

$$\frac{G(\tau, T)}{G_{\text{rec}}(\tau, T; T')} = \frac{\int_0^\infty d\omega \rho(\omega, T) K(\omega, \tau, T)}{\int_0^\infty d\omega \rho(\omega, T') K(\omega, \tau, T)}$$

different
same
equals to unity at all τ

if the spectral function doesn't change at each temperature

S. Datta *et al.*, PRD 69 (2004) 094507

$$\frac{\cosh[\omega(\tau - N_\tau/2)]}{\sinh[\omega N_\tau/2]} = \sum_{\tau'=\tau; \Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} \frac{\cosh[\omega(\tau' - N'_\tau/2)]}{\sinh[\omega N'_\tau/2]}$$

$$T = 1/(N_\tau a) \quad N'_\tau = m N_\tau \quad m = 1, 2, 3, \dots$$

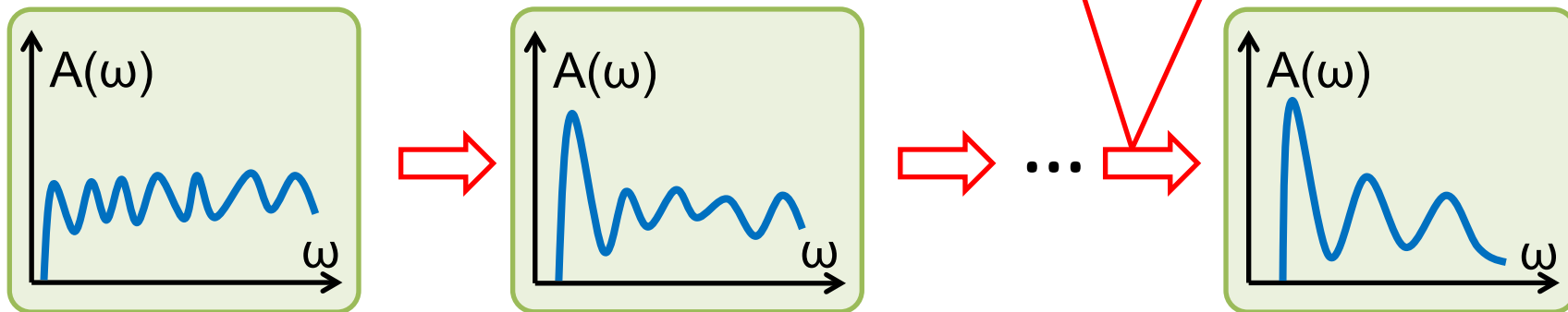
$$G_{\text{rec}}(\tau, T; T') = \sum_{\tau'=\tilde{\tau}; \Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} G(\tau', T')$$

H.-T. Ding *et al.*, PRD 86 (2012) 014509

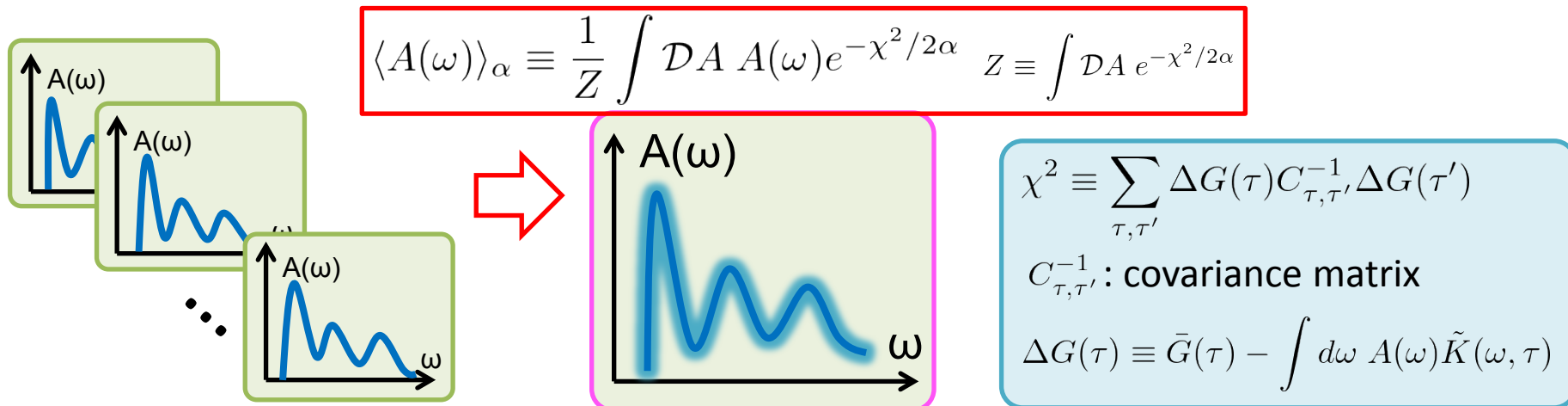
Stochastic method: basic idea

For given α (fictitious temperature, regularization parameter),

1. generate SPFs stochastically



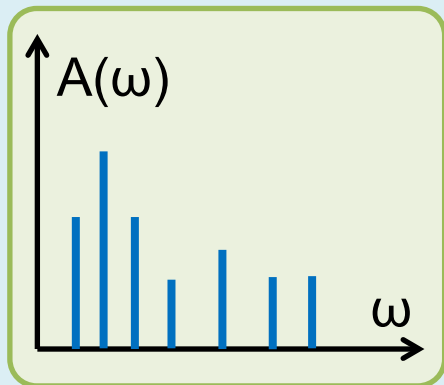
2. average over all possible spectra



Stochastic method: basis

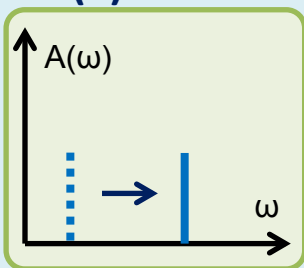
δ functions

$$A(\omega) = \sum_i r_i \delta(\omega - a_i)$$

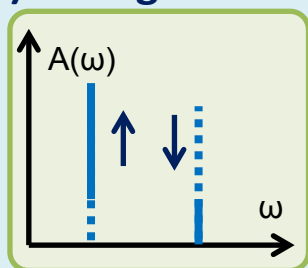


- Update schemes

(a) Shift



(b) Change residues



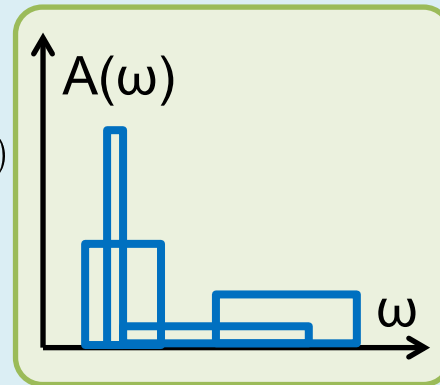
- Constraint

$$\sum_i r_i = \bar{G}(\tau_0)$$

Boxes

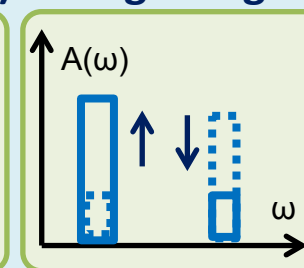
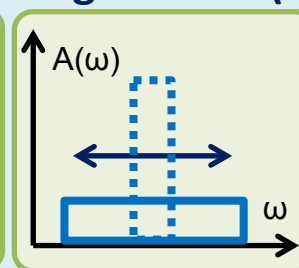
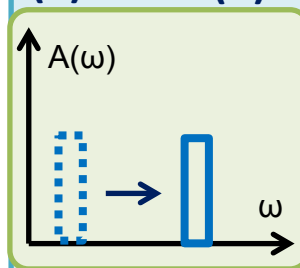
$$A(\omega) = \sum_i h_i R(\omega, a_i, w_i)$$

$$R(\omega, a_i, w_i) \equiv \begin{cases} 1 & (a_i - \frac{w_i}{2} \leq \omega \leq a_i + \frac{w_i}{2}) \\ 0 & (\text{otherwise}) \end{cases}$$



- Update schemes

(a) Shift (b) Change width (c) Change heights



- Constraint

$$\sum_i h_i w_i = \bar{G}(\tau_0)$$

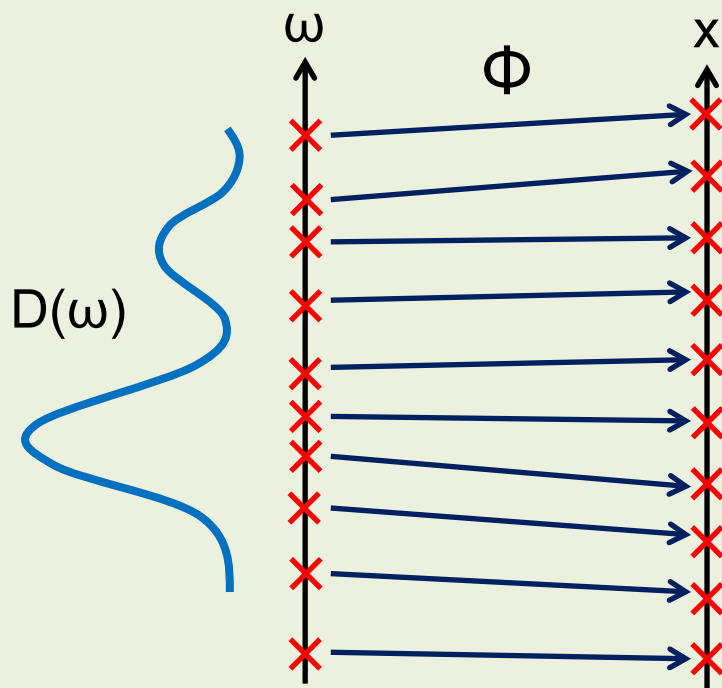
Update schemes which change the number of the basis are also possible.

Stochastic method: default model

K.S.D. Beach, arXiv:cond-mat/0403055

$$x \equiv \phi(\omega) = \frac{1}{\mathcal{N}} \int_{-\infty}^{\omega} d\omega' D(\omega')$$

$D(\omega)$: Default model (prior information)



$$G(\tau) = \int d\omega A(\omega) \tilde{K}(\omega, \tau)$$

$$= \int dx n(x) \tilde{K}(\phi^{-1}(x), \tau)$$

$$n(x) \equiv \frac{A(\phi^{-1}(x))}{D(\phi^{-1}(x))}$$

$$\bar{G}(\tau_0) - \int dx n(x) = 0$$

$$\langle A(\omega) \rangle_{\alpha} = \langle n(\phi(\omega)) \rangle_{\alpha} D(\omega)$$

Stochastic method: comparison with MEM(1)

Maximum entropy method (MEM)

$$F \equiv \chi^2 / 2 - \alpha S \quad S \equiv - \int d\omega A(\omega) \ln \left(\frac{A(\omega)}{D(\omega)} \right) : \text{entropy}$$

Minimizing F

$$\left. \frac{\delta F}{\delta A} \right|_{A=\bar{A}} = 0 \quad \Rightarrow \quad \bar{A}(\omega) : \text{the most likely solution}$$

Stochastic method

K.S.D. Beach, arXiv:cond-mat/0403055

$$H[n] \equiv \chi^2 = \int dx \epsilon(x)n(x) + \frac{1}{2} \int dx dy V(x, y)n(x)n(y) : \text{Hamiltonian}$$

Mean field treatment

$$(n(x) - \bar{n}(x))(n(y) - \bar{n}(y)) \approx 0 \quad \Rightarrow \quad \bar{A}(\omega) = \bar{n}(\phi(\omega))D(\omega)$$

Stochastic method: comparison with MEM(1)

Maximum entropy method (MEM)

$$F \equiv \chi^2/2 - \alpha S \quad S \equiv - \int d\omega A(\omega) \ln \left(\frac{A(\omega)}{D(\omega)} \right) : \text{entropy}$$

Minimizing F

$$\left. \frac{\delta F}{\delta A} \right|_{A=\bar{A}} = 0 \quad \Rightarrow \quad \bar{A}(\omega) : \text{the most likely solution}$$

Stochastic method

K.S.D. Beach, arXiv:cond-mat/0403055

$$H[n] \equiv \chi^2 = \int dx \epsilon(x)n(x) + \frac{1}{2} \int dx dy V(x,y)n(x)n(y) : \text{Hamiltonian}$$

Equivalent!

Mean field treatment

$$(n(x) - \bar{n}(x))(n(y) - \bar{n}(y)) \approx 0 \quad \Rightarrow \quad \bar{A}(\omega) = \bar{n}(\phi(\omega))D(\omega)$$

Stochastic method: comparison with MEM(2)

S. Fuchs *et al.*, PRE81, 056701 (2010)

MEM

$$P[A|\bar{G}] = \frac{P[\bar{G}|A]P[A]}{P[\bar{G}]}$$

- Prior probability

$$P[A] \propto \exp(\alpha S)$$

- Likelihood function

$$P[\bar{G}|A] \propto \exp(-\chi^2/2)$$

- Posterior probability

$$P[A|\bar{G}] \propto e^{-F}$$

$$\Rightarrow \max P[A|\bar{G}] \leftrightarrow \min F \leftrightarrow \frac{\delta F}{\delta A} = 0$$

Stochastic method

$$P[n|\bar{G}] = \frac{P[\bar{G}|n]P[n]}{P[\bar{G}]}$$

- Prior probability

$$P[n] \propto \Theta[n] \delta \left(\int dx n(x) - \bar{G}(\tau_0) \right)$$

- Likelihood function

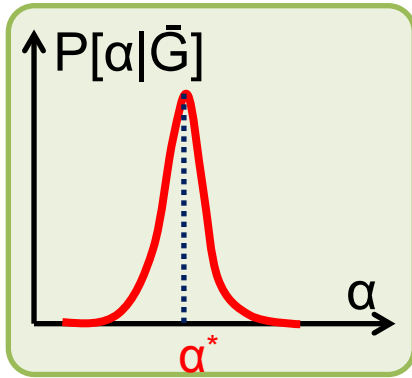
$$P[\bar{G}|n] \propto \exp(-\chi^2/2\alpha)$$

- Posterior probability

$$P[n|\bar{G}] = \Theta[n] \delta \left(\int dx n(x) - \bar{G}(\tau_0) \right) e^{-\chi^2/2\alpha}$$

$$\Rightarrow \langle n(x) \rangle_\alpha = \int \mathcal{D}n n(x) P[n|\bar{G}]$$

Stochastic method: eliminating α

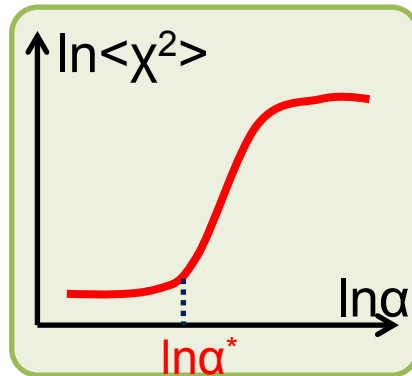


(a) By using the posterior probability $P[\alpha | \bar{G}]$

$$P[\alpha | \bar{G}] \propto P[\alpha] \int \mathcal{D}A e^{-\chi^2/2\alpha} \quad P[\alpha] = 1, 1/\alpha$$

Choosing α at the peak location of $P[\alpha | \bar{G}]$ or

Taking average $\langle\langle A(\omega) \rangle\rangle \equiv \int d\alpha \langle A(\omega) \rangle_\alpha P[\alpha | \bar{G}]$



(b) By using the log-log plot of α vs $\langle \chi^2 \rangle$

Flat region at large α : default model dominant

Crossover region: both χ^2 -fitting and the default model are important

Flat region at small α : χ^2 -fitting dominant, overfitting

Choosing α at the kink of $\ln \langle \chi^2 \rangle$

K.S.D. Beach, arXiv:cond-mat/0403055

Stochastic method : summary

MEM

- Most likely solution

$$\max P[A|\bar{G}] \leftrightarrow \min F \leftrightarrow \frac{\delta F}{\delta A} = 0$$

where

$$P[A|\bar{G}] \propto e^{-F} \quad F \equiv \chi^2/2 - \alpha S$$

- Prior information

$$S \equiv - \int d\omega A(\omega) \ln \left(\frac{A(\omega)}{D(\omega)} \right)$$

- Eliminating α

$$P[\alpha|\bar{G}] \propto P[\alpha] \int \mathcal{D}A e^{-F[A]}$$

where

$$P[\alpha] = 1, 1/\alpha$$



$$\langle A(\omega) \rangle = \int d\alpha A_\alpha(\omega) P[\alpha|\bar{G}]$$

Stochastic methods

- Most likely solution $\mathcal{D}'n \equiv \mathcal{D}n \Theta[n] \delta \left(\int dx n(x) - \bar{G}(\tau_0) \right)$

Stochastically evaluate

$$\langle n(x) \rangle_\alpha = \int \mathcal{D}'n n(x) e^{-\chi^2/2\alpha} \quad \text{for } n(x) \equiv \frac{A(\phi^{-1}(x))}{D(\phi^{-1}(x))}$$

SAI (Stochastic Analytical Inference)

S. Fuchs *et al.*, PRE81, 056701 (2010)
HO, PoS LATTICE 2015, 175 (2016)

- Prior information

$$x \equiv \phi(\omega) = \frac{1}{\mathcal{N}} \int_{-\infty}^{\omega} d\omega' D(\omega')$$

- Basis

δ functions

- Eliminating α

$$P[n|\bar{G}] \propto P[\alpha] \int \mathcal{D}'n e^{-\chi^2[n]/2\alpha}$$

$$\rightarrow \langle \langle n(x) \rangle \rangle = \int d\alpha \langle n(x) \rangle_\alpha P[n|\bar{G}]$$

SOM (Stochastic Optimization Method)

A. S. Mishchenko *et al.*, PRB62, 6317 (2000)
H.-T. Shu *et al.*, PoS LATTICE 2015, 180 (2016)

- Prior information

None ($x = \omega$)

dose not rely on DM!

- Basis

Boxes

- Eliminating α

Choosing α

at a critical point of $\langle \chi^2 \rangle_\alpha$

Default models

- Free SPF

$$\rho(\omega) = \frac{N_c}{L^3} \sum_{\mathbf{k}} \sinh\left(\frac{\omega}{2T}\right) \left[b^{(1)} - b^{(2)} \frac{\sum_{i=1}^3 \sin^2 k_i}{\sinh^2 E_{\mathbf{k}}(m)} \right] \times \frac{\delta(\omega - 2E_{\mathbf{k}}(m))}{(1 + \mathcal{M}_{\mathbf{k}}(m))^2 \cosh^2 E_{\mathbf{k}}(m)}$$

$$\cosh E_{\mathbf{k}}(m) = 1 + \frac{\mathcal{K}_{\mathbf{k}}^2 + \mathcal{M}_{\mathbf{k}}^2(m)}{2(1 + \mathcal{M}_{\mathbf{k}}(m))}$$

$$\mathcal{K}_{\mathbf{k}} = \sum_{i=1}^3 \gamma_i \sin k_i$$

$$\mathcal{M}_{\mathbf{k}}(m) = \sum_{i=1}^3 (1 - \cos k_i) + m$$

$b^{(1)} = 3, b^{(2)} = 1$ for the V channel

- A resonance peak

$$\rho(\omega) = \frac{\Gamma M}{(\omega^2 - M^2)^2 + M^2 \Gamma^2} \frac{\omega^2}{\pi}$$

- A transport peak

$$\rho(\omega) = \frac{\omega \eta}{\omega^2 + \eta^2}$$