Plasmon mass scale and linearized gauge field fluctuations in classical Yang-Mills theory

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Introduction & initial conditions [1]

Classical Yang-Mills (CYM) calculations have been used to model the pre-thermal evolution of the strongly interacting matter created in ultrarelativistic heavy-ion collisions. Our aim here is to study the limits of the quasiparticle picture in real time classical Yang-Mills theory on a lattice in 3 spatial dimensions.
We sample our gauge fields so that the initial quasiparticle spectrum satisfies:

$$f(k, t=0) = \frac{n_0}{g^2}\frac{k}{\Delta}\exp\left(\frac{-k^2}{2\Delta^2}\right).$$

Extracting plasmon mass, 3 methods [1]

DR Effective dispersion relation with Coulomb gauge fields

$$\omega^2(k) = \frac{\left\langle \left|\dot{E}_i^a(k)\right|^2\right\rangle}{\left\langle \left|E_i^a(k)\right|^2\right\rangle},$$

fit as $\omega^2 = ak^2 + \omega_{pl}^2$.

UE Add a uniform electric field at $t = t_0$, measure oscillations of electric and magnetic energy vs t.

HTL Perturbation theory, Hard Thermal Loops

$$\omega_{pl}^2 = \frac{4}{3}g^2 N_c \int \frac{d^3k}{(2\pi)^3}\frac{f(k)}{k}.$$

Quantum fluctuations [2]

Due to plasma instabilities quantum fluctuations can have a very dramatic impact on the equilibration of the classical field system. The fluctuation spectrum is very UV-divergent, which makes the numerical treatment difficult. To control these problems we explicitly linearize the fluctuations on the lattice on mode by mode basis.
Continuum equations of motion of linearized fluctuations:

$$\begin{aligned}\dot{a}_i &= e^i\\ \dot{e}^i &= [D_j,[D_j,a_i]] - [D_j,[D_i,a_j]] + ig[a_j,F_{ji}].\end{aligned}$$

Gauss's law (for the fluctuations) reads

$$c(\mathbf{x},t) = \left[D_i, e^i\right] + ig\left[a_i, E^i\right] = 0.$$

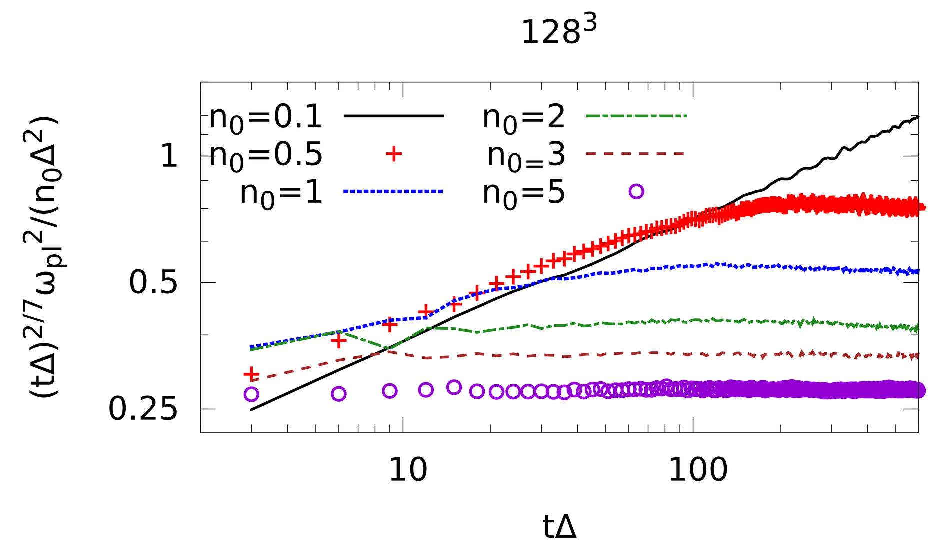
Conclusions

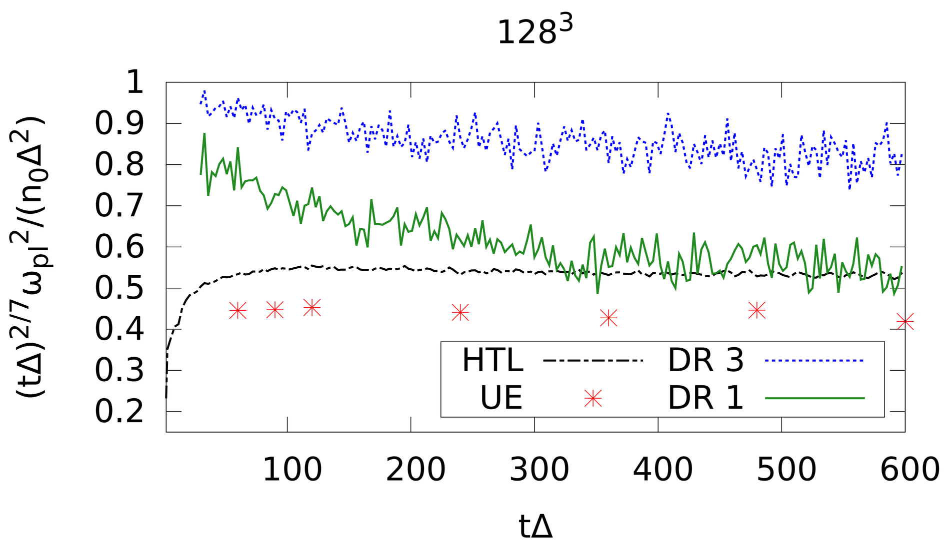
- We have studied the plasmon mass scale in pure glue QCD using 3 methods. The DR method agrees with the other methods within a factor of two.
- The UE and HTL methods agree in the continuum limit.
- The UE method is the most stable against varying ultraviolet and infrared cutoffs.
- We have also derived and implemented linearized lattice equations for fluctuations in CYM, conserving Gauss's law exactly.

References

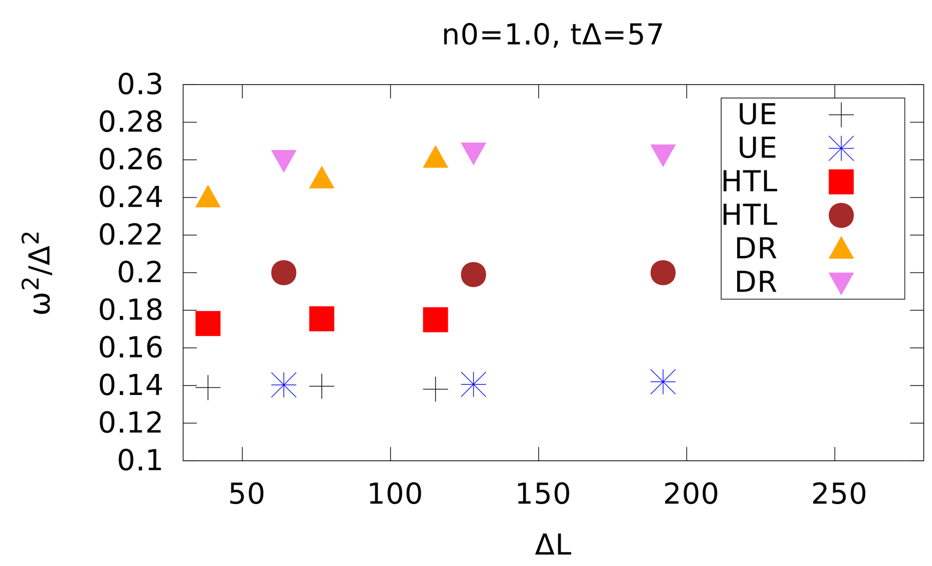
[1] T. Lappi, J. Peuron, Phys. Rev. D 95, 014025
[2] A. Kurkela, T. Lappi, J. Peuron, Eur. Phys. J. C76 (2016) no. 12, 688
[3] A. Kurkela, G. D. Moore, Phys.Rev. D89 (2014) no.7, 074036

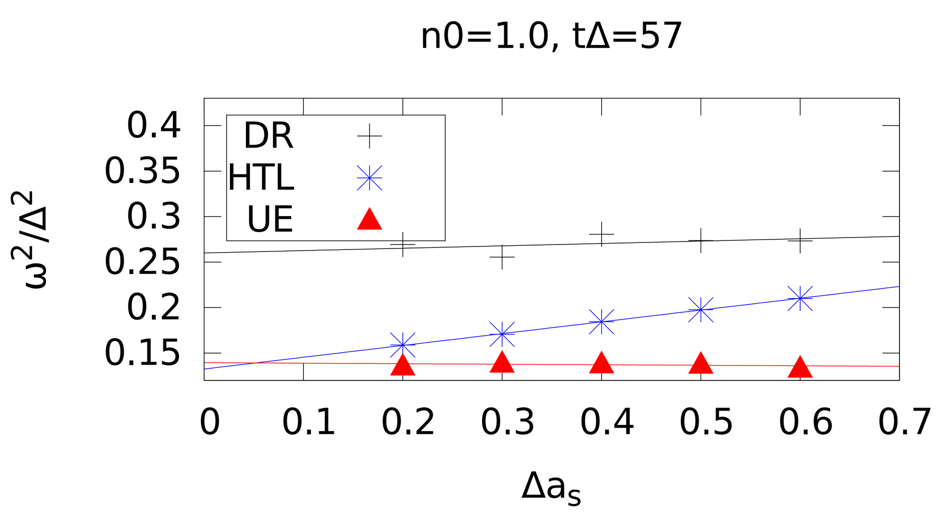
Results & dependence on the lattice cutoffs[1]





- More highly occupied systems enter the asymptotic regime faster.
- The late-time evolution is consistent with a $t^{-2/7}$ power law [3].
- The DR method depends on maximum k^2/Δ^2 in fit (DR 1 vs. DR 3).





- Left: infrared cutoff (lattice size $L\Delta$) dependence with two different ultraviolet cutoffs (lattice spacings $a_s\Delta = 0.3$ [up], $a_s\Delta = 0.5$ [down]). **We observe no significant IR-cutoff dependence.**
- Right: ultraviolet cutoff dependence. **The HTL and UE methods seem to converge to same continuum limit.**

Fluctuations on the lattice [2]

The time-evolution equation for the fluctuation of the electric field is straightforward

$$\begin{aligned}a_s^2 e^i(t+\mathrm{d}t) = a_s^2 e^i(t) - \mathrm{d}t\sum_{j\neq i}\Big[i\Big(&a_i(\mathbf{x})\square_{i,j}(\mathbf{x}) + a_j(\mathbf{x}+\hat{\boldsymbol{\imath}}\to\mathbf{x})\square_{i,j}(\mathbf{x}) + a_i(\mathbf{x})\square_{i,-j}(\mathbf{x})\\ &-\square_{i,j}(\mathbf{x})a_i(\mathbf{x}+\hat{\boldsymbol{\jmath}}\to\mathbf{x}) - \square_{i,j}(\mathbf{x})a_j(\mathbf{x}) + \square_{i,-j}(\mathbf{x})a_j(\mathbf{x}-\hat{\boldsymbol{\jmath}}\to\mathbf{x})\\ &-a_j(\mathbf{x}+\hat{\boldsymbol{\imath}}-\hat{\boldsymbol{\jmath}}\to\mathbf{x}+\hat{\boldsymbol{\imath}}\to\mathbf{x})\square_{i,-j}(\mathbf{x}) - \square_{i,-j}(\mathbf{x})a_i(\mathbf{x}-\hat{\boldsymbol{\jmath}}\to\mathbf{x})\Big)\Big]_{\mathrm{ah}},\end{aligned}$$

with the background field plaquette $\square_{i,j}(\mathbf{x})$ and the parallel transported fluctuation

$$a_j(\mathbf{x}+\hat{\boldsymbol{\imath}}\to\mathbf{x}) \equiv U_i(\mathbf{x})a_j(\mathbf{x}+\hat{\boldsymbol{\imath}})U_i^\dagger(\mathbf{x}).$$

The subscript ah stands for the antihermitean traceless part.

Problem The naive discretization of $\dot{a}_i = e^i$ does not conserve the discretized Gauss's law:

$$c(\mathbf{x},t) = \sum_i \frac{1}{a^2}\left\{e^i(\mathbf{x}) - e^i(\mathbf{x}-\hat{\boldsymbol{\imath}}\to\mathbf{x}) + i[a_i(\mathbf{x}-\hat{\boldsymbol{\imath}}\to\mathbf{x}), E^i(\mathbf{x}-\hat{\boldsymbol{\imath}}\to\mathbf{x})]\right\}.$$

Solution We construct the time-evolution equation of a_i by explicitly requiring that it conserves Gauss's law.

Result The equation (in the fundamental representation of SU(2)) is

$$a_i(t+\mathrm{d}t) = \frac{i}{2\,\mathrm{Tr}\left[E^iE^i\right]}\left[E^i, -i\left(\square_{0i}e^{i\perp}\square_{0i}^\dagger - e^{i\perp}\right) + \left[E^i, \square_{0i}a_i^\perp(t)\square_{0i}^\dagger\right]\right] + \mathrm{d}t e^{i\parallel} + a_i^\parallel(t).$$

Here $\square_{0i} = e^{iE^i\mathrm{d}t}$ is the "timelike plaquette". The notations $\perp$ and $\parallel$ refer to components of a_i and e^i that are perpendicular or parallel to E^i in color space.

We have also tested a numerical implementation of these equations and verfied that the Gauss's law is conserved within machine precision and that the linearization works.