Chiral Shock Waves

Srimoyee Sen, Naoki Yamamoto (arXiv : 1609.07030v1) Shockwaves

Chiral hydrodynamics

Ideal and dissipative hydrodynamics

- Energy momentum conservation and particle number conservation.

$$\partial_\mu T^{\mu\nu} = 0, \qquad T^{\mu\nu} = h u^\mu u^\nu - p g^{\mu\nu}$$
$$\partial_\mu j^\mu = 0. \qquad j^\mu = n u^\mu$$

- Dissipative processes require additional terms in the conserved quantities --- to be constrained by the second law of thermodynamics.

Background field and anomaly

- Consider massless fermions of single chirality
- In the presence of background electromagnetic field

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda, \quad \partial_\mu j^\mu = C E^\mu B_\mu$$

- External fields add further dissipative terms and anomaly adds nondissipative terms to constitutive relations.

Chiral transport

- Ignoring dissipation and in the absence of a magnetic field in Landau frame

$$j^\mu = n u^\mu + \xi \omega^\mu$$
$$T^{\mu\nu} = h u^\mu u^\nu - p g^{\mu\nu}$$
$$\xi = C\mu^2\left(1 - \frac{2}{3}\frac{n\mu}{h}\right) + DT^2\left(1 - \frac{2n\mu}{h}\right)$$

Shock waves in nonchiral matter

Nonchiral shock waves

- Consider fluid flowing along x axis and there is a surface of discontinuity perpendicular to the propagation.
- Denote the two sides across the surface by 1 and 2.

$$j_1^x = j_2^x, \qquad T_1^{xx} = T_2^{xx}$$
$$T_1^{0x} = T_2^{0x}$$

- Rankine-Hugoniot relations:

$$\frac{v_1\gamma_1}{V_1} = \frac{v_2\gamma_2}{V_2} \qquad h_1 v_1 \gamma_1^2 = h_2 v_2 \gamma_2^2$$
$$h_1 v_1^2 \gamma_1^2 + p_1 = h_2 v_2^2 \gamma_2^2 + p_2$$

Pressure-volume relation

$$v_1 = \sqrt{\frac{(p_2 - p_1)(\epsilon_2 + p_1)}{(\epsilon_2 - \epsilon_1)(\epsilon_1 + p_2)}} \qquad v_2 = \sqrt{\frac{(p_2 - p_1)(\epsilon_1 + p_2)}{(\epsilon_2 - \epsilon_1)(\epsilon_2 + p_1)}}$$

- The adiabatic

$$h_1^2 V_1^2 - h_2^2 V_2^2 + (p_2 - p_1)(h_1 V_1^2 + h_2 V_2^2) = 0$$

Weak shockwaves:

$$\epsilon_2 \to \epsilon_1 \qquad p_2 \to p_1$$

From $c_s^2 = \lim_{2\to 1} \frac{p_2 - p_1}{\epsilon_2 - \epsilon_1} > 0$

$\epsilon_2 > \epsilon_1 \longrightarrow p_2 > p_1$
→ compression shock wave

$\epsilon_2 < \epsilon_1 \longrightarrow p_1 > p_2$
→ rarefaction shock wave

Entropy considerations

Define $\Delta H = H_2 - H_1$ $\quad \Delta V = V_2 - V_1$
$\Delta S = S_2 - S_1$

Expand the pressure volume relation using :

$$\Delta H = T\Delta S + V_1 \Delta p + \frac{1}{2}\frac{\partial V}{\partial p}\bigg|_1 (\Delta p)^2 + \frac{1}{6}\frac{\partial^2 V}{\partial p^2}\bigg|_1 (\Delta p)^3$$
$$\Delta V = \frac{\partial V}{\partial p}\bigg|_1 \Delta p + \frac{1}{2}\frac{\partial^2 V}{\partial p^2}\bigg|_1 (\Delta p)^2 + \frac{1}{6}\frac{\partial^3 V}{\partial p^3}\bigg|_1 (\Delta p)^3 + \frac{\partial V}{\partial S}\bigg|_1 \Delta S + \cdots,$$

We obtain: $\Delta S = \frac{1}{12 H_1 T}\frac{\partial^2 (HV)}{\partial p^2}\bigg|_1 (\Delta p)^3 + O((\Delta p)^4)$

For any realistic EOS, $\partial^2(HV)/\partial p^2 > 0$
Hence from second law of thermodynamics

$S_2 > S_1 \longrightarrow p_2 > p_1$

$p_1 > p_2$

Shock waves in chiral matter

Chiral shock waves

- Vorticity chosen to be along x
- Hydro makes sense when $\omega \ll \mu$
- Also we have to consider $|\omega|\rho \ll 1$

where $\rho = \sqrt{y^2 + z^2}$ is the distance from the axis of rotation.

- The speed perpendicular to the x axis is given by $v_\perp = \omega\rho(1 - v_x^2)$
- Corresponding continuity equation demands

$$\omega_1\left(1 - (v_1^x)^2\right) = \omega_2\left(1 - (v_2^x)^2\right)$$

- **The new pressure-volume relation**

$$\frac{v_1}{V_1\sqrt{1 - v_1^2}} - \frac{v_2}{V_2\sqrt{1 - v_2^2}} = -(\xi_1\omega_1 - \xi_2\omega_2)$$

In the limit $|\omega|\rho \ll 1$ the expressions for v_1 and v_2 remain unchanged.

A change of variables

- We don't know the expansion of the RHS of the new pressure-volume Relation in terms of Δp and ΔS.
- The RHS is known as a function of chemical potential and temperature.
- Use the following to express chemical potential and temperature in Terms of entropy and volume per particle

$$n = \frac{\mu^3}{6\pi^2} + \frac{\mu T^2}{6} \qquad S = \frac{\pi^2 T}{\mu}$$

Entropy discontinuity

- The new pressure-volume relation expanded

$$\Delta S \approx \frac{216\pi^6}{\mu_1^{11} T_1}(\Delta p)^3 - \frac{\omega_1\lambda}{T_1}\frac{36\sqrt{2}\pi^4}{\mu_1^8}(\Delta p)^2$$

dominates for $\omega_1 < \frac{\Delta p}{\mu_1^3} 3\sqrt{2}\pi^2$ with $\lambda \equiv 4\pi^2 C = \pm 1$

---- and we are back to nonchiral shock waves.

For $\omega_1 > \frac{\Delta p}{\mu_1^3} 3\sqrt{2}\pi^2$ however, we have $\Delta S \approx -\frac{\omega_1\lambda}{T_1}\frac{36\sqrt{2}\pi^4}{\mu_1^8}(\Delta p)^2$

Clarification about the LHS of the pressure-volume relation

- The expansion of the LHS looks like

$$\frac{T_1\left(\Delta S - \frac{1}{12 H_1 T_1}\frac{d^2(HV)}{dp^2}(\Delta p)^3\right)}{H_1\sqrt{-V_1^2 - H_1\frac{dV}{dp}}}$$

- $-V_1^2 - H_1\frac{dV}{dp}$ is positive and

$$\frac{1}{12 H_1 T_1}\frac{d^2(HV)}{dp^2} \approx \frac{216\pi^6}{\mu_1^{11} T_1}$$

Conclusion

- **Both compression and rarefaction shockwaves** are **allowed** in the presence of a vorticity in chiral matter.
- Depending on the chirality of the Fermion species, **the wave can travel either along the vorticity or opposite to it**, but not both.