Baryon diffusion coefficient and electric conductivity in a hot hadron gas

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H-QM Helmholtz Research School Quark Matter Studies

Abstract

In baryon-rich systems, spatial gradients in baryon chemical potential μ_B or net-baryon density n_B generate a diffusion current J_B,

$$J_B^x = \sigma_{\mathbf{B}} \nabla^x \frac{\mu_B}{T} = \mathbf{D}\, \nabla^x n_B. \quad (1)$$

We compute the baryon diffusion constant $\mathbf{D}$ for a realistic hadron resonance gas using the Boltzmann equation. Electric fields $\vec{E}$ accelerate electric charges, such that a static electric current J_e develops,

$$J_e^x = \sigma_{\mathrm{el}} E^x. \quad (2)$$

We compute the electric conductivity σ_{el} for hot hadron resonance gases. Based on [1].

Motivation

Different motivations for calculating σ_{el} and $\mathbf{D}$:

- Self-induction (Lenz effect) for *decaying magnetic fields* in heavy-ion collisions: Large conductivity $\rightarrow$ slow decay of magnetic fields.
- Thermal dilepton rate is related to σ_{el}
- Baryon diffusion constant $\mathbf{D}$ required for explanation of baryon diffusion currents in baryon-rich experiments
- Hydro models will need $\mathbf{D}$ as input
- Transport coefficients show extremum at phase transition

The method

We describe the evolution of the single particle distribution function $f^i_{\mathbf{k}}(x)$ of each species i by the Boltzmann equation,

$$k^\mu \frac{\partial}{\partial x^\mu} f^i_{\mathbf{k}} + k_\nu q_i F^{\mu\nu} \frac{\partial}{\partial k^\mu} f^i_{\mathbf{k}} = \sum_{j=1}^{N_{\text{species}}} C_{ij}(x^\mu, k^\mu), \quad (3)$$

In equilibrium, $f^i_{0,\mathbf{k}} = \exp(-u^\mu k_\mu/T + q_i \mu_q/T + \lambda_i \mu_B/T)$. Here, q_i (λ_i) is electric (baryon) charge. We now suddenly ...
...switch on an electric field: $F^{\mu\nu} \rightarrow \delta F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu$
...or apply a gradient in baryon chemical potential $\nabla_\nu \mu_B/T$,
so $f^i_{\mathbf{k}} = f^i_{0,\mathbf{k}} + \delta f^i_{\mathbf{k},e/B}$. We aim to solve for the linear response $\delta f^i_{\mathbf{k},e/B}$, which is from (3) of the form

$$\delta \tilde{f}^i_{\mathbf{k},e} \equiv B^\nu(q^\mu, k^\mu) \tilde{E}_\nu \text{ or } \delta \tilde{f}^i_{\mathbf{k},B} \equiv B^\nu(q^\mu, k^\mu) \widetilde{\nabla_\nu \mu_B/T}. \quad (4)$$

The net charge diffusion currents then are

$$j_e^\mu = N_q^{\langle\mu\rangle} = \sum_{i=1}^{N_{\text{species}}} q_i \int dK\, \delta f^i_{\mathbf{k},e} k^{\langle\mu\rangle}$$

$$j_B^\mu = N_B^{\langle\mu\rangle} = \sum_{i=1}^{N_{\text{species}}} \lambda_i \int dK\, \delta f^i_{\mathbf{k},B} k^{\langle\mu\rangle},$$

where we use the notation $dK \equiv \mathrm{d}^3k / \left[(2\pi)^3 k^0\right]$, and $\langle\cdot\rangle$ is the spatial projection. We want zero frequency transport coefficients, so $q^\mu \equiv 0$. Then $B^\nu \sim k^{\langle\nu\rangle}$, so we expand $B^\nu(k^\mu)$ in a power series in energy,

$$B_i^\alpha(k^\mu) = f^i_{0,\mathbf{k}} k^{\langle\alpha\rangle} \sum_{n=0}^{\infty} a_n^{(i)} E^n_{\mathbf{k}}, \quad (5)$$

where $a_n^{(i)}$ are expansion coefficients. Using (5) in (4) and (3), we can solve for $a_n^{(i)}$ by inverting (fixing the Landau frame)

$$\sum_{n=0}^{\infty} \sum_{j=1}^{N_{\text{species}}} \left[\mathcal{A}^i_{mn} \delta^{ij} + \mathcal{C}^{ij}_{mn}\right] a_n^{(j)} = b^i_m,$$

where every matrix element is a collision integral.

Collisions

We use a linearized collision operator for **elastic 2-to-2 collisions**. This enables the use of s-dependent **resonance cross sections** (instead of, e.g., simple relaxation times). The linearized collision term can be written as,

$$C_{ij}(x^\mu, k^\mu) = \int \mathrm{d}K' \mathrm{d}P \mathrm{d}P' \gamma_{ij} W^{ij}_{\mathbf{k}\mathbf{k}'\rightarrow\mathbf{p}\mathbf{p}'} f^i_{0,\mathbf{k}} f^j_{0,\mathbf{k}'} \times \left(\frac{\delta f^i_{\mathbf{p}}}{f^i_{0,\mathbf{p}}} + \frac{\delta f^j_{\mathbf{p}'}}{f^j_{0,\mathbf{p}'}} - \frac{\delta f^i_{\mathbf{k}}}{f^i_{0,\mathbf{k}}} - \frac{\delta f^j_{\mathbf{k}'}}{f^j_{0,\mathbf{k}'}}\right)$$

where $W^{ij}_{\mathbf{k}\mathbf{k}'\rightarrow\mathbf{p}\mathbf{p}'} = s\sigma_{ij}(s,\Theta)(2\pi)^6 \delta^{(4)}(k^\mu + k'^\mu - p^\mu - p'^\mu)$ and $\gamma_{ij} = 1 - 1/2\delta_{ij}$. The total cross section $\sigma_{\text{tot},ij}(s)$ is an integral of the differential cross section $\sigma_{ij}(s,\Theta)$. In principle, any choice of differential elastic cross section is possible.

Hadron resonance gas

We study a hadronic gas with interactions among $\pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \bar{K}^0, p, n, \bar{p}, \bar{n}$. We use Breit-Wigner shaped cross sections when available, otherwise approx. constant values. These values are similar to parametrisations used, e.g., in UrQMD or GiBUU. Error estimation by scaling the constant cross sections by factor 2 and 1/2.

	π^+	π^-	π^0	K^+	K^-	K^0	$\bar{K}^0$	p	n	$\bar{p}$	$\bar{n}$
π^+	10	ρ	ρ	10	10	$K^\star$	10	Δ	10	10	Δ
π^-		10	ρ	$K^\star$	10	10	$K^\star$	10	Δ	Δ	10
π^0			5	$K^\star$	10	$K^\star$	$K^\star$	Δ	Δ	Δ	Δ
K^+				10	10	10	50	6	10	20	10
K^-					10	50	10	20	10	6	10
K^0						10	50	6	6	20	20
$\bar{K}^0$							10	8	20	6	6
p								20	20	100	20
n									20	20	100
$\bar{p}$										10	10
$\bar{n}$											10

Table 1: The cross sections we used among all species. Numbers are in mb, $\rho, K^\star$ and Δ denote Breit-Wigner shaped cross sections with those resonances.

Results

In the following we compare the electric conductivity with different other calculations: BAMPS (pQCD, $N_f = 3$) Ref. [2]. PHSD Ref. [3], DQPM Ref. [4]. Quenched continuum lattice Ref. [5]. Unquenched lattice Ref. [6]. Non-conformal holographic model Ref. [7]. Chiral perturbation theory [8]. We expect a minimum of σ_{el}/T at T_c. First we consider a gas with all species, but only constant cross sections:

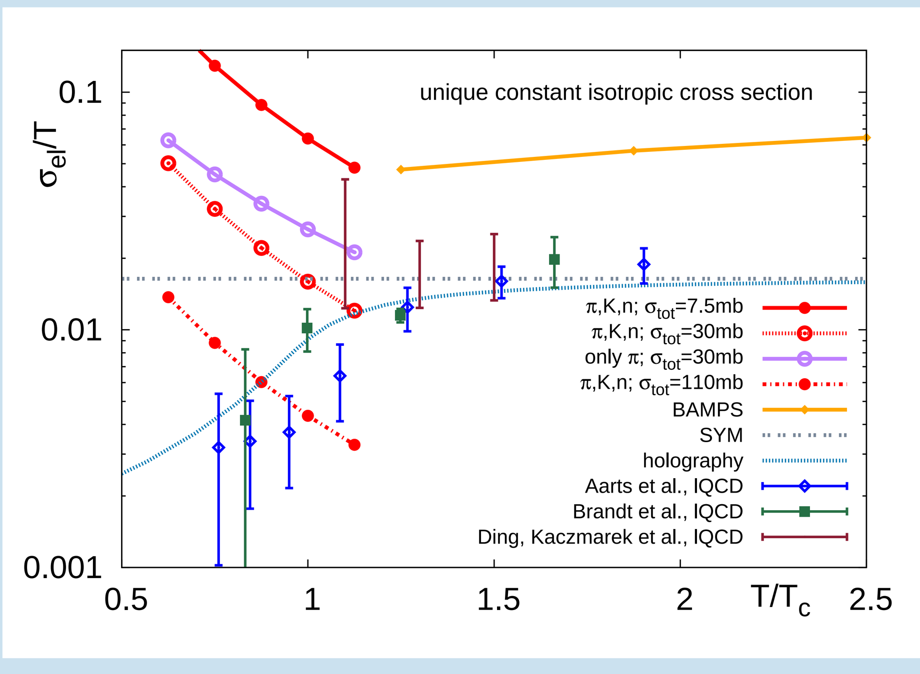

Figure 1: Using a constant isotropic cross section for all 11 species, we can extract effective cross sections of various other models. Lattice [6] requires 110 mb, pQCD [2] 7.5 mb.

Then we use a gas of π^+, π^-, π^0, with resonance cross sections, as in Tab. 1.:

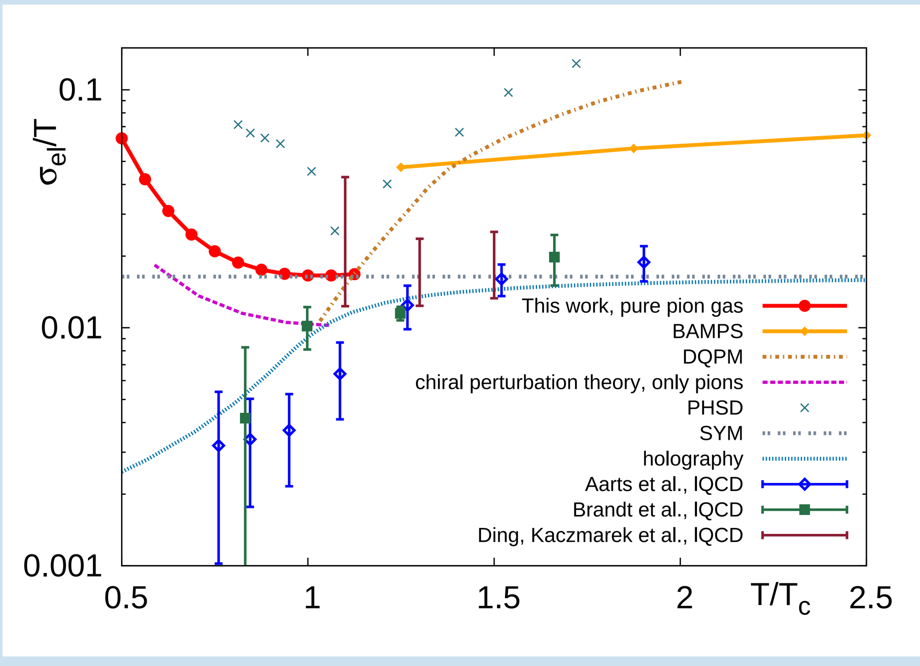

Figure 2: Result for a pure pion gas including the ρ resonance (red line). Chiral perturbation theory [8] is very similar.

Using all species and cross sections from Tab. 1 our result matches lattice QCD around $T_c \approx 160$ MeV:

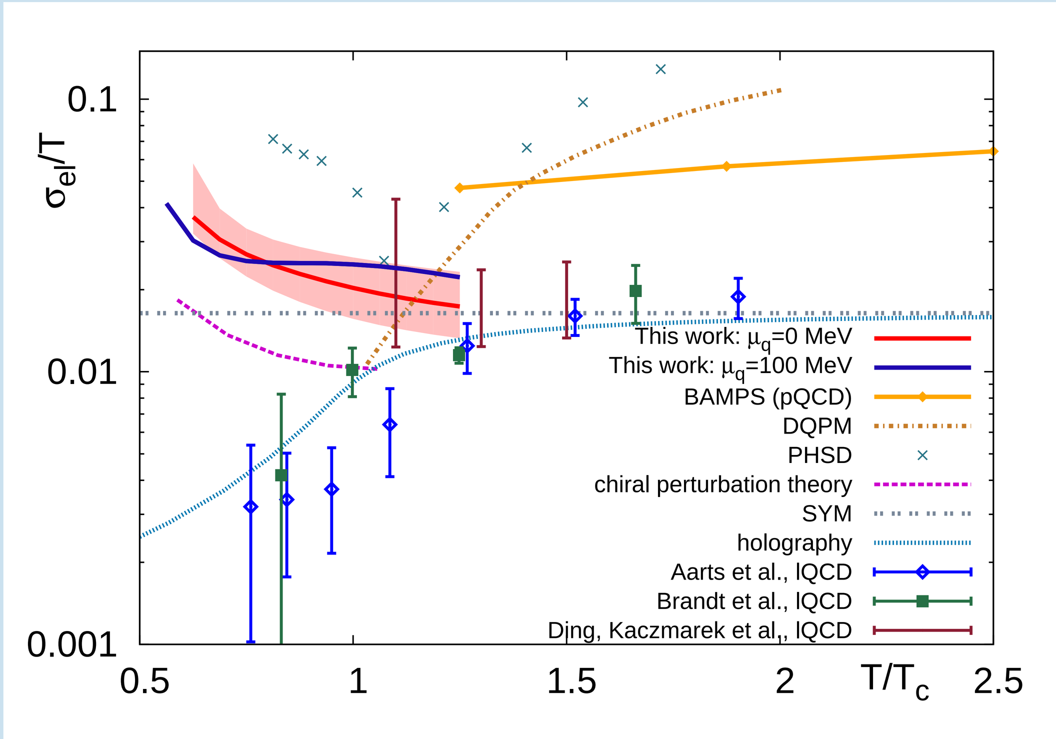

Figure 3: Full result with all 11 species from Tab. 1 (red line). Errorbands correspond to variation of all *constant* cross-sections by a factor 2 and 1/2. Within errors, we agree very good with quenched continuum lattice [5]. Using a electro chemical potential $\mu_q = 100$ MeV (blue line), the T dependence changes.

The first hadronic calculation of the baryon diffusion constant, using all cross sections from Tab. 1. Again, a chemical potential ($\mu_B = 100$ MeV) changes the slope slightly. We compare to holographic models Ref. [7, 9] and lattice, Ref. [6].

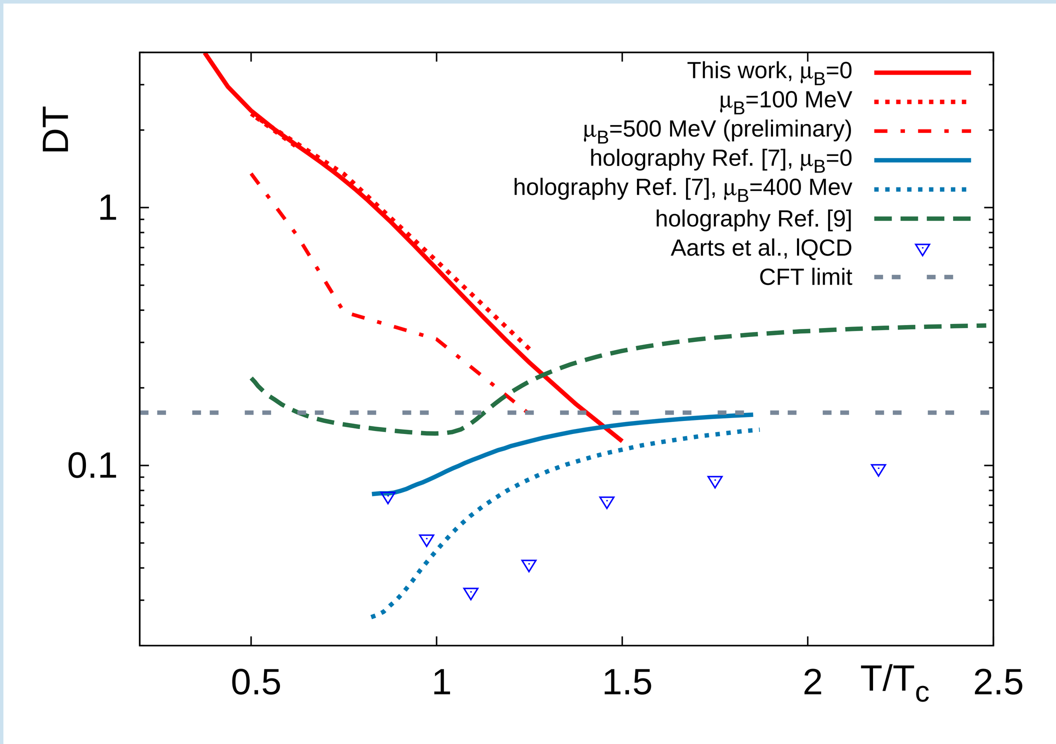

Figure 4: Full result for baryon diffusion with all 11 species from Tab. 1 (red line).

Conclusions

First kinetic calculation of the hadronic transport coefficients *electric conductivity* and *baryon diffusion constant*, using realistic scattering matrix elements. The coefficients drop for higher T and match lattice QCD results at around T_c. We show results for finite chemical potentials. The formalism can be extended to more complicated cross sections.
Outlook: What is the interplay of various chemical potentials (μ_q, μ_B, μ_S) for these coefficients ?

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